Sri Chaitanya IIT Academy., India

✪ A.P ✪ T.S ✪ Chandigarh ✪ Karnataka ✪ Tamilnadu ✪ Maharastra ✪ Delhi ✪ Ranchi

Jee Main 2020(Sep)

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Question Paper, Key and Solutions

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Physics

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. Amount of solar energy received on the earth's surface per unit area per unit time is defined a solar constant. Dimension of solar constant is :

1) MLT^{-2}
2) $M^2L^0T^{-1}$
3) ML^2T^{-2}
4) ML^0T^{-3}

Key : 4

Sol : Solar constant is defined as . Solar energy received on earth's surface per unit Area per unit time

So it has dimensions of intensity $I = \frac{E}{tA} = \frac{ML^2T^{-2}}{TL^2}$

Dimensions of solar constant = ML^0T^{-3}

2. Which of the following will NOT be observed when a multimeter (operating in resistance measuring mode) probes connected across a component, are just reversed ?

1) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes if the chosen component is metal wire.

2) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes if the chosen component is capacitor

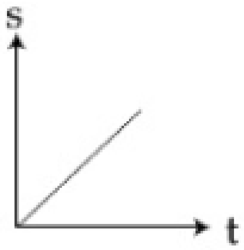
3) Multimeter shows an equal deflection in both cases i.e. before and after reversing the probes if the chosen component is resistor

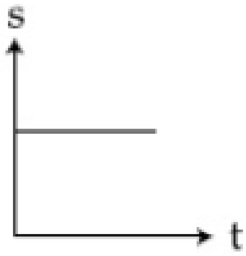
4) Multimeter shows a deflection, accompanied by a splash of light out of connected component in one direction and NO deflection on reversing the probes if the chosen component is LED.

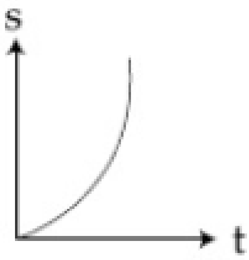
Key : 2

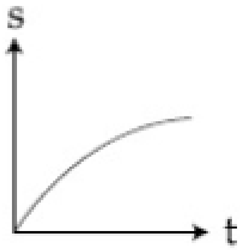
Sol : Multimeter when used in resistance mode across a capacitor it shows some deflection which is in between milli ohm to mega ohm .

3. A particle is moving unidirectionally on a horizontal plane under the action of a constant power supplying energy source. The displacement (s) – time (t) graph that describes the motion of the particle is (graphs are drawn schematically and are not to scale) :

1) 

2) 

3) 

4) 

Key : 3

Sol : Work done = change in K.E

$$Pt = \frac{1}{2}mv^2 - \frac{1}{2}mv^2 \ \textbf{if v = 0}$$

$$V = \sqrt{\frac{2pt}{m}}$$

$$\frac{ds}{dt} = \sqrt{\frac{2p}{m}} . t^{\frac{1}{2}}$$

$$\int ds = \int \sqrt{\frac{2p}{m}} t^{\frac{1}{2}} dt$$

$$S = \frac{2}{3}\sqrt{\frac{2p}{m}} t^{\frac{3}{2}}$$

4. If a semiconductor photodiode can detect a photon with a maximum wavelength of 400nm, then its band gap energy is :

Planck's constant $h = 6.63 \times 10^{-34}$ J.s. Speed of light $c = 3 \times 10^{8}$ m / s

1) 1.1eV 2) 3.1eV 3) 2.0eV 4) 1.5eV

Key : 2

Sol : Bond gap = Energy of photon = $\frac{hc}{\lambda}$

Bond gap = $\frac{4.14 \times 10^{-15} \times 3 \times 10^{8}}{4 \times 10^{-7}}$ ev = 3.1ev

5. Concentric metallic hollow spheres of radii R and 4R hold charges Q_1 and Q_2 respectively. Given that surface charge densities of the concentric spheres are equal, the potential difference V(R) – V(4R) is :

1) $\frac{3Q_1}{4\pi\varepsilon_0 R}$ 2) $\frac{Q_2}{4\pi\varepsilon_0 R}$ 3) $\frac{3Q_1}{16\pi\varepsilon_0 R}$ 4) $\frac{3Q_2}{4\pi\varepsilon_0 R}$

Key : 1

Sol : Surface charge densities are equal

$\therefore \frac{Q_1}{4\pi R^2} = \frac{Q_2}{4\pi(4R)^2}$

$\Rightarrow Q_2 = 16Q_1$

Potential of inner shell $= \frac{kQ_1}{R} + \frac{kQ_2}{4R}$

Potential of outer shell $= \frac{kQ_1}{4R} + \frac{kQ_2}{4R}$

$\therefore$ Potential difference between shells = $\frac{3kQ_1}{4R}$ = $\frac{3Q_1}{16\pi\varepsilon_0 R}$

6. The electric field of a plane electromagnetic wave propagating along the x direction in vacuum is $\vec{E} = E_0\hat{j}\cos(\omega t - kx)$. The magnetic field $\vec{B}$, at the moment t = 0 is

1) $\vec{B} = E_0\sqrt{\mu_0 \epsilon_0}\cos(kx)\hat{k}$ 2) $\vec{B} = \frac{E_0}{\sqrt{\mu_0 \epsilon_0}}\cos(kx)\hat{j}$

3) $\vec{B} = E_0\sqrt{\mu_0 \epsilon_0}\cos(kx)\hat{j}$ 4) $\vec{B} = \frac{E_0}{\sqrt{\mu_0 \epsilon_0}}\cos(kx)\hat{k}$

Key : 1

Sol : Amplitude of magnetic field $B_0 = \frac{E_0}{C}$

$B_0 = E_0\sqrt{\mu_0\varepsilon_0}$

$\vec{E}\times\vec{B}$ is along direction of propagation

$\therefore \hat{j}\times\hat{B}$ must be parallel to $\hat{i}$ $\therefore \vec{B}$ should be along $\hat{k}$

$\vec{E},\vec{B}$ are in same phase

$\therefore \vec{B} = B_0\hat{k}\cos\omega t - kx$

$\vec{B} = E_0\sqrt{\mu_0\varepsilon_0}\cos(\omega t - kx)\hat{k}$

At $t = 0$ $\vec{B} = E_0\sqrt{\mu_0\varepsilon_0}\cos kx\,\hat{k}$

7. The radius R of a nucleus of mass number A can be estimated by the formula $R = \left(1.3\times10^{-15}\right)A^{1/3}m$. It follows that the mass density of a nucleus is of the order of : $\left(M_{prot} \cong M_{neut} = 1.67\times10^{-27}kg\right)$

1) $10^{17}kgm^{-3}$
2) $10^{10}kgm^{-3}$
3) $10^{3}kgm^{-3}$
4) $10^{24}kgm^{-3}$

Key : 1

Sol : density = $\frac{M_P}{V_P}$

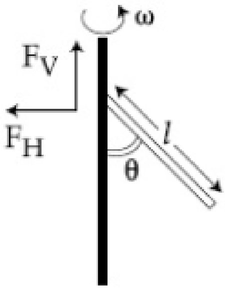
$$= \frac{1.67\times10^{-27}}{\frac{4}{3}\pi\left(1.3\times10^{-15}\right)^3(1)} kg/m^3$$

$$= \frac{3\times1.67}{4\pi\times(1.3)^3}\times10^8 kg/m^3$$

$$= \frac{30\times1.67}{4\pi(1.3)^3}\times10^{17} kg/m^3$$

$\therefore d = 1.81\times10^{17} kg/m^3$

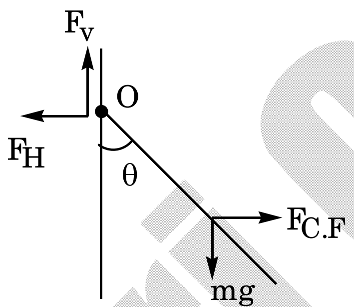
8.



A uniform rod of length 'ℓ' is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed ω the rod makes an angle θ with it (see figure). To find θ equate the rate of change of angular momentum (direction going into the paper) $\frac{m\ell^2}{12}\omega^2 \sin\theta\cos\theta$ about the centre of mass (CM) to the torque provided by the horizontal and vertical forces F_H and F_V about the CM. The value of θ is then such that :

1) $\cos\theta = \frac{3g}{2\ell\omega^2}$
2) $\cos\theta = \frac{2g}{3\ell\omega^2}$
3) $\cos\theta = \frac{g}{2\ell\omega^2}$
4) $\cos\theta = \frac{g}{\ell\omega^2}$

Key : 1



Sol :

About point 'O'

$\tau_{C.F} = \tau_{mg}$ $\qquad \tau_{mg} = mg\frac{\ell}{2}\sin\theta$

$\tau_{C.F} = \int dm\,(x\cos\theta)\omega^2 x\sin\theta \quad = \int \frac{m}{\ell}x^2dx\ \omega^2\cos\theta\sin\theta$

$\frac{m\ell^2}{3}\omega^2\cos\theta\sin\theta$

$\therefore mg\frac{\ell}{2}\sin\theta = \frac{m\ell^2\omega^2\cos\theta\sin\theta}{3} \qquad \cos\theta = \frac{3g}{2\ell\omega^2}$

9. A block of mass m attached to a massless spring is performing oscillatory motion of amplitude 'A' on a frictionless horizontal plane. If half of the mass of the block breaks

off when it is passing through its equilibrium point, the amplitude of oscillation for the remaining system become fA. The value of f is :

1) $\frac{1}{\sqrt{2}}$ 2) 1 3) $\frac{1}{2}$ 4) $\sqrt{2}$

Key : 1

Sol : If the block breaks off when it passes through mean position it will carry its own momentum with it. Which means its velocity of block still remains same

$A\omega = A_f\omega_f$

$A\sqrt{\frac{k}{m}} = A_f\sqrt{\frac{2k}{m}} \quad A_f = \frac{A}{\sqrt{2}}$

10. To raise the temperature of a certain mass of gas by 50^{0}C at a constant pressure, 160 calories of heat is required. When the same mass of gas is cooled by 100^{0}C at constant volume, 240 calories of heat is released. How many degrees of freedom does each molecule of this gas have (assume gas to ideal)?

