Sri Chaitanya IIT Academy., India

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Question Paper, Key and Solutions

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Physics

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. When the wavelength of radiation falling on a metal is changed from 500 nm to 200 nm, the maximum kinetic energy of the photoelectrons becomes three times larger. The work function of the metal is close to :

1) 0.61 eV 2) 0.81 eV 3) 0.52 eV 4) 1.02 eV

Key: 1

Sol: $KE = h\upsilon - \phi$

$$h\upsilon_1 = \frac{12420}{5000} = 2.484$$

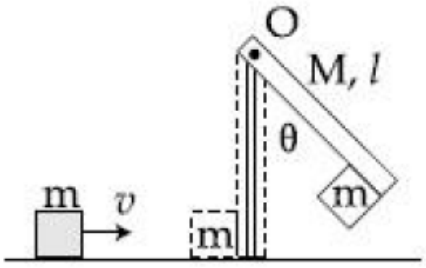
$$h\upsilon_2 = \frac{12420}{2000} = 6.21$$

$$K = 2.484 - \phi$$

$$3K = 6.21 - \phi$$

$$\phi = 0.621$$

2. A block of mass m = 1 kg slides with velocity $\upsilon = 6 m/s$ on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about O and swings as a result of the collision making angle before momentarily coming to rest. If the rod has mass M = 2 kg, and length l = 1 m, the value of θ is approximately: $\left(\text{takeg} = 10 m/s^2\right)$



1) 69° 2) 55° 3) 63° 4) 49°

Key: 3

Sol: Conservation of angular momentum about hinge $mvL = L$

Energy conservation $\frac{L^2}{2I} = mgl(1-\cos\theta) + Mg\frac{1}{2}(1-\cos\theta)$

$$\frac{(mvL)^2}{2\left(mL^2 + \frac{ML^2}{3}\right)} = \left(m + \frac{M}{2}\right)gL(1-\cos\theta)$$

$$\cos\theta = \frac{23}{50}$$

3. When a diode is forward biased, it has a voltage drop of 0.5 V. The safe limit of current through the diode is 10 mA. If a battery of emf 1.5 V is used in the circuit, the value of minimum resistance to be connected in series with the diode so that the current does not exceed the safe limit is :

1) 200Ω 2) 50Ω 3) 100Ω 4) 300Ω

Key: 3

Sol: $1.5 - 0.5 = iR$

$$R = \frac{1}{10 \times 10^{-3}} = 100\Omega$$

4. Moment of inertia of a cylinder of mass M, length L and radius R about an axis passing through its centre and perpendicular to the axis of the cylinder is $I = M\left(\frac{R^2}{4} + \frac{L^2}{12}\right)$. If such a cylinder is to be made for a given mass of material, the ratio L/R for it to have minimum possible I is :

1) $\sqrt{\frac{3}{2}}$ 2) $\sqrt{\frac{2}{3}}$ 3) $\frac{2}{3}$ 4) $\frac{3}{2}$

Key: 1

Sol: $I = \frac{MR^2}{4} + \frac{ML^2}{12}$

$V = \pi R^2 L$

$R^2 = \frac{V}{\pi L}$

$I = \frac{MV}{4\pi L} + \frac{ML^2}{12}$

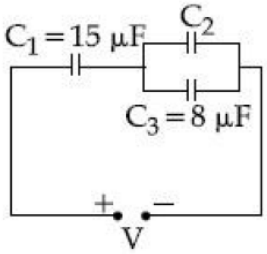
$\frac{dI}{dL} = \frac{MV}{4\pi}\left[-\frac{1}{L^2}\right] + M\left[\frac{2L}{12}\right]$

$\frac{dI}{dL} = 0$

$\frac{\pi R^2 L}{4\pi L^2} = \frac{2L}{12}$

$\left[\frac{R}{L}\right]^2 = \frac{2}{3}$

5. In the circuit show in the figure, the total charge is $750\mu C$ and the voltage across capacitor C_2 is 20 V. Then the charge on capacitor C_2 is :



1) $160\mu C$ 2) $590\mu C$ 3) $650\mu C$ 4) $450\mu C$

Key: 2

Sol: $750 = Q_2 + 8(20)$

$Q_2 = 590$

6. Pressure inside two soap bubbles are 1.01 and 1.02 atmosphere, respectively. The ratio of their volumes is :

1) 8 : 1 2) 0.8 : 1 3) 2 : 1 4) 4 : 1

Key: 1

Sol: $P_0 + \frac{4T}{R_2} = 1.02$

$$\frac{V_1}{V_2} = \left(\frac{R_1}{R_2}\right)^3 = \left(\frac{0.02}{0.01}\right)^2 = \frac{8}{1}$$

7. A charged particle carrying charge $1\mu C$ is moving with velocity $\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) ms^{-1}$. If an external magnetic field of $\left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) \times 10^{-3} T$ exists in the particle is moving then the force on the particle is $\vec{F} \times 10^{-6} N$. The vector $\vec{F}$ is :

1) $-300\hat{i} + 320\hat{j} - 90\hat{k}$ 2) $-3.0\hat{i} + 3.2\hat{j} - 0.9\hat{k}$

3) $-0.30\hat{i} + 0.32\hat{j} - 0.09\hat{k}$ 4) $-30\hat{i} + 32\hat{j} - 9\hat{k}$

NTA Key: 4

Sol: $\vec{F} = q\vec{V} \times \vec{B}$

$$\vec{F} \times 10^{-9} = 10^{-6}\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) \times \left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) \times 10^{-3}$$

$$\vec{F} = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 5 & 3 & -6 \end{vmatrix} = -30\hat{i} + 32\hat{j} - 9\hat{k}$$

8. In a Young's double slit experiment, light of 500 nm is used to produce an interference pattern. When the distance between the slits is 0.05 mm, the angular width (in degree) of the fringes formed on the distance screen is close to :

1) 0.17° 2) 0.07° 3) 1.7° 4) 0.57°

Key: 4

Sol: $\beta = \frac{\lambda}{d} = \frac{500 \times 10^{-9}}{0.05 \times 10^{-3}} = 10^{-2} rad = 0.57 deg$

9. A balloon filled with helium (32ºC and 1.7 atm.) bursts. Immediately afterwards the expansions of helium can be considered as:

1) Irreversible adiabatic
2) Irreversible isothermal
3) Reversible adiabatic
4) Reversible isothermal

Key: 1

Sol: Bursting of balloon is irreversible adiabatic

10. In a radioactive material, fraction of active material remaining after time t is 9/16. The fraction that was remaining after t/2 is :

1) $\frac{7}{8}$ 2) $\frac{3}{5}$ 3) $\frac{4}{5}$ 4) $\frac{3}{4}$

Key: 4

Sol: If fraction after $\frac{'t'}{2}$ is x then after 't', it is x^2

$x^2 = \frac{9}{16} \Rightarrow x = \frac{3}{4}$

11. The magnetic field of a plane electromagnetic wave is

$\vec{B} = 3\times10^{-8}\sin\left[200\pi\left(y+ct\right)\right]\hat{i}T$

where $c = 3\times10^{8}\,ms^{-1}$ is the speed of light. The corresponding electric field is :

1) $\vec{E} = -9\sin\left[200\pi\left(y+ct\right)\right]\hat{k}V/m$

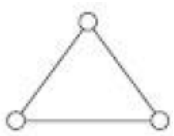
2) $\vec{E} = 3\times10^{-8}\sin\left[200\pi\left(y+ct\right)\right]\hat{k}V/m$

3) $\vec{E} = 10^{-6}\sin\left[200\pi\left(y+ct\right)\right]\hat{k}V/m$

4) $\vec{E} = 9\sin\left[200\pi\left(y+ct\right)\right]\hat{k}V/m$

Key: 1

Sol: $E_0 = B_0 C$ $E_0 = 3 \times 10^{-8} \times 3 \times 10^8 = 9$

12. 

Consider a gas of triatomic molecules. The molecules are assumed to be triangular and made of massless rigid rods whose vertices are occupied by atoms. The internal energy of a mole of the gas at temperature T is :

1) $\frac{5}{2}RT$ 2) $\frac{3}{2}RT$ 3) $3RT$ 4) $\frac{9}{2}RT$

Key: 3

Sol: Degrees of freedom for a non linear triation molecule = 6

$$U = \frac{f}{2}RT = 3RT$$

13. A 750 Hz, 20 V (rms) source is connected to a resistance of 100Ω, an inductance of 0.1803 H and a capacitance of 10 μF all in series. The time in which the resistance (heat capacity 2 J/ºC) will get heated by 10ºC. (assume no loss of heat to the surroundings) is close to :

1) 418 s 2) 348 s 3) 365 s 4) 245 s

Key: 2

Sol: $\omega = 2\pi \times 750$

$$Z = \sqrt{100^2 + \left(LW - \frac{1}{CW}\right)^2} = 834$$

$$I_{rms} = \frac{V_{rms}}{Z} = 0.024$$

$$P = V_{rms} i_{rms} \cos\theta$$

$$Pt = S\theta$$

$$t = \frac{S\theta}{P} = \frac{2 \times 10}{0.058} = 348S$$

14. Magnitude of magnetic field (in SI units at the centre of a hexagonal shape coil of side 10 cm, 50 turns and carrying current I (Ampere) in units of $\frac{\mu_0 I}{\pi}$ is :

