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Question Paper, Key and Solutions

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Physics**(SINGLE CORRECT KEYWER TYPE)**

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its Keywer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct Keywer, 0 if not attempted and -1 in all other cases.

1. A hollow spherical shell at outer radius R floats just submerged under the water surface.

The inner radius of the shell is r . If the specific gravity of the shell material is $\frac{27}{8}$ w.r.t water, the value of r is:

- 1) $\frac{2}{3}R$ 2) $\frac{8}{9}R$ 3) $\frac{4}{9}R$ 4) $\frac{1}{3}R$

Key: No Keywer

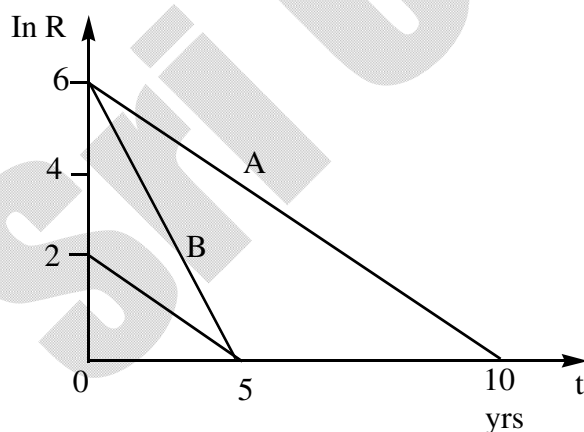
$$\text{Sol: } \frac{4}{3}\pi R^3 \rho_w g = \frac{4}{3}\pi (R^3 - r^3) \frac{27}{8} \rho_w g$$

$$R^3 = (R^3 - r^3) \left(\frac{27}{8}\right)$$

$$8R^3 = 27R^3 - 27r^3$$

$$r = \frac{19^{\frac{1}{3}}}{3} R$$

2. Activities of three radioactive substances A, B and C are represented by the curves A, B and C in the figure. Then their half lives $T_{\frac{1}{2}}(A):T_{\frac{1}{2}}(B):T_{\frac{1}{2}}(C)$ are in the ratio :



- 1) 2 : 1 : 3 2) 4 : 3 : 1 3) 3 : 2 : 1 4) 2 : 1 : 1

Key: 1

Sol:

$$\lambda_A = 6/10 \text{ (from slope)}$$

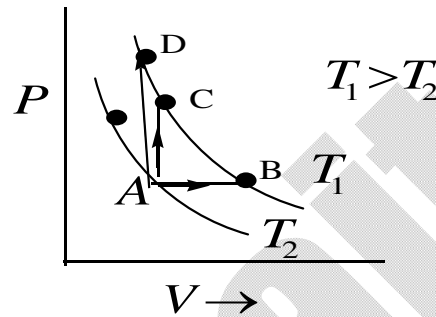
$$\lambda_B = 6/5$$

$$\lambda_C = 2/5$$

$$T_A : T_B : T_C = \frac{10}{6} : \frac{5}{6} : \frac{5}{2}$$

$$\Rightarrow 20 : 10 : 30$$

3. Three different processes that can occur in an ideal monoatomic gas are shown in the P - V diagram. The paths are labeled as $A \rightarrow B$, $A \rightarrow C$ and $A \rightarrow D$. The change in internal energies during these processes are taken as E_{AB} , E_{AC} and E_{AD} and the work done as W_{AB} , W_{AC} and W_{AD} . The correct relation between these parameters are:



- 1) $E_{AB} = E_{AC} = E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} > 0$
- 2) $E_{AB} > E_{AC} > E_{AD}$, $W_{AB} < W_{AC} < W_{AD}$
- 3) $E_{AB} < E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} > W_{AD}$
- 4) $E_{AB} = E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} < 0$

Key: 1

Sol: Clearly $E_A = E_B = E_C$ as change in temperature is same

$$W_{AC} = 0$$

$$W_{AB} > 0 \because V \text{ Increases}$$

$$W_{AD} < 0 \because V \text{ Decreases}$$

4. A bullet of mass 5g, Travelling with a speed of 210 m/s, strikes a fixed wooden target. One half its kinetic energy is converted into heat in the bullet while the other half is converted into heat in the wood. The rise of temperature of the bullet if the specific heat of the material is $0.030 \text{ cal/(g}^\circ\text{C)}$ ($1 \text{ cal} = 4.2 \times 10^7 \text{ ergs}$) close to:

- 1) 87.5°C
- 2) 119.2°C
- 3) 83.3°C
- 4) 38.4°C

Key: 1

Sol: $\frac{1}{2} \left(\frac{1}{2} mv^2 \right) = ms\Delta T$
 $(\Delta T = 87.5^\circ \text{C})$

5. Two capacitors of capacitance C and $2C$ are charged to Potential differences V and $2V$, Respectively. These are then connected in Parallel in such a manner that the Positive terminal of one is connected to the negative terminal of one is connected to the negative terminal of the other. The final energy of this configuration is :

- 1) $\frac{25}{6} CV^2$ 2) $\frac{9}{2} CV^2$ 3) Zero 4) $\frac{3}{2} CV^2$

Key: 4

Sol: $Q_{\text{final}} = 4CV - CV$

$$\frac{1}{2} \frac{(Q_{\text{final}})^2}{C_{\text{net}}} = U_{\text{final}}$$

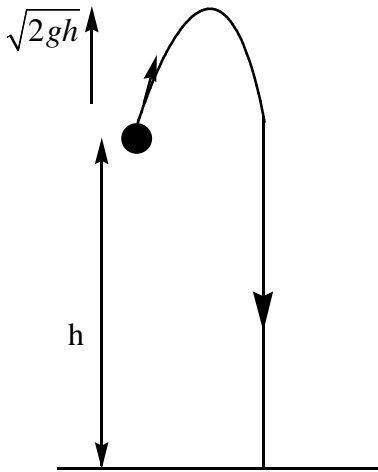
$$\frac{1}{2} \frac{(3CV)^2}{3C} = U_{\text{final}}$$

$$\frac{3}{2} CV^2 = U_{\text{final}}$$

6. The helicopter rises from rest on the ground vertically upwards with a constant acceleration g . A food packet is dropped from the helicopter when it is at a height h . time taken by the packet to reach the ground is closed to [g is the acceleration due to gravity]

- 1) $t = 3.4 \sqrt{\left(\frac{h}{g}\right)}$ 2) $t = \frac{2}{3} \sqrt{\left(\frac{h}{g}\right)}$ 3) $t = 1.8 \sqrt{\frac{h}{g}}$ 4) $t = \sqrt{\frac{2h}{3g}}$

Key : 1



$$-h = \sqrt{2gh} t - \frac{1}{2}gt^2$$

Sol:

$$\text{On Solving } t = (1 + \sqrt{2}) \sqrt{\frac{h}{g}}$$

7. In a resonance tube experiment when the tube is filled with water up to a height of 17.0 cm from bottom, it resonates with a given tuning fork. When the level is raised the next resonance with the same tuning fork occurs at a height of 24.5 cm. If the velocity of sound in air is 330 m/s, the tuning fork frequency is :

- 1) 1100Hz 2) 550Hz 3) 2200Hz 4) 3300 Hz

Key: 3

$$\begin{aligned} \text{Sol : } L_2 - L_1 &= \frac{\lambda}{2} \\ \lambda &= 15\text{cm} \\ f &= \frac{330}{15(10^{-2})} = 2200 \end{aligned}$$

8. Assume that the displacement (s) of air is proportional to the pressure difference (Δp) created by a sound wave. Displacement (s) further depends on the speed of sound (v), density of air (ρ) and the frequency (f). If $\Delta p \sim 10\text{Pa}$, $v \sim 30\text{m/s}$, $\rho \sim 1\text{kg/m}^3$ and $f \sim 1000\text{Hz}$ then s will be of the order of (take multiplicative constant to be 1)

- 1) 1mm 2) $\frac{1}{10}$ mm 3) 10mm 4) $\frac{3}{100}$ mm

Key: No Option

Sol: $\Delta p = BK S_0$, $B = \text{Bulk modulus} = \rho V^2$

$$\Delta p = \frac{\rho V^2 (1) f S_0}{V}$$

$$\frac{\Delta p}{\rho V f} = S_0,$$

$$S_0 = \frac{1}{30} \text{ mm}$$

9. An electrical power line, having a total resistance of 2Ω , delivers 1 kW at 220 V . The efficiency of the transmission line is approximately.