1) 3 2) 6 3) 5 4) 7

Key : 2

Sol : At constant pressure $dQ = nC_p dT$

$160 = nC_P(50)$

At constant volume

$dQ = nC_v dT$

$240 = nC_v(100)$

$\frac{C_P(50)}{C_v(100)} = \frac{160}{240}$

$\frac{\gamma}{2} = \frac{2}{3}$

$\gamma = \frac{4}{3}$

$f = \frac{2}{\gamma - 1} = 6$

11. Two sources of light emit X – rays of wavelength 1nm and visible light of wavelength 500nm, respectively. Both the sources emit light of the same power 200W. The ratio of the number density of photons of X – rays to the number density of photons of the

visible light of the given wavelength is :

1) $\frac{1}{250}$ 2) 250 3) $\frac{1}{500}$ 4) 500

Key : 3

Sol : $P=\left(\frac{n}{t}\right)\frac{hc}{\lambda}$ $\Rightarrow\left(\frac{n}{t}\right)=\frac{P\lambda}{hc}$

$\therefore\frac{n_1}{n_2}=\frac{\lambda_1}{\lambda_2}$

$\frac{n_1}{n_2}=\frac{1}{500}$

12. A uniform magnetic field B exists in a direction perpendicular to the plane of a square loop made of a metal wire. The wire has a diameter of 4mm and a total length of 30 cm. The magnetic field changes with time at a steady rate dB/ dt=0.032Ts^{-1}. The induced current in the loop is close to

(Resistivity of the metal wire is $1.23\times10^{-8}\Omega m$)

1) 0.34A 2) 0.43A 3) 0.61A 4) 0.53A

Key : 3

Sol : $\varepsilon=\frac{dB}{dt}\ell^2$ $i=\frac{\varepsilon}{R}$

$$i=\frac{dB}{dt}\frac{\ell^2}{\frac{16(\rho\ell)}{A}}$$

$$i=\frac{dB}{dt}\frac{\ell A}{16\rho}$$

$$i=\frac{0.032\times0.3\times\pi\times2^2\times10^{-6}}{16\times1.23\times10^{-8}}=0.61A$$

13. Two light waves having the same wavelength λ in vacuum are in phase initially. Then the first wave travels a path L_1 through a medium of refractive index n_1 while the second wave travels a path of length L_2 through a medium of refractive index n_2. After this the phase difference between the two waves is :

1) $\frac{2\pi}{\lambda}\left(n_1L_1 - n_2L_2\right)$ 2) $\frac{2\pi}{\lambda}\left(\frac{L_1}{n_1} - \frac{L_2}{n_2}\right)$

3) $\frac{2\pi}{\lambda}\left(\frac{L_2}{n_1} - \frac{L_1}{n_2}\right)$ 4) $\frac{2\pi}{\lambda}\left(n_2L_1 - n_1L_2\right)$

Key : 1

Sol : Optical path = $\mu\ell$

$\therefore$ Phase difference = $\frac{2\pi}{\lambda}(\Delta x)$

$$\phi = \frac{2\pi}{\lambda}\left(n_1L_1 - n_2L_2\right)$$

14. A calorimeter of water equivalent 20 g contains 180g of water at 25^0C. 'm' grams of steam at 100^0C is mixed in it till the temperature of the mixture is 31^0C. The value of 'm' is close to (Latent heat of water = 540 cal g^{-1}, specific heat of water = 1 cal g^{-1} $^0C^{-1}$)

1) 2 2) 4 3) 3.2 4) 2.6

Key : 1

Sol : Heat gained by cold body = heat lost by hot body

$$200 \times 16 = m(540) + m \times 1 \times 69$$

$$1200 = m(609)$$

$$m \approx 2\text{gms}$$

15. A metallic sphere cools from 50^0C to 40^0C in 300s. If atmospheric temperature around is 20^0C, then the sphere's temperature after the next 5 minutes will be close to :

1) 35^0C 2) 33^0C 3) 28^0C 4) 31^0C

Key : 2

Sol : $-\frac{d\theta}{dt} = k\left(\theta_{avg} - \theta_{sur}\right)$

$$\frac{10}{300} = K(45 - 20)$$

$$\frac{40 - \theta_1}{300} = k\left(\frac{40 + \theta_1}{2} - 20\right)$$

$$\frac{10}{40-\theta_f}=k\left(\frac{40+\theta_1}{2}-20\right)$$

$$\frac{10}{40-\theta_f}=\frac{25}{\theta_f}\times 2$$

$$\theta_f=\frac{100}{3}=33^0C$$

16. The mass density of a planet of radius R varies with the distance r from its centre as $\rho(r)=\rho_0\left(1-\frac{r^2}{R^2}\right)$. Then the gravitational field is maximum at :

1) $r=\frac{1}{\sqrt{3}}R$ 2) $r=\sqrt{\frac{5}{9}}R$ 3) r = R 4) $r=\sqrt{\frac{3}{4}}R$

Key : 2

Sol : For points to inside the planet, at distance x from the centre $(x<r)$ g is given by

$$g=\frac{G\int_0^x r_0\left(1-\frac{x^2}{R^2}\right)4\pi r^2dr}{x^2}$$

$$g=Gr_0 4\pi\left(\frac{x}{3}-\frac{x^3}{5R^2}\right)$$

For max g $\frac{dg}{dx}=0$

$$\therefore \frac{1}{3}-\frac{3x^2}{5R^2}=0$$

$$x=\sqrt{\frac{5}{9}}R$$

17. A perfectly diamagnetic sphere has a small spherical cavity at its centre, which is filled with a paramagnetic substance. The whole system is placed in a uniform magnetic field $\vec{B}$. Then the field inside the paramagnetic substance is :

1) $\vec{B}$

2) much large than $|\vec{B}|$ but opposite to $\vec{B}$

3) much large than $|\vec{B}|$ and parallel to $\vec{B}$

4) zero

Key : 4

Sol : Diamagnetic substances does not allow any magnetic field lines to pass through them. Therefore magnetic field inside it must be zero

18. Hydrogen ion and singly ionized helium atom are accelerated, from rest, through the same potential difference. The ratio of final speeds of hydrogen and helium ions is close to :

1) 5 : 7 2) 2 : 1 3) 10 : 7 4) 1 : 2

Key : 2

Sol : $\frac{1}{2}mv^2 = q\ \Delta v$

$$v = \sqrt{\frac{2q\ \Delta v}{m}}$$

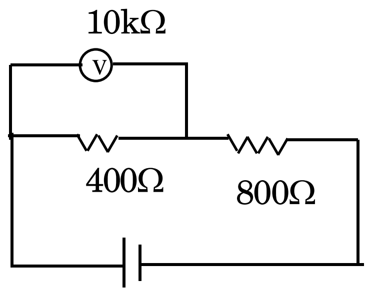
$$v \alpha \sqrt{\frac{q}{m}}$$

$$\therefore \frac{v_1}{v_2} = \frac{\sqrt{\frac{1}{1}}}{\sqrt{\frac{1}{4}}} = \frac{2}{1}$$

19. Two resistors 400Ω and 800Ω are connected in series across a 6V battery. The potential difference measured by a voltmeter of $10k\Omega$ across 400Ω resistor is close is :

1) 1.95V 2) 1.8V 3) 2V 4) 2.05V

Key : 1



Sol :

Let voltmeter measure v volts

$\therefore$ i in $10k\Omega = \dfrac{v}{10000}A$

i in $400\Omega = \dfrac{v}{400}A$

i in $600\Omega = \dfrac{v}{10000} + \dfrac{v}{400}A$

Applying kirchoff's law $v + \left(\dfrac{v}{10000} + \dfrac{v}{400}\right)800 = 6$

$v = \dfrac{600}{308}$

$v = 1.95$volts

20. A block of mass 1.9 kg is at rest at the edge of a table, of height 1m. A bullet of mass 0.1kg collides with the block and sticks to it. If the velocity of the bullet is 20m/ s in the horizontal direction just before the collision then the kinetic energy just before the combined system strikes the floor, is [Take g = 10m/ s^2. Assume there is no rotational motion and loss of energy after the collision is negligible]

1) 20J 2) 19J 3) 23J 4) 21J

Key : 4

Sol : Conserving momentum

0.120 = 2v

V = 1

Then conserving kinetic energy when it hits the ground $= \dfrac{1}{2}2(1)^2 + 2(10)1$