1) $50\sqrt{3}$ 2) $500\sqrt{3}$ 3) $250\sqrt{3}$ 4) $5\sqrt{3}$

Key: 2

Sol: Magnetic field due to one side at centre is

$$B = \frac{\mu_0 i}{4\pi r}\sin\alpha + \sin\beta$$

$$= \frac{\mu_0 i}{4\pi\left(\frac{\sqrt{3}a}{2}\right)}\left(\sin 30^\circ + \sin 30^\circ\right) \qquad = \frac{\mu_0 i}{2\pi\sqrt{3}a}$$

There are 6 sides and 50 turns $B = \frac{\mu_0 i}{2\pi\sqrt{3}(0.1)} \times 6 \times 50 = 500\sqrt{3}\frac{\mu_0 i}{\pi}$

15. Using screw gauge of pitch 0.1 cm and 50 divisions on its circular scale, the thickness of an object is measured. It should correctly be recorded as :

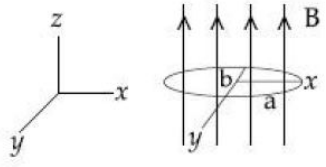
1) 2.125 cm 2) 2.124 cm 3) 2.121 cm 4) 2.123 cm

Key: 2

Sol: $LC = \frac{0.1cm}{50} = 0.002cm$

Reading should be multiple of C

16. An elliptical loop having resistance R, of semi major axis a, and semi minor axis b is placed in a magnetic field as shown in the figure. If the loop is rotated about the x-axis with angular frequency ω, the average power loss in the loop due to Joule heating is:



1) $\frac{\pi^2 a^2 b^2 B^2 \omega^2}{R}$ 2) $\frac{\pi^2 a^2 b^2 B^2 \omega^2}{2R}$ 3) $\frac{\pi abB\omega}{R}$ 4) Zero

Key: 2

Sol: $\phi = BA\sin\omega t = B(\pi ab)\sin\omega t$

$$\upsilon = \left|\frac{d\phi}{dt}\right| = B\pi ab\omega\cos\omega t$$

$$V_0 = B\pi ab\omega \qquad P = \frac{V_{rms}^2}{R} = \left(\frac{B\pi ab\omega}{\sqrt{2}}\right)^2\frac{1}{R}$$

17. A satellite is moving in a low nearly circular orbit around the earth. Its radius is roughly equal to that of the earth's radius $4R_e$. By firing rockets attached to it, its speed is instantaneously increased in the direction of its motion so that it become $\sqrt{\frac{3}{2}}$ times larger. Due to this the farthest distance from the centre of the earth that the satellite reaches is R. Value of R is :

1) $4R_e$ 2) $2R_e$ 3) $25R_e$ 4) $3R_e$

Key: 4

Sol: $U_i = \frac{-GMm}{2R}$

$$U_g = \frac{-GMm}{2R} + \frac{1}{2}m\left(\sqrt{\frac{3}{2}}\sqrt{\frac{GM}{R}}\right)^2$$

$$= \frac{-GMm}{4R} = \frac{-GMm}{2a} \qquad 2a = r_{min} + r_{max}$$

$$4R = R + r_{max} \qquad r_{max} = 3R$$

18. A uniform thin rope of length 12 m and mass 6 kg hangs vertically from a rigid support and a block of mass 2 kg is attached to its free end. A transverse short wave-train of wavelength 6 cm is produced at the lower end of the rope. What is the wavelength of wavetrain (in cm) when it reaches the top of the rope?

1) 12 2) 6 3) 3 4) 9

Key: 1

Sol: $V = f\lambda$

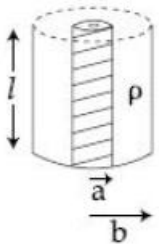
$V \propto \lambda$

$\sqrt{T} \propto \lambda$

$$\frac{\sqrt{2}}{\sqrt{2+6}} = \frac{6}{\lambda}$$

$$\lambda = 6(2) = 12\text{cm}$$

19. Model a torch battery of length l to be made up of a thin cylindrical bar of radius 'a' and a concentric thin cylindrical shell of radius 'b' filled in between with an electrolyte of resistivity ρ (see figure). If the battery is connected to a resistance of value R, the maximum Joule heating in R will take place for :



1) $R = \frac{\rho}{2\pi l}\left(\frac{b}{a}\right)$ 2) $R = \frac{\rho}{2\pi l}\ln\left(\frac{b}{a}\right)$ 3) $R = \frac{2\rho}{\pi l}\ln\left(\frac{b}{a}\right)$ 4) $R = \frac{\rho}{\pi l}\ln\left(\frac{b}{a}\right)$

Key: 2

Sol: Maximum joule heating n R is possible when internal resistance = external resistance

$$R_{internal} = \int_a^b \frac{Pdx}{2\pi xl} = \frac{P}{2\pi l}\ln\frac{b}{a} = R$$

20. Two isolated conducting spheres S_1 and S_2 of radius $\frac{2}{3}R$ and $\frac{1}{3}R$ have $12\,\mu C$ and $-3\,\mu C$ charges, respectively, and are at a large distance from each other. They are now connected by a conducting wire. A long time after this is done the charges on S_1 and S_2 are respectively :

1) $6\,\mu C$ and $3\,\mu C$
2) $4.5\,\mu C$ on both
3) $+4.5\,\mu C$ and $-4.5\,\mu C$
4) $3\,\mu C$ and $6\,\mu C$

Key: 1

Sol: After connected, Both spheres are at same potential.

$$\frac{q_1}{C_1}=\frac{q_2}{C_2}\times C\propto R$$

$$\frac{q}{2}=\frac{(12-3)-q}{1}$$

$$q=18-2q\Rightarrow q=6$$

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value(in decimal notation, truncated/ rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. A Bakelite beaker has volume capacity of 500 cc at at 30ºC. When it is partially filled with V_m volume (at 30ºC) of mercury, it is found that the unfilled volume of the beaker remains constant as temperature is varies. If $\gamma_{(beaker)}=6\times10^{-6}{}^{\circ}C^{-1}$ and $\gamma_{(mercury)}=1.5\times10^{-4}{}^{\circ}C^{-1}$, where γ is the coefficient of volume expansion, then V_m (in cc) is close to _____.

Key: 20

Sol: $\Delta V_{container}=\Delta V_{liq}$

$$\gamma_{beaker}V_{beaker}=\gamma_C V_l$$

$$\left(6\times10^{-6}\right)500=\left(1.5\times10^{-4}\right)V_m$$

$$V_m=20$$

22. A person of 80 kg mass is standing on the rim of a circular platform of mass 200 kg rotating about its axis at 5 revolutions per minute (rpm). The person now starts moving towards the centre of the platform. What will be the rotational speed (in rpm) of the platform when the person reaches its centre _____.

Key: 9

Sol: Conservation of angular momentum $\left(80R^2+200\frac{R^2}{2}\right)5=\left(80(0)^2+\frac{200R^2}{2}\right)\omega$

$$\omega=\frac{5\times180}{100}=9$$

23. When a long glass capillary tube of radius 0.015 cm is dipped in a liquid, the liquid rises to a height of 15 cm within it. If the contact angle between the liquid and glass to close to 0º, the surface tension of the liquid, in milliNewton m^{-1}, is $\left[\rho_{(liquid)}=900\,kgm^{-3}, g=10ms^{-2}\right]$ (Give answer in closest integer)

Key: 101

Sol: $h=\frac{2T\cos\theta}{r\rho g}$

$$15\times10^{-2}=\frac{2\times T\times1}{0.015\times10^{-2}\times900\times10}$$

$$T=101.25\,mN/m$$

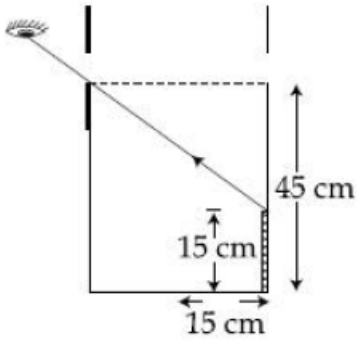
24. A cricket ball of mass 0.15 kg is thrown vertically up by a bowling machine so that it rises to a maximum height of 20 m after leaving the machine. If the part pushing the ball applies a constant force F on the ball and moves horizontally a distance of 0.2 m while launching the ball, the value of F (in N) is $\left(g=10ms^{-2}\right)$_____

Key: 150

Sol: Work energy theorem $F(0.2)=0.15\times10\times20$

$$F=150N$$

25. An observer can see through a small hole on the side of a jar (radius 15 cm) at a point at height of 15 cm from the bottom (see figure). The hole is at a height of 45 cm. When the jar is filled with a liquid up to a height of 30 cm the same observer can see the edge at the bottom of the jar. If the refractive index of the liquid is N/100, where N is an integer, the value of N is _____



Key: 158

Sol: $\text{Tani} = \frac{15}{15} = 1$

$i = 45^\circ$

$\text{Tanr} = \frac{15}{30} = \frac{1}{2}$

$\sin r = \frac{1}{\sqrt{5}}$

$1\sin i = \mu \sin r$

$1\frac{1}{\sqrt{2}} = \mu\frac{1}{\sqrt{5}}$

$\mu = \frac{\sqrt{5}}{\sqrt{2}}$

CHEMISTRY

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. Tyndall effect is observed when :

1) The diameter of dispersed particles is much smaller than the wavelength of light used.

2) The diameter of dispersed particles is much larger than the wavelength of light used

3) The refractive index of dispersed phase is greater than that of the dispersion medium

4) The diameter of dispersed particles is similar to the wavelength of light used.