- 1) 72% 2) 85% 3) 96% 4) 91%

Key: 3

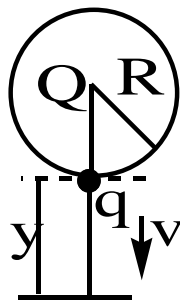
Sol: $I = \frac{1000}{220} = \text{power upon voltage}$

$$\text{loss} = \left(\frac{100}{22} \right)^2 (2)$$

$$= \frac{10000}{242}$$

$$n = \left(\frac{10000}{10000 + \frac{10000}{242}} \right) \times 100\% = 96\%$$

10. A Solid sphere of radius R carries a charge $Q + q$ distributed uniformly over its volume. A very small point like piece of it of mass m gets detached from the bottom of the sphere and falls down vertically under gravity. This piece carries charge q . if it acquires a speed v when it has fallen through a vertical height y (see I figure) then : (assume the remaining portion to be spherical)



$$1) v^2 = y \left[\frac{qQ}{4\pi \epsilon_0 R(R+y)m} + g \right]$$

$$2) v^2 = y \left[\frac{qQ}{4\pi \epsilon_0 R^2 y m} + g \right]$$

$$3) v^2 = 2y \left[\frac{qQ}{4\pi \epsilon_0 R(R+y)m} + g \right]$$

$$4) v^2 = 2y \left[\frac{QqR}{4\pi \epsilon_0 (R+y)^3 m} + g \right]$$

Key: 3

Sol: $\frac{1}{2}mv^2 = mgy + kQq \left[\frac{1}{R} - \frac{1}{(R+y)} \right]$, where $K = 1/4\pi\epsilon_0$

$$V^2 = 2y \left[g + \frac{Qq}{4\pi \epsilon_0 R(R+y)m} \right]$$

11. With increasing biasing voltage of a photodiode, the photocurrent magnitude:

- 1) Increases initially and saturates finally
- 2) increases initially and after attaining certain value, it decreases
- 3) remains constant
- 4) increases linearly

Key: 3

Sol: After certain potential electron Hole pair Production Will be nearly constant

12. A wheel is rotating freely with an angular speed ω on a shaft. The moment of inertia of the wheel is I and the moment of inertia of the shaft is negligible. Another wheel of moment of inertia $3I$ initially at rest is suddenly coupled to the same shaft. The resultant fractional loss in the kinetic energy of the system is:

- 1) 0
- 2) $\frac{1}{4}$
- 3) $\frac{5}{6}$
- 4) $\frac{3}{4}$

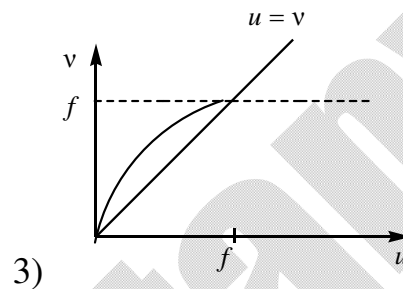
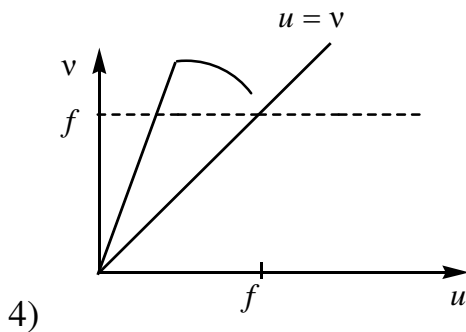
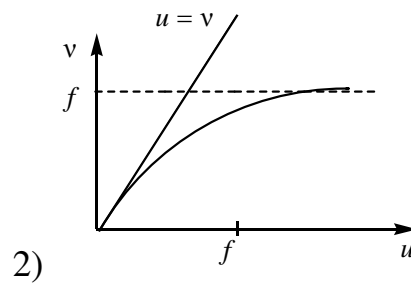
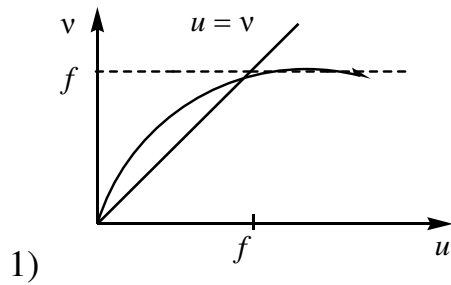
Key: 4

Sol: $I\omega = 4I\omega^1$

$$\frac{\omega}{4} = \omega^1$$

$$Loss = \frac{1}{2}I\omega^2 - \frac{1}{2}4I\left(\frac{\omega}{4}\right)^2$$

13. For a concave lens of focal length f , the relation between object and image distances u and v , respectively, from its pole can best be represented by ($u = v$ is the reference line):



Key: 2

Sol: as u tends towards infinity v tends towards focus and all other distance will be less than f and it will never intersect $u = v$ line

14. Number of molecules in a volume of 4cm^3 of a perfect mono atomic gas at some temperature T and at a pressure of 2cm of mercury is close to? (Given, mean kinetic energy of a molecule (at T) is $4 \times 10^{-14}\text{ erg}$, $g = 980\text{ cm/s}^2$, density of mercury = 13.6 g/cm^3)

- 1) 5.8×10^{16} 2) 4.0×10^{16} 3) 4.0×10^{18} 4) 5.8×10^{18}

Key: 3

Sol: $n = \frac{PV}{KT}$ { $K = \text{Boltzman Constant}$ }

$$n = \frac{2(13.6)(980)(4)}{KT}$$

$$n = \frac{2(13.6)(980)(4)}{\frac{2}{3}(KT)}, \frac{3}{2}KT = KE$$

$$= \frac{3 \times 13.6 \times (980)(4)}{4 \times 10^{-14}}$$

15. A square loop of side $2a$, and carrying current i , is kept in XZ plane with its centre at origin a long wire carrying the same current i is placed parallel to the z-axis and passing through the point $(0, b, 0)$, ($b \gg a$) the magnitude of the torque on the loop about Z-axis is given by :

1) $\frac{2\mu_0 I^2 a^2}{\pi b}$ 2) $\frac{\mu_0 I^2 a^3}{2\pi b^2}$ 3) $\frac{2\mu_0 I^2 a^3}{\pi b^2}$ 4) $\frac{\mu_0 I^2 a^2}{2\pi b}$

Key: 1

Sol: Torque = $\vec{M} \times \vec{B}$

$$\vec{M} = I(2a)^2 \text{ and } \vec{B} = \frac{\mu_0 I}{2\pi b}$$

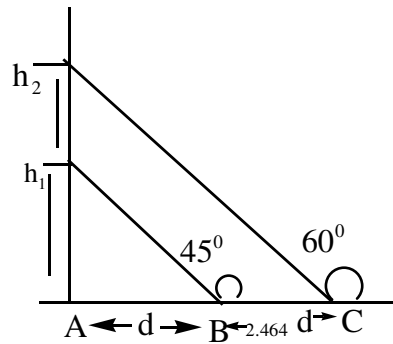
16. A galvanometer of resistance G is converted into a voltmeter of range $0-1V$ by connecting a resistance R_1 in series with it. The additional resistance that should be connected in series with R_1 to increase the range of the voltmeter to $0-2V$ will be:

1) G 2) R_1 3) $R_1 - G$ 4) $R_1 + G$

Key: 4

Sol: $I(R_1 + G) = 1$
 $I(R_1 + x + G) = 2$
 $x = R_1 + G$

17. A balloon is moving up in air vertically above a point on the ground when it is at height h_1 a girl standing at a distance d (point B) from a (see figure) sees it at an angle 45° with respect to the vertical. when the balloon climbs up a further height h_2 , it is seen at an angle 60° with respect to the vertical if the girl moves further by a distance $2.464d$ (Point C). Then the height h_2 is (Given $\tan 30^\circ = 0.5774$):



- 1) 0.464 d 2) d 3) 0.732 d 4) 1.464 d

Key: 2

Sol: $h_1 = d$

$$\tan 30^\circ = \frac{h_2 + d}{3.464d}$$

$$h_2 = d$$

18. An Electron is constrained to move along the y-axis with a speed of $0.1c$ (c is the speed of light) in the presence of electronic wave, whose electric field