$K.E_{final} = 21J$

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value(in decimal notation, truncated/ rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

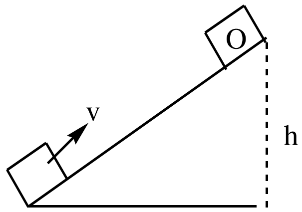
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. A block starts moving up an inclined plane of inclination 30^0 with an initial velocity of υ_0. It comes back to its initial position with velocity $\dfrac{\upsilon_0}{2}$. The value of the coefficient of kinetic friction between the block and the inclined plane is close to $\dfrac{1}{1000}$. The nearest

integer to I is ___

Key : 346

Sol :



Work done by all fores = ΔK.E

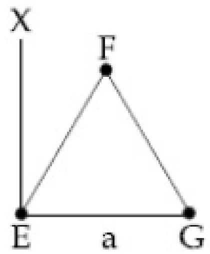
$$-mgh - \mu mg(h)\sqrt{3} = -\frac{1}{2}mv^2$$

$$mgh - \mu mg\left(\sqrt{3}n\right) = \frac{1}{2}m\left(\frac{v}{2}\right)^2$$

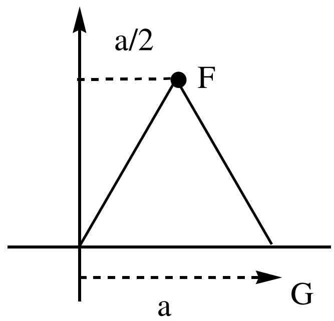
Dividing equations $\dfrac{1+\sqrt{3}\mu}{1-\sqrt{3}\mu} = 4$

$\mu = \dfrac{\sqrt{3}}{5}$ $\qquad$ $\mu = \dfrac{346}{1000}$ $\qquad$ I= 346

22. An massless equilateral triangle EFG of side ‘a’ (As shown in figure) has three particles of mass m situated at its vertices. The moment of inertia of the system about the line EX perpendicular to EG in the plane of EFG is $\frac{N}{20}ma^2$ where N is an integer. The value of N is ___



Key : 25



Sol :

$$I = m\left(\frac{a}{2}\right)^2 + ma^2$$

$$= \frac{5}{4}ma^2$$

$$I = \frac{25}{20}ma^2$$

$$\therefore N = 25$$

23. A galvanometer coil has 500 turns and each turn has an average area of $3 \times 10^{-4} m^2$. If a torque of 1.5Nm is required to keep this coil parallel to a magnetic field when a current of 0.5A is flowing through it, the strength of the field (in T) is ____

Key : 20

Sol : $\tau = NiAB$

$$1.5 = 500 \times 105 \times 3 \times 10^{-4} B$$

$$B = 20T$$

24. When an object is kept at a distance of 30cm from a concave mirror, the image is formed at a distance of 10 cm from the mirror. If the object is moved with a speed of 9 cms^{-1}, the speed (in cms^{-1}) with which image moves at the instant is ___

Key : 1

Sol : $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v^2}\frac{dv}{dt} = -\frac{1}{u^2}\frac{dv}{dt}$$

$$\frac{dv}{dt} = -\frac{v^2}{u^2}\frac{dv}{dt}$$

$$\therefore \frac{dv}{dt} = -\left(\frac{10}{30}\right)^2 9$$

$$\left|\frac{dv}{dt}\right| = 1cm/\ sec$$

25. If minimum possible work is done by a refrigerator in converting 100 grams of water at 0^0C to ice, how much heat (in calories) is released to the surroundings at temperature 27^0C (Latent heat of ice = 80 cal/ gram) to the nearest integer ?

Key : 8791

Sol : $Q_{abs} = mL$

$$Q_{abs} = 100(80)cal$$

$$Q_{abs} = 8000cal$$

$$CoP = \frac{Q_{abs}}{\omega} = \frac{Q_{abs}}{Q_{rej} - Q_{abs}} = \frac{273}{27}$$

$$\frac{8000}{Q_{rej} - 8000} = 10.11$$

$$Q_{rej} = 8791.21$$

$$Q_{rej} \approx 8791$$

CHEMISTRY

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. The incorrect statement (s) among (a) –(d) regarding acid rain is (are) :

   (a) It can corrode water pipes.

   (b) It can damage structures made up of stone

   (c) It cannot cause respiratory ailments in animals

   (d) It is not harmful for trees

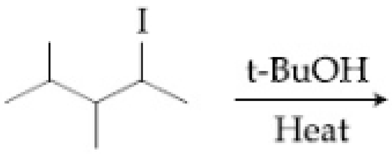
   1) (a), (c) and (d) 2) (a), (b) and (d)

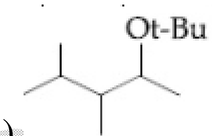
   3) (c) and (d) 4) (c) only

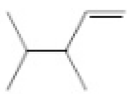
Key: 3

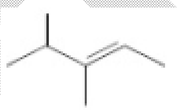
Sol: (C) Aquatic animals as well as human lungs will suffer by acid rain. Acid rain also causes respiratory ailments
(D) Acid rain also effect trees and agriculture

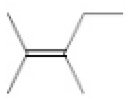
2. The major product in the following reaction is:



1) 

2) 

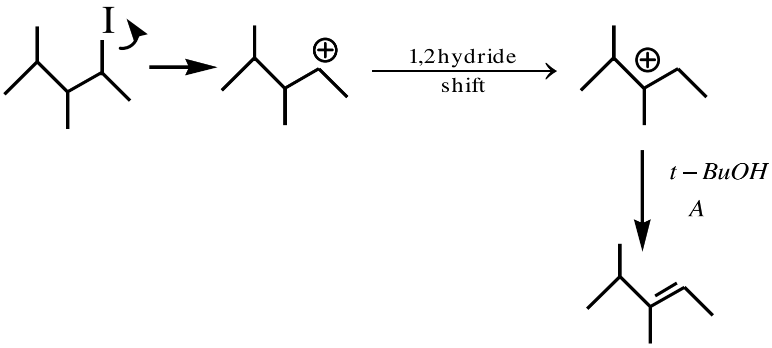
3) 

4) 

Ans: 4

Suggested Key: 3

Sol: Reaction follows E_1 mechanism as weak base (ROH) is taken



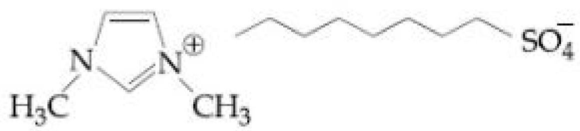
Majar product

Bulky base will remove less hindered hydrogen

3. An ionic micelle is formed on the addition of :

1) liquid diethyl ether to aqueous NaCl solution

2) excess water to liquid 

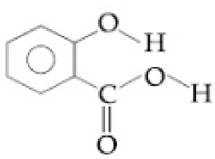
3) excess water to liquid 

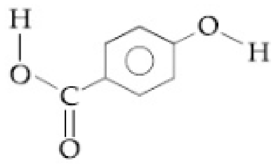
4) Sodium stearate to pure toluene

Key : 3

Sol. Ionic micells are formed by (3).

4. Consider the following molecules and statements related to them:

A) 

B) 

a) (B) is more likely to be crystalline than (A)

b) (B) has higher boiling point than (A)

c) (B) dissolves more readily than (A) in water

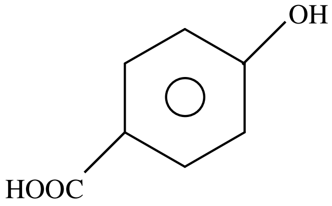
Identify the correct option from below:

1) (b) and (c) are true   2) only (a) is true

3) (a) and (c) are true   4) (a) and (b) are true

Key : 4

Sol: (a) B is likely to be more crystalline as it may form parallel polymeric chains which helps it crystallize.



(b) (B) will have higher boiling point due to effective intermolecular H – bonding

(C) (B) will be more soluble due to more H- bonding with water molecules H – bonding with water will be less in (A) due to intermolecular H – bonding

5. Complex A has a composition of $H_{12}O_6Cl_3Cr$. If the complex on treatment with conc. H_2SO_4 loses 13.5 % of its original mass, the correct molecular formula of A is

[Given : atomic mass of Cr = 52 amu and Cl = 35 amu]

1) $[Cr(H_2O)_5Cl]Cl_2 . H_2O$ 2) $[Cr(H_2O)_6]Cl_3$

3) $[Cr(H_2O)_4Cl_2]Cl. 2H_2O$ 4) $[Cr(H_2O)_3Cl_3] . 3H_2O$

Key : 3

Sol: M.wt of $H_{12}O_6C\ell_3Cr = 265$ % loss is 13.5. So amount of water lost per 1 mol ≈ 36. This is equal to 2 moles of water. The number of H_2O molecules lost should present in outer sphere of the hydrate isomer. So the correct formula of compound is $\left[Cr(H_2O)_4 C\ell_2\right]C\ell.2H_2O$

6. A mixture of one mole each of H_2, He and O_2 each are enclosed in a cylinder of volume V at temperature T. If the partial pressure of H_2 is 2 atm, the total pressure of the gases in the cylinder is:

1) 38 atm 2) 22 atm 3) 6 atm 4) 14 atm

Key : 3

Sol.
$$n_{H_2} = 1 mole, n_{He} = 1 mole, n_{O_2} = 1 mole$$
$$P_{H_2} = 2 atm \quad P_{total} = P_{H_2} + P_{He} + P_{O_2}$$
$$P_{H_2} = P_{He} = P_{O_2}$$
$$\therefore P_{total} = 2 + 2 + 2 = 6 atm$$

7. Three isomers A, B and C (mol. Formula $C_8H_{11}N$) give the following results:

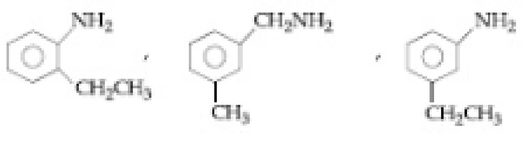
$$A \text{ and } C \xrightarrow{Diazotization} P + Q \xrightarrow[(ii)Oxidation]{(i)Hydrolysis} \begin{matrix} R\,(\text{Product of } A) \\ + \\ S\,(\text{Product of } C) \end{matrix}$$

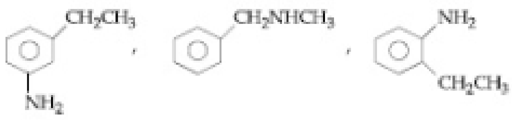
$$(KMnO_4 + H^+)$$

R has lower boiling point than S

$B \xrightarrow{C_6H_5SO_2Cl}$ alkali –insoluble product

A, B and C, respectively are:

1) 

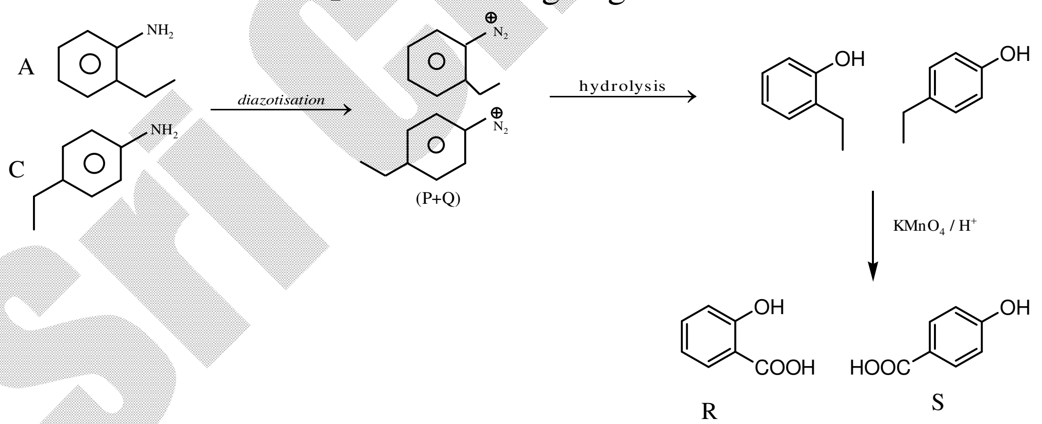
2) 

3) 

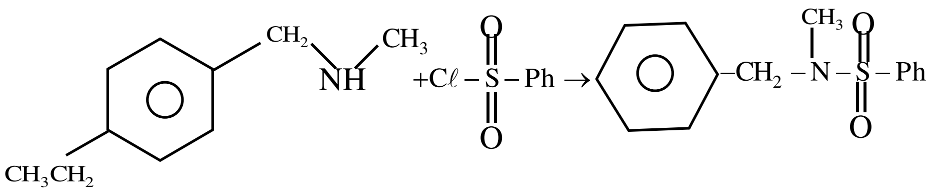
4) 

Key: 3

Sol. A and C must have $-NH_2$ as it is undergoing diazotization

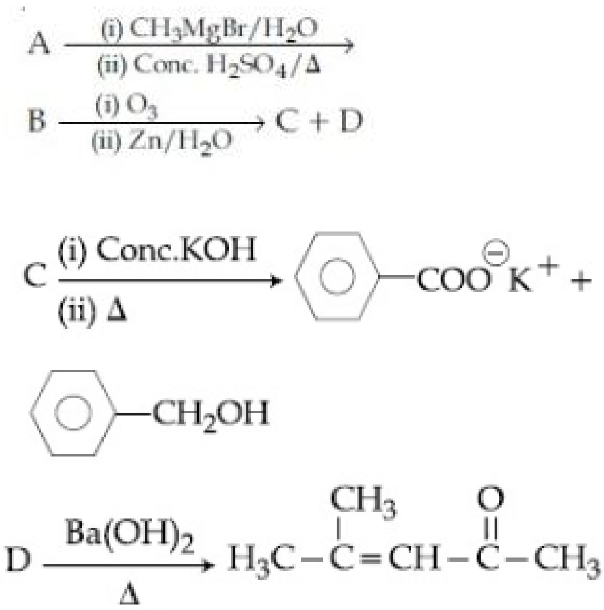


(R has low boiling point than S due to intermolecular H-bonding)



Alkali insoluble due to absence of acidic hydrogen

8. The compound A in the following reactions is:



1) $C_6H_5-\overset{O}{\overset{\|}{C}}-CH_3$

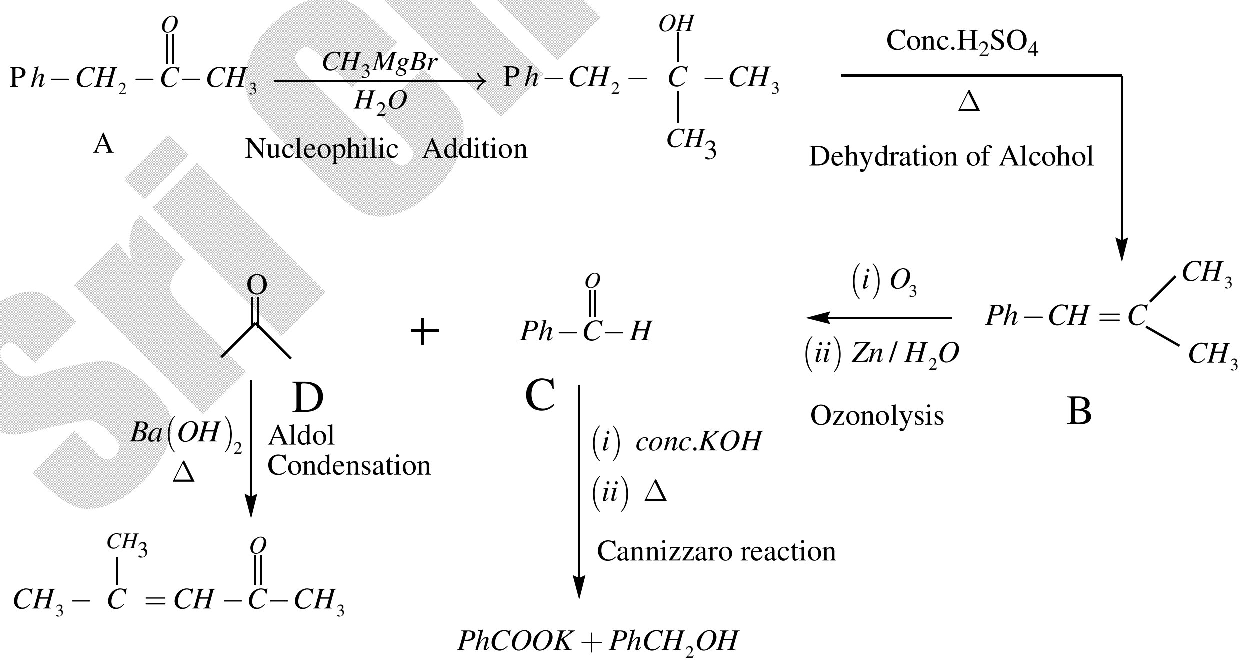
2) $C_6H_5-\overset{O}{\overset{\|}{C}}-CH\begin{matrix}CH_3\\CH_3\end{matrix}$

3) $C_6H_5-CH_2-\overset{O}{\overset{\|}{C}}-CH_3$

4) $C_6H_5-\overset{O}{\overset{\|}{C}}-CH_2CH_3$

Key: 3

Sol:



9. The strength of 5.6 volume hydrogen peroxide (of density 1 g/mL) in terms of mars

percentage and molarity (M), respectively are :

(Take molar mass of hydrogen peroxide as 34 g/mol)

1) 0.85 and 0.25 2) 0.85 and 0.5

3) 1.7 and 0.25 4) 1.7 and 0.5

Key:4

Sol. Molarity $= \frac{5.6}{11.2} = 0.5\,\text{M}$

$0.5\,\text{moles of}\,H_2O_2\,\text{in}\,1\text{L}$ solution

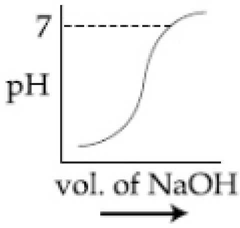
As d= 1gmcc

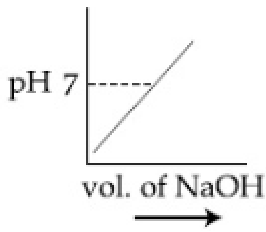
$(0.5 \times 34)\,\text{gms of}\,H_2O_2\,\text{in}\,1\text{kg}$ solution

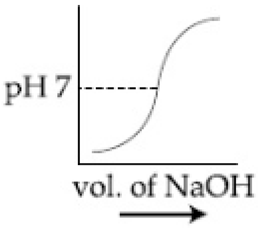
Mass percentage of $H_2O_2 \frac{0.5 \times 34}{1000} \times 100$

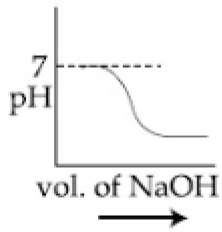
= 1.7%

10. 100 mL of 0.1 M HCl is taken in a beaker and to it 100 mL of 0.1 M NaOH is added in steps of 2 mL and the pH is continuously measured. Which of the following graphs correctly depicts the change in pH?