Key: 4

Sol: a) The 'd' of D.P. 'X' of light used

b) The intensity of scattered light depends on difference between refractive indox of DP & DM.

c) In tyophillic colloids, the difference is appreciable and hence, they show tyhdal effect (T.E)

d) In tyophillic colloids the difference is very small and hence T.E. is very less.

e) T.E. $\propto$ difference in R.I of DP & DM

2. Of the species, NO, NO^+, NO^{2+} and NO^- ,the one with minimum bond strength is :

1) NO 2) NO^{2+} 3) NO^+ 4) NO^-

Key: 4

Sol: Bond strength is directly proportional to Bond order [Bond strength α bond order]

Species: $NO^{\oplus}$

According to M.O.T configuration of NO^+ is

$\overleftarrow{1s^2}\overleftarrow{1s^2}^*\overleftarrow{2s^2}\overleftarrow{2s^2}^*\ \pi 2py^2 = \pi 2p_z^2\overleftarrow{2p_x^2}$

Bond order = Bonding electrons – antibonding electrons

$= \frac{10-4}{2} = \frac{6}{2} = 3$

Species NO^{+2}

$\overleftarrow{1s^2}\,\overleftarrow{1s^2}^{*}\,\overleftarrow{2s}\,\overleftarrow{2s^2}^{*}\;\pi 2p_y^2 = \pi 2p_z^2\,\overleftarrow{2p_x^1}$

Bond order $= \frac{9-4}{2} = \frac{5}{2} = 2.5$

Species : $NO^{-}\;\overleftarrow{1s^2}\,\overleftarrow{1s^2}^{*}\;\overleftarrow{2s^2}\,\overleftarrow{2s^2}^{*}\;\pi 2p_y^2 = \pi 2p_z^2\,\overleftarrow{2p_x^2}\,\overleftarrow{2p_x^2}^{*}$

Bond order $= \frac{10-6}{2} = \frac{4}{2} = 2$

Species no $\overleftarrow{1s^2}\,\overleftarrow{1s^2}^{*}\;\overleftarrow{2s^2}\,\overleftarrow{2s^2}^{*}\;\pi 2p_y^2 = \pi 2p_z^2\,\overleftarrow{2p_x^2}\,\overleftarrow{2p_x^1}^{*}$

Bond order $= \frac{10-5}{2} = \frac{5}{2} = 2.5$

Low bond order having minimum strength

$Alc\,HCN, I_2 / OH^{-} NaOCl$

3. The atomic number of the element unnilennium is :

1) 109 2) 119 3) 102 4) 108

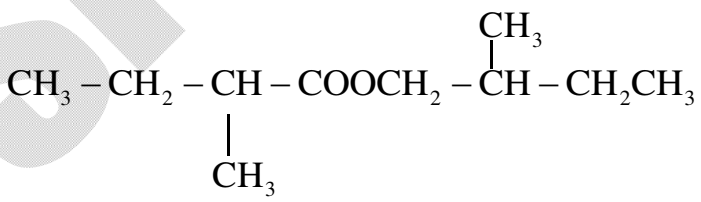
Key: 1

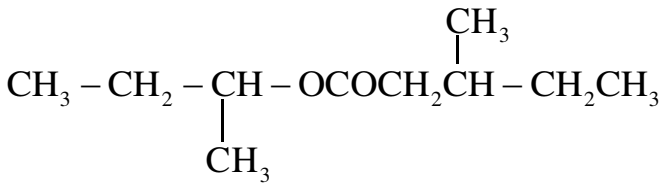
Sol: The atomic number of unnilennium is 109

Word root according to IUPAC cm-1

4. An organic compound [A], molecular formula $C_{10}H_{20}O_2$ was hydrolyzed with dilute sulphuric acid to give a carboxylic aicd [B] and an alcohol [C]. Oxidation of [C] with $CrO_3 - H_2SO_4$ produced [B]. Which of the following structures are not possible for [A]?

1)





2)

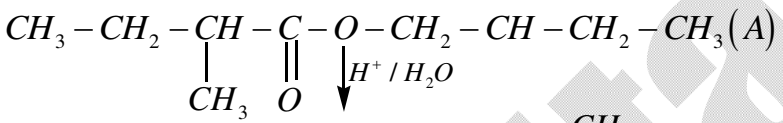
3) $CH_3CH_2CH_2COOCH_2CH_2CH_2CH_3$

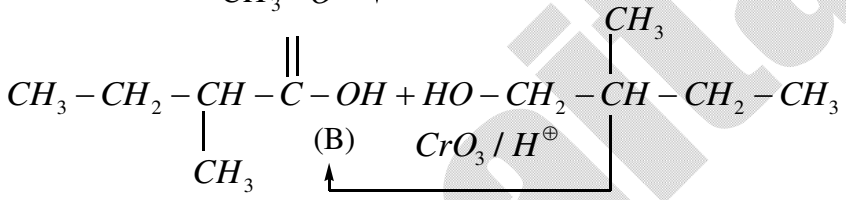
4) $(CH_3)_3-C-COOCH_2C(CH_3)_3$

Key: 2

Sol: 1) $CH_3-CH_2CH_2-COOCH_2-CH_2-CH_2-CH_3[C_8H_6O_2]$

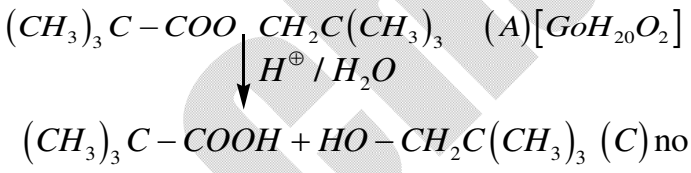
$M.F$ of A is $(C_6H_{20}O_2)$



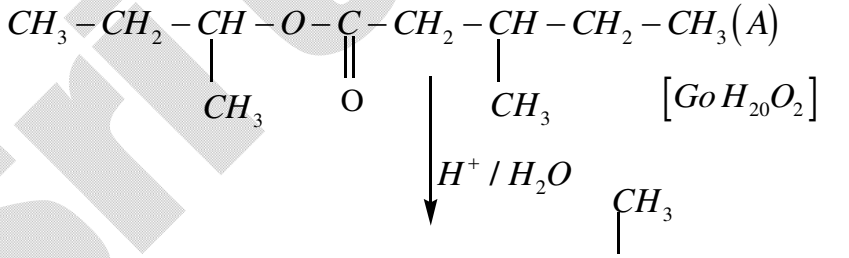


2)

C gives B



3) $(CH_3)_3C-COOH+HO-CH_2C(CH_3)_3$ (C) no oxidation

4) 

C) on oxidation with CrO_3 +H_2SO_4 does not give acid

5. The electronic spectrum of $\left[Ti\left(H_2O\right)_6\right]^{3+}$ shows a single broad peak with a maximum at $20,300\,cm^{-1}$. The crystal field stabilization energy (CFSE) of the complex ion, in kJ mol^{-1} , is : $\left(1\,kJ\,mol^{-1}=83.7\,cm^{-1}\right)$

1) 242.5 2) 83.7 3) 145.5 4) 97

Key: 4

Sol: complex $\left[Ti\left(H_2O\right)_6\right]^{+3}$ gives a peak at 20,300 Gm^{-1}

The crystal field strabilization energy is $\Delta_0 = hc\bar{r}$

$\therefore \Delta_0 = 6.625\times10^{-34}\times3\times10^{8}\times20,300$

$= 4\times10^{-19}\,J\,/\,molecule$

$= 6.023\times10^{23}\times4\times10^{-19}\,J\,/\,mole$

$= 243005\,J\,/\,mole = 243kJ\,/\,mole$

$Ti-22\left[Ar\right]4s^2 3d^2$

$Ti^{+3}-\left[Ar\right]4s^{o}3d^{1}$

$CFSE = 0.4\Delta_0 = 0.4\times243kJ\,/\,mole = 97kJ\,/\,mole$

6. The mechanism of SN^1 reaction is given as :

$$R-X \longrightarrow \underset{\text{Ion pair}}{R^{\oplus}X^{\ominus}} \longrightarrow \underset{\substack{\text{Solvent}\\ \text{Separated ion}\\ \text{pair}}}{R^{\oplus}\,\|\,X^{\ominus}} \xrightarrow{Y^{\ominus}} R-Y+X^{\ominus}$$

A student writes general characteristics based on the given mechanism as :

a) The reaction is favoured byweak nucleophiles

b) $R^{(+)}$ would be easily formed if the substituents are bulky

c) The reaction is accompained by recemization.

d) The reaction is favoured by non-polar solvents.

Which observations are correct?

1) (a), (b) and (c) 2) (b) and (d) 3) (a) and (c) 4) (a) and (b)

Key: 1

Sol: Given reaction is SN1

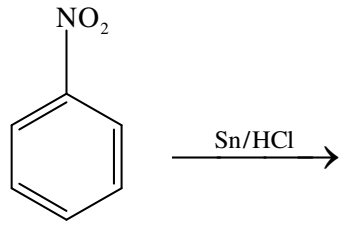
*SN1 reaction is independent of strength of nucleophiles. Generally strong nucleophiles favoured the SN2 but in SN1 weak nucleophiles are favored

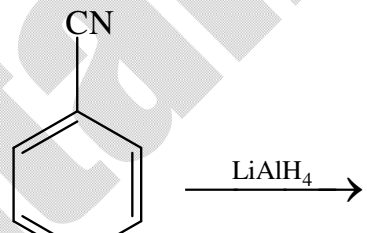
Order of stability of Alkyl carbocations $R^{\oplus}$ is $3^{o} > 2^{o} > 1^{o}$. Bulky carbocation (3°) more stable and forms easily increase the bulkyness of carbocation increase the stability of carbocation.

