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✪ A.P ✪ T.S ✪ Chandigarh ✪ Karnataka ✪ Tamilnadu ✪ Maharastra ✪ Delhi ✪ Ranchi

Jee Main 2020(Sep)

04-Sep-2020 (Morning Shift)

Question Paper, Key and Solutions

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Physics

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. For a transverse wave travelling along a straight line, the distance between two peaks (crests) is 5 m, while the distance between one crest and one trough is 1.5 m. The possible wavelengths (in m) of the waves are :

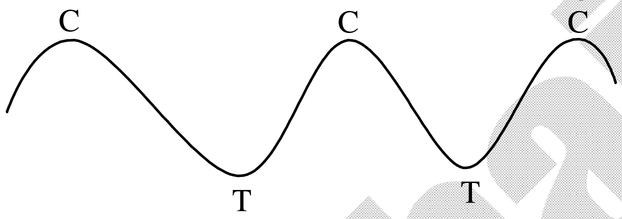
1) 1, 2, 3, ... 2) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, ...$ 3) 1, 3, 5, ... 4) $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}, ...$

Key: 4

Sol: Distance between crests $= n\lambda = 5$

So $\lambda = \frac{5}{n} m \, ... (1)$

Also distance between a crest and trough is given as 1.5 m



$\Rightarrow (2k-1)\frac{\lambda}{2} = 1.5$

So $\lambda = \frac{3}{2k-1} \, ... (2)$

$(1), (2)$ satisfy, $\frac{n}{2k-1} = \frac{5}{3} = \frac{15}{9} = \frac{25}{15}$

$\Rightarrow n \in \{5, 15, 25, ...\}$

So $\lambda \in \left\{1, \frac{1}{3}, \frac{1}{5}, ...\right\}$

2. The specific het of water $= 4200 \, J kg^{-1} K^{-1}$ and the latent heat of ice $= 3.4 \times 10^5 \, J \, kg^{-1}$. 100 grams of ice at $0°C$ is placed in 200 g of water at $25°C$. The amount of ice that will melt as the temperature of water reaches $0°C$ is close to (in grams) :

1) 61.7 2) 64.6 3) 63.8 4) 69.3

Key: 1

Sol:

200 g of water at 25°C $\xrightarrow{ms\Delta T}$ 200 g of water at 0°C

Heat extracted out of system $=(0.2)(4200)(25)=21000\,J$

So amount of ice melt $=\dfrac{Q}{L}=\dfrac{21000}{3.4\times10^{5}}$ kg

$=\dfrac{210}{3.4}g=61.76$ gm

3. A battery of 3.0 V is connected to a resistor dissipating 0.5 W of power. If the terminal voltage of the battery is 2.5 V, the power dissipated with in the internal resistance is :

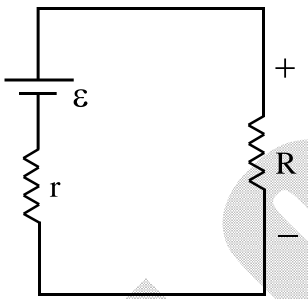
1) 0.072 W 2) 0.50 W 3) 0.125 W 4) 0.10 W

Key: 4

Sol: Given,

$\varepsilon=3V$

Terminal potential difference = 2.5 V



The same potential difference is across external load "R" $=V_R=2.5V$

So $i=\dfrac{P_R}{V_R}=\dfrac{0.5}{2.5}=0.2A$

So power supplied $=\varepsilon i=3(0.2)=0.6W$

Hence power dissipated in $r=0.6-0.5=0.1W$

4. On the x-axis and at a distance x from the origin, the gravitational is given by $\frac{Ax}{(x^2+a^2)^{3/2}}$ in the x-direction. The magnitude of gravitational potential on the x-axis at a distance x, taking its value to be zero at infinity, is :

1) $A(x^2+a^2)^{3/2}$ 2) $\frac{A}{(x^2+a^2)^{3/2}}$ 3) $\frac{A}{(x^2+a^2)^{1/2}}$ 4) $A(x^2+a^2)^{1/2}$

Key: 3

Sol: Given $E_x = \frac{Ax}{(x^2+a^2)^{3/2}}$

$$V(x)-V(\infty) = -\int_{\infty}^{x} \frac{Ax}{(x^2+a^2)^{3/2}}dx$$

Given $V(\infty)=0 \Rightarrow V(x) = -\frac{A}{2}\int_{\infty}^{x} \frac{d(x^2+a^2)}{(x^2+a^2)^{3/2}}$

$$= -\frac{A}{2}\frac{(x^2+a^2)^{-1/2}}{\left(-\frac{1}{2}\right)}\Bigg|_{\infty}^{x} = A(x^2+a^2)^{-1/2}$$

5. Starting from the origin at time t = 0, with initial velocity $5\hat{j}\,ms^{-1}$, a particle moves in the $x-y$ plane with a constant acceleration of $(10\hat{j}+4\hat{j})ms^{-2}$. At time t, its coordinates are $(20m,\ y_0 m)$. The value of t and y_0 are, respectively :

1) 5 s and 25 m 2) 2 s and 24 m 3) 2 s and 18 m 4) 4 s and 52 m

Key: 3

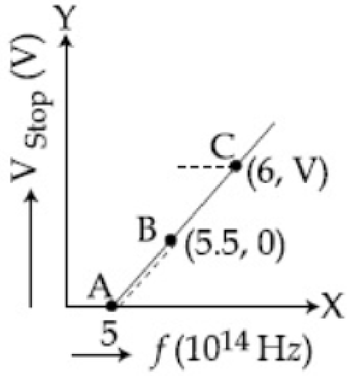
Sol: $\bar{S} = t\bar{u} + \frac{1}{2}t^2\bar{a}$ $\vec{r}_f - \bar{r}_i = t\bar{u} + \frac{1}{2}t^2\bar{a}$

$$20\hat{i} + y_0\hat{j} = t_0(5\hat{j}) + \frac{1}{2}t_0^2(10\hat{i}+4\hat{j})$$

Equating components wise,

$20 = 5t_0^2 \Rightarrow t_0 = 2\sec$ and $y_0 = 5t_0^2 \Rightarrow t_0 = 2\sec$ and $y_0 = 5t_0 + 2t_0^2 \Rightarrow y_0 = 18m$

6. Given figure shows few data points in a photo electric effect experiment for a certain metal. The minimum energy for ejection of electron from its surface is : (Plancks constant $h = 6.62\times10^{-34}\ J\cdot s$)



1) 2.59 eV 2) 2.10 eV 3) 2.27 eV 4) 1.93 eV

Key: 3

Sol: $hf = \phi + eV_{stop}$

ϕ is work function, which by definition is the minimum energy for the surface electron's ejection.

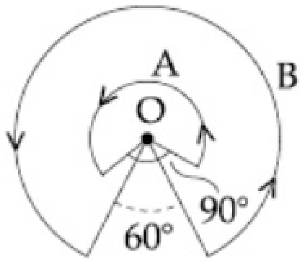
Re-arranging, $V_{stop} = \frac{h}{e}f - \frac{\phi}{e}$

x intercept $= \frac{\phi}{h} = 5.5\times10^{14}$ Hz

$\therefore\ \phi = \left(6.62\times10^{-34}\right)\left(5.5\times10^{14}\right)$J

$= \frac{\left(6.62\times10^{-34}\right)\left(5.5\times10^{14}\right)}{1.6\times10^{-19}}$eV =2.276 eV

7. A wire A, bent in the shape of an arc of a circle, carrying a current of 2 A and having radius 2 cm and another wire B, also bent in the shape of arc of a circle, carrying a current of 3 A and having radius of 4 cm, are placed as shown in the figure. The ratio of the magnetic fields due to the wires A and B at the common centre O is :



1) 6 : 5 2) 4 : 6 3) 6 : 4 4) 2 : 5

Key: 1

Sol: Magnetic field due to arc of circular current $=\left(\frac{\theta}{360°}\right)\frac{\mu_0 i}{2R}$ at the center. Here required

$$\text{ratio} = \frac{\theta_4 i_A / R_A}{\theta_8 i_B / R_B}$$

$$= \frac{270}{300}\times\frac{2}{3}\times\frac{4}{2}=\frac{6}{5}$$

8. A air bubble of radius 1 cm in water has an upward acceleration 9.8 cm s^{-2}. The density of water is 1 gm cm^{-3} and water offers negligible drag force on the bubble. The mass of the bubble is $\left(g = 980 \text{ cm}/\text{s}^2\right)$

1) 3.15 gm 2) 4.15 gm 3) 4.51 gm 4) 1.52 gm

Key: 2

Sol: Buoyant force dominates weight

So acceleration $a=\frac{F_B - mg}{m}$

$$\frac{F_B}{m} = a + 5 = 989.8 \text{ cm}/\text{s}^2$$

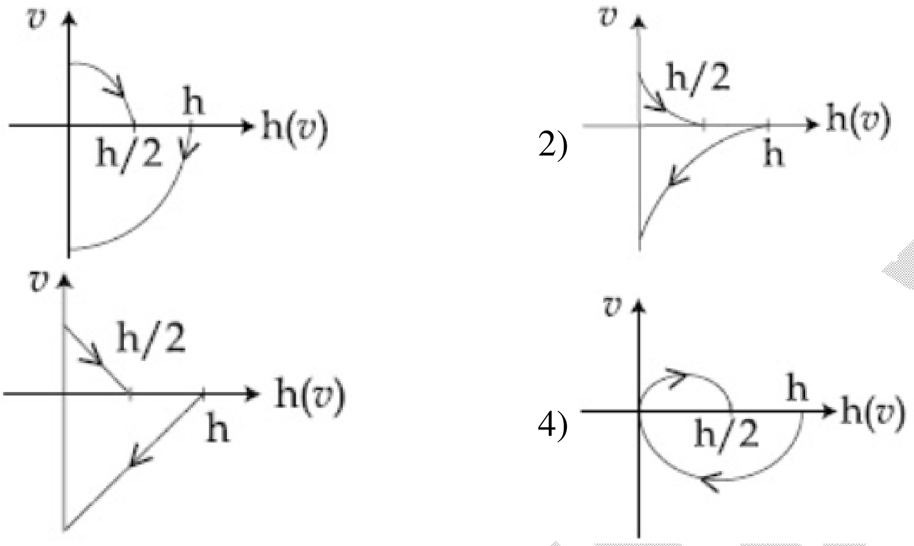
Also $F_B = V\rho_{liq}\, g$

$$=\frac{4}{3}\pi(1)^3(1)(980)$$

$$\text{So } m = \frac{\left(\frac{4\pi}{3}\right)(980)}{989.8} = 4.145 \text{ gm}$$

9. A Tennis ball is released from a height h and after freely falling on a wooden floor it rebounds and reaches height $\frac{h}{2}$. The velocity versus height of the ball during its motion may be represented graphically by :
(graph are drawn schematically and on not to scale)

1) 2) 3) 4)



Key: 1

Sol:
We can see from options that downward direction is answered negative.
Also velocity 'V' changes sign when it hits the floor with a velocity.
Using $v^2 - u^2 = 2aS$, with $u = 0$ and $S = \pm h$ we can conclude that graph has to be parabolic.