1) 

2) 

3) 

4) 

Key:3

Sol. At neutralization of strong acid and base $P^H = 7$. During addition of NaOH solution, P^H gradually increases lost not linear

11. The five successive ionization enthalpies of an element are 800, 2427, 3658, 25024 and 32824 kJ mol^{-1}. The number of valence electrons in the element is:

1) 5 2) 3 3) 4 4) 2

Key:2

Sol : Since there is large difference between 3rd and 4th ionization enthalpies, there shoud be 3 electrons in the valence shell

12. For the reaction $2A + 3B + \frac{3}{2}C \rightarrow 3P$, Which statement is correct ?

1) $\frac{dn_A}{dt} = \frac{dn_B}{dt} = \frac{dn_C}{dt}$ 2) $\frac{dn_A}{dt} = \frac{3}{2}\frac{dn_B}{dt} = \frac{3}{4}\frac{dn_C}{dt}$

3) $\frac{dn_A}{dt} = \frac{2}{3}\frac{dn_B}{dt} = \frac{4}{3}\frac{dn_C}{dt}$ 4) $\frac{dn_A}{dt} = \frac{2}{3}\frac{dn_B}{dt} = \frac{3}{4}\frac{dn_C}{dt}$

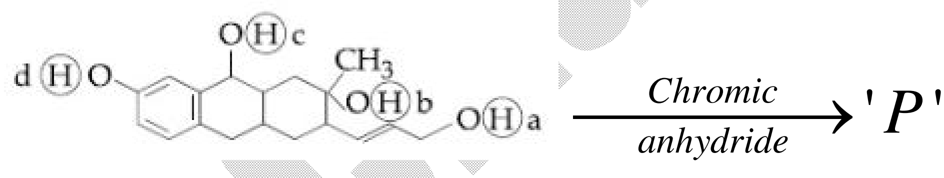
Key. 3

Sol. $2A + 3B + \frac{3}{2}C \rightarrow 3P$

$$\frac{1}{2}\frac{dn_A}{dt} = \frac{1}{3}\frac{dn_B}{dt} = \frac{2}{3}\frac{dn_C}{dt} = \frac{1}{3}\frac{dn_s}{dt}$$

$$\therefore \frac{dn_A}{dt} = \frac{2}{3}\frac{dn_B}{dt} = \frac{4}{3}\frac{dn_C}{dt}$$

13. Consider the following reaction:



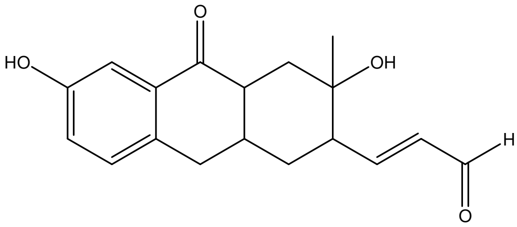
The product 'P' gives positive ceric ammonium nitrate test. This is because of the presence of which of these –OH group(s)?

1) (b) and (d) 2) (b) only

3) (c) and (d) 4) (d) only

Key: 2

Sol: Primary and secondary alcohol will get oxidized with chromic anhydride. So product P will be



3^0 alcohols give red colour with ceric ammonium nitrate.

14. Match the following drugs with their therapeutic actions:

(i) Ranitidine	(a) Antidepressant
(ii) Nardil (Phenelzine)	(b) Antibiotic
(iii) Chloramphenicol	(c) Antihistamine
(iv) Dimetane (Brompheniramine)	(d) Antacid
	(e) Analgesic

1) (i) –(d), (ii) – (a), (iii) – (b), (iv) – (c)
2) (i) –(a), (ii) – (c), (iii) – (b), (iv) – (e)
3) (i) –(e), (ii) – (a), (iii) – (c), (iv) – (d)
4) (i) –(d), (ii) – (c), (iii) – (a), (iv) – (e)

Key: 1

Sol: (i) Ranitidine - Antacid
(ii) Nardil (Phenelzine) - Antidepressant
(iii) Chloramphenicol - Antibiotic
(iv) Dimetane (Brompheniramine) - Antihistamine

15. Consider the hypothetical situation where the azimuthal quantum number, l, takes values 0,1,2, ………… n+1, where n is the principal quantum number. Then, the element with atomic number:

1) 8 is the first noble gas
2) 9 is the first alkali metal
3) 6 has a 2p-valence subshell
4) 13 has a half-filled valence subshell

Key. 4

Sol: ‘*l*’ can have values (n-1) to (n+1) total n+2 energy levels
1s 2p 1d
2s 2p 2d 2f

1) Atomic number 18 is having electrnic configuration $1s^21p^61d^{10}$ and is noble gas.
2) Atomic number ‘19’ is first alkali metal
3) Atomic number ‘6’ has $1s^21p^4$, i.e. 1s, 1p valence subshell.
4) atomic number 16 has $i1s1p^61d^5$ halffilled d-configuration

16. Among the statements (I-IV) the correct ones are:

I) Be has smaller atomic radius compared to Mg

II) Be has higher ionization enthalpy than Al.

III) Charge /radius ratio to Be is greater than that of Al.

IV) Both Be and Al form mainly covalent compounds

1) (II), (III) and (IV) 2) (I), (II) and (IV)

3) (I), (II) and (III) 4) (I), (III) and (IV)

Key:2

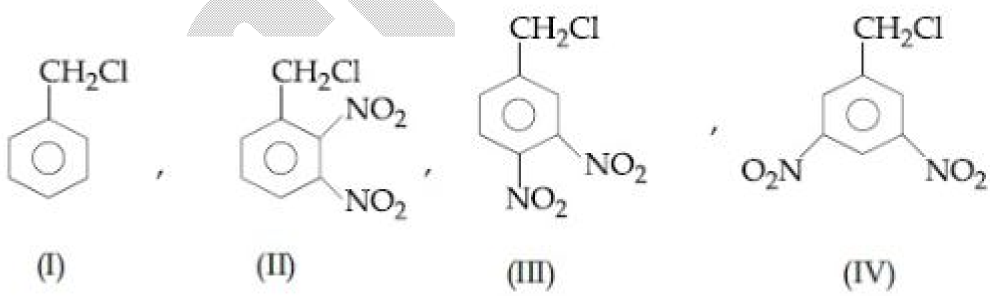
Sol: I. The atomic radius of Be is smaller than that of Mg as we move downward in a group atomic size decreases

II. Ionization enthalpy of Be is more than that of $A\ell$ because Be is smaller than $A\ell$. Further from Be electron is removed from 2s which is nearer to nucleus. In $A\ell$ the eletron is removed from 3p which is away from the nucleus

III. Charge/radius ratio of Be is less than that of $A\ell$ because Be carries 2+ charges and $A\ell$ carries 3+ charges. But ionic radii of Be^{2+} and $A\ell^{3+}$ are almost equal.

IV. Because of more polarizing power compounds of Be and $A\ell$ are covalent.

17. The decreasing order of reactivity of the following compounds towards nucleophilic substitution (S_N2) is:

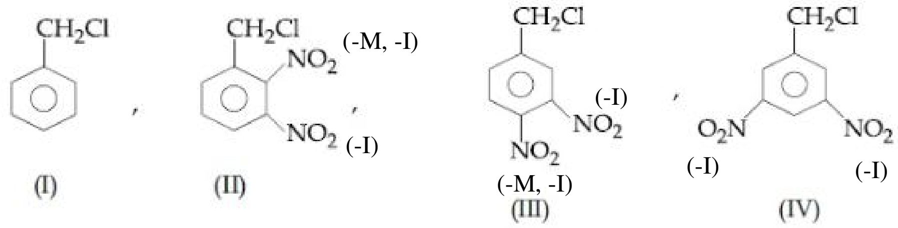


1) (IV) > (II) > (III) > (I) 2) (II) > (III) > (IV) > (I)

3) (II) > (III) > (I) > (IV) 4) (III) > (II) > (IV) > (I)

Key: 2

Sol: Electron with drawing group increases the rate of SN^2 reaction



Net electron withdrawing will be maximum in (II) as – NO_2 is closer w.r.t III.In (IV) only –I of – NO_2 will be applicable as it is present on meta position

18. The incorrect statement is:

1) Manganate and permanganate ions are paramagnetic

2) Manganate ion is green in colour and permanganate ion is purple in colour

3) In manganate and permanganate ions, the π - bonding takes place by overlap of p-orbitals of oxygen and d-orbitals of manganese

4) Manganate and permanganate ions are tetrahedral

Key:1

Sol: Manganate is paramagnetic while permanganate is diamagnetic so statement I is incorrect

19. The increasing order of the reactivity of the following compounds in nucleophilic addition reaction is:

Propanal, benzaldehyde, Propanone, Butanone.