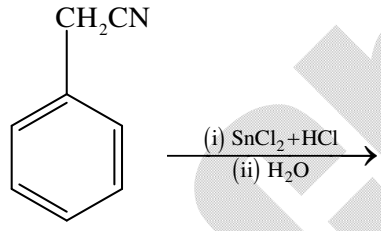
In SN1 both inversion and retension can takes place in equal ratio gives the Racemic mixture if R-X is optically active.

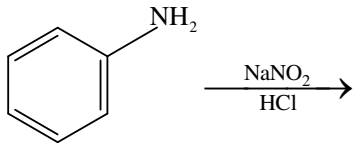
SN1 is favourable in polar solvent due to salvation of carbocation increase the stability of carbocation.

7. The Kjeldahl method of Nitrogen estimation fails for which of the following reaction products?

a) 

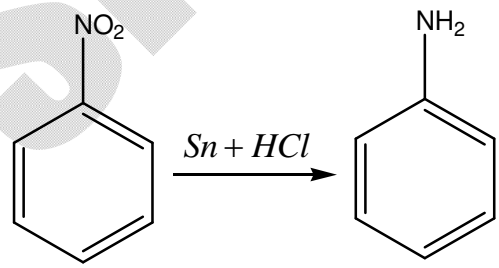
b) 

c) 

d) 

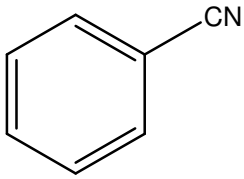
1) (b) and (c) 2) (a), (c) and (d) 3) (c) and (d) 4) (a) and (d)

Key: 3

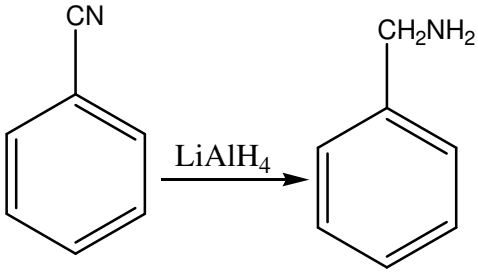


Sol: a)

Nitro benzene on reduction with Sn + HCl gives Aniline. Nitrogen is attached to carbon and hydrogen. Nitrogen can be estimated by using kjeldahl's method

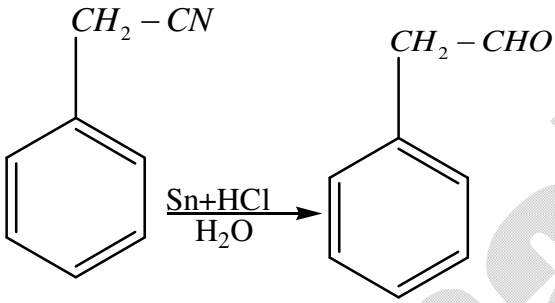


b) on reduction with $LiAlH_4$ gives benzyl amine.



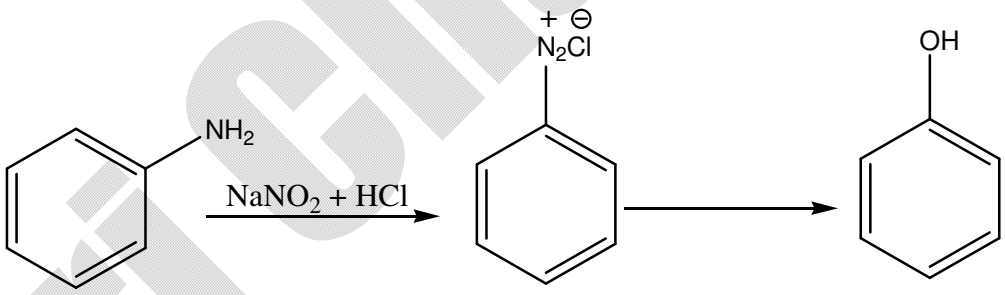
In a compound Nitrogen attached to both carbon and hydrogen, nitrogen can be estimated by Kjeldahl's method.

C) Benzyl cyanide on reduction with Sn+HCl and followed by hydrolysis gives aldehyde.



Resultant compound no nitrogen

d)



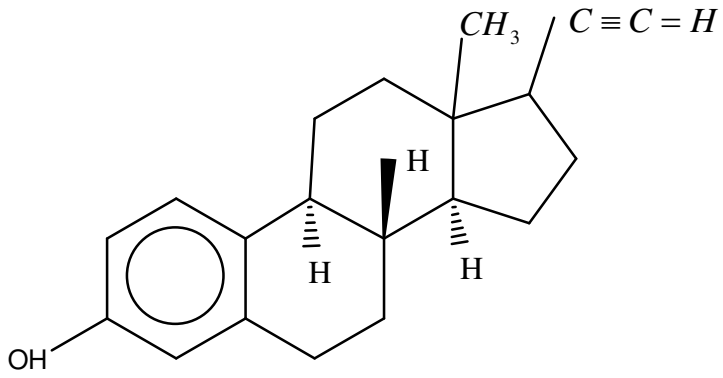
only at ice cold temperature gives diazonium in this reaction final product is phenol no nitrogen in the resultant compound.

8. The antifertility drug "Novestrol" can react with :

1) Alcoholic HCN; NaOCl; $ZnCl_2$ / HCl
2) Br_2 / water; $ZnCl_2$ / HCl; NaOCl
3) $ZnCl_2$ / HCl; $FeCl_3$; Alcoholic HCN
4) Br_2 / water; $ZnCl_2$ / HCl; $FeCl_3$

Key: 4

Sol:



Novestral [anti futility drug]

Which de colourised $Br_2 + H_2O$ due to $-C \equiv C^-$ and reacts with $ZnCl_2 + HCl$ presence of 3° alcohol group, and also gives +ve neutral $FeCl_3$ test presence of phenolic group (Does not give I_2 / OH^-)

9. Aqua regia is used for dissolving noble metals (Au, Pt, etc.). The gas evolved in this process is :

1) NO 2) N_2 3) N_2O_3 4) N_2O_5

Key: 1

Sol: $Au + 4H^{(+)} + NO_3^{(-)} + 4Cl^{(-)} \rightarrow [AuCl_4]_2^{(-)} + NO + 2H_2O$

$3Pt + 16H^{(+)} + 4NO_3^{(-)} + 18Cl^{(-1)} \rightarrow 3[PtCl_6]_2^{(-)} + 4NO + 8H_2O$

So Aquaregia is used to dissolve Au, Pt

Here, NO gas releases.

10. Thermal power plants can lead to :

1) Eutrophication 2) Blue baby syndrome

3) Acid rain 4) Ozone layer depletion

Key: 2

Sol: During burning of fossil fuels (Coal, oil), the gases like SO_2, NO_2 release and these form acid rain with rain – water.

11. Henry's constant (in kbar) for four gases α, β, γ and δ in water at 298 K is given below :

	α	β	γ	δ
K_H	50	2	2×10^{-5}	0.5

(density of water $= 10^3 \text{kg m}^{-3}$ at 298 K)

This table implies that :

1) The pressure of a 55.5 molal solution of δ is 250 bar

2) Solubility of γ at 308 K is lower than at 298 K

3) The pressure of a 55.5 molal solution of γ is 1 bar

4) α has the highest solubility in water at a given pressure

Key: 1

Sol: 1) Solubility $\propto \left(\frac{1}{KH}\right)$

So, V – has highest solubility

2) It is not related to data of the table

3) $(P)_8 = (K_H)_8 \times (x)_8$

$$(P)_8 = 0.5\times10^3 \times \left(\frac{55.5}{55.5+\frac{1000}{18}}\right)$$

= 250 bar

4) FUV :

$(P)V = (K_H)_V \times (x)_V$

$$(P)V = 2\times10^{-5}\times10^3 \times \left(\frac{55.5}{55.5+\frac{1000}{18}}\right)$$

= 0.01 bar

12. Glycerol is separated in soap industries by :

1) Steam distillation
2) Fractional distillation
3) Distillation under reduced pressure
4) Differential extraction

Key: 3

Sol: Glycerol is separated in soap industries by distillation under reduced pressure
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13. An acidic buffer is obtained on mixing :

1) 100 mL of 0.1 M HCl and 200 mL of 0.1 M NaCl
2) 100 mL of 0.1 CH_3COOH and 100 mL of 0.1 M NaOH
3) 100 mL of 0.1 M CH_3COOH and 200 mL of 0.1 M NaOH
4) 100 mL of 0.1 M HCl and 200 mL of 0.1 M CH_3COONa

Key: 4

Sol: 1) $CH_3COONa + HCl \rightarrow CH_3COOH + NaCl$

200×0.1	100×0.1	0
$(20 - x)$	$(10 - x)$	0
$(1 = 10)(20 - 10)$	$(10 - 10)$	10
$= 10$	$= 0$	10

R.S $= CH_3COONa + CH_3COOH = 10 : 10$

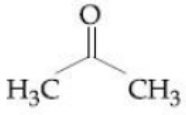
$= 1 : 1$ ratio $\Rightarrow$ A.B.S

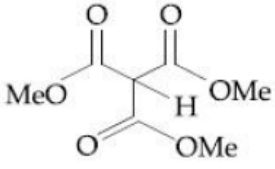
2) $HCl + NaCl \Rightarrow$ Not buffer

3) $CH_3COOH + NaOh = 1 : 1 \Rightarrow$ Not buffer

4) $CH_3COOH + NaOH = 1 : 2 \Rightarrow$ Not buffer

14. Which one of the following compounds possesses the most acidic hydrogen?

1) 

2) 

3) $H_3C-C\equiv C-H$

4) 

Key: 2

Sol:

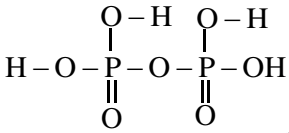
Conjugate base of the compound is more stable because of carbanion is conjugate with three carbonyl groups.