10. Match the C_P / C_V ratio for ideal gases with different type of molecules :

Molecule Type	C_P / C_V
(A) Monatomic	(I) 7/5
(B) Diatomic rigid molecules	(II) 9/7
(C) Diatomic non-rigid molecules	(III) 4/3
(D) Triatomic rigid molecules	(IV) 5/3

1) $(A)-(II), (B)-(III), (C)-(I), (D)-(IV)$

2) $(A)-(IV), (B)-(II), (C)-(I), (D)-(III)$

3) $(A)-(III), (B)-(IV), (C)-(II), (D)-(I)$

4) $(A)-(IV), (B)-(I), (C)-(II), (D)-(III)$

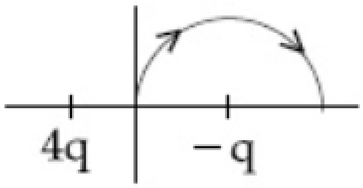
Key: 4

Sol: $C_V = \frac{fR}{2}$, $C_P = \frac{fR}{2} + R$

$\Rightarrow \frac{C_P}{C_V} = \frac{f+2}{f}$, f is degrees of freedom.

Gas	Translation	Rotation	Vibration	$\frac{4}{C_V}$
Monatomic	3	0	0	5/3
Diatomic Rigid	3	2	0	7/5
Diatomic non-Rigid	3	2	2	9/7
Triatomic rigid	3	3	0	4/3

11. A two point charges 4q and $-q$ are fixed on the x-axis at $x = -\frac{d}{2}$ and $x = \frac{d}{2}$, respectively. If a third point charge 'q' is taken from the origin to x = d along the semicircle as shown in the figure, the energy of the charge will :



1) increase by $\frac{3q^2}{4\pi \in_0 d}$ 2) decrease by $\frac{q^2}{4\pi \in_0 d}$

3) decrease by $\frac{4q^2}{3\pi \in_0 d}$ 4) increase by $\frac{2q^2}{3\pi \in_0 d}$

Key: 3

Sol:

The change is only due to 4q
Charge as distance from "$-q$" is not changing.

$$\text{So } \Delta U = \frac{(4q)(q)}{4\pi \in_0}\left(\frac{1}{(3d/2)} - \frac{1}{(d/2)}\right)$$

$$= -\frac{4q^2}{3\pi \in_0 d}$$

12. Dimensional formula for thermal conductivity is (here K denotes the temperature) :

1) $MLT^{-2}K^{-2}$ 2) $MLT^{-3}K^{-1}$ 3) $MLT^{-2}K$ 4) $MLT^{-3}K$

Key: 2

Sol: $\frac{dQ}{dt}=\frac{kA\Delta T}{\Delta x}$

$$k=\frac{\left(\frac{dQ}{dt}\right)(\Delta x)}{(\Delta T)A}$$

$$[k]=\frac{(\text{work})}{(\text{Time})(\text{Temp})(\text{length})}$$

$$=\frac{\text{Force}}{(\text{Time})(\text{Temp})}$$

$$=\frac{\left[MLT^{-2}\right]}{Tk} \qquad =\left[MLT^{-3}k^{-1}\right]$$

13. A beam of plane polarised light of large cross-sectional area and uniform intensity of $3.3\ W\,m^{-2}$ falls normally on a polariser (cross sectional area $3\times10^{-4}\ m^2$) which rotates about its axis with an angular speed of 31.4 rad/s. The energy of light passing through the polariser per revolution, is close to :

1) $1.5\times10^{-4}\ J$ 2) $1.0\times10^{-4}\ J$ 3) $1.0\times10^{-5}\ J$ 4) $5.0\times10^{-4}\ J$

Key: 2

Sol: Halus law for polarised light $\Rightarrow I=I_0\cos^2\theta$

Here $I_0=3.3\ w/m^2$

$\theta=\omega t$ where $\omega=10\pi\, rad/s$

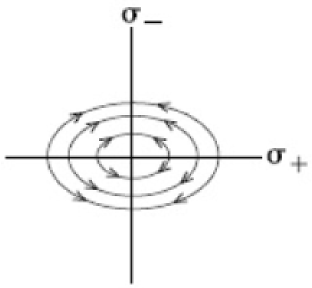
Energy $E=\int IA\,dt$

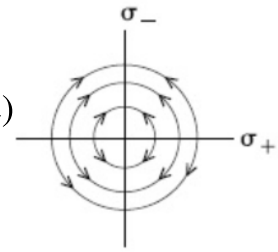
$$=I_0A\int\cos^2\omega t\,dt \qquad =\frac{I_0A}{\omega}\int_0^{2\pi}\cos^2\omega t\,d(\omega t)$$

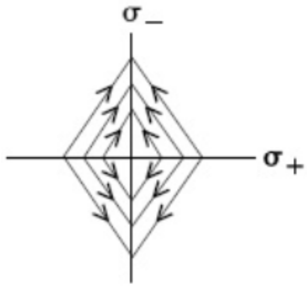
$$=\frac{I_0A}{\omega}\pi \qquad =\frac{(3.3)\left(3\times10^{-4}\right)}{10}$$

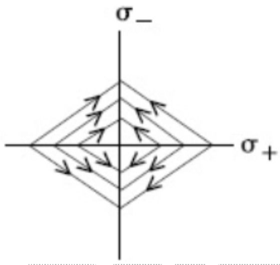
$$=0.99\times10^{-4}\ J$$

14. Two charged thin infinite plane sheets of uniform surface charge density σ_+ and σ_-, where $|\sigma_+| > |\sigma_-|$, intersect at right angle. Which of the following best represents the electric field lines for this system :

1) 

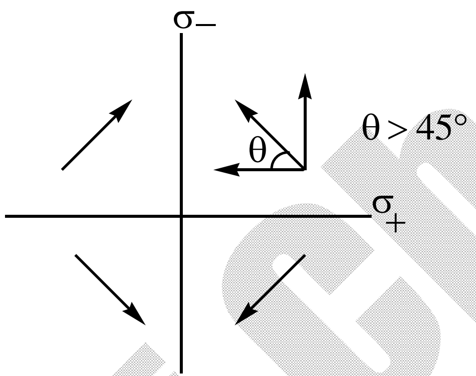
2) 

3) 

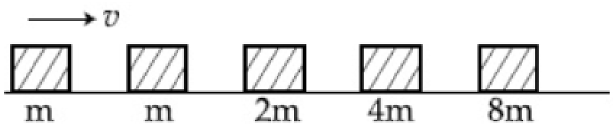
4) 

Key: 3

Sol:



15. Blocks of masses m, 2m, 4m and 8m are arranged in a line on a frictionless floor. Another block of mass m, moving with speed v along the same line (see figure) collides with mass m in perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. By the time the last block of mass 8m starts moving the total energy loss is p% of the original energy. Value of 'p' is close to :



1) 37 2) 87 3) 94 4) 77

Key: 3

Sol: We can combine all initial masses at rest into one unit.

So $m_1 = m, m_2 = 15m$

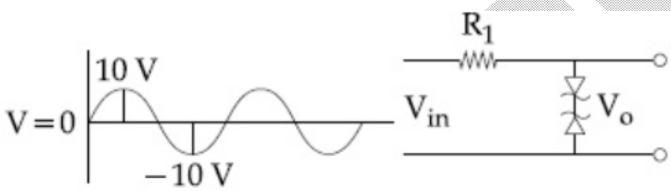
Kinetic energy in CM frame is lost

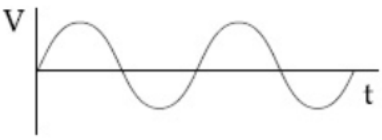
$= \frac{1}{2}\mu v_{rel}^2$, μ is reduced mass $= \frac{1}{2}\frac{15m}{16}v^2$

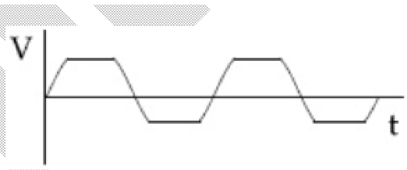
Loss percentage $= \frac{15}{16} \times 100$

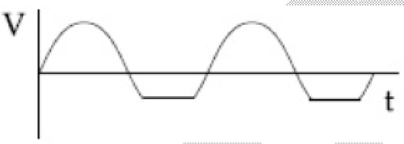
16. Take the breakdown voltage of the zener diode used in the given circuit as 6V. For the input voltage shown in figure below, the time variation of the output voltage is :

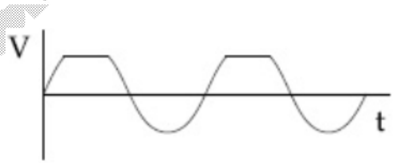
(Graphs drawn are schematic and not to scale)



1)


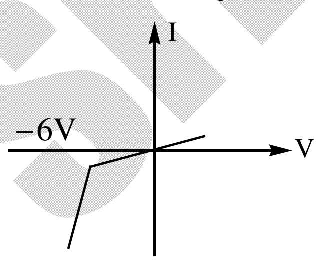
2)


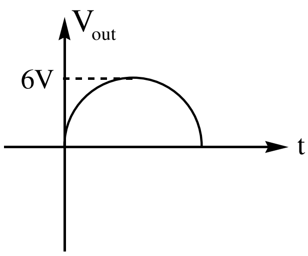
3)


4)


Key: 2

Sol: During each half cycle, when V_{in} builds from 0 to 6V, one of the diodes is in reverse biased but not yet reached break down.