$\vec{E} = 30 \hat{j} \sin(1.5 \times 10^7 t - 5 \times 10^{-2} x) \text{ V/m}$. The maximum magnetic force experienced by the electron will be :

(Given $C = 3 \times 10^8 \text{ ms}^{-1}$ and electron charge $= 1.6 \times 10^{-19} \text{ C}$)

- 1) $4.8 \times 10^{-19} \text{ N}$ 2) $3.2 \times 10^{-18} \text{ N}$ 3) $2.4 \times 10^{-18} \text{ N}$ 4) $1.6 \times 10^{-19} \text{ N}$

Key: 1

Sol: $B_{\max} = \frac{E_{\max}}{C}$

$$F_{\max} = 1.6 \times 10^{-19} (0.1 \times 3 \times 10^8) \times 10^{-7} \text{ N}$$

19. A physical quantity z depends on four observables a, b, c and d , as $z = \frac{a^2 b^3}{\sqrt{cd^3}}$. The

percentages of error in the measurement of a, b, c and d are 2%, 1.5%, 4% and 2.5% respectively. The percentage of error in z is:

- 1) 14.5% 2) 16.5% 3) 13.5% 4) 12.25%

Key: 1

Sol:

$$\begin{aligned} \% \text{error in } Z &= 2(\% \text{error in } a) + \left(\frac{2}{3}\right)(\% \text{error in } b) + \frac{1}{2}(\% \text{error in } c) + 3(\% \text{error in } d) \\ &= 2 \times 2 + \frac{2}{3}(1.5) + \frac{1}{2} \times 4 + 3(2.5) = 14.5\% \end{aligned}$$

20. The value of the acceleration due to gravity is g_1 at a height $h = \frac{R}{2}$ (R = radius of the earth) from the surface of the earth. It is again equal to g_1 at a depth d below the surface of the earth. The ratio $\left(\frac{d}{R}\right)$ equals

- 1) $\frac{4}{9}$ 2) $\frac{5}{9}$ 3) $\frac{1}{3}$ 4) $\frac{7}{9}$

Key: 2

$$\text{Sol: } \frac{GM}{(R+H)^2} = \frac{GM}{R^3}(R-d)$$

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value (in decimal notation, truncated/ rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct Keywer , 0 if not attempted and 0 in all other cases.

21. A compound microscope consists of an objective lens of focal length 1 cm and an eye piece of focal length 5 cm with a separation of 10 cm . The distance between an object and the objective lens, at which the strain on the eye is minimum is $\frac{n}{40}$. The value of n is _____

Key: 50

Sol: Strain is minimum this implies intermediate image is at focal length of eye piece that means image distance from objective is 5 cm so

$$\frac{1}{5} + \frac{1}{x} = \frac{1}{10}$$

$$\frac{1}{x} = \frac{4}{5}$$

$$x = 50$$

22. A force $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})\text{ N}$ acts at a point $(4\hat{i} + 3\hat{j} - \hat{k})\text{ m}$. Then the magnitude of torque about the point $(\hat{i} + 2\hat{j} + \hat{k})\text{ m}$ will be $\sqrt{x}\text{ N-m}$. The value of x is _____

Key: 195

$$\text{Sol: } \vec{r} = (4-1)\hat{i} + (3-2)\hat{j} - 2\hat{k}$$

$$\vec{F} = \hat{i} + 2\hat{j} + 3\hat{k} \quad \vec{\tau} = \vec{r} \times \vec{F}$$

23. A particle of mass $200 MeV/c^2$ collides with a hydrogen atom at rest. Soon after the collision the particle comes to rest, and the atom recoils and goes to its first excited state. The initial kinetic energy of the particle (in eV) is $\frac{N}{4}$. The value of N is: (Given the mass of the hydrogen atom to be $1 GeV/c^2$) _____

Key: 51

Sol: Mass of hydrogen atom is 5 times mass of colliding particle

$$\text{Loss} = \frac{1}{2}mv^2 - \frac{1}{2}(5m)\left(\frac{v}{5}\right)^2$$

$$\text{Loss excitation energy} \quad 10.2 \text{ eV} = \frac{1}{2}mv^2 \left(\frac{4}{5}\right) \quad \frac{51 \text{ eV}}{4} = k$$

24. A beam of electrons of energy E scatters from a target having atomic spacing of 1 \AA . The first maximum intensity occurs at $\theta = 60^\circ$. Then E (in eV) is _____. (Planck constant $h = 6.64 \times 10^{-34} \text{ Js}$, $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$, electron mass $m = 9.1 \times 10^{-31} \text{ kg}$)

Key: 50

$$\text{Sol: } 2d \times \sin 60^\circ = \lambda, d\sqrt{3} = \lambda$$

$$\lambda = \sqrt{3} \text{ \AA}$$

$$\lambda = \sqrt{\frac{150}{V}}$$

$$\text{Energy} = 50 \text{ eV}$$

25. Two concentric circular coils, C_1 and C_2 are placed in the XY plane. C_1 has 200 turns, and a radius of 1 cm . C_2 has 200 turns and radius of 20 cm . C_2 carries a time dependent current $I(t) = (5t^2 - 2t + 3) \text{ A}$ where t is in s . The emf induced in C_1 (in mV), at the instant $t = 1 \text{ s}$ is $\frac{4}{x}$. The value of x is _____

Key: 5.06

$$\text{Sol: } \phi = (200) \left(\frac{\mu_0 I}{2R_2} \right) \pi R_1^2$$

$$500 \frac{d\phi}{dt} = \frac{200 \times 500 \times 4\pi \times 10^{-7} \times \pi \times 10^{-4}}{2 \times 20 \times 10^{-2}} \times \frac{dI}{dt}$$

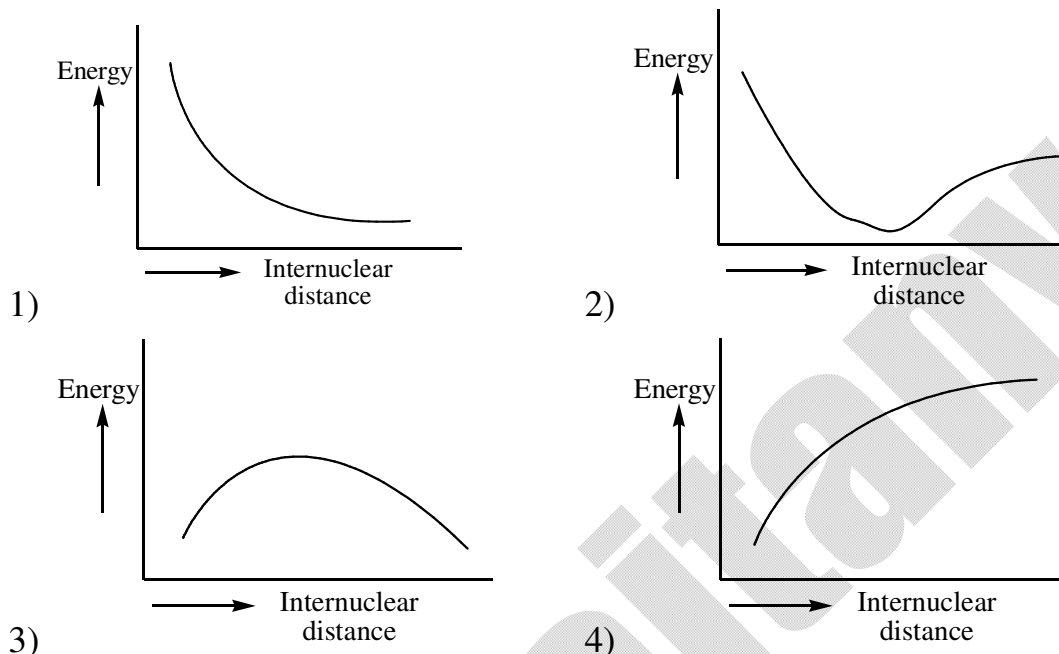
$$\frac{4}{x} = \frac{200 \times 500 \times \pi^2 \times 10^{-7} \times 10^{-4}}{10^{-1}} (10t - 2)$$

CHEMISTRY**(SINGLE CORRECT KEYWER TYPE)**

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its Keywer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct Keywer, 0 if not attempted and -1 in all other cases.