15. In a molecule of pyrophosphoric acid, the number of $P-OH, P=O$ and $P-O-P$ bonds/moiety (ies) respectively are :

1) 4, 2 and 0 2) 4, 2 and 1 3) 3, 3 and 3 4) 2, 4 and 1

Key: 2

Sol: $H_4P_2O_7$



$(P-OH)=4$

$(P=O)=2$

$(P-O-P)=1$

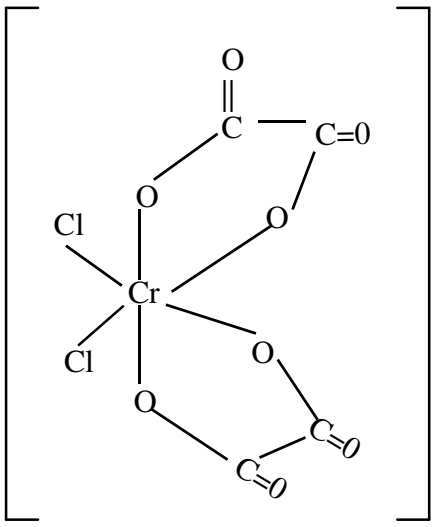
16. The complex that can shown optical activity is :

1) trans $-\left[Fe(NH_3)_2(CN)_4\right]^-$ 2) cis $-\left[Fe(NH_3)_2(CN)_4\right]^-$

3) trans $-\left[Cr(Cl_2)(ox)_2\right]^{3-}$ 4) cis $-\left[CrCl_2(ox)_2\right]^{3-}$ $(ox = oxalate)$

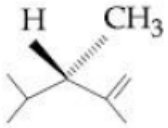
Key: 4

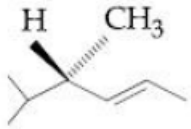
Sol: Cis $\left[CrCl_2(OX)_2\right]^{-3}$ is optically active because the compound does not contain any symmetry



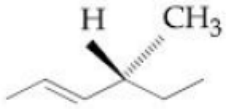
Trans $\left[CrCl_2(OX)_2\right]^{-3}$ is OIA due to POS

17. Which of the following compounds produces an optically inactive compound on hydrogenation?

1) 

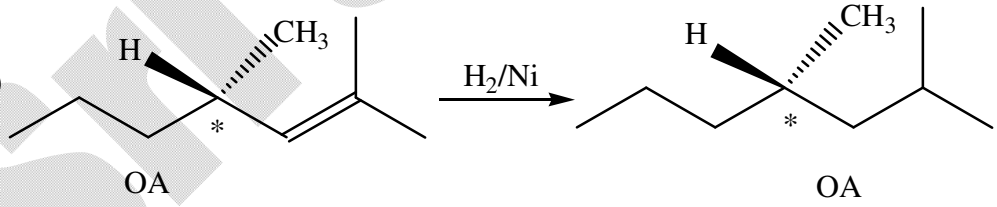
2) 

3) 

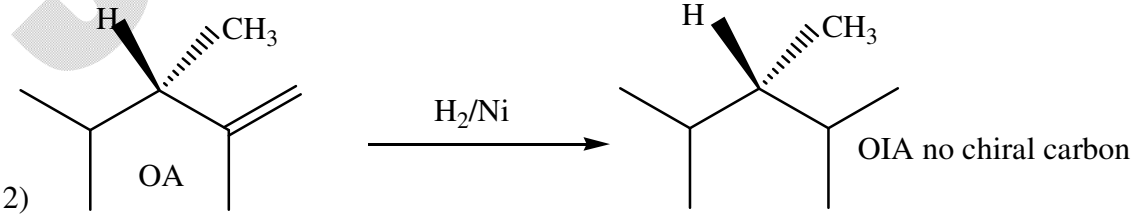
4) 

Key: 1

Sol:

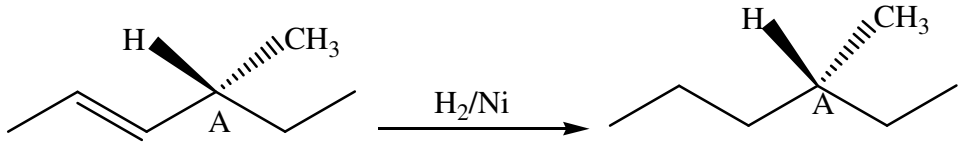
1) 

OA OA

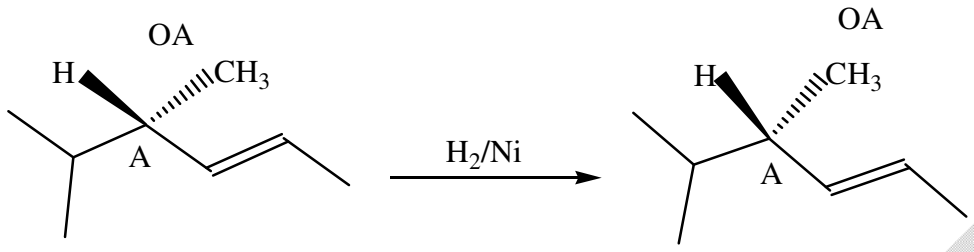
2) 

OIA no chiral carbon

3)



4)



18. Let C_{NaCl} and C_{BaSO_4} be the conductance (in S) measured for saturated aqueous solutions of NaCl and $BaSO_4$, respectively, at a temperature T.

Which of the following is false ?

1) $C_{BaSO_4}(T_2) > C_{BaSO_4}(T_1)$ for $T_2 > T_1$

2) $C_{NaCl}(T_2) > C_{NaCl}(T_1)$ for $T_2 > T_1$

3) $C_{NaCl} >> C_{BaSO_4}$ at a given T

4) Ionic mobilities of ions from both salts increase with T.

Key: 2

Sol: Here, NaCl is strong electrolyte so it ions completely at room temperature. So its ionisation no changes (V-U less extent charge) with increase of temperature. But $BaSO_4$ is weak electrolyte and hence its conductance increases with temperature.

19. It is true that :

1) A zero order reaction is a single step reaction

2) A first order reaction is always a single step reaction

3) A second order reaction is always a multistep reaction

4) A zero order reaction is a multistep reaction

Key: 4

Sol: A zero circle reaction is complex and takes place in many steps. For each step molecularity cannot be zero.

20. If the boiling point of H_2O is 373 K, the boiling point of H_2S will be :

1) Equal to 373 K

2) More than 373 K

3) Greater than 300 K but less than 373 K

4) Less than 300 K

Key: 4

Sol: At room temperature water is liquid and has high Boiling point 373K due to inter molecular hydrogen bonding where as H_2S is gas it has no hydrogen bonding. Hence is less than 300K. [BP if H_2S is -60ºC]

b.p. of $H_2S \le H_2O$

$bp(H_2O) = 373K$

$bp(H_2S) = 213K$

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value (in decimal notation, truncated/ rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer , 0 if not attempted and 0 in all other cases.

21. An element with molar mass $2.7 \times 10^{-2}\,kg\,mol^{-1}$ forms a cubic unit cell with edge length 405 pm. If its density is $2.7 \times 10^{3}\,kg\,m^{-3}$, the radius of the element is approximately $\_\_\_\_\_ \times 10^{-12}\,m$ (to the nearest integer).

Key: 143.00

Sol: $d = \dfrac{Z \times M}{No \times \forall} \Rightarrow$

$$2.7 \times 10^{3} = \frac{Z \times 2.7 \times 10^{-2}}{6 \times 10^{23} \times \left(405 \times 10^{-12}\right)^{3}}$$

$Z = 4 \Rightarrow fcc$ So $\sqrt{2}a = 4r$

$$r = \left(\frac{\sqrt{2} \times 405}{4}\right) = 143.1675\,pm$$

22. The mole fraction of glucose $\left(C_6H_{12}O_6\right)$ in an aqueous binary solution is 0.1. The mass percentage of water in it, to the nearest integer, is _____.