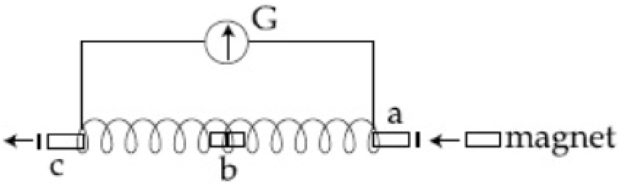




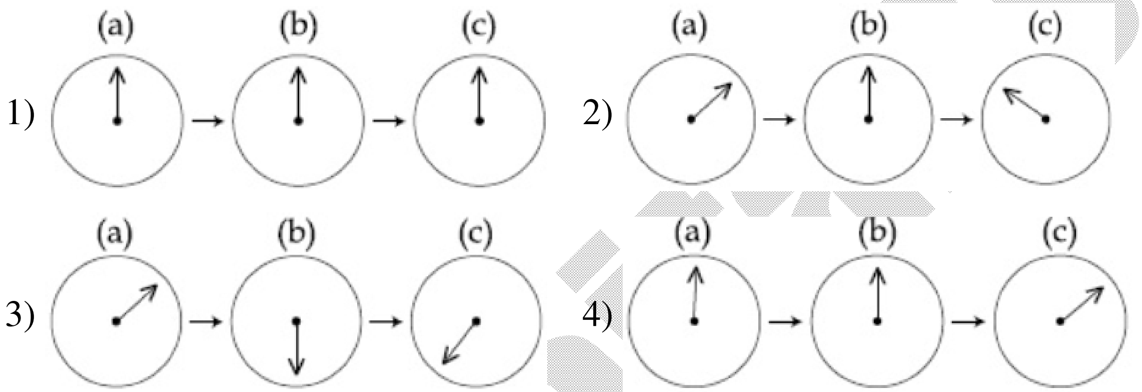
So, the diode branch is open and bypasses the V_{in}.

Once V_{in} builds from 6V to 10V, then one zener achieves breakdown and other in forward bias $(\Delta V = 0)$ produces constant voltage.

17. A small bar magnet is moved through a coil at constant speed from one end to the other. Which of the following series of observations will be seen on the galvanometer G attached across the coil ?



Three positions shown describe : (a) the magnet's entry (b) magnet is completely inside and (c) magnet's exit.



Key: 2

Sol: Rate of change of flux is maximum at entry and exit and also in the opposite direction. Also when in middle, it should be zero.

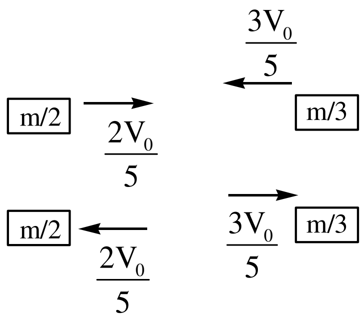
18. Particle A of mass $m_A = \frac{m}{2}$ moving along the x-axis with velocity v_0 collides elastically with another particle B at rest having mass $m_B = \frac{m}{3}$. If the particles move along the x-axis after the collision, the change $\Delta\lambda$ in de-Broglie wavelength of particle A, in terms of its de-Broglie wavelength (λ_0) before collision is :

1) $\Delta\lambda = 2\lambda_0$ 2) $\Delta\lambda = \frac{5}{2}\lambda_0$ 3) $\Delta\lambda = 4\lambda_0$ 4) $\Delta\lambda = \frac{3}{2}\lambda_0$

Key: 3

Sol: $V_{cm} = \dfrac{\frac{m}{2}V_0}{\frac{m}{2}+\frac{m}{3}} = \dfrac{3V_0}{5}$

In cm frame,



In ground frame, $\frac{m}{2}$ moves forward with $\frac{3V_0}{5}-\frac{2V_0}{5}=\frac{V_0}{5}$

So $\lambda_f = 5\lambda_0 \Rightarrow \Delta\lambda = 4\lambda_0$

19. A small bar magnet placed with its axis at $30°$ with an external field of 0.06 T experiences a torque of $0.018\ \text{Nm}$. The minimum work required to rotate it from its stable to unstable equilibrium position is :

1) $11.7\times10^{-3}\ \text{J}$ 2) $7.2\times10^{-2}\ \text{J}$ 3) $9.2\times10^{-3}\ \text{J}$ 4) $6.4\times10^{-2}\ \text{J}$

Key: 2

Sol: $\tau = \overline{m}\times\overline{B}$

$U = -\overline{m}\cdot\overline{B}$

$W = \Delta U\Big|_{\text{stable}}^{\text{unstable}} = 2mB$

Given $\tau_{\text{at } 30°} = mB\sin30° = 0.018$

$\Rightarrow mB = 0.036$ So $W = 2mB = 0.072\,\text{J}$

20. Choose the correct option relating wavelengths of different parts of electromagnetic wave spectrum :

1) $\lambda_{\text{visible}} > \lambda_{x-\text{rays}} > \lambda_{\text{radio waves}} > \lambda_{\text{micro waves}}$

2) $\lambda_{\text{visible}} < \lambda_{\text{micro waves}} < \lambda_{\text{radio waves}} < \lambda_{x-\text{rays}}$

3) $\lambda_{\text{radio waves}} > \lambda_{\text{micro waves}} > \lambda_{\text{visible}} > \lambda_{x-\text{rays}}$

4) $\lambda_{x-\text{rays}} < \lambda_{\text{micro waves}} < \lambda_{\text{radio waves}} < \lambda_{\text{visible}}$

Key: 3

Sol:

X-rays are most energetic and radio waves are least.

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value(in decimal notation, truncated/ rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. In a compound microscope, the magnified virtual image is formed at a distance of 25 cm from the eye-piece. The focal length of its objective lens is 1 cm. If the magnification is 100 and the tube length of the microscope is 20 cm, then the focal length of the eye-piece lens (in cm) is ____________ .

Sol:

$$m = \frac{V_0}{u_0}\frac{D}{u_e}$$

Far point ↙ ↘ Near point

$$= \frac{\ell}{f_0}\frac{D}{f_e} \qquad \approx \frac{\ell}{f_0}\left(1+\frac{D}{f_e}\right)$$

Here, $\frac{\ell}{f_0}\left(1+\frac{D}{f_e}\right)=100$

$$\frac{20}{1}\left(1+\frac{25}{f_e}\right)=100$$

$f_e = 6.25$ cm Answer : 6.25 cm

NTA key is wrongly given for far point adjustment.

22. A circular disc of mass M and radius R is rotating about its axis with angular speed ω_1. If another stationary disc having radius $\frac{R}{2}$ and same mass M is dropped co-axially on to the rotating disc. Gradually both discs attain constant angular speed ω_2. The energy lost in the process is p% of the initial energy. Value of p is __________ .

Key: 20.00

Sol: Energy lost form would be similar to the calculation in inelastic collision. There, we used reduced mass. Here we use reduced inertia.

$$K \cdot E_{lost} = \frac{1}{2}\frac{I_1 I_2}{I_1+I_2}\omega^2 \qquad \text{Percentage lost} = \frac{I_2}{I_1+I_2}\times 100$$

$$= \frac{(R/2)^2}{R^2+\left(\frac{R}{2}\right)^2}\times 100 = 20\%$$

23. In the line spectra of hydrogen atom, difference between the largest and the shortest wavelenths of the Lyman series is 304 $\overset{0}{A}$. The corresponding difference for the Paschan series in $\overset{0}{A}$ is : ___________ .

Key: 10553.14

Sol: Largest wavelength $\Rightarrow$ minimum energy and vice-versa.

Lyman

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) \Rightarrow \lambda_{max} = \frac{4}{3R}$$

$$\frac{1}{\lambda_{min}} = R\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) \Rightarrow \lambda_{min} = \frac{1}{R}$$

$$\lambda_{max} - \lambda_{min} = \frac{1}{3R} = 304\theta^{\circ}$$

Paschen

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right) \Rightarrow \lambda_{max} = \frac{16\times 9}{7R}$$

$$\frac{1}{\lambda_{min}} = R\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right) \Rightarrow \lambda_{min} = \frac{9}{R}$$

$$\lambda_{max} - \lambda_{min} = \frac{81}{7R} = \frac{81}{7}\times 3\times 304\,A^{\circ}$$

$$= 10553.14\,A^{\circ}$$

24. A closed vessel contains 0.1 mole of a monatomic ideal gas at 200 K. If 0.05 mole of the same gas at 400 K is added to it, the final equilibrium temperature (in K) of the gas in the vessel will be close to ___________ .

Key: 266.67

Sol: Assuming $Q = 0,\ W = 0$

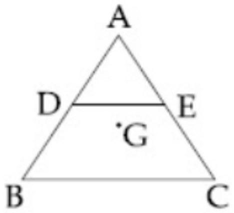
We have $\Delta U = 0 \quad \Rightarrow n_1C_{V_1}\left(T_f - T_1\right) + n_2C_{V_2}\left(T_f - T_2\right) = 0$

Same gas is added, so, $C_{V_1} = C_{V_2}$

$$\Rightarrow T_f = \frac{n_1T_1 + n_2T_2}{n_1 + n_2} = \frac{(0.1)(200) + (0.05)(400)}{0.1 + 0.05}$$

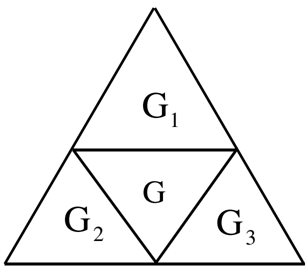
$$= \frac{20 + 20}{0.15} = 266.6666$$

25. ABC is a plane lamina of the shape of an equilateral triangle. D, E are mid points of AB, AC and G is the centroid of the lamina. Moment of inertia of the lamina about an axis passing through G and perpendicular to the plane ABC is I_0. If part ADE is removed, the moment of inertia of the remaining part about the same axis is $\frac{NI_0}{16}$ where N is an integer. Value of N is __________ .