1. The potential energy curve for the H_2 molecule as a function of internuclear distance is



Key: 2

Sol. As the internuclear distance decreases the energy of the molecule is also decreased upto and length. and then repulsion forces dominates then the energy of the system is increased.

2. The vlaues of the crystal field stabilization energies for a high spin d^6 metal ion in octahedral and tetrahedral fields, respectively, are.

- 1) $-0.4\Delta_0$ and $-0.6\Delta_t$ 2) $-0.4\Delta_0$ and $-0.27\Delta_t$
 3) $-1.6\Delta_0$ and $-0.4\Delta_t$ 4) $-2.4\Delta_0$ and $-0.6\Delta_t$

Key: 1

Sol. d^6 configuration in octahedral complex – high spin.

Configuration is $t_{2g}^4 e_g^2$

C.F.S.E = $-0.4\Delta_0 t_{2g}$ electrons + $0.6\Delta_0 e_g$ electrons

$$= -0.4\Delta_0 \times 4 + 0.6 \times \Delta_0 \times 2$$

$$= -1.6\Delta_0 + 1.2\Delta_0$$

$$= 0.4\Delta_0$$

In tetrahedral complex.

Configuration is $e_g^3 t_{2g}^3$

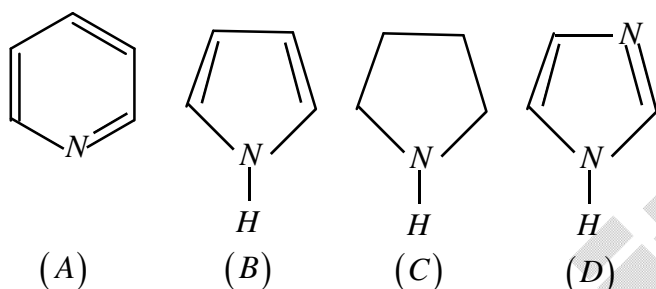
$$C.F.S.E = -0.6\Delta_f \times 3 + 0.4 \times \Delta_f \times 3$$

$$= 0.6 \times \Delta_f \times 3 + 0.4 \times \Delta_f \times 3$$

$$= -1.8 \times \Delta_f + 1.2\Delta_f$$

$$= -0.6\Delta_f$$

3. The increasing order of basicity of the following compounds is



1) (A) < (B) < (C) < (D)

2) (B) < (A) < (D) < (C)

3) (B) < (A) < (C) < (D)

4) (D) < (A) < (B) < (C)

Key:

Sol. Aliphatic amines are more basic than aromatic amines

\therefore c is more basic.

'B' is least basic since the *l.p* electron on 'N' atom involved in aromaticity.

4. The most appropriate reagent for conversion of C_2H_5CN into $CH_3CH_2CH_2NH_2$ is.

1) $NaBH_4$

2) $LiAlH_4$

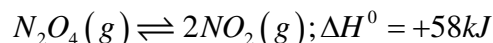
3) $Na(CN)BH_3$

4) CaH_2

Key:

Sol. $LiAlH_4$ powerful reducing agent.

5. Consider the following reaction:



For each of the following cases (a,b), the direction in which the equilibrium shifts is.

a) Temperature is decreased

b) Pressure is increased by adding N_2 at constant T .

1) (a) towards product, (b) towards reactant

2) (a) towards reactant, (b) towards product

3) (a) towards reactant, (b) no change

4) (a) towards product, (b) no change

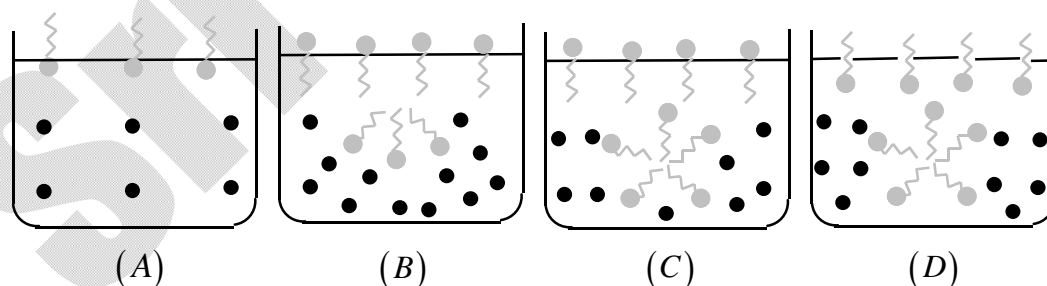
Key: 3

Sol. Increase in temperature favour forward reaction i.e. endothermic, Decrease in temperature favour backward reaction

Adding increases (N_2) not participating in given reaction shifts the equilibrium towards more number of moles side at constant pressure.

At constant volume or with increase in pressure number of effect on equilibrium state.

6. Identify the correct molecular picture showing what happens at the critical micellar concentration (CMC) of an aqueous solution of a surfactant (● polar head; ~ non-polar tail; ■ water).



1) C

2) D

3) A

4) B

Key:

Sol.

7. Which of the following is not an essential amino acid?

- 1) Lysine 2) Tyrosine 3) Leucine 4) Valine

Key: 2

Sol. Tyrosine – can not be synthesized by the body

8. The structure of PCl_5 in the solid state is

- 1) trigonal bipyramidal
2) square pyramidal
3) square planar $[PCl_4]^+$ and octahedral $[PCl_6]^+$
4) tetrahedral $[PCl_4]^+$ and octahedral $[PCl_6]^+$

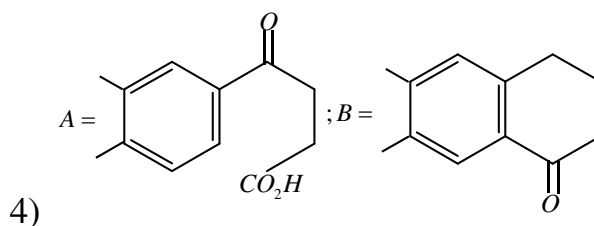
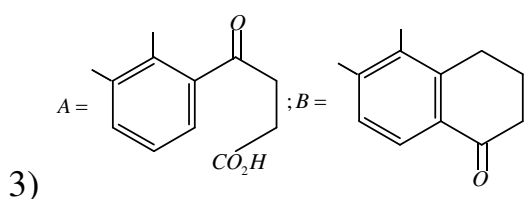
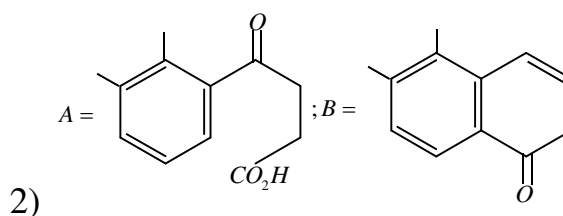
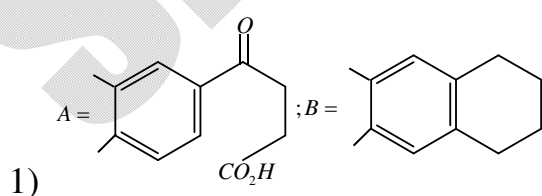
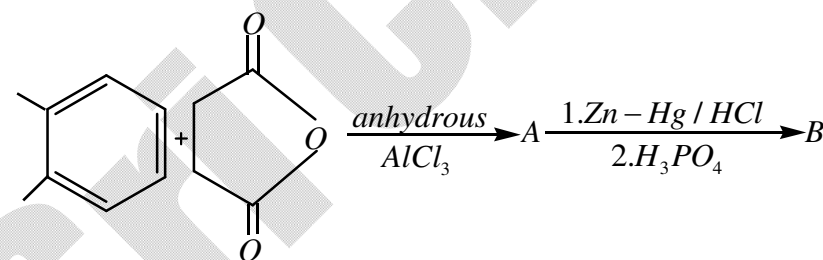
Key: 4

Sol. PCl_5 in solid state exist as $PCl_4^+ PCl_6^-$

$PCl_4^+ \rightarrow$ Tetrahedral

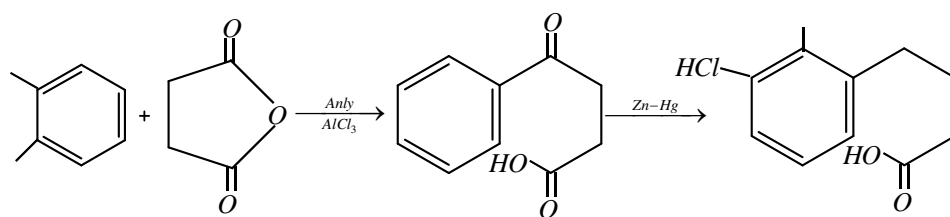
$PCl_6^- \rightarrow$ octahedral

9. In the following reaction sequence the major products A and B are.

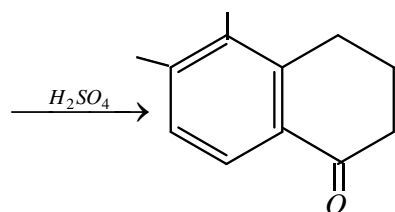


Key: 3

Sol.