1) Benzaldehyde < Propanal < Propanone < Butanone

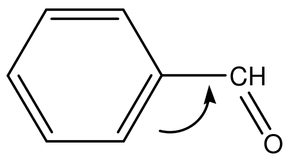
2) Propanal < Propanone < Butanone < Benzaldehyde

3) Butanone < Propanone < Benzaldehyde < Propanal

4) Benzaldehyde < Butanone < Propanone < Propanal

Key: 3

Sol: In general, aldehydes are more reactive than ketone towards nucleophilic addition,due to less sterric hindrance and more electrophilic carbon.
Carbonyl carbon of propanal is more electrophilic than benzaldehyde due to delocalization of π - electron of phenyl ring towards carbonyl group



20. The d-electron configuration of $[Ru(en)_3]Cl_2$ and $[Fe(H_2O)_6]\ Cl_2$ respectively are:

1) $t_{2g}^4\, e_g^2$ *and* $t_{2g}^6\, e_g^0$ 2) $t_{2g}^6\, e_g^0$ *and* $t_{2g}^4\, e_g^2$

3) $t_{2g}^{4}\, e_{g}^{2}$ *and* $t_{2g}^{4}\, e_{g}^{2}$ 4) $t_{2g}^{6}\, e_{g}^{0}$ *and* $t_{2g}^{6}\, e_{g}^{0}$

Key: 2

Sol: Fe and Ru belong to same group and their ions Fe^{2+} and Ru^{2+} in the given complex have d^6 configuration. In $\left[Ru(en)_3\right]C\ell_2$, the electrons rearrange having $t_{2g}^{6}eg^{0}$. In $\left[Fe(H_2O)_6\right]^{2+}$ since H_2O is wea
k lignad electrons do not rearrange. So have $t_{2g}^{4}eg^{2}$.

(NUMERICAL VALUE TYPE)

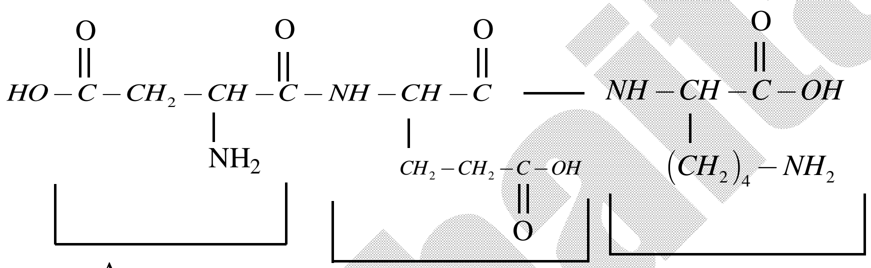
This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer , 0 if not attempted and 0 in all other cases.

21. The number of $>C=O$ groups present in a tripeptide Asp – Glu – Lys is _____

Key: 5.00

Sol : $Asp - Glu - Lys$



Asp
Aspartic acid

Glu
Glutamic acid

Lys
Lysine

22. If 250 cm^3 of an aqueous solution containing 0.73 g of a protein A is isotonic with one litre of another aqueous solution containing 1.65 g of a protein B, at 298 K, the ratio of the molecular masses of A and B is ______ x 10^{-2} (to the nearest integer)

Key. 177

Sol. For isotonic situations

$\pi_A = \pi_B$

Vant half factor $= 1$ for protein

$$C_A = C_B \Rightarrow \frac{0.73}{m_A \times 250} \times 1000 = \frac{1.65}{m_B \times 1000} \times 1000$$

$$\frac{m_A}{m_B} = \frac{0.73 \times 4}{1.65} = 177$$

23. 6.023 x 10^{22} molecules are present in 10 g of a substance ‘x’. The molarity of a solution containing 5 g of substance ‘x’ in 2 L solution is ______ x 10^{-3}.

Key. 25

Sol. 6.023×10^{22} molecules in grams

6.023×10^{23} molecules are present in 100 gms

Molecular mass of substance 'x' = 100 gms

$$M=\frac{5}{100\times2}=0.025 \qquad =2510^{-3}$$

24. The volume (in mL) of 0.1 N NaOH required to neutralise 10 mL of 0.1 N phosphinic acid is ______

Key. 10

Sol. Phosphoinic acid is H_3PO_3

$$xNaOH+H_3PO_3\rightarrow Na_xH_{3-x}PO_3+H_2O$$

$$n_1M_1v_1=n_2N_2v_2 \quad n_2M_2=N_2=0.1N$$

$$1\times0.1\times V_{NaOH}=0.1\times10$$

$$V_{NaOH}=10\,ml$$

25. An acidic solution of dichromate is electrolyzed for 8 minutes using 2A current. As per the following equation $Cr_2O_7^{2-}+14H^{+}+6e^{-}\rightarrow2Cr^{3+}+7H_2O$.

The amount of Cr^{3+} obtained was 0.104 g. The efficiency of the process (in%) is (Take : F=96000 C, At. Mass of chromium =52)_____

Key. 60.31

Sol. Let 'x' be the efficiency

$$96500C\rightarrow\frac{52}{3}gms$$

$$?\rightarrow0.104gm$$

$$\frac{0.104}{52/3}\times96500C$$

$$\text{Efficiency}=\frac{\frac{0.104}{52/3}\times96500}{8\times60\times2}\times100=60.31$$

MATHEMATICS

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. Let $a,b,c \in R$ be such that $a^2+b^2+c^2=1$.If

$$a\cos\theta = b\cos\left(\theta+\frac{2\pi}{3}\right)=c\cos\left(\theta+\frac{4\pi}{3}\right)$$

where $\theta=\frac{\pi}{9}$,then the angle between the

vectors $a\hat{i}+b\hat{j}+c\hat{k}$ and $b\hat{i}+c\hat{j}+a\hat{k}$ is :

1)0 2) $\frac{2\pi}{3}$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{9}$

Key : 3

Sol : Let a $Cos\theta = b\ Cos(\theta+\frac{2\pi}{3}) = c\,Cos(\theta+\frac{4\pi}{3}) = K(Say)$

$$\Rightarrow \frac{k}{a}+\frac{k}{b}+\frac{k}{c}=Cos\theta+Cos(\theta+\frac{2\pi}{3})+Cos(\theta+\frac{4\pi}{3})$$

$$=Cos\theta+2Cos(\theta+\pi)Cos\left(\frac{\pi}{3}\right) \qquad =0$$

$$\Rightarrow\sum ab=0\Rightarrow(\bar{a}i+b\bar{j}+ck).(bi+c\bar{j}+ak)=0$$

$\Rightarrow$ the two vectors are perpendicular to each other

$\therefore$ Angle = $\frac{\pi}{2}$

2. Suppose f(x) is a polynomial of degree four having critical points at -1,0,1.If $T=\{x\in R/f(x)=f(0)\}$,then the sum of squares of all the elements of T is

1)8 2) 2 3) 4 4)6

Key: 3

Sol : $f^1(x)=\lambda x(x^2-1)=\lambda(x^3-x)(\lambda\neq 0)$

$$f(x)=\lambda\left(\frac{x^4}{4}-\frac{x^2}{2}\right)+k=\lambda\left(\frac{x^4}{4}-\frac{x^2}{2}\right)+f(0)$$

If $f(x)=f(0)\Rightarrow\lambda\left(\frac{x^4}{4}-\frac{x^2}{2}\right)=0$

$\Rightarrow x^2=0\ or\ x^2=2 \qquad \Rightarrow x=0,\ x=\pm\sqrt{2}$

$\therefore$ Sum of the squares of the values of x in T is 0 + 2 + 2 = 4.

3. The set of all real values of λ for which the quadratic equations,

$(\lambda^2+1)x^2-4\lambda x+2=0$ always have exactly one root in the interval (0,1) is :

1) (2,4] 2)(-3, -1) 3) (1,3] 4)(0,2)

Key: 3

Sol: Let $f(x)=(\lambda^2+1)x^2-4\lambda x+2$

If f(x) = 0 has exactly one root in (0,1) then

$f(0)\,f(1)\le 0 \Rightarrow 2(\lambda^2+1-4\lambda+2)\le 0$

$\Rightarrow(\lambda-1)(\lambda-3)\le 0$

$\lambda\in[1,3]$

But for $\lambda=1\Rightarrow n=1$ is a repeat root

$\therefore\ \lambda\in(1,3]$

4. If $x^3dy+xy\,dx=x^2dy+2y\,dx$; $y(2)$ =e and x > 1, then y(4) is equal to :

1) $\frac{3}{2}\sqrt{e}$ 2) $\frac{\sqrt{e}}{2}$ 3) $\frac{3}{2}+\sqrt{e}$ 4) $\frac{1}{2}+\sqrt{e}$

Key: 1

Sol: $x^3dy+xydx=x^2dy+2y\,dx$

$\Rightarrow(x^3-x^2)dy=y(2-x)dx$

$\Rightarrow\frac{dy}{y}=\frac{2-x}{x^2(x-1)}dx=\left(\frac{1}{x-1}-\frac{1}{x}-\frac{2}{x^2}\right)dx$