Key: 11.00

Sol: Divide the larger plate into four equal plates as shown.
If I_0 is moment of inertia of bigger plate, we may write
$I_0 = \beta ma^2$
Where β is a shape factor.



So each small plate will have moment of inertia about its own centroid as
$\beta \frac{m}{4}\left(\frac{a}{2}\right)^2 = \frac{I_0}{16}$.

So their contribution to I_0 can be written as $I_0 = 4\left(\frac{I_0}{16}\right) + 3x$

$\Rightarrow x = \frac{I_0}{4}$

We require $I_0 - \{\text{single contribution}\}$

$= I_0 - \left\{\frac{I_0}{16} + x\right\} = \frac{11I_0}{16}$

CHEMISTRY

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

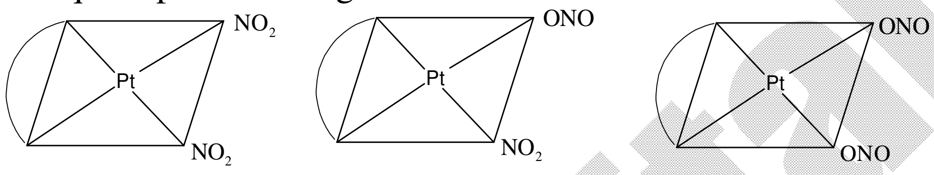
1. The number of isomers possible for $\left[Pt(en)(NO_2)_2\right]$ is :

1) 1 2) 2 3) 3 4) 4

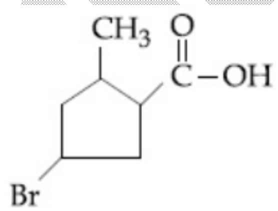
Key: 3

Sol: The number of isomers possible for $\left[Pt(en)(NO_2)_2\right]$

$\Rightarrow$Square planar arrangement



2. The IUPAC name of the following compound is :



1) 5-Bromo-3-methylcyclopentanoic acid

2) 3-Bromo-5-methylcyclopentane carboxylic acid

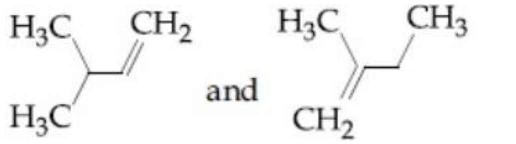
3) 4-Bromo-2-methylcyclopentane carboxylic acid

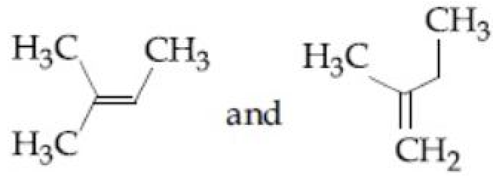
4) 3-Bromo-5-methylcyclopentanoic acid

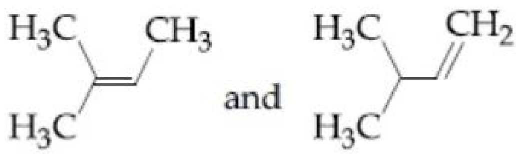
Key: 3

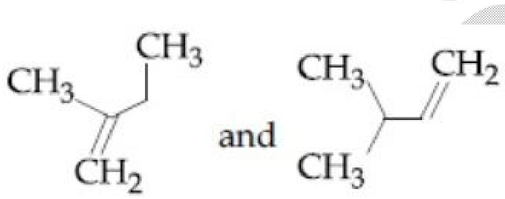
Sol: Preferred functional group is –COOH; so it will get lowest numbering. i.e. position 1 & then we will take lowest locant set of the substituents i.e., 3-Methyl & 4-Bromo.

3. When neopentyl alcohol is heated with an acid, it slowly converted into an 85 : 15 mixture of alkenes A and B, respectively. What are these alkenes ?

1) 

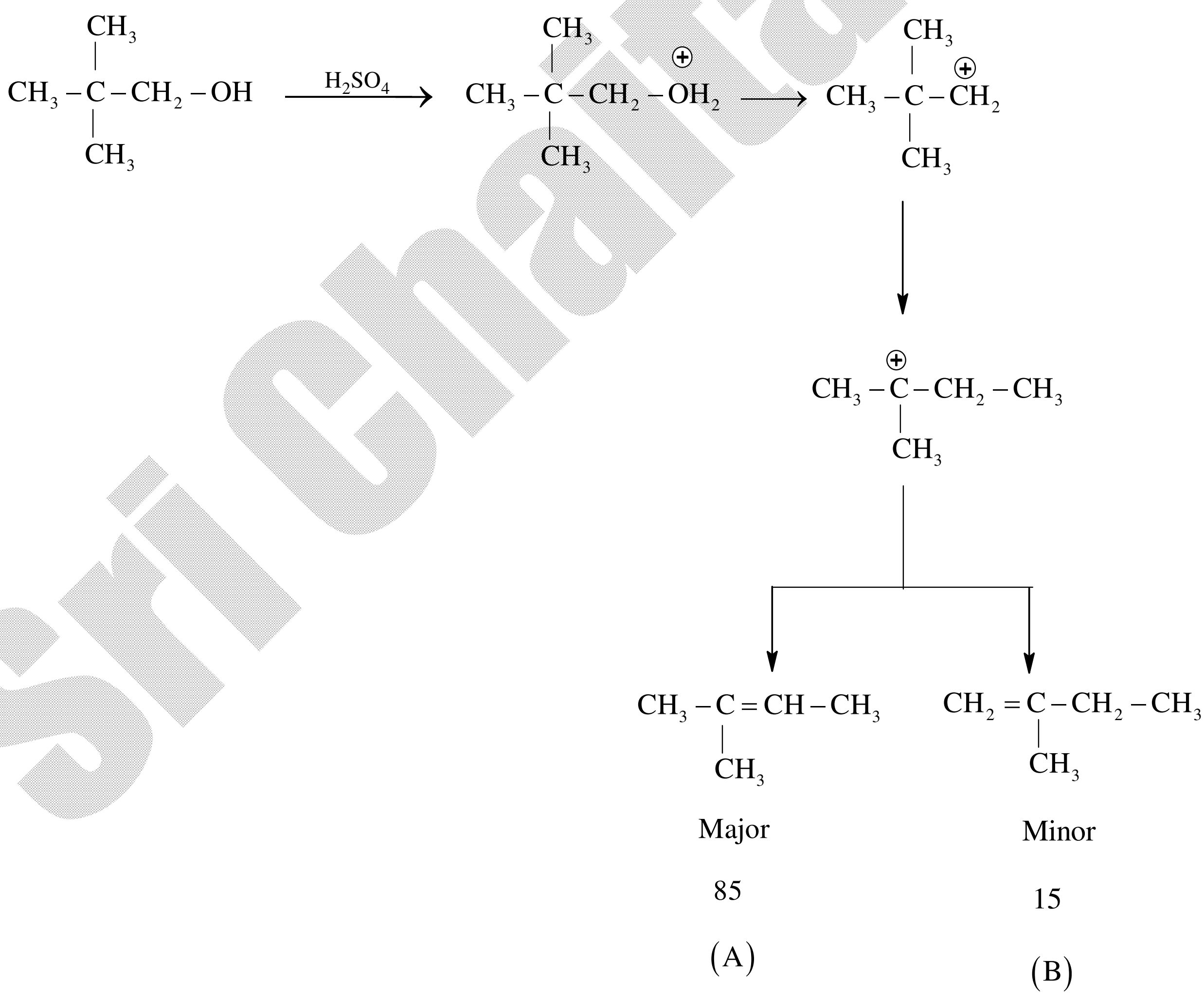
2) 

3) 

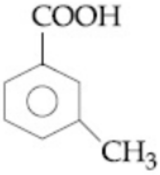
4) 

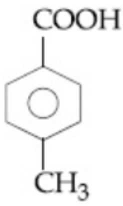
Key: 2

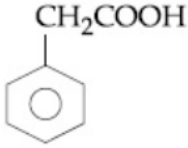
Sol:

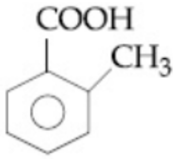


4. [P] on treatment with $Br_2/FeBr_3$ in CCl_4 produced a single isomer $C_8H_7O_2Br$ while heating [P] with soda lime gave toluene. The compound [P] is :

1) 

2) 

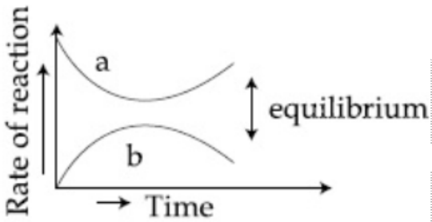
3) 

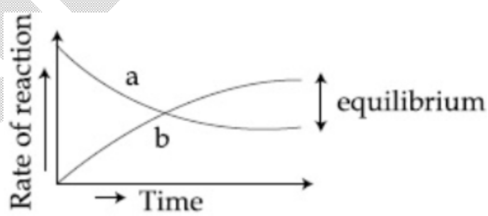
4) 

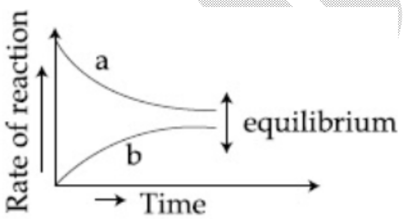
Key: 2

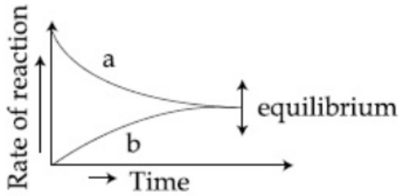
Sol: 1,2 & 4 on heating with soda lime gives toluene. But on reaction with $Br_2/FeBr_3$ in CCl_4
1 gives 3 products of formula $C_8H_7O_2Br$
2gives 1 products of formula $C_8H_7O_2Br$
& 4 gives 2 products of formula $C_8H_7O_2Br$

5. For the equilibrium $A \rightleftharpoons B$, the variation of the rate of the forward (a) and reverse (b) reaction with time is given by :

1) 

2) 

3) 

4) 

Key: 4

Sol: At equilibrium rate of forward reaction = rate of backward reaction only '4' option is satisfying the condition therefore answer is 4.