Key: 47.00

Sol: m. f. of glucose $= \dfrac{\text{moles of glucose}}{\left(\text{moles of glucose} + \text{moles of } H_2O\right)}$

$$0.1 = \left(\frac{1}{10}\right) = \left(\frac{1}{1+9}\right)$$

So glucose moles = 1, (h glucose) $\qquad hH_2O = 9$

W glucose $= 1 \times 180\,\text{grm}$ $\qquad WH_2O = 9 \times 18\,\text{grmas}$

$$\text{Mass \% of } H_2O = \left(\frac{\omega H_2O \times 100}{\omega H_2O + \omega\text{glucose}}\right)$$

$$= \left(\frac{9 \times 18 \times 100}{(9 \times 18) + 180}\right) \qquad = \left(\frac{162 \times 100}{342}\right) = 47.368$$

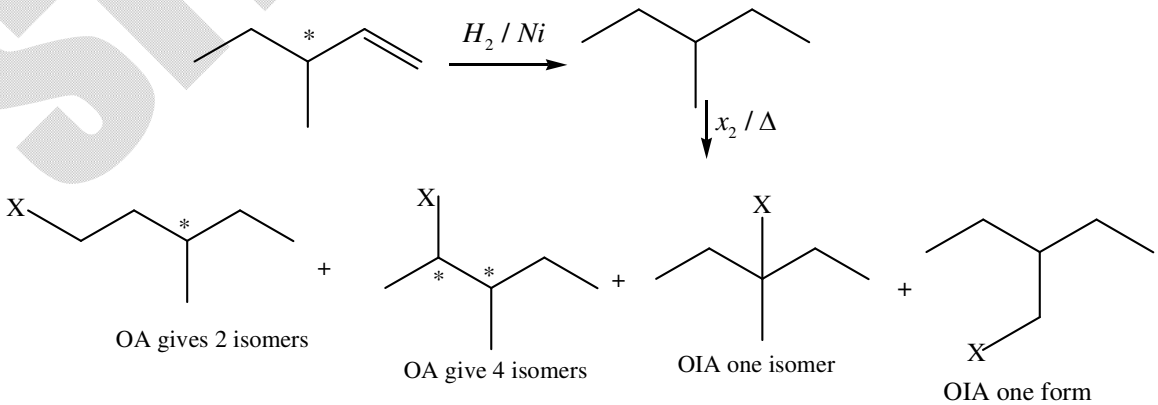
23. The total number of monohalogenated organic products in the following (including steroisomers) reaction is _____.

A $\xrightarrow[\text{(ii) } X_2/\Delta]{\text{(i) } H_2/Ni/\Delta}$

(Simplest optically active alkene)

Key: 8.00

Sol: Simplest optically active Alkene is



24. The volume strength of 8.9 M H_2O_2 solution calculated at 273 K and 1 atm is _____. ($R = 0.0821\ L\ atm\ K^{-1}mol^{-1}$) (rounded off to the nearest integer)

Key: 100.00

Sol: 10 Volumes st of $H_2O_2 \longrightarrow 0.893M$ of H_2O_2

$Y = ? \longrightarrow 8.9M \qquad y = \dfrac{10 \times 8.9}{0.893} = 99.664 \cong 100$

25. The photoelectric current from Na (work function, $w_0 = 2.3eV$) is stopped by the output voltage of the cell $Pt(s)\left|H_2(g,1bar)\right|HCl(aq.,pH=1)\left|AgCl(s)\right|Ag(s)$.

The pH of aq. HCl required to stop the photoelectric current from $K(w_0 = 2.25eV)$, all other conditions remaining the same, is _____ $\times 10^{-2}$ (to the nearest integer).

$2.303\dfrac{RT}{F} = 0.06V; E^0_{AgCl|Ag|Cl^{-1}} = 0.22V$

Key: 58.00

Sol: $H_2 \rightarrow 2H^+ + 2e^-$

$2e^- + 2AgCl \rightarrow 2Ag + 2Cl^-$

$E = E^0_{AgCl/Ag/Cl^-} - \dfrac{0.06}{2}\log\left[H^+\right]^2[Cl]^2$

$E^\circ - \dfrac{0.06}{2}\log\left(10^{-1}\right)^2\left(10^{-1}\right)^2 = p - 2.3$

$(-)E^\circ - \dfrac{0.06}{2}\log\left[H^+\right]^2[Cl]^2 = p - 2.25$

$\dfrac{0.06}{2}\left[\log\left[H^+\right]^4 - \log 10^{-4}\right] = -0.05$

$\dfrac{0.06}{2}\left[-4pH + 4\right] = -0.05 \quad (0.06)(2)[1 - pH] = -0.05$

$1 - pH = \dfrac{-5}{12} \qquad pH = \dfrac{14}{12}$

$pH = \dfrac{1700}{12} \times 10^{-2}$

$pH = 141.66 \times 10^{-2}$

MATHEMATICS

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. If $y^2+\log_e\left(\cos^2 x\right)=y, x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ then :

1) $|y'(0)|+|y''(0)|=1$ 2) $|y''(0)|=2$

3) $|y'(0)|+|y''(0)|=3$ 4) $y''(0)=0$

Key: 2

Sol: $2yy'-2\tan x=y'$ _____ (1)

$2yy''+2(y')^2-2\sec^2 x=y''$ _____ (2)

when x=0 $\Rightarrow$ y=0,1

if x=0 and y=0 $\Rightarrow y'(0)=0$

if x=0 and y=1 $\Rightarrow y'(0)=0$

equation $(2)\,2yy''+2(y')^2-2\sec^2 x=y''$

$at\ x=0\Rightarrow 2yy''-2=y''\Rightarrow y''(2y-1)=2$

$\Rightarrow y''(0)=\frac{2}{2y-1}=\begin{cases}-2 & ;y=0\\ 2 & y=1\end{cases}$ $\Rightarrow |y''(0)|=2$

2. The proposition $p\rightarrow\sim(p\wedge\sim q)$ is equivalent to :

1) $(\sim p)\vee q$ 2) $(\sim p)\wedge q$ 3) $(\sim p)\vee(\sim q)$ 4) q

Key: 1

Sol: $\equiv p\Rightarrow(\sim(p\wedge\sim q))$

$\equiv\sim p\vee(\sim(p\wedge\sim q))$

$\equiv\sim p\vee(\sim p\vee q)$

$\equiv\sim p\vee q$

3. If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is :

1) $\frac{1}{6}$ 2) $\frac{1}{5}$ 3) $\frac{1}{4}$ 4) $\frac{1}{7}$

Key: 1

Sol: $S_{25} = S_{15}^{1} = S_{40} - S_{25} \Rightarrow 2.S_{25} = S_{40} \Rightarrow 2.\frac{25}{3}\left[2a+24d\right] = \frac{40}{2}\left[2a+39d\right]$

$\Rightarrow 25\left[6+24d\right] = 20\left[6+39d\right] \Rightarrow 25\times 6\left(1+4d\right) = 20\times 3\left(2+13d\right) \Rightarrow 10+40d = 8+52d \Rightarrow d = \frac{1}{6}$

4. For the frequency distribution :

$\text{Variate}(x)$: x_1 x_2 $x_3 \ldots\ldots x_{15}$

$\text{Frequency}(f)$: f_1 f_2 $f_3 \ldots\ldots f_{15}$

where $0 < x_1 < x_2 < x_3 < \ldots\ldots < x_{15} = 10$ and

$\sum_{i=1}^{15} f_i > 0$, the standard deviation cannot be :

1) 1 2) 4 3) 6 4) 2

Key: 3

Sol: variance $\sigma^2 = \frac{\sum fixi^2}{\sum fi} - \left(\frac{\sum fixi}{\sum fi}\right)^2$

Given $f_1 + f_2 + \ldots + f_{15} > 0$ and $x_1, x_2 \ldots\ldots x_{15}$ are positive and variance $\leq$ range

$\therefore$ Variance cannot be 6

5. $2\pi - \left(\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65}\right)$ is equal to :

1) $\frac{\pi}{2}$ 2) $\frac{7\pi}{4}$ 3) $\frac{3\pi}{2}$ 4) $\frac{5\pi}{4}$

Key: 3

Sol: $2\pi-\left[\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{16}{63}\right]$

$$\Rightarrow 2\pi-\left[\tan^{-1}\left(\frac{\frac{4}{5}+\frac{5}{12}}{1-\frac{4}{3}.\frac{5}{12}}\right)+\tan^{-1}\frac{16}{63}\right]$$

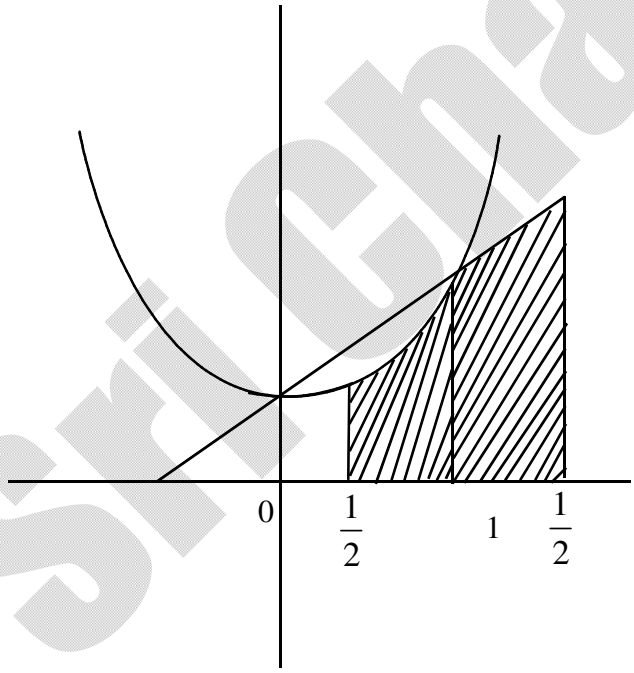
$$\Rightarrow 2\pi-\left[\tan^{-1}\frac{63}{16}+\cot^{-1}\frac{63}{16}\right]=2\pi-\frac{\pi}{2}=\frac{3\pi}{2}$$