$\Rightarrow\log y=\log(x-1)-\log x+\frac{2}{x}+C\ (\because x>1)$

Given that y (2) = e $\Rightarrow 1=0-\log_e^2+1+C\Rightarrow C=\log 2$

$\Rightarrow\log y=\log\left(\frac{x-1}{x}\right)+\frac{2}{x}+\log 2$

For $x=4\Rightarrow\log(y(4))=\log\left(\frac{2.3}{4}\right)+\frac{1}{2}$ $=\log\left(\frac{3\sqrt{e}}{2}\right)$ $\Rightarrow y(4)=\frac{3\sqrt{e}}{2}$

5. Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25}+\frac{y^2}{b^2}=1(b<5)$ and the hyperbola, $\frac{x^2}{16}-\frac{y^2}{b^2}=1$ respectively satisfying $e_1e_2=1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively ,then the ordered pair (α,β) is equal to :

1) $\left(\frac{24}{5},10\right)$ 2) $\left(\frac{20}{3},12\right)$ 3) $(8,10)$ 4) $(8,12)$

Key: 3

Sol: $e_1 = \sqrt{\frac{25-b^2}{25}}, e_2 = \sqrt{\frac{16-b^2}{16}}$

Given that , $e_1 e_2 = 1 \Rightarrow e_1^2 e_2^2 = 1 \Rightarrow (25-b^2)(16+b^2) = 25.16$

$\Rightarrow b^2 = 9$

$\therefore \alpha = 2\sqrt{25-9} = 8, \beta = 2\sqrt{16+9} = 10$

$\therefore (\alpha, \beta) = (8,10)$

6. If the value of the integral $\int_0^{1/2} \frac{x^2}{(1-x^2)^{3/2}} dx$ is $\frac{k}{6}$,then k is equal to :

1) $3\sqrt{2}+\pi$ 2) $2\sqrt{3}-\pi$ 3) $3\sqrt{2}-\pi$ 4) $2\sqrt{3}+\pi$

Key: 2

Sol: $I = \int_0^{\frac{1}{2}} \frac{x^2}{(1-x^2)^{3/2}} dx, put\ x = Sin\theta \Rightarrow dx = Cos\theta\, d\theta$

$$I = \int_0^{\frac{\pi}{6}} \frac{Sin^2\theta}{(1-Sin^2\theta)^{3/2}} Cos\theta\, d\theta = \int_0^{\frac{\pi}{6}} Tan^2\theta\, d\theta$$

$$= \int_0^{\frac{\pi}{6}} (Sec^2\theta - 1)\, d\theta = Tan^2\theta - \theta\Big|_0^{\frac{\pi}{6}}$$

$$\therefore \frac{k}{6} = \frac{1}{\sqrt{3}} - \frac{\pi}{6} = \frac{6-\sqrt{3}\pi}{6\sqrt{3}} = 2\sqrt{3}-\pi$$

7. Let R_1 *and* R_2 be two relations defined as follows :

$R_1 = \{(a,b) \in R^2 :\ a^2 + b^2 \in Q\}$ and

$R_2 = \{(a,b) \in R^2\ :\ a^2 + b^2 \notin Q\}$,where Q is the set of all rational numbers .Then :

1) R_1 and R_2 are both transitive.

2) Neither R_1 nor R_2 is transitive.

3) R_2 is transitive but R_1 is not transitive.

4) R_1 is transitive but R_2 is not transitive

Key : 2

Sol: $R_1 = \{ (a,b) \in R^2 : a^2+b^2 \in Q \}$ and

$R_2 = \{ (a,b) \in R^2 : a^2+b^2 \notin Q \}$

For R_1 if $a = 1 + \sqrt{2}$, $b = 1 - \sqrt{2}$, $c = 2^{3/4}$

$(a,b) \in R_1$, $(b,c) \in R$, but $(a,c) \notin R_1$

$\therefore$ R_1 is not transitive

For R_2, if $a = 1+\sqrt{2}$, $b = 1 + 2\sqrt{2}$, $c = 1-\sqrt{2}$

$(a,b) \in R_2$ $(b,c) \in R_2$ but, $(a,c) \notin R_2$

$\therefore$ R_2 is not transitive

8. $\lim\limits_{x-a} \dfrac{(a+2x)^{\frac{1}{3}} - (3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}} - (4x)^{\frac{1}{3}}}$ $(a \neq 0)$ (a=0) is equal to:

1) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$ 2) $\left(\frac{2}{3}\right)^{\frac{4}{3}}$ 3) $\left(\frac{2}{9}\right)^{\frac{4}{3}}$ 4) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$

Key: 4

Sol : $\underset{x \to a}{Lt} \dfrac{(a+2x)^{1/3} - (3x)^{1/3}}{(3a+x)^{1/3} - (4x)^{1/3}} (a \neq 0) = \underset{x \to a}{Lt} \dfrac{\frac{1}{3}(a+2x)^{-2/3} - \left[\frac{-1}{3}(3x)^{-2/3}\right]}{\frac{1}{3}(3a+x)^{-2/3} - \frac{1}{3}(4x)^{-2/3}.4}$

$$= \frac{+(3a)^{-2/3}}{+3(4a)^{-2/3}} = \frac{1}{3}\left(\frac{4}{3}\right)^{2/3} = \frac{2}{3}\left(\frac{2}{9}\right)^{1/3}$$

9. If the term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is k then 18 k is equal to ;

1)11 2) 9 3) 5 4) 7

Key : 4

Sol : $r = \dfrac{nP}{p+q} = \dfrac{9.2}{2+1} = 6$

Independent term = k = ${}^9C_6 \left(\frac{3}{2}\right)^3 \left(\frac{-1}{3}\right)^6 = \frac{7}{18}$

10. Let p, q, r be three statements such that the truth value of (p ^ q) $\rightarrow$ $(\sim q \vee r)$ is F. Then the truth values of p, q, r are respectively:

1) T ,F,T 2) T,T,F 3) T,T,T 4)F,T,F

Key : 2

Sol : $(p \wedge q)$ should be TRUE and $(\sim q \vee r)$ should be false

11. Let A be a 3×3 matrix such that adj $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$ and

B = adj (adj A).

If $|A|=\lambda$ and $\left|(B^{-1})^T\right|=\mu$ then the ordered pair, $(|\lambda|,\mu)$ is equal to :

1) $\left(9,\frac{1}{9}\right)$ 2) $(3,81)$ 3) $\left(3,\frac{1}{81}\right)$ 4) $\left(9,\frac{1}{81}\right)$

Key: 3

Sol : $|AdJ(A)|$=2(4)+(-1)+2=9=$|A|^2$

$\therefore\ |\lambda|=3$ and

AdJ(AdJ(A)} = $|A|^{n-2}$ A for A is a square matrix of order n .

$\therefore$B=$|A|A$

$|B|=|A|^4$ $\Rightarrow\left|(B^{-1})^T\right|=\frac{1}{|B|}=\frac{1}{|A|^4}=\frac{1}{81}=\mu$ $\therefore\ (|\lambda|,\mu)=\left(3,\frac{1}{81}\right)$

12. The plane which bisects the line joining the points $(4,-2,3)$ and (2, 4,-1)at right angles also passes through the point :

1)(0, -1, 1) 2) (4,0,-1) 3) (0,1,-1) 4) (4,0,1)

Key : 2

Sol : The equation of the perpendicular bisecting plane of the segment joining (9,-2,3) and (2,4,-1) is $2x(2)+2y(-6)+2z(4)=(16+4+9)-(4+16+1)=8$

$\Rightarrow x-3y+2z=2$

It also passes through (4, 0, -1)

13. The probability that a randomly chosen 5-digit number is made from exactly two digits is :

1) $\frac{135}{10^4}$ 2) $\frac{150}{10^4}$ 3) $\frac{121}{10^4}$ 4) $\frac{134}{10^4}$

Key: 1

Sol: The required probability $=\frac{{}^9C_2\left(2^5-2\right)+{}^9C_1\left(2^4-1\right)}{9.10^4}=\frac{135}{10^4}$

14. $\int \sin^{-1}\left(\sqrt{\frac{x}{1+x}}\right)$ dx = A(x) $\tan^{-1}(\sqrt{x})$ + B(x)+c, where C is a constant of integration, then the ordered pair (A(x), B(x)} can be :

1) (x+1 , $\sqrt{x}$) 2) (x+1 , -$\sqrt{x}$) 3) (x-1 , $\sqrt{x}$) 4)(x-1, $\sqrt{x}$)

Key: 2

Sol: $\int \sin^{-1}\sqrt{\frac{x}{1+x}}dx=\int Tan^{-1}\sqrt{x}dx$

By using by parts,

$$\int Tan^{-1}\sqrt{x}dx=x.\tan^{-1}\sqrt{x}-\int\frac{x}{1+x}.\frac{1}{2\sqrt{x}}dx$$

$$=x.\tan^{-1}\sqrt{x}-\int\left(\frac{1}{2\sqrt{x}}-\frac{1}{\left(1+\left(\sqrt{x}\right)^2\right)}.\frac{1}{2\sqrt{x}}\right)dx$$

$$=x.\tan^{-1}\sqrt{x}-\sqrt{x}+Tan^{-1}\sqrt{x}+c$$

$$=(x+1)Tan^{-1}\sqrt{x}-\sqrt{x}+c$$

$$\Rightarrow A(x)=x+1, B(x)=-\sqrt{x}$$

15. If a $\triangle$ABC has vertices A(-1 , 7), B(-7,1) and C(5,-5),then its orthocentre has coordinates :

1) $(-3,3)$ 2) $\left(-\frac{3}{5},\frac{3}{5}\right)$ 3) $(3,-3)$ 4) $\left(\frac{3}{5},-\frac{3}{5}\right)$