6. The region in the electromagnetic spectrum where the Balmer series lines appear is :

1) Infrared 2) Microwave 3) Visible 4) Ultraviolet

Key: 3

Sol: for hydrogen atom
Lyman series $\rightarrow$ Vltravoilet range
Balmer series $\rightarrow$ visible range
Paschen series $\rightarrow$ Infrared range

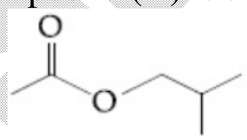
7. For one mole of an ideal gas, which of these statements must be true ?
(a) U and H each depends only on temperature
(b) Compressibility factor z is not equal
(c) $C_{P,m} - C_{V,m} = R$
(d) $dU = C_V dT$ for any process

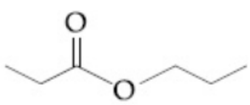
1) (b), (c) and (d) 2) (a), (c) and (d) 3) (a) and (c) 4) (c) and (d)

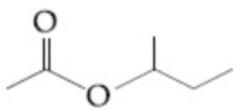
Key: 2

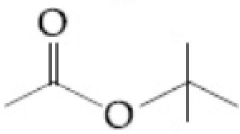
Sol: (1) Ideal gas participating in any proces $dU = nCv\Delta T$ and $\Delta H = nCp\Delta T$ for 1 mol of gas change in U & H depends only on temperature
(2) for an Ideal gas Z=1 (False)
(3) $C_{P,M} - C_{V,M} = R$ is true for an ideal gas
(4) $dU = nC_V dT$
$dU = C_V dT$ is true for 1 mol of Ideal gas participating in any process

8. An organic compound (A) (molecular formula $C_6H_{12}O_2$) was hydrolysed with dil. H_2SO_4 to give a carboxylic acid (B) and an alcohol (C). 'C' gives white turbidity immediately when treated with anhydrous $ZnCl_2$ and conc. HCl. The organic compound (A) is :

1) 

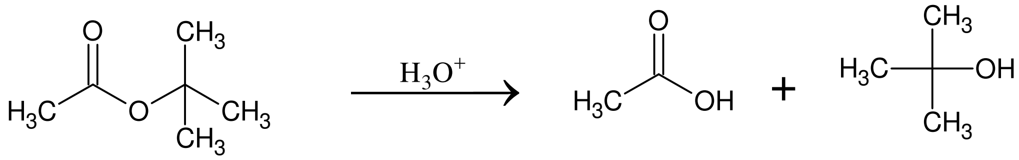
2) 

3) 

4) 

Key: 4

Sol: "C" is an alcohol which gives immediate turbidity with Lucas reagent so it must be $3°$ alcohol



9. The pair in which both the species have the same magnetic moment (spin only) is :

1) $\left[Mn(H_2O)_6\right]^{2+}$ and $\left[Cr(H_2O)\right]^{2+}$ 2) $\left[Cr(H_2O)_6\right]^{2+}$ and $\left[Fe(H_2O)_6\right]^{2+}$

3) $\left[Cr(H_2O)_6\right]^{2+}$ and $[CoCl_4]^{2-}$ 4) $\left[Cr(OH)_4\right]^{2-}$ and $\left[Fe(NH_3)_6\right]^{2+}$

Key: 2

Sol: $\left[Cr(H_2O)_6\right]^{2+}$ and $\left[Fe(H_2O)_6\right]^{2+}$ have same magnetic moment as both Ce^{2+} and Fe^{2+} have same number of unpaired electrons.

10. On combustion of Li, Na and K in excess of air, the major oxides formed, respectively, are :

1) Li_2O_2, Na_2O_2 and K_2O_2 2) Li_2O, Na_2O and K_2O_2

3) Li_2O, Na_2O_2 and K_2O 4) Li_2O, Na_2O_2 and KO_2

Key: 4

Sol: On combustion

$Li \longrightarrow Li_2O$

$Na \longrightarrow Na_2O_2$

$K \longrightarrow KO_2$

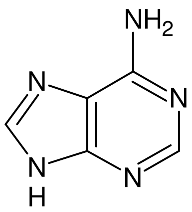
11. Which of the following will react with $CHCl_3 + alc.KOH$?

1) Adenine and lysine 2) Thymine and proline

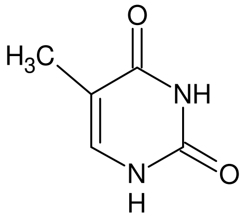
3) Adenine and proline 4) Adenine and thymine

Key: 1

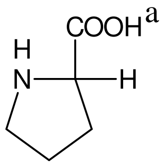
Sol:



Adenine(A)



Thymine(T)



Proline

$H_2N-(CH_2)_4-\underset{\underset{NH_2}{|}}{CH}-COOH$:Lysine

Free $-NH_2$ group is required to give reaction with $CHCl_3+alc.KOH$. So ans is 1

Reaction is carbylamines test.

12. The elements with atomic numbers 101 and 104 belong to, respectively, :

1) Actinoids and Group 4
2) Group 11 and Group 4
3) Group 6 and Actinoids
4) Actinoids and Group 6

Key: 1

Sol: Element with atomic numbe 101 is Nobilium which is actinoid and the atomic number 104 is Rutherfordium which belongs to group 4

13. Among statements (a) – (d), the correct ones are :

(a) Lime stone is decomposed to CaO during the extraction of iron from its oxides.

(b) In the extraction of silver, silver is extracted as an anionic complex.

(c) Nickel is purified by Mond's process.

(d) Zr and Ti are purified by Van Arkel method.

1) (a), (b), (c) and (d)
2) (a), (c) and (d) only
3) (b), (c) and (d) only
4) (c) and (d) only

Key: 1

Sol: (a) $CaCO_3 \longrightarrow CaO + CO_2$ during the extraction of iron in blast furnace reduction

(b) During extraction of silver, leaching of Ag_2S with $NaCN$ gives $Na\left[Ag(CN)_2\right]$ which is anionic complex of silver.

(c) Nickel is purified by Mond's process

(d) Zr and Ti are purified by Van Arkel method

14. On heating, lead (II) nitrate gives a brown gas (A). The gas (A) on cooling changes to a colourless solid/liquid (B). (B) on heating with NO changes to a blue solid (C). The oxidation number of nitrogen in solid (C) is :

1) +3 2) +5 3) +4 4) +2

Key: 1

Sol:

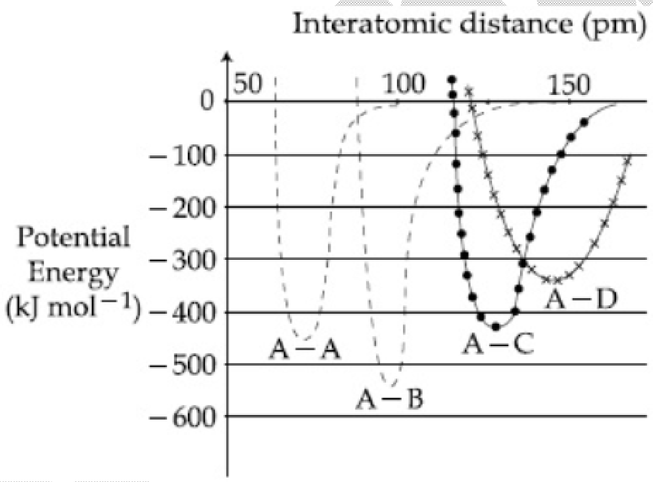
$$Pb(NO_3)_2(s) \xrightarrow{\Delta} PbO(s) + \underset{\substack{\text{Brown gas}\\(A)}}{NO_2(g)} + O_2(g)$$

$$\underset{(A)}{2NO_2(g)} \xrightarrow{\text{Cooling}} \underset{\text{colorless}(B)}{N_2O_4}$$

$$\underset{\text{Color less}(B)}{N_2O_4} + NO \longrightarrow \underset{\text{Blue solid}(C)}{N_2O_3}$$

Oxidation number of nitrogen in solid (C) is +3

15. The intermolecular potential energy for the molecules A, B, C and D given below suggests that :



1) A-D has the shortest bond length

2) D is more electronegative than other atoms

3) A-B has the stiffest bond

4) A-A has the largest bond enthalpy

Key: 3

Sol: From the given graphs, we can understand that A-B bond is strongest and is stiffest bond.

16. Match the following :

(i) Foam (a) smoke

(ii) Gel (b) cell fluid

(iii) Aerosol (c) jellies

(iv) Emulsion (d) rubber

(e) froth

(f) milk

1) $(i)-(d), (ii)-(b), (iii)-(a), (iv)-(e)$ 2) $(i)-(e), (ii)-(c), (iii)-(a), (iv)-(f)$

3) $(i)-(b), (ii)-(c), (iii)-(e), (iv)-(d)$ 4) $(i)-(d), (ii)-(b), (iii)-(e), (iv)-(f)$

Key: 2

Sol: Foam means DP is gas and DM is liquid
Examples: Froath, whipped cream, soap lather $(i)\rightarrow(e)$

Gel means DP is liquid and DM is solid. Examples: Cheese, butter, jellies $(ii)\rightarrow(C)$

Aerosols means DP is solid and DM is gas examples smoke, dust or Aersols menas DP is liquid & DM is gas examples fog, mist, cloud $(iii)\rightarrow(a)$

Emulsion means DP is liquid and DM is liquid
Examples: milk, hair cream

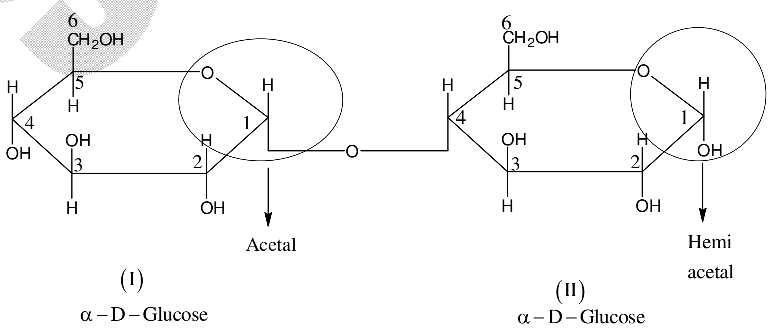
17. What are the functional groups present in the structure of maltose ?