(A)



(B)

10. A diatomic molecule X_2 has a body-centred cubic (bcc) structure with a cell edge of 300 pm. The density of the molecule is 6.17 g cm^{-3} . The number of molecules present in 200g of X_2 is.

(Avogadro constant (N_A) = $6 \times 10^{23} \text{ mol}^{-1}$)1) $8N_A$ 2) $40N_A$ 3) $2N_A$ 4) $4N_A$

Key: 4

Sol.
$$e = \frac{2 \times M}{N_A} \times \frac{1}{a^3}$$

$$6.17 = \frac{2 \times M}{N_A \times (300 \times 10^{-10})^3} \quad Z = 2$$

$$M = \frac{6.17 \times N_A \times 27 \times 10^{-24}}{2}$$

$$\text{Number of molecules} = \frac{200}{M} = \frac{200 \times 2}{6.17 \times N_A \times 27 \times 10^{-24}} \times N_A$$

$$= \frac{400}{6.17} \times \frac{1}{27} \times 20^{24} = \frac{4000}{6.17} \times \frac{1}{27} \times 10^{23}$$

$$= \frac{4000}{6.17} \times \frac{1}{27} \times \frac{1}{6} \times N_A = \frac{648.29}{27} \times \frac{1}{6} \times N_A$$

$$= 24 \times \frac{1}{6} N_A = 4N_A$$

11. The condition that indicates a polluted environment is.

- 1) pH of rain water to be 5.6 2) eutrophication
3) 0.03% of CO_2 in the atmosphere 4) BOD value of 5 ppm

Key: 2

Sol. Eutrophication is causing water pollution water overly enriched with minerals nutrients Which induce excessive growth of algae this process may result in oxygen content of water ($D - O$) decreased

12. The difference between the radii of 3rd and 4th orbits of Li^{2+} is ΔR_1 . The difference between the radii of 3rd and 4th orbits of He^+ is ΔR_2 . Ratio $\Delta R_1 : \Delta R_2$ is.

- 1) 3:2 2) 8:3 3) 2:3 4) 3:8

Key: 3

Sol. $r = 0.0529 \times \frac{n^2}{z}$

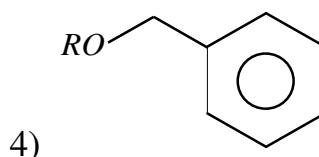
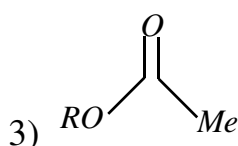
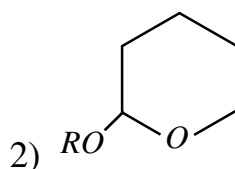
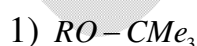
$$\Delta R_1 = \left(0.0529 \times \frac{4^3}{3} - 0.0529 \times \frac{3^3}{3} \right) = 0.529$$

$$\Delta R_1 = 0.529 \left(\frac{4^2}{3} - \frac{3^2}{3} \right) = 0.529 \times \left(\frac{16-9}{3} \right) = \frac{7}{3}$$

$$\Delta R_2 = 0.529 \left(\frac{4^2}{3} - \frac{3^2}{3} \right) = 0.529 \times \left(\frac{16-9}{3} \right) = \frac{7}{2}$$

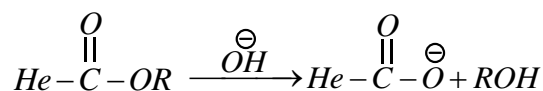
$$\Delta R_1 : \Delta R_2 = 2 : 3$$

33. Which of the following derivatives of alcohols is unstable in an aqueous base?



Key: 3

Sol. Esters are getting hydrolysed in basic medium into ROH and $RCOO^{\ominus}$



14. In the sixth period, the orbitals that are filled are.

- 1) $6s, 6p, 6d, 6f$ 2) $6s, 5d, 5f, 6p$ 3) $6s, 5f, 6d, 6p$ 4) $6s, 4f, 5d, 6p$

Key: 4

Sol. In the 6th period the orbitals filled are.

$6s, 4f, 5d, 6p$

15. A flask contains a mixture of compounds A and B. Both compounds decompose by first-order kinetics. The half-lives for A and B are 300s and 180s, respectively. If the concentrations of A and B are equal initially, the time required for the concentration of A to be four times that of B (in s) is. (Use $\ln 2 = 0.693$)

- 1) 180 2) 300 3) 900 4) 120

Key: 3

Sol. $A_k = A_0 \cdot e^{-k_1 t}$

$$4x = A_0 \cdot e^{-k_1 t}$$

$$A_0 = B_0$$

$$x = 80 \cdot e^{-k_2 t}$$

$$4 = e^{-k_1 t + k_2 t}$$

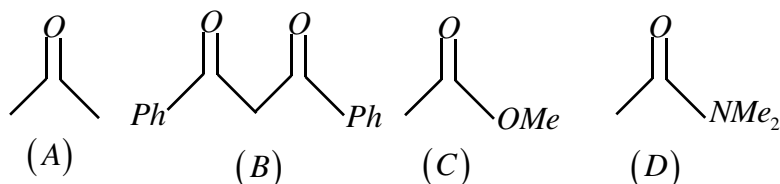
$$\ln 4 = (k_2 - k_1) t$$

$$t = \frac{\ln y}{k_2 - k_1} = \frac{2 \times 0.683}{0.693 \left(\frac{1}{180} - \frac{1}{300} \right)}$$

$$= \frac{2 \times 180 - 300}{300 - 180}$$

$$= \frac{2 \times 100 \times 100}{120} = 900$$

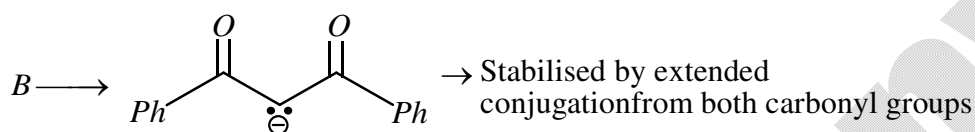
16. The increasing order of the acidity of the α -hydrogen of the following compounds is.



- 1) (B) < (C) < (A) < (D) 2) (C) < (A) < (B) < (D)
 3) (A) < (C) < (D) < (B) 4) (D) < (C) < (A) < (B)

Key: 4

Sol: $B > A > C > D$ Acidity of α stability of carbnion α -hydrogen



17. The correct electronic configuration and spin-only magnetic moment (BM) of Gd^{3+} ($Z = 64$), respectively, are

- 1) $[Xe]4f^7$ and 8.9 2) $[Xe]5f^7$ and 8.9
 3) $[Xe]4f^7$ and 7.9 4) $[Xe]5f^7$ and 7.9

Key: 3

Sol. For, $[Gd] = [Xe]4f^7 5d^1 6s^2$

$$[Gd^{3+}] = [Xe]4f^7$$

Number of unpaired electrons = 7

$$\mu = \sqrt{n(n+2)} = \sqrt{7(7+2)} = \sqrt{63}$$

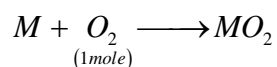
$$= 7.93 \text{ B.M}$$

18. An Ellingham diagram provides information about.

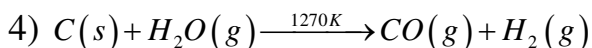
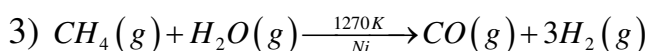
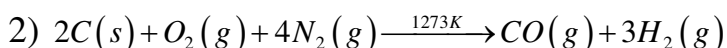
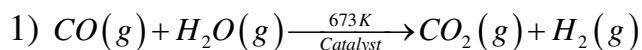
- 1) the temperature dependence of the standard Gibbs energies of formation of some metal oxides.
- 2) the conditions of pH and potential under which a species is thermodynamically stable.
- 3) the pressure dependence of the standard electrode potentials of reduction reactions involved in the extraction of metals.
- 4) the kinetics of the reduction process.