Key : 1

Sol : $A(-1,7),B(-7,1),C(5,-5)$

From the given data, A, B, C are equidistant from (0, 0). Hence S(0, 0) is the circumcentre of the $\Delta^{le}ABC$

Centrid of $\Delta^{le} ABC$ is $G(-1,1)$

$\therefore 3G = 2S + H \Rightarrow H = (-3,3)$

16. If Z_1 , Z_2 are complex numbers such that Re (Z_1) = $|Z_1 - 1|$, Re (Z_2)=$|Z_2 - 1|$ and $\arg(z_1 - z_2) = \frac{\pi}{6}$, then $\text{Im}(z_1 + z_2)$ is equal to :

1) $\frac{1}{\sqrt{3}}$ 2) $\frac{\sqrt{3}}{2}$ 3) $\frac{2}{\sqrt{3}}$ 4) $2\sqrt{3}$

Key:4

Sol: Let $|z-1| = \text{Re}(z)$, Let $z_1 = x_1 + iy_1, z_2 = x_2 + iy_2$

$\Rightarrow (x-1)^2 + y^2 = x^2$

$\Rightarrow y^2 = 2x - 1$

$\therefore (x_1, y_1), (x_2, y_2)$ lies on $y^2 = 2x - 1$

$\Rightarrow y_1^2 - y_2^2 = 2(x_1 - x_2)$

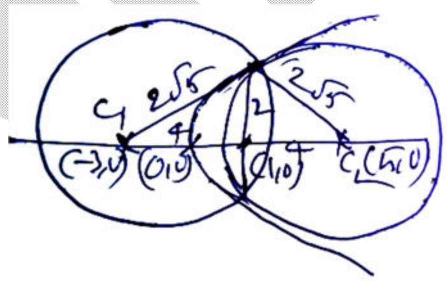
$\Rightarrow \frac{y_1 - y_2}{x_1 - x_2} = \frac{1}{\sqrt{3}} = \frac{2}{y_1 + y_2} \Rightarrow y_1 + y_2 = \text{Im}(z_1 + z_2) = 2\sqrt{3}$

17. Let the latusrectum of the parabola y^2 = 4x be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$ .Then the distance between the centres of the circles C_1 and C_2 is:

1) 8 2) $4\sqrt{5}$ 3)12 4) $8\sqrt{5}$

Key: 1

Sol:



$C_1C_2 = 8$

18. Let $x_i (1 \le i \le 10)$ be ten observations of a random variable X. If $\sum_{i=1}^{10}(x_i - p) = 3$ and $\sum_{i=1}^{10}(x_i - p)^2 = 9$ where $0 \neq$ p $\in$R, then the standard devation of these observations is :

1) $\frac{4}{5}$ 2) $\sqrt{\frac{3}{5}}$ 3) $\frac{7}{10}$ 4) $\frac{9}{10}$

Ans: 4

Sol: $\sum x_i = 10P + 3$

$\sum x_i^2 - 2p\sum x_i + 10p^2 = 9$

$\sum x_i^2 = 2p(10p+3) - 10p^2 + 9 = 10p^2 + 6p + 9$

$\sigma^2 = \frac{\sum x_i^2}{10} - \left(\frac{\sum x_i}{10}\right)^2 = p^2 + \frac{6p}{10} + \frac{9}{10} - \left(p^2 + \frac{9}{100} + \frac{6p}{10}\right) = \frac{81}{100}$ $\sigma = \frac{9}{10}$

19. If the surface area of a cube is increasing at a rate of 3.6 cm^2 /sec, retaining its shape : then the rate of change of its volume (in cm^3 /sec) ,when the length of a side of the cube is 10 cm is :

1)10 2) 20 3) 9 4) 18

Key: 3

Sol: Let x be the length of the side of the cube

$\therefore$ surface area, s = $6x^2$ $\Rightarrow \frac{ds}{dt} = 12x\frac{dx}{dt}$

$\Rightarrow \frac{dx}{dt} = \frac{3.6}{12\times 10} = 0.03$ for x= 10

$v = x^3 \Rightarrow \frac{dv}{dt} = 3x^2\frac{dx}{dt} = 300\times 0.03 = 9$

20. If the sum of the series

$20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + \ldots$ up to n^{th} term is 488 and the n^{th} term is negative, then :

1)n =41 2) n=60 3) n^{th} term is -4 4) n^{th} term is $-4\frac{2}{5}$

Key: 3

Sol: $S = 20 + \left(19 + \frac{3}{5}\right) + \left(19 + \frac{1}{5}\right) + \ldots n\ terms$

Here term are in A.P with common difference $d = \frac{-2}{5}$

$\therefore 488 = \frac{n}{2}\left[2(20) + (n-1)\left(\frac{-2}{5}\right)\right] = n\left(20 + \frac{1-n}{5}\right)$

$\Rightarrow n^2 - 101n + 5 \times 488 = 0 \qquad \Rightarrow n = 40 \ (or) \ 61$

But, $t_n < 0 \Rightarrow n = 61 \ \& \ t_n = -4$

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. The total number of 3- digit numbers, whose sum of digits is 10,is __________

Key: 54

Sol: Let abc be three digital number

$\therefore a+b+c=10, a \geq 1 \ \& \ b, c \geq 0$

Let $a-1=x \Rightarrow a=x+1 \ \& \ n \geq 0$

$\therefore x+1+b+c=10 \Rightarrow x+b+c=9$

Total number of solutions = ${}^{9+3-1}C_{3-1} = {}^{11}C_2 = 55$

But in these solutions (10, 0, 0) is one of the solution eliminate it.

22. Let S be the set of all integer solutions ,

(x,y,z), of the system of equations

x-2y+5z =0

-2x+4y+z=0

-7x+14y+9z=0

Such that 15 $\leq$ $x^2+y^2+z^2 \leq 150$ .Then the number of elements in the set S is equal to__

Key: 8

Sol: x – 2y +5z =0

-2x + 4y + z =0 and

$\Rightarrow$ -7x +14y +9z =0

$\Rightarrow$ z=0 and x=2y and also 15 $\leq$ $x^2 + y^2 + z^2 \leq 150$

$\Rightarrow 15 \leq 5y^2 \leq 150$

$\Rightarrow 3 \leq y^2 \leq 30$

$\Rightarrow y = \pm 2, \pm 3, \pm 4, \pm 5$

23. If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3

and 243 such that 4^{th} A.M, is equal to 2^{nd} G.M, then m is equal to________

Key: 39

Sol: Let 3, x_1,x_2,x_3,x_4 .....x_n, 243 are in A.P.and
3,a,b,c,243 are in G.P. with common radio r
$\therefore 243 = 3.r^4 \Rightarrow r=3 \Rightarrow b=3.r^2 =27$
Given that x_4=b=27=3+4d
$\Rightarrow d = 6$
$243 = 3 +(m+1).6 \Rightarrow m+1=40 \Rightarrow m=39$

24. If the tangent to the curve, $y = e^x$ at a point (c,e^c) and the normal to the parabola, $y^2 = 4x$ at the point (1,2) intersect at the same point on the x-axis , then the value of c is_______

Key: 4

Sol: $\frac{dy}{dx} = e^x \Rightarrow \left(\frac{dy}{dx}\right)_{(c,\, e^c)} = e^c$

Eq. of the tangent at $(c, e^c)$to the curve $y=e^x$ is

$y - e^c = e^c (x-c)$

If meets the x-axis at x= c-1

Eq. of the normal to $y^2 = 4x$ at (1,2) is

y+x =2+1 (t=1) $\Rightarrow$ x+y=3

it meets x axis at x=3

$\therefore c - 1 = 3 \Rightarrow c = 4$

25. Let a plane P contain two lines

$\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j}), \lambda \in R$ and $\vec{r} = -\hat{j} + \mu(\hat{j} - \hat{k}), \mu \in R$

If Q (α, β, γ) is the foot of the perpendicular drawn from the point M(1,0,1) to P, then $3(\alpha + \beta + \gamma)$ equals_____

Key: 5

Sol: Equation of the plane P is $\begin{vmatrix} x-1 & y & z \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = 0$

$\Rightarrow$ (x-1) (-1) - y(-1) + z(1) = 0

$\Rightarrow$ x-1-y-z=0

Given that $Q(\alpha,\beta,\gamma)$ is the foot of the perpendicular drawn from M(1,0,1) to the plane p.

$\therefore \frac{\alpha-1}{1}=\frac{\beta-0}{1}=\frac{\gamma-1}{-1}=\frac{-(1-0-1-1)}{1+1+1}=\frac{1}{3}$

$\Rightarrow \alpha=\frac{4}{3},\beta=\frac{-1}{3},\gamma=\frac{2}{3}$

$\Rightarrow 3(\alpha+\beta+\gamma)=5$

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