1) Two acetals 2) One ketal and one hemiketal

3) One acetal and one ketal 4) One acetal and one hemiacetal

Key: 4

Sol:



Maltose

18. The ionic radii of O^{2-}, F^{-}, Na^{+} and Mg^{2+} are in the order :

1) $F^{-} > O^{2-} > Na^{+} > Mg^{2+}$ 2) $O^{2-} > F^{-} > Mg^{2+} > Na^{+}$

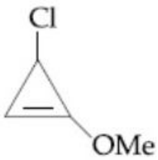
3) $Mg^{2+} > Na^{+} > F^{-} > O^{2-}$ 4) $O^{2-} > F^{-} > Na^{+} > Mg^{2+}$

Key: 4

Sol: Correct
Order of ionic radii is
$O^{2-} > F^{-} > Na^{+} > Mg^{2+}$
As in case of isoelectronic species, with increase of effective nuclear charge, size decreases

19. The decreasing order of reactivity of the following organic molecules towards $AgNO_3$ solution is


(A)


(B)


(C)

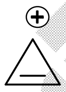

(D)

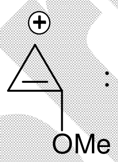
1) $(C) > (D) > (A) > (B)$ 2) $(A) > (B) > (D) > (C)$

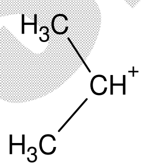
3) $(A) > (B) > (C) > (D)$ 4) $(B) > (A) > (C) > (D)$

Key: 4

Sol: With increase in stability of carbocation involved reactivity with $AgNO_3$ will increase.

(A)  : Aromatic

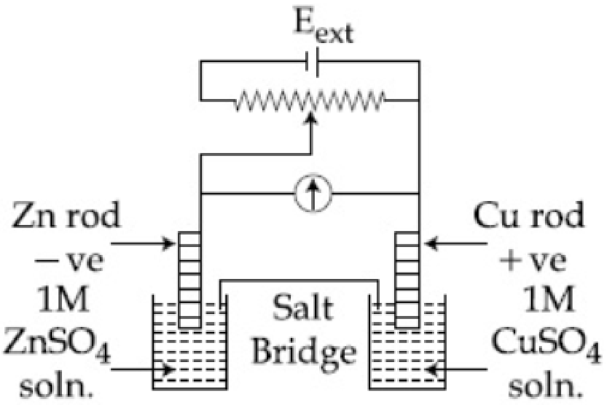
(B)  : Aromatic; further $+R$ if $-OMe$

(C)  : $2°$ with $6 \propto 4$

(D) $CH_3 - \overset{\oplus}{C}H - CH_2 - NO_2$: $2°$ with $5 \propto 4$ & strong $-I$ of $-NO_2$

So reactivity order is $B > A > C > D$

20.



$E^0_{Cu^{2+}|Cu} = +0.34\ V$

$E^0_{Zn^{2+}|Zn} = -0.76\ V$

Identify the incorrect statement from the options below for the above cell :

1) If $E_{ext} = 1.1\ V$, no flow of e^- or current occurs

2) If $E_{ext} < 1.1\ V$, Zn dissolves at anode and Cu deposits at cathode

3) If $E_{ext} > 1.1\ V$, Zn flows from Cu to Zn

4) If $E_{ext} > 1.1\ V$, Zn dissolves at Zn electrode and Cu deposits at Cu electrode

Key: 4

Sol: Since Anode and cathode are in standard conditions $E_{cell} = E^o_{cell} = 1.1V$

1) Since E_{ext} is opposiving the emf of galvanic cell if $E_{ext} = 1.1V$ then no net current will flow (correct)

2) If $E_{ext} < 1.1V$ then zinc will act as cathode and Cu as anode. Zn will dissolve and Cu will deposit (correct)

3) If $E_{ext} > 1.1V$ then oxidation takes place at Cu and reduction takes place at Zn. Electron will move from copper to zinc. (correct)

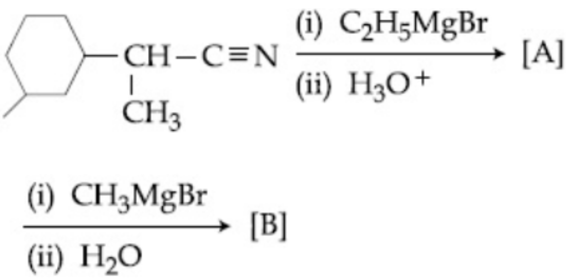
4) Since (2) is correct (4) is wrong

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value (in decimal notation, truncated/ rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

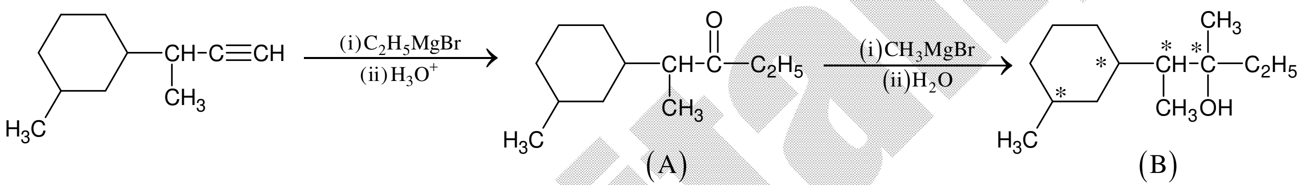
Marking scheme: +4 for correct answer , 0 if not attempted and 0 in all other cases.

21. The number of chiral centres present in [B] is ____________ .



Key: 4

Sol:



22. A 20.0 mL solution containing 0.2 g impure H_2O_2 reacts completely with 0.316 g of $KMnO_4$ in acid solution. The purity of H_2O_2 (in %) is __________ (mol. wt. of $H_2O_2 = 34$; mol. wt. of $KMnO_4 = 158$)

Key: 85

Sol: $e_{H_2O_2} = e_{KMnO_4}$

$n_{H_2O_2}(2) = n_{KMnO_4(s)}$

$n_{H_2O_2} = \left(\frac{5}{2}\right)\left(\frac{0.316}{158}\right)$

$n_{H_2O_2} = \left(\frac{10^{-2}}{2}\right)$

$W_{H_2O_2} = 17\times10^{-2}$

Mass % of $H_2O_2 = \frac{17\times10^{-2}\times100}{0.2} = 85$

23. At 300 K, the vapour pressure of a solution containing 1 mole of n-hexane and 3 moles of n-heptane is 550 mm of Hg. At the same temperature, if one more mole of n-heptane is added to this solution, the vapour pressure of the solution increases by 10 mm of Hg. What is the vapour pressure in mm Hg of n-heptane in its pure state ____________ ?

Key:

Sol: Sol: $550 = P^o_{Hexane} \times \frac{1}{4} + P^o_{Heptane} \times \frac{3}{4}$

$P^o_{Hexane} + 3P^o_{heptane} = 2200 \text{-----}(1)$

$560 = P^o_{Hexane} \times \frac{1}{5} + P^o_{heptane} \times \frac{4}{5}$

$P^o_{Hexane} + P^o_{heptane} = 2800 \text{--------}(2)$

$(2) - (1)$

$P^o_{heptane} = 600 \text{ mm of Hg}$

24. If 75% of a first order reaction was completed in 90 minutes, 60% of the same reaction would be completed in approximately (in minutes) ______________ .

(Take : log 2 = 0.30; log 2.5 = 0.40)

Key: 60

Sol: $t_{3/4} = 90 \text{ minutes}$ $\quad t_{1/2} = \frac{t_{3/4}}{2} = 45 \text{ minutes}$

$\frac{\ell n2}{45} t = \ell n\left(\frac{100}{40}\right)$ $\quad t = 45\frac{\ell n(2.5)}{\ell n 2}$ $\quad t = 45 \times \frac{\log 2.5}{\log 2} = \frac{45 \times 0.4}{0.3} = 60$

25. The mass of ammonia in grams produced when 2.8 kg of dinitrogen quantitatively reacts with 1 kg of dihydrogen is __________ .

Key:

Sol: $n_{N_2} = 100$ $\quad n_{N_2} = \frac{10^3}{2} = 500$

$$N_2 + 3H_2 \longrightarrow 2NH_3$$

t =0 100 500 -

Since N_2 is limiting reagent moles of NH_3 formed is 200.

$W_{NH_3(gm)} = 3400$

MATHEMATICS

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is :

1) 9 2) 7 3) 3 4) 5

Key: 2

Sol: 63 + a + b = 8 $\Rightarrow$ $a+b=17$ ______(1)

$\sigma^2 = 13.5 = \dfrac{25+49+144+100+225+196+a^2+b^2+800}{8}$

$\Rightarrow a^2+b^2=169$ ______(2) from (1) and (2) ab = 60

$(a-b)^2=(a+b)^2-4ab$

$(a-b)^2=49 \Rightarrow (a-b)=7$

2. The integral $\int\left(\dfrac{x}{x\sin x+\cos x}\right)^2 dx$ is equal to

(where C is a constant of integration) :

1) $\tan x-\dfrac{x\sec x}{x\sin x+\cos x}+C$ 2) $\sec x+\dfrac{x\tan x}{x\sin x+\cos x}+C$

3) $\sec x-\dfrac{x\tan x}{x\sin x+\cos x}+C$ 4) $\tan x+\dfrac{x\sec x}{x\sin x+\cos x}+C$

Key: 1

Sol: $\int\dfrac{x}{\cos x}\cdot\dfrac{x\cos x}{(x\sin x+\cos x)^2}dx$

$=\dfrac{x}{\cos x}\cdot\dfrac{-1}{(x\sin x+\cos x)}+\int\dfrac{\cos x+x\sin x}{\cos^2 x}\cdot\dfrac{1}{(x\sin x+\cos x)}dx$

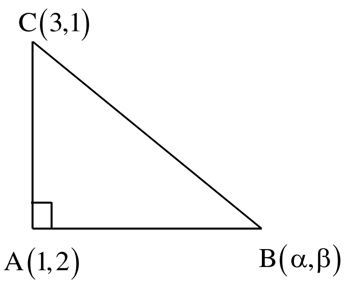
$=\dfrac{-x}{\cos x(x\sin x+\cos x)}+\tan x+c$