6. The area (in sq. units) of the region $\left\{(x,y):0\le y\le x^2+1,0\le y\le x+1,\frac{1}{2}\le x\le 2\right\}$ is :

1) $\frac{23}{16}$ 2) $\frac{23}{6}$ 3) $\frac{79}{24}$ 4) $\frac{79}{16}$

Key: 3

Sol: $y\le x^2+1, y\le x+1, \frac{1}{2}\le x\le 2$



$$Area=\int_{1/2}^{1}\left(x^2+1\right)+\int_{1}^{2}\left(x+1\right)$$

$$=\frac{79}{24}Sq.units$$

7. The solution curve of the differential equation, $\left(1+e^{-x}\right)\left(1+y^2\right)\frac{dy}{dx}=y^2$, which passes through the point (0,1), is :

1) $y^2+1=y\left(\log_e\left(\frac{1+e^{-x}}{2}\right)+2\right)$ 2) $y^2=1+y\log_e\left(\frac{1+e^{x}}{2}\right)$

3) $y^2+1=y\left(\log_e\left(\frac{1+e^{x}}{2}\right)+2\right)$ 4) $y^2=1+y\log_e\left(\frac{1+e^{-x}}{2}\right)$

Key: 2

Sol: Given equation is $\frac{1+y^2}{y^2}dy=\frac{e^x}{1+e^x}dx\Rightarrow\left(\frac{1}{y^2}+1\right)dy=\frac{e^x dx}{1+e^x}\Rightarrow-\frac{1}{y}+y=\ln\left(1+e^x\right)+C$

$\therefore$ passes through $(0,1)$

$\therefore -1+1=\ln 2+C\Rightarrow C=-\ln 2$

Equation of curve $y-\frac{1}{y}=\ln\left(1+e^x\right)-\ln 2$

$\Rightarrow\left(y^2-1\right)=y\ln\left(\frac{1+e^x}{2}\right)$

8. Let [t] denote the greatest integer $\le$ t. If for some $\lambda\in R-\{0,\ 1\}$, $\lim\limits_{x\to0}\left|\frac{1-x+|x|}{\lambda-x+[x]}\right|=L$, then L is equal to :

1) $\frac{1}{2}$ 2) 0 3) 1 4) 2

Key: 4

Sol: $LHL=RHL$

$\Rightarrow\lim\limits_{h\to0}\left(\frac{1+h+h}{1-1-\lambda}\right)=\lim\limits_{h\to0}\left(\frac{1-h+h}{1+0-\lambda}\right)$ $\Rightarrow\frac{1}{\lambda}=\frac{1}{1-\lambda}\Rightarrow1-\lambda=\lambda=\frac{1}{2}$

$\Rightarrow L=\lim\limits_{x\to0}\left|\frac{1-x+|x|}{\frac{1}{2}+[x]}\right|=\frac{1}{\frac{1}{2}}=2$

9. A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse $3x^2+4y^2=12$, then this hyperbola does not pass through which of the following points?

1) $\left(1,-\frac{1}{\sqrt{2}}\right)$ 2) $\left(\sqrt{\frac{3}{2}},\frac{1}{\sqrt{2}}\right)$ 3) $\left(\frac{1}{\sqrt{2}},0\right)$ 4) $\left(-\sqrt{\frac{3}{2}},1\right)$

Key: 2

Sol: ellipse $\frac{x^2}{4}+\frac{y}{3}=1$ $a_1=2$ $b_1=\sqrt{3}$ $e_1=\sqrt{1-\frac{3}{2}}=\frac{1}{2}$

Focus $=(\pm1,0)$

For hyperbola $a_2=\frac{1}{\sqrt{2}}$ and $a_2e_2=1\Rightarrow e_2=\sqrt{2}\Rightarrow b_2=a_2=\frac{1}{\sqrt{2}}$

Hyperbola $\frac{x^2}{a_2^2}-\frac{y^2}{b_2^2}=1$

$\Rightarrow x^2-y^2=\frac{1}{2}$

10. A die is thrown two times and the sum of the scores appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is :

1) $\frac{1}{8}$ 2) $\frac{1}{4}$ 3) $\frac{1}{3}$ 4) $\frac{1}{9}$

Key: 4

Sol: $A\rightarrow$ multiple of 4, $B\rightarrow$ at least one dice shows 4

$(1,3),(2,2),(3,1)$

$(6,2)(2,6),(5,3)(3,5)(4,4)\rightarrow$ favourable

$(6,6)$

$P\left(\frac{B}{A}\right)=\frac{1}{9}$

11. Consider the two sets :

$A=\{m\in R:\text{both the roots of } x^2-(m+1)x+m+4=0 \text{ are real}\}$ and $B=[-3,5)$

Which of the following is not true ?

1) $A\cup B=R$ 2) $A\cap B=\{-3\}$

3) $A-B=(-\infty,-3)\cup(5,\infty)$ 4) $B-A=(-3,5)$

Key: 3

Sol: $x^2-(m+1)x+(m+4)=0$

Roots are real $\Rightarrow \Delta\geq 0$

$\Rightarrow A=(-\infty,-3]\cup[5,\infty)$

$B=[-3,5)$

$A-B=(-\infty,-3)\cup[5,\infty)$

$\therefore$ option 3 is wrong

12. $\int_{-\pi}^{\pi}|\pi-|x||dx$ is equal to :

1) π^2 2) $\frac{\pi^2}{2}$ 3) $\sqrt{2}\pi^2$ 4) $2\pi^2$

Key: 1

Sol: $2\int_0^{\pi}|\pi-x|dx=2\int_0^{\pi}(\pi-x)dx=2\left[\pi x-\frac{x^2}{2}\right]_0^{\pi}=2\left(\pi^2-\frac{\pi^2}{2}\right)=2.\frac{\pi^2}{2}=\pi^2$

13. If α and β are the roots of the equation $x^2+px+2=0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2+2qx+1=0$, then $\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right)\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$ is equal

1) $\frac{9}{4}(9-p^2)$ 2) $\frac{9}{4}(9-q^2)$ 3) $\frac{9}{4}(9+q^2)$ 4) $\frac{9}{4}(9+p^2)$

Key: 1

Sol: $\alpha+\beta=-p,\alpha\beta=2$

$\frac{1}{\alpha}+\frac{1}{\beta}=q,\frac{1}{\alpha\beta}=\frac{1}{2}$

Now $\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right)=\left(\alpha\beta+2+\frac{1}{\alpha\beta}\right)\left(\alpha\beta-\frac{\alpha}{\beta}-\frac{\beta}{\alpha}+\frac{1}{\alpha\beta}\right)$

$=\left(2+2+\frac{1}{2}\right)\left(2-\frac{\alpha^2+\beta^2}{\alpha\beta}+\frac{1}{2}\right)=\frac{9}{2}\left(\frac{5}{2}-\frac{(\alpha-\beta)^2-2\alpha\beta}{\alpha\beta}\right)$

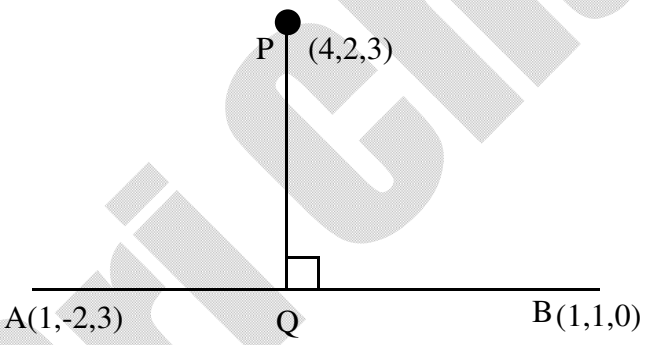
$=\frac{9}{2}\left(\frac{5}{2}-\frac{p^2-4}{2}\right)=\frac{9}{2}\left(\frac{5-p^2+4}{2}\right)=\frac{9}{4}\left(9-p^2\right)$

14. The foot of the perpendicular drawn from the point (4, 2, 3) to the line joining the points (1, -2, 3) and (1, 1, 0) lies on the plane:

1) $x+2y-z=1$ 2) $x-2y+z=1$ 3) $x-y-2z=1$ 4) $2x+y-z=1$

Key: 4

Sol: Line AB is



$\frac{x-1}{0}=\frac{y+2}{3}=\frac{z-3}{-3}$

$\Rightarrow\frac{x-1}{0}=\frac{y+2}{1}=\frac{z-3}{-1}=t$

$Q=(1,t-2,-t+3)$

DR's of $\overrightarrow{PQ}=(-3,t-4,-t)$

But $PQ\perp AB\Rightarrow t=2\Rightarrow Q=\text{foot}=(1,0,1)$

So, Option (4) is correct

15. The lines $\vec{r}=\left(\hat{i}-\hat{j}\right)+l\left(2\hat{i}+\hat{k}\right)$ and $\vec{r}=\left(2\hat{i}-\hat{j}\right)+m\left(\hat{i}+\hat{j}-\hat{k}\right)$