Key: 1

Sol. Eilingham diagram represents plot of ΔG_f^0 vs T for any metal oxide formation for 1 mole of O_2 used

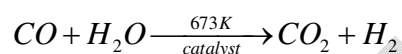


19. The equation that represents the water-gas shift reaction is



Key: 1

Sol. Water gas shift reaction is a less expensive and more efficient method for industrial preparation of H_2 gas.



20. If a person is suffering from the deficiency of nor-adrenaline, what kind of drug can be suggested?

1) Antidepressant

2) Analgesic

3) Anti-inflammatory

4) Antihistamine

Key: 1

Sol. Noradrenaline is a hormone which functions as a neurotransmitter. Its deficiency results in decline in signal-sending activity in the central nervous system. To treat such situation, anti-depressant drugs like Prozac, Zoloft etc. are prescribed.

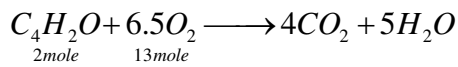
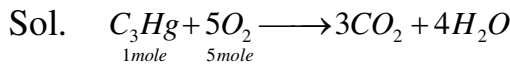
(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value (in decimal notation, truncated/ rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -30, 30.27, -127.30).

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. The minimum number of moles of O_2 required for complete combustion of 1 mole of propane and 2 moles of butane is _____.

Key. 18



Total moles of O_2 required = 5+13
=18 moles

22. A soft drink was bottled with a partial pressure of CO_2 of 3 bar over the liquid at room temperature. The partial pressure of CO_2 over the solution approaches a value of 30 bar when 44g of CO_2 is dissolved in 1 kg of water at room temperature. The approximate pH of the soft drink is _____ $\times 10^{-1}$.

(First dissociation constant of $H_2CO_3 = 4.0 \times 10^{-7}$; $\log 2 = 0.3$; density of the soft drink = 1 g mL^{-1})

Key: 37

Sol: Henry's law :- $\omega g \propto P$ (for 1 kg of solvent)

I_n 100rg of H_2O

$$\frac{\omega g_1}{\omega g_2} = \frac{P_1}{P_2} \Rightarrow \frac{\Delta_4}{\omega} = \frac{30}{3} \Rightarrow \omega = 4.4 \text{ g of } \omega_2$$

= 0.1 moles of CO_2

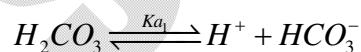
Wt. of solution = 1000 + 4.4

= 1004.4g

$$\therefore V_{sol} = \frac{1004.4}{1} = 1004.4 \text{ ml}$$

= 1.0044l

$$\therefore [CO_2] = \frac{0.1}{1.0044} \approx 0.3 \text{ M} = [H_2CO_3]$$



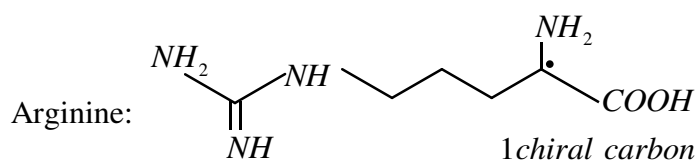
$$[H^+] = \sqrt{K_{a1} \times C} = \sqrt{\Delta \times 10^{-7} \times 10^{-1}}$$

= 2×10^{-4}

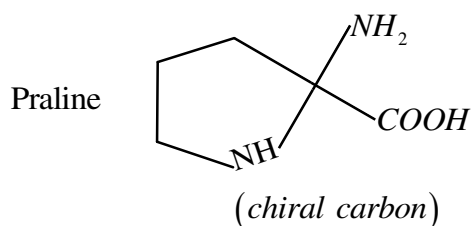
$$\Rightarrow pH = \Delta - \log 2 = 3.7 = 37 \times 10^{-1}$$

23. The number of chiral carbon(s) present in peptide, Ile-Arg-Pro, is _____.

Key. 4



Sol.



$$\text{Total chiral carbons} = 2 + 1 + 1 = 4$$

24. An oxidation-reduction reaction in which 3 electrons are transferred has a ΔG^0 of $17.37 \text{ kJ mol}^{-1}$ at 25°C . The value of E_{cell}^0 (in V) is _____ $\times 10^{-2}$. ($1 F = 96,500 \text{ C mol}^{-1}$)

Key: -6

Sol: $\Delta G^0 = -nFE_{\text{cell}}^0$

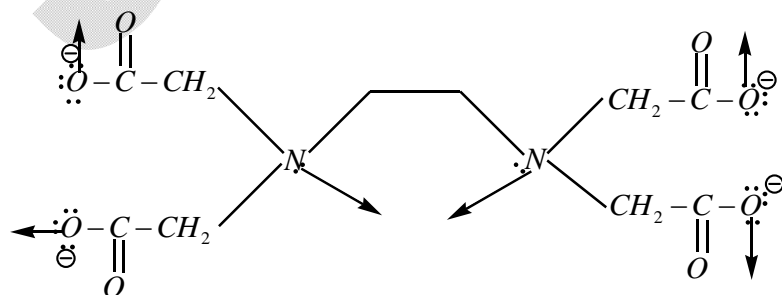
$$E_{\text{cell}}^0 = -\frac{\Delta G^0}{nF} = -\frac{17.37 \times 10^3}{3 \times 96500}$$

$$= -0.06 = -6 \times 10^{-2} \text{ V}$$

25. The total number of coordination sites in ethylenediaminetetraacetate (EDTA^{4-}) is

Key: 6

Sol. EDTA^{4-} is a hexadentate ligand.



MATHEMATICS**(SINGLE CORRECT KEYWER TYPE)**

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its Keywer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct Keywer, 0 if not attempted and -1 in all other cases.

1. Let $\lambda \in R$, The system of linear equations

$$2x_1 - 4x_2 + \lambda x_3 = 1,$$

$$x_1 - 6x_2 + x_3 = 2,$$

$$\lambda x_1 - 10x_2 + 4x_3 = 3$$

Is inconsistent for:

- 1) exactly one positive value of λ 2) every value of λ
 3) exactly two values of λ 4) exactly one negative value of λ

Key: 4

Sol: For inconsistent system, we use the condition $\Delta = 0$ and at least one of $\Delta_1, \Delta_2, \Delta_3$ will be non-zero

$$\Delta = \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix} = 0$$

$$\Rightarrow 2(-24+10) + 4(4-\lambda) + \lambda(-10+6\lambda) = 0$$

$$\Rightarrow -28+16-4\lambda-10\lambda+6\lambda^2 = 0 \Rightarrow 6\lambda^2 - 14\lambda - 12 = 0 \Rightarrow 3\lambda^2 - 7\lambda - 6 = 0$$

$$\Rightarrow (\lambda - 3)(3\lambda + 2) = 0 \Rightarrow \lambda = 3, -\frac{2}{3}$$

$$\Delta_1 = \begin{vmatrix} 1 & -4 & \lambda \\ 2 & -6 & 1 \\ 3 & -10 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 0 & \lambda \\ 2 & 2 & 1 \\ 3 & 2 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 0 & \lambda \\ 2 & 2 & -3 \\ 3 & 2 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 & \lambda \\ -1 & 0 & -3 \\ 3 & 2 & 0 \end{vmatrix} = -2(\lambda - 3)$$

$$\Delta_2 = \begin{vmatrix} 2 & 1 & \lambda \\ 1 & 2 & 1 \\ \lambda & 3 & 4 \end{vmatrix} = 2(5) - 1(4 - \lambda) + \lambda(3 - 2\lambda) = -2\lambda^2 + 4\lambda + 6$$

$$\Delta_3 = \begin{vmatrix} 2 & -4 & 1 \\ 1 & -6 & 2 \\ \lambda & -10 & 3 \end{vmatrix} = -2 \begin{vmatrix} 2 & 2 & 1 \\ 1 & 3 & 2 \\ \lambda & 5 & 3 \end{vmatrix} = -2[2(-1) - 2(3 - 2\lambda) + 1(5 - 3\lambda)] = -2[\lambda - 3]$$