3. A triangle ABC lying in the first quadrant has two vertices as A(1, 2) and B(3, 1). If $\angle BAC = 90°$, and ar $(\Delta ABC) = 5\sqrt{5}$ sq. units, then the abscissa of the vertex C is :

1) $1+\sqrt{5}$ 2) $2\sqrt{5}-1$ 3) $2+\sqrt{5}$ 4) $1+2\sqrt{5}$

Key: 4

Sol:



$m_{AC}.m_{AB} = \left(\frac{\beta-2}{\alpha-1}\right)\left(-\frac{1}{2}\right) = -1$ $\Rightarrow \beta = 2\alpha$ ____(1)

$[ABC] = 5\sqrt{5} = \frac{1}{2}AB.AC$ ____(2)

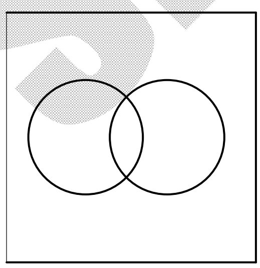
Solve (1) & (2) $|\alpha-1| = 2\sqrt{5} \Rightarrow \alpha = 1 \pm 2\sqrt{5}$

4. A survey shows that 63% of the people in a city read news paper A whereas 76% read news paper B. If x% of the people read both the news papers, then a possible value of x can be :

1) 29 2) 55 3) 65 4) 37

Key: 2

Sol: $n(A) = 63; n(B) = 76; n(A \cap B) = x\%$:



$\Rightarrow x \geq 63+76-100$ $n \geq 39$

Also $n(A \cap B) \leq n(A) = 63$

Ans: 2

5. Let $f(x)=\int \frac{\sqrt{x}}{(1+x)^2}dx\,(x\ge 0)$. Then $f(3)-f(1)$ is equal to :

1) $-\frac{\pi}{12}+\frac{1}{2}+\frac{\sqrt{3}}{4}$ 2) $\frac{\pi}{6}+\frac{1}{2}-\frac{\sqrt{3}}{4}$ 3) $-\frac{\pi}{6}+\frac{1}{2}+\frac{\sqrt{3}}{4}$ 4) $\frac{\pi}{12}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

Key: 4

Sol:

$$\int \frac{\sqrt{x}}{(1+x)^2}dx\ ;\ x\ge 0$$

Sol: $x=\tan^2\theta;\ dx=2\tan\theta\sec\theta d\theta$

$$\int 2\tan^2\theta.\cos^2\theta d\theta=\int 2\sin^2\theta=\int(1-\cos 2\theta)d\theta$$

$$=\theta-\frac{\sin 2\theta}{2}=\tan^{-1}\sqrt{x}-\frac{\sqrt{x}}{1+x}+C$$

6. Let x_0 be the point of local maxima of $f(x)=\vec{a}\cdot(\vec{b}\times\vec{c})$, where $\vec{a}=x\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=-2\hat{i}+x\hat{j}-\hat{k}$ and $\vec{c}=7\hat{i}-2\hat{j}+x\hat{k}$. Then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ at $x=x_0$ is :

1) 14 2) – 4 3) – 22 4) – 30

Key: 3

Sol:

$$f(x)=\begin{vmatrix} x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix}$$

$= x(x^2-2)+2(-2x+7)+3(4-7x)$

$= x^3-2x-4x+14+12-21x=x^3-27x+26$

$\Rightarrow f'(x)=3x^2-27\Rightarrow f'(x)=0\Rightarrow x=\pm 3$

Max.at x = -3

$\bar{a}=-3\hat{i}-2\hat{j}+3\hat{k};\ \bar{b}=-2\hat{i}-3\hat{j}-\hat{k};\bar{c}=7\hat{i}-2\hat{j}-3\hat{k}$

$\bar{a}.\bar{b}=6+6-3=9$

$\bar{b}\cdot\bar{c}=-14+6+3=-5$

$\bar{c}.\bar{a}=-21+4-9=-26$

7. If $\left(a+\sqrt{2}\,b\cos x\right)\left(a-\sqrt{2}\,b\cos y\right)=a^2-b^2$, where $a>b>0$, then $\frac{dx}{dy}$ at $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$ is :

1) $\frac{a-2b}{a+2b}$ 2) $\frac{2a+b}{2a-b}$ 3) $\frac{a-b}{a+b}$ 4) $\frac{a+b}{a-b}$

Key: 4

Sol:

$$\left(a+\sqrt{2}b\cos x\right)\left(+\sqrt{2}b\sin y\right)y'+\left(a-\sqrt{2}b\cos y\right)\left(-\sqrt{2}b\sin x\right)=0$$

$$y'(a+b)(b)+(a-b)(-b)=0\Rightarrow\frac{dy}{dx}=\frac{a-b}{a+b}$$

8. Let $u=\frac{2z+i}{z-ki}$, $z=x+iy$ and $k>0$. If the curve represented by $\mathrm{Re}(u)+\mathrm{Im}(u)=1$ intersects the y-axis at the points P and Q where PQ = 5, then the value of k is :

1) 1/2 2) 3/2 3) 4 4) 2

Key: 4

Sol:

$$u=\frac{2z+i}{2-ki}=\frac{2(x+iy)+i}{x+iy-ki}=\frac{2x+i(2y+1)}{x+i(y-k)}\times\frac{x-i(y-k)}{x-i(y-k)}$$

$$\mathrm{Re}\,u=\frac{2x^2+(2y+1)(y-k)}{x^2+(y-k)^2}$$

$$\mathrm{Ing}\,u=\frac{x(2y+1)-2x(y-k)}{x^2+(y-k)^2}$$

$$\mathrm{Re}\,u+\mathrm{Ing}\,u=1\_\_\_\_\_\_(1)$$

Put on y-axis $\Rightarrow x=0$ put in (1) $\frac{(2y+1)(y-k)}{(y-k)^2}=1$

$$\Rightarrow(y-k)(y+1+k)=0\Rightarrow y=k,-(1+k)$$

P (0, k), Q(0, –1 – k) PQ = 5 $\Rightarrow$ 12K + 11 = 5

$2K=4,-6\Rightarrow$ K = 2, -3; K > 0 $\Rightarrow$ K = 2.

9. Let P(3, 3) be a point on the hyperbola, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If the normal to it at P intersects the x-axis at (9, 0) and e is its eccentricity, then the ordered pair $\left(a^2, e^2\right)$ is equal to :

1) (9, 3) 2) $\left(\frac{3}{2},2\right)$ 3) $\left(\frac{9}{2},2\right)$ 4) $\left(\frac{9}{2},3\right)$

Key: 4

Sol:

P(3, 3) lies on $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \Rightarrow \frac{1}{a^2}-\frac{1}{b^2}=9$ ______(1)

N^2 at $P(3,3)$: $\frac{a^2}{3}x+\frac{b^2}{3}y=a^2+b^2$

(9, 0) lies $\Rightarrow$ $2a^2=b^2$ _______(2)

From (1) & (2) $a^2=\frac{9}{2};b^2=9$

$e^2=1+\frac{b^2}{a^2}=1+2=3$

10. The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to :

1) ${}^{51}C_7-{}^{30}C_7$ 2) ${}^{50}C_6-{}^{30}C_6$ 3) ${}^{50}C_7-{}^{30}C_7$ 4) ${}^{51}C_7+{}^{30}C_7$

Key: 1

Sol:

$\sum_{r=0}^{20} {}^{50-r}C_6 = {}^{50}C_6+{}^{49}C_6+......+{}^{31}C_6+{}^{30}C_7-{}^{30}C_7$

$={}^{51}C_7-{}^{30}C_7$

11. Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b)$ be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t)=\frac{5}{12}+t-t^2$, then a^2+b^2 is equal to :

1) 145 2) 126 3) 116 4) 135

Key: 2

Sol:

$10=\frac{2b^2}{a} \Rightarrow b^2=\sqrt{a}$ $f(t)=\frac{2}{3}-\left(t-\frac{1}{2}\right)^2 \Rightarrow C=\frac{2}{3}$

$b^2=a^2\left(1-c^2\right) \Rightarrow 5a=a^2\left(1-\frac{4}{9}\right)$

$\Rightarrow a=9; b^2=45 \Rightarrow a^2+b^2=81+45=126$

12. If $A=\begin{bmatrix}\cos\theta & i\sin\theta\\ i\sin\theta & \cos\theta\end{bmatrix}$, $\left(\theta=\frac{\pi}{24}\right)$ and $A^5=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, where $i=\sqrt{-1}$, then which one of the following is not true ?

1) $0\le a^2+b^2\le 1$ 2) $a^2-b^2=\frac{1}{2}$ 3) $a^2-d^2=0$ 4) $a^2-c^2=1$

Key: 2

Sol:

$A=\begin{bmatrix}\cos\theta & i\sin\theta\\ i\sin\theta & \cos\theta\end{bmatrix}$

A^5 means rotation 5 times $\Rightarrow A^5=\begin{bmatrix}\cos 5\theta & i\sin 5\theta\\ i\sin 5\theta & \cos 5\theta\end{bmatrix}$

$=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$

Compare $a^2-b^2=\cos^2 5\theta+\sin^2 5\theta=1$

(2) option is not true

13. Let α and β be the roots of $x^2-3x+p=0$ and γ and δ be the roots of $x^2-6x+q=0$. If $\alpha,\beta,\gamma,\delta$ form a geometric progression. Then ratio $(2q+p):(2q-p)$ is :

1) 3 : 1 2) 33 : 31 3) 5 : 3 4) 9 : 7

Key: 4

Sol: $x^2-3x+p=0\left\langle\begin{matrix}\alpha\\ \beta=\alpha r\end{matrix}\right.$; $x^2-6x+q=0\left\langle\begin{matrix}\gamma=\alpha r^2\\ \delta=\alpha r^3\end{matrix}\right.$

$\alpha+\alpha r=3$

$\alpha(\alpha+\alpha r)=6\Rightarrow r^2=2\Rightarrow r=\pm\sqrt{2}$

$\frac{2q+p}{2q-p}=\frac{2\alpha^2r^5+\alpha^2r}{2\alpha^2r^5-\alpha^2r}=\frac{2r^4+1}{2r^7-1}=\frac{9}{7}$

14. Let f be a twice differentiable function on $(1,6)$. If $f(2)=8,\ f'(2)=5,\ f'(x)\ge 1$ and $f''(x)\ge 4$, for all $x\in(1,6)$, then :