1) Intersect when $l = 1$ and $m = 2$

2) Intersect for all value of l and m

3) Intersect when $l = 2$ and $m=\frac{1}{2}$

4) Do not intersect for any value of l and m

Key: 4

Sol: $\vec{r}=\left(\vec{i}-\vec{j}\right)+l\left(2\vec{i}+\vec{k}\right)$

$\vec{r}=\left(2\vec{i}-\vec{j}\right)+m\left(\vec{i}+\vec{j}-\vec{k}\right)$

$2r+1=m+2\Rightarrow 2l-m=1\rightarrow(1)$

$-1=-1+m\Rightarrow m=0\rightarrow 2$

$\Rightarrow l=\frac{1}{2}$

Also $l=-m$ which is not true

So lines do not intersect $\Rightarrow$ option 4

16. If $\Delta=\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix}=Ax^2+Bx^2+Cx+D$, then B+ C is equal to :

1) -3 2) 9 3) -1 4) 1

Key: 1

Sol: Using $R_2\rightarrow R_2-2R_1$

$R_3\rightarrow R_3-3R_1$ and then expand

We get $Ax^3+Bx^2+Cx+D=-3x^3+12x^2-15x+6$

$\Rightarrow A=-3,B=12,C=-15,D=6$

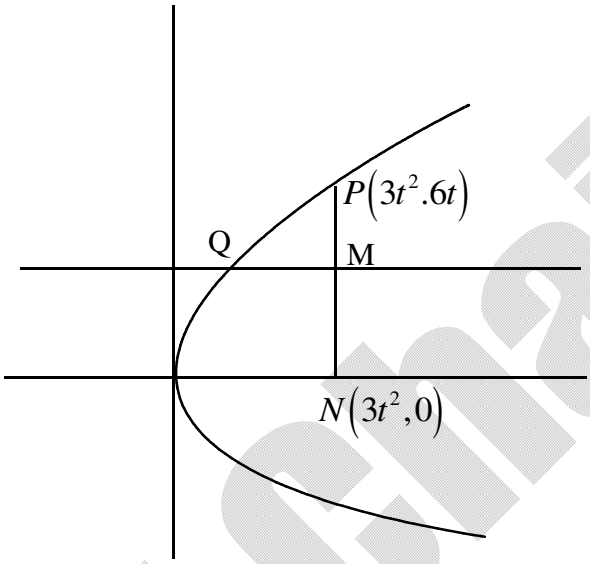
$B+C=-3$

17. Let P be a point on the parabola, $y^2 = 12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid-point M of PN, parallel to its axis which meets the parabola at Q. If the y-intercept of the line NQ is $\frac{4}{3}$, then :

1) $MQ = \frac{1}{4}$ 2) PN = 3 3) PN = 4 4) $MQ = \frac{1}{3}$

Key: 1

Sol: $M = \left(3t^2, 3t\right)$



MQ line is y=3t

Solving $y^2 = 12x, y = 3t$

$\Rightarrow x = \frac{3t^2}{4}$

$Q = \left(\frac{3t^2}{4}, 3t\right), N = \left(3t^2, 0\right)$

Equation of NQ is $4x + 3ty = 12t^2$

$x = 0 \Rightarrow y = 4t = \frac{4}{3}\left(given\right)$

$\Rightarrow t = \frac{1}{3}$

$MQ = \frac{1}{4}$

18. If the number of integral terms in the expansion of $\left(3^{\frac{1}{2}}+5^{\frac{1}{8}}\right)^n$ is exactly 33, then the least value of n is :

1) 248 2) 264 3) 128 4) 256

Key: 4

Sol: $T_{r+1}={}^nC_r\,2^{\frac{n-r}{2}}5^{\frac{r}{8}}$

For integral terms

$\frac{r}{8}$ and $\frac{n-r}{2}$

Then $r=8,16,24,.....$

$n=t_{33}=0+32\times 8=256$

19. The function, $f(x)=(3x-7)x^{2/3}, x\in R$ is increasing for all x lying in :

1) $\left(-\infty,\frac{14}{15}\right)\cup(0,\infty)$ 2) $\left(-\infty,\frac{14}{15}\right)$

3) $(-\infty,0)\cup\left(\frac{14}{15},\infty\right)$ 4) $(-\infty,0)\cup\left(\frac{3}{7},\infty\right)$

Key: 3

Sol: $f(x)=(3x-7)x^{2/3}$

$f'(x)>0\Rightarrow x^{2/3}\left(\frac{15x-14}{3x}\right)>0$

$\Rightarrow x(15x-14)>0$

$\therefore$ f increases for all $n\in(-\infty,0)\cup\left(\frac{14}{15},\infty\right)$

20. The value of $\left(2.{}^1P_0-3.{}^2P_1+4.{}^3P_2-.....\text{upto}\,51^{\text{th}}\text{ term}\right)+\left(1!-2!+3!-.....\text{up to}\,51^{\text{th term}}\right)$ is equal to :

1) $1-51(51)!$ 2) $1+(52)!$ 3) 1 4) $1+(51)!$

Key: 2

Sol: $S=\left(2^1p_0-3^2P_1+4^3P_2+.....51\text{terms}\right)+\left(1!-2!+3!-4!+.....51\text{terms}\right)$

$=2!-3!+4!-5!+.....+52!+\left(1!-2!+3!-4!+.....+51!\right)=1+52!$

(NUMERICAL VALUE TYPE)

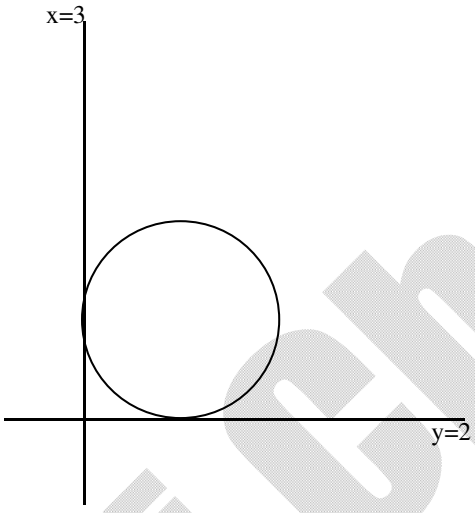
This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value (in decimal notation, truncated/ rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. The diameter of the circle, whose centre lies on the line x + y = 2 in the first quadrant and which touches both the lines x = 3 and y = 2, is _____

Key: 3.00

Sol: Center lies on $x+y=2$



Centre $=(\alpha,2-\alpha)$

Radius $=|\alpha-3|=|\alpha|$

$\Rightarrow\alpha-3=-\alpha$

$\Rightarrow\alpha=\frac{3}{2}$

Radius $=\frac{3}{2}\Rightarrow$ diameter $=3$

22. The value of $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+....\text{to }\infty\right)}$ is equal to _____

Key: 4.00

Sol: $0.16^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\infty\right)}$

$=0.16^{\log_{2.5}\left(\frac{1}{2}\right)}=0.16^{\log_5\frac{1}{2}}=\left(\frac{2}{5}\right)^{2\log_{\left(\frac{5}{2}\right)}\left(\frac{1}{2}\right)}=\left(\frac{5}{2}\right)^{-2\log_{\left(\frac{5}{2}\right)}\left(\frac{1}{2}\right)}$

$=\left(\frac{5}{2}\right)^{2\log_{\frac{5}{2}}2}$

$=\left(\frac{5}{2}\right)^{\log_{\frac{5}{2}}4}=4$

23. If $\left(\frac{1+i}{1-i}\right)^{m/2}=\left(\frac{1+i}{i-1}\right)^{n/3}=1,(m,n\in N)$ then the greatest common divisor of the least values of m and n is _____.

Key: 4.00

Sol: $\left(\frac{(1+i)^2}{2}\right)^{\frac{m}{2}}=\left(\frac{(1+i)^2}{2}\right)^{\frac{m}{3}}=1\Rightarrow\left(\frac{2i}{2}\right)^{m/2}=\left(\frac{2i}{2}\right)^{n/3}=1\Rightarrow i^{m/2}=i^3=1$

$M_{least}=8$, $n_{least}=12$

HCM(8,12)= 4

24. Let $A=\begin{bmatrix}x & 1\\ 1 & 0\end{bmatrix}, x\in R$ and $A^4=\begin{bmatrix}a_{ij}\end{bmatrix}$. If $a_{11}=109$, then a_{22} is equal to_____

Key: 10.00

Sol: $A^2=\begin{bmatrix}x & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}x & 1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}x^2+1 & x\\ x & 1\end{bmatrix}$

$A^4=\begin{bmatrix}x^2+1 & x\\ x & 1\end{bmatrix}\begin{bmatrix}x^2+1 & x\\ x & 1\end{bmatrix}=\begin{bmatrix}(x^2+1)^2+x^2 & -\\ - & x^2+1\end{bmatrix}$

$(x^2+1)^2+x^2=109$

$x^4+3x^2-108=0 \quad\Rightarrow(x^2+12)(x^2-9)=0$

$x^2=9 \quad\Rightarrow\quad x^2+1=10$

25. If $\lim\limits_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\}=2^{-k}$, then the value of k is_____

Key: 8.00

Sol: $\lim\limits_{x\to 0}\left(\frac{1-\cos\frac{x^2}{2}}{x^4}\right)\left(\frac{1-\cos\frac{x^2}{4}}{x^4}\right)=2^{-k}$

$\lim\limits_{x\to 0}\left(\frac{1-\cos\frac{x^2}{2}}{\left(\frac{x^2}{2}\right)^2}\right)\frac{1}{4}\left(\frac{1-\cos\frac{x^2}{4}}{\left(\frac{x^2}{4}\right)^2}\right)\frac{1}{16}\Rightarrow\frac{1}{2}.\frac{1}{4}.\frac{1}{2}.\frac{1}{16}=\frac{1}{2^8}=2^{-8}$

Prepared by

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