We can see for $\lambda = 3$, we get all $\Delta_1, \Delta_2, \Delta_3$ as zero

So, correct Keywer is option (4)

2. If the volume of parallelepiped, formed by the vectors

$\vec{a} = \hat{i} + \hat{j} + n\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - n\hat{k}$ and $\vec{c} = \hat{i} + n\hat{j} + 3\hat{k}$ ($n \geq 0$), is 158 cu.unuts, then:

- 1) $\vec{a} \cdot \vec{c} = 17$ 2) $\vec{b} \cdot \vec{c} = 10$ 3) $n=9$ 4) $n=7$

Key: 2

Sol: $Volume = \left[\vec{a} \vec{b} \vec{c} \right] = \begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix} = 158$

$$|3n^2 - 5n + 6| = 158 \Rightarrow (3n^2 - 5n + 164)(3n^2 - 5n - 152) = 0$$

$$\Rightarrow 3n^2 - 5n - 152 = 0 \Rightarrow n = \frac{5+43}{6} = 8$$

So, we get $\vec{b} \cdot \vec{c} = 2+n=10$, which give option(2) as correct Keywer

3. If the common tangent to the parabolas, $y^2 = 4x$ and $x^2 = 4y$, also ouches the circle

$x^2 + y^2 = c^2$ then c is equal to

- 1) $\frac{1}{\sqrt{2}}$ 2) $\frac{1}{4}$ 3) $\frac{1}{2\sqrt{2}}$ 4) $\frac{1}{2}$

Key: 1

Sol: Slope forms of tangents for given parabolas will be $y = mx + \frac{1}{m}$ and $y = mx - m^2$

As we need common tangent, it occurs when $\frac{1}{m} = -m^2 \Rightarrow m = -1$

So, common tangent is $x + y + 1 = 0$, which will be tangent to $x^2 + y^2 = c^2$,

where c will be $\frac{1}{\sqrt{2}}$

So, option (1) is the correct Keywer

4. If $y(x)$ is the solution of the differential equation $\left(\frac{5+e^x}{2+y} \right) \frac{dy}{dx} + e^x = 0$ satisfying $y(0) =$

1, then the value of $y(\log_e 13)$ is :

- 1) -1 2) 2 3) 1 4) 0

Key: 1

Sol: $\frac{dy}{2+y} + \frac{e^x}{5+e^x} dx = 0$

Integrating on both sides, we get $(y+2)(5+e^x) = C$

Given $y(0) = 1 \Rightarrow C = 18$

So, curve is $(y+2)(5+e^x) = 18$

$x = \ln 13 \Rightarrow y+2 = 1 \Rightarrow y = -1$

So, option(1) is the correct Keywer

5. If $\int (e^{2x} + 2e^x - e^{-x} - 1)e^{(e^x+e^{-x})} dx = g(x)e^{(e^x+e^{-x})} + c$, where c is constant of integration, then $g(0)$ is equal to:

- 1) 2 2) 1 3) e 4) e^2

Key: 1

Sol: $\int (e^{2x} + 2e^x - e^{-x} - 1)e^{(e^x+e^{-x})} dx = g(x)e^{(e^x+e^{-x})} + c$
 $\Rightarrow (e^{2x} + 2e^x - e^{-x} - 1)e^{(e^x+e^{-x})} = g'(x)e^{(e^x+e^{-x})} + g(x)e^{(e^x+e^{-x})}(e^x - e^{-x})$
 $\Rightarrow (e^{2x} + 2e^x - e^{-x} - 1) = g'(x) + g(x)(e^x - e^{-x})$

So, $g(x) = e^x + 1$, which gives $g(0) = 2$

So, the correct Keywer is option (1)

6. The product of the roots of the equation, $9x^2 - 18|x| + 5 = 0$, is :

- 1) $\frac{25}{9}$ 2) $\frac{5}{9}$ 3) $\frac{5}{27}$ 4) $\frac{25}{81}$

Key: 4

Sol: Let $|x| = t$

We get product of positive roots of $9t^2 - 18t + 5 = 0$ is $\frac{5}{9}$

So, If $t = \alpha, \beta$ are the roots, then the roots of x will be $\alpha, -\alpha, \beta, -\beta$

which makes product equal to $\frac{25}{81}$

So, the correct Keywer is option (4)

7. The mean and the variance of 7 observations are 8 and 16, respectively. If five of the observations are 2, 4, 10, 12 and 14, then the absolute difference of the remaining two observations is:

- 1) 4 2) 1 3) 2 4) 3

Key: 3

Sol: Let the other 2 observations be x, y

$$\text{Mean is } 8 \Rightarrow \frac{x + y + 2 + 4 + 10 + 12 + 14}{7} = 8 \Rightarrow x + y = 14$$

$$\text{Variance is } 16 \Rightarrow \sigma^2 = \frac{\sum x_i^2}{7} - \left(\frac{\sum x_i}{7}\right)^2 \Rightarrow 16 = \frac{x^2 + y^2 + 4 + 16 + 100 + 144 + 196}{7} - 64$$

$$\Rightarrow x^2 + y^2 = 100$$

$$\text{So, } (x - y)^2 + (x + y)^2 = 2(x^2 + y^2) \Rightarrow |x - y| = \sqrt{200 - 196} = 2$$

So, option(3) is the correct Keywer

8. If $2^{10} + 2^9 3^1 + 2^8 3^2 + \dots + 2^1 3^9 + 3^{10} = S$ then S is equal to :

- 1) $\frac{3^{11}}{2} + 2^{10}$ 2) $3^{11} - 2^{12}$ 3) 3^{11} 4) $2 \cdot 3^{11}$

Key: 3

$$\text{Sol: } S - 2^{11} = 2^{10} \frac{\left(1 - \left(\frac{3}{2}\right)^{11}\right)}{1 - \frac{3}{2}} \Rightarrow S - 2^{11} = 3^{11} - 2^{11} \Rightarrow S = 3^{11}$$

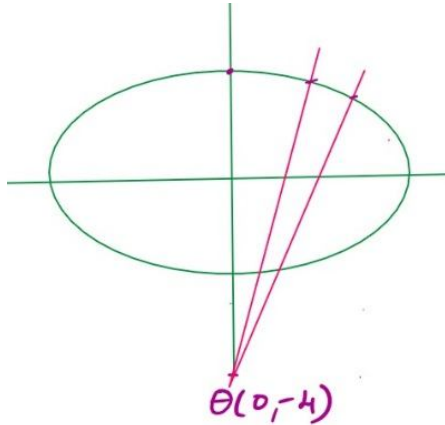
So, Option (3) is the correct Keywer

9. If the point P on the curve, $4x^2 + 5y^2 = 20$ is farthest from the point Q(0, -4), then $PQ^2 =$

- 1) 48 2) 29 3) 36 4) 21

Key: 3

Sol: Checking geometrically, we can say P will be co-vertex which is $(0, 2)$ and $PQ^2 = 36$



So, option(3) is the correct Keywer

10. If the co-ordinates of two points A and B are $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$ respectively and P is any point on the conic, $9x^2 + 16y^2 = 144$, then $PA+PB$ is equal to:

- 1) 8 2) 16 3) 6 4) 9

Key: 1

Sol: Ellipse equation is $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and foci will be $(\sqrt{7}, 0), (-\sqrt{7}, 0)$ with $e = \frac{\sqrt{7}}{4}$

So, $PA+PB = 2a = 8$

So, option (1) is the correct Keywer

11. The negation of the Boolean expression $x \leftrightarrow \sim y$ is equivalent to:

- 1) $(x \wedge y) \vee (\sim x \wedge \sim y)$ 2) $(x \wedge y) \wedge (\sim x \wedge \sim y)$
 3) $(x \wedge \sim y) \vee (\sim x \wedge y)$ 4) $(\sim x \wedge y) \vee (\sim x \wedge \sim y)$

Key: 1

Sol: $\sim(x \leftrightarrow \sim y) \equiv \sim[(x \Rightarrow \sim y) \wedge (\sim y \Rightarrow x)] \equiv \sim[\sim(\sim x \vee \sim y) \vee \sim(y \vee x)] \equiv [(x \wedge y) \vee (\sim y \wedge \sim x)]$

So, option(1) is the correct Keywer

12. If the function $f(x) = \begin{cases} k_1(x - \pi)^2, & x \leq \pi \\ k_2 \cos x, & x > \pi \end{cases}$ is twice differentiable, then the ordered pair