1) $f(5)\le 10$ 2) $f'(5)+f''(5)\le 20$

3) $f(5)+f'(5)\le 26$ 4) $f(5)+f'(5)\ge 28$

Key: 4

Sol:

$f'(x) \geq 1 \Rightarrow \int_2^5 f'(x)dx \geq \int_2^5 dx$

$f(5) - f(2) \geq 3 f(5) \geq 11$

Also $f''(x) \geq 4 \Rightarrow \int_2^5 f''(x)dx \geq \int_2^5 xdx$

$f'(5) - f'(2) \geq 12$

$f'(5) \geq 17$ ____(2)

(1)+ (2) $\Rightarrow f(5) + f'(5) \geq 28$

15. If $1+(1-2^2\cdot 1)+(1-4^2\cdot 3)+(1-6^2\cdot 5)+......+(1-20^2\cdot 19)=\alpha-220\beta$, then an ordered pair (α, β) is equal to :

1) (11, 97) 2) (10, 103) 3) (10, 97) 4) (11, 103)

Key: 4

Sol:

$$S = 1 + \sum_{r=1}^{10}\left(1-(2r)^2(2r-1)\right) = 1 + 10 - \sum_{r=1}^{10}\left(8r^3 - 4r^2\right)$$

$$= 11 - \left[8\left(\frac{10\times 11}{2}\right)^2 - 4\left(\frac{10\times 11\times 21}{6}\right)\right] = 11 - 220(103) \Rightarrow \alpha = 11, \beta = 103.$$

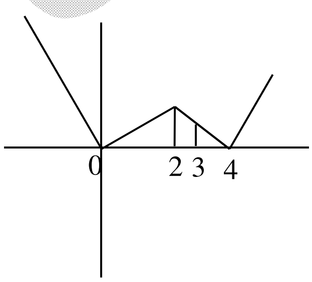
16. Let $f(x) = |x-2|$ and $g(x) = f(f(x))$, $x \in [0, 4]$. Then $\int_0^3 (g(x) - f(x))dx$ is equal to :

1) 0 2) $\frac{1}{2}$ 3) 1 4) $\frac{3}{2}$

Key: 3

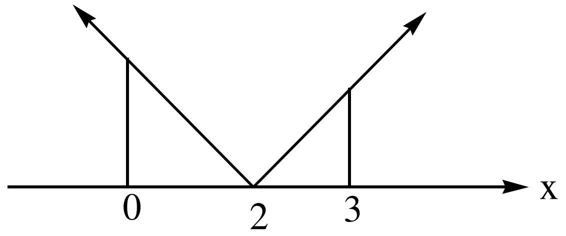
Sol:

$\int_0^3 f(f(x))dx - \int_0^3 f(x)dx$ $\qquad f(f(x)) = \left||x-2|-2\right|$



$A_1 = \frac{1}{2}\times 2\times 2+\frac{1}{2}[3]$ $= 2+\frac{3}{2}=\frac{7}{2}$

Reg Area = $\frac{7}{2}-\frac{5}{2}=1$



$A_2 = \frac{1}{2}\times 2\times 2+\frac{1}{2}\times 1\times 1$

$= 2+\frac{1}{2}=\frac{5}{2}$

17. Let [t] denote the greatest integer $\le t$. Then the equation in x, $[x]^2+2[x+2]-7=0$ has :

1) infinitely many solutions
2) exactly four integral solutions
3) exactly two solutions
4) no integral solution

Key: 1

Sol:

$[x]^2+2[x]+4-7=0$

$[x]^2+2[x]-3=0 \Rightarrow ([x]+3)([x]-1)=0$

$-3\le [x]\le 1$

18. Let $y = y(x)$ be the solution of the differential equation,

$xy'-y = x^2(x\cos x+\sin x)$, $x>0$. If $y(\pi)=\pi$, then $y''\left(\frac{\pi}{2}\right)+y\left(\frac{\pi}{2}\right)$ is equal to :

1) $1+\frac{\pi}{2}+\frac{\pi^2}{4}$
2) $1+\frac{\pi}{2}$
3) $2+\frac{\pi}{2}$
4) $2+\frac{\pi}{2}+\frac{\pi^2}{4}$

Key: 3

Sol:

$$y' - \frac{1}{x}y = x(x\cos + \sin x)$$

I.F = $\frac{1}{x}$

Sol: $$y.\frac{1}{x} = \int \frac{1}{x}x(x\cos x + \sin x)dx + C$$

$$\frac{y}{x} = x\sin x + C \Rightarrow C = 1$$

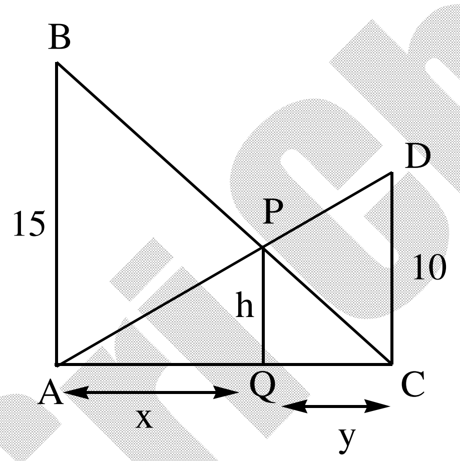
$$y = x^2 \sin x + x$$

19. Two vertical poles AB = 15 m and CD = 10 m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is :

1) 6 2) 10/3 3) 5 4) 20/3

Key: 1

Sol:



$\Delta CPQ \sim \Delta CBA$

$$\frac{y}{x+y} = \frac{h}{15} \text{\_\_\_\_(1)}$$

$\Delta APQ \sim \Delta ADC$

$$\frac{x}{x+y} = \frac{h}{10} \text{\_\_\_\_(2)}$$

(1) + (2) $1 = \frac{h}{15} + \frac{h}{10}$

$$h\left(\frac{2+3}{30}\right) = 1 \Rightarrow h = 6$$

20. Given the following two statements :

$(S_1):(q \vee p) \rightarrow (p \leftrightarrow \sim q)$ is a tautology.

$(S_2): \sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy. Then :

1) only (S_2) is correct
2) both (S_1) and (S_2) are not correct
3) only (S_1) is correct
4) both (S_1) and (S_2) are correct

Key: 2

Sol:

p	**q**	**~p**	**~q**	**(I)** **q v q**	**(II)** **p ⟷ ~q**	**(I) →(II)**
T	T	F	F	T	R	F
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	F	F	T

s_1 is not tautology

Similarly s_2 is not fallacy

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value (in decimal notation, truncated/ rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. The probability of a man hitting a target is $\frac{1}{10}$. The least number of shots required, so that the probability of his hitting the target at least once is greater than $\frac{1}{4}$, is ________ .

Key: 3

Sol:

$$p = \frac{1}{10}; q = \frac{9}{10}$$

$$p(x \geq 1) = 1 - p(x = 1) = 1 - \left(\frac{9}{10}\right)^n \geq \frac{1}{4}$$

$$\Rightarrow \left(\frac{9}{10}\right)^n \leq \frac{3}{4}$$ Now verity to get n = 3

22. Suppose a differntiable function $f(x)$ satisfies the identity

$f(x+y)=f(x)+f(y)+xy^2+x^2y$, for all real x and y. If $\lim_{x\to 0}\frac{f(x)}{x}=1$, then $f'(3)$ is equal to ___________ .

Key: 10

Sol: $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{f(x)+f(h)+xh^2+x^2h-f(x)}{h}$

$f'(x)=\lim_{h\to 0}\frac{f(h)}{h}+x^2=1+x^2$

$f'(3)=3^2+1=10$

23. If the system of equations

$x-2y+3z=9$

$2x+y+z=b$

$x-7y+az=24$, has infinitely many solutions, then $a-b$ is equal to _________ .

Key: 5

Sol:

$$\begin{vmatrix}1 & -2 & 3 & 9\\ 2 & 1 & 1 & b\\ 1 & -7 & a & 24\end{vmatrix}$$

$R_3\to R_3-R_1; R_2\to R_2-2R_1$

$$\begin{vmatrix}1 & -2 & 3 & 9\\ 0 & 5 & -5 & b-18\\ 0 & -5 & a-3 & 15\end{vmatrix} R_3\to R_3+R_2$$

$$\begin{vmatrix}1 & -2 & 3 & 9\\ 0 & 5 & -5 & b-18\\ 0 & 0 & a-8 & b-3\end{vmatrix} \text{for do sol}$$

$a-8=0$ and $b-3=0$

$a=8$, $b=3$

$a-b=5$

24. If the equation of a plane P, passing through the intersection of the planes, $x+4y-z+7=0$ and $3x+y+5z=8$ is $ax+by+6z=15$ for some $a, b \in R$, then the distance of the point $(3, 2, -1)$ from the plane P is ___________ .

Key: 3

Sol:

Req. plane : $x+4y-z+7+\lambda(3x+y+5z-8)=0$

$(1+3\lambda)x+(4+\lambda)y+(-1+5\lambda)z+7-8\lambda=0$

$\frac{-1+5\lambda}{6}=\frac{8\lambda-7}{15} \Rightarrow -5+25\lambda=16\lambda-14$

$9\lambda=-9 \Rightarrow \lambda=-1$

$-2x+3y-6z+15=0 \Rightarrow 2x-3y+6z-15=0$

$d=\frac{|6-6-6-15|}{\sqrt{4+9+36}}=\frac{21}{7}=3$

25. Let $\left(2x^2+3x+4\right)^{10}=\sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{13}}$ is equal to ___________ .

Key: 8

Sol:

$a_7=\frac{10!}{3!1!6!}2^3.3.4^6+\frac{10!}{2!3!5!}.2^2.3^3.4^5+\frac{10!2.3^5.4^4}{1!5!4!}+\frac{10!3^4.4^3}{7!3!}$

$a_{13}=\frac{10!2^6.3.4^3}{6!1!3!}+\frac{10!.2^5.3^3.4^2}{5!3!2!}+\frac{10!2^4.3^5.4}{4!5!1!}+\frac{10!2^3.3^7.4^0}{3!7!}$

$\frac{a_7}{a_{13}}=2^3=8$

Prepared by

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