(k_1, k_2) is equal to

- 1) $(1, 1)$ 2) $\left(\frac{1}{2}, 1\right)$ 3) $(1, 0)$ 4) $\left(\frac{1}{2}, -1\right)$

Key: 2

Sol: $f(x)$ is twice differentiable meKey, $f(x)$ and $f'(x)$ are continuous at $x = \pi$ and

$$f''(\pi^-) = f''(\pi^+)$$

$$f(x) = \begin{cases} k_1(x-\pi)^2 - 1, & x \leq \pi \\ k_2 \cos x, & x > \pi \end{cases}$$

$$f(\pi) = f(\pi^+) \Rightarrow -1 = -k_2 \Rightarrow k_2 = 1$$

$$f'(x) = \begin{cases} 2k_1(x-\pi), & x < \pi \\ -k_2 \sin x, & x > \pi \end{cases}$$

$$f'(\pi^-) = f'(\pi^+) \Rightarrow 0 = 0$$

$$f''(x) = \begin{cases} 2k_1, & x < \pi \\ -k_2 \cos x, & x > \pi \end{cases}$$

$$f''(\pi^-) = f''(\pi^+) \Rightarrow 2k_1 = k_2 \Rightarrow k_1 = \frac{1}{2}$$

So, option(2) is the correct Keywer

13. A survey shows that 73% of the persons working in an office like coffee, where as 5% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be:

1) 36

2) 63

3) 54

4) 38

Key: 1

Sol; Let C and T denote the set of people liking coffee and tea and total employees be 100Given, $n(C) = 73$ and $n(T) = 65$ We have $73 \leq n(C \cup T) \leq 100$

$$\Rightarrow 73 \leq n(C) + n(T) - n(C \cap T) \leq 100$$

$$\Rightarrow -65 \leq -n(C \cap T) \leq -38$$

$$\Rightarrow 38 \leq n(C \cap T) \leq 65$$

So, option (1) will be the correct Keywer

14. The value of $\int_{\pi/6}^{\pi/4} \frac{dx}{1 + e^{\sin x}}$ is:

1) π 2) $\frac{3\pi}{2}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{2}$

Key: 4

Sol: $I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}} dx$

$$\Rightarrow I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{-\sin x}} dx \left(\because \int_{-a}^a f(x) dx = \int_{-a}^a f(-x) dx \right)$$

So, $2I = \int_{-\pi/2}^{\pi/2} dx = \pi \Rightarrow I = \frac{\pi}{2}$. So, Option(4) is the correct Keywer

15. If (a, b, c) is the image of the point (1, 2, -3) in the line, $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$ then

a + b + c is equal to:

- 1) 2 2) 3 3) -1 4) 1

Key: 1

Sol: So, foot of perpendicular will be $\left(\frac{a+1}{2}, \frac{b+2}{2}, \frac{c-3}{2}\right)$

Let foot of perpendicular on $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$ be $F(2t-1, 3-2t, -t)$ from $A(1, 2, -3)$

AF will be perpendicular to line with d.r.s $\langle 2, -2, -1 \rangle$, which gives the relation

$$2(2t-1-1) - 2(3-2t-2) - 1(-t+3) = 0 \Rightarrow 9t-9=0 \Rightarrow t=1$$

So, foot of perpendicular will be $F(1, 1, -1)$ which should be same as $\left(\frac{a+1}{2}, \frac{b+2}{2}, \frac{c-3}{2}\right)$

$\Rightarrow a=1, b=0, c=1$, So, option(1) is the correct Keywer

16. If the minimum and the maximum values of the function $f : \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \rightarrow R$, defined by

$$f(\theta) = \begin{vmatrix} -\sin^2 \theta & -1 - \sin^2 \theta & 1 \\ -\cos^2 \theta & -1 - \cos^2 \theta & 1 \\ 12 & 10 & -2 \end{vmatrix} \text{ are } m \text{ and } M \text{ respectively, then order pair}$$

(m, M) is equal to:

- 1) (-4, 0) 2) (-4, 4) 3) (0, 4) 4) $(0, 2\sqrt{2})$

Key: 1

$$\text{Sol: Given, } f(\theta) = \begin{vmatrix} -\sin^2 \theta & -1 - \sin^2 \theta & 1 \\ -\cos^2 \theta & -1 - \cos^2 \theta & 1 \\ 12 & 10 & -2 \end{vmatrix} = \begin{vmatrix} 11 & 7 & 0 \\ -\cos^2 \theta & -1 - \cos^2 \theta & 1 \\ 12 & 10 & -2 \end{vmatrix}$$

$$= \begin{vmatrix} 11 & 7 & 0 \\ -\cos^2 \theta & -1 - \cos^2 \theta & 1 \\ 1 & 3 & -2 \end{vmatrix} = \begin{vmatrix} 4 & 7 & 0 \\ 0 & -1 - \cos^2 \theta & 1 \\ 0 & 3 & -2 \end{vmatrix}$$

$$= 4(2 + 2\cos^2 \theta - 3) = 4\cos 2\theta$$

$$2\theta \in \left[\frac{\pi}{2}, \pi \right] \Rightarrow \cos 2\theta \in [-1, 0] \Rightarrow 4\cos 2\theta \in [-4, 0] \quad m = -4 \quad M = 0$$

So, option(1) will be the correct Keywer

17. If S is the sum of the first 10 terms of the series

$$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \tan^{-1}\left(\frac{1}{21}\right) + \dots, \text{ then } \tan(S) \text{ is equal to:}$$

- 1) $\frac{-6}{5}$ 2) $\frac{5}{11}$ 3) $\frac{5}{6}$ 4) $\frac{10}{11}$

Key: 3

Sol: General term of sequence 3,7,13,21,... will be of the type $t_n = an^2 + bn + c$

$$t_1 = 3 = a + b + c$$

$$t_2 = 7 = 4a + 2b + c$$

$$t_3 = 13 = 9a + 3b + c$$

So, we take $t_2 - t_1$ and $t_3 - t_2$

$$\Rightarrow 3a + b = 4 \quad \text{and} \quad 5a + b = 6$$

$$\Rightarrow 2a = 2 \quad \text{or} \quad a = 1$$

So, we get $a = b = c = 1$

$$\tan^{-1}\left(\frac{1}{n^2 + n + 1}\right) = \tan^{-1}\left(\frac{n+1-n}{1+n(n+1)}\right) = \tan^{-1}(n+1) - \tan^{-1}n$$

So, sum of first 10 terms will be $\tan^{-1}11 - \tan^{-1}1 = S$

$$\text{So, } \tan S = \frac{11-1}{1+11} = \frac{10}{12} = \frac{5}{6}$$

So, option(3) is the correct Keywer

18. If $3^{2\sin 2\alpha - 1}$, 14 and $3^{4-2\sin 2\alpha}$ are the first three terms of an A.P for some α , then the sixth term of this A.P. is:

- 1) 66 2) 65 3) 78 4) 81

Key: 1

Sol: Let $3^{2\sin 2\alpha} = t, t \in \left[\frac{1}{9}, 9\right]$

We get $\frac{t}{3}, 14, \frac{81}{t}$ are in A.P.

$$\Rightarrow 28 = \frac{t}{3} + \frac{81}{t} \Rightarrow t^2 - 84t + 243 = 0 \Rightarrow t = 3, 81$$

So, $t = 3$

So, common difference will be 13

So, 6th term will be $14 + 4(13) = 66$

So, option(1) will be the correct Keywer

19. If α is positive root of the equation $\mathbf{P(x) = x^2 - x - 2 = 0}$ then $\lim_{x \rightarrow \alpha^+} \frac{\sqrt{1 - \cos(P(x))}}{x + \alpha - 4}$

is equal to:

- 1) $\frac{1}{2}$ 2) $\frac{1}{\sqrt{2}}$ 3) $\frac{3}{\sqrt{2}}$ 4) $\frac{3}{2}$

Key: 3

Sol: Positive root of $x^2 - x - 2 = 0$ is 2

So, $\alpha = 2$

$$\lim_{x \rightarrow \alpha^+} \frac{\sqrt{1 - \cos(p(x))}}{x + \alpha - 4} = \lim_{x \rightarrow 2^+} \frac{\sqrt{1 - \cos(x^2 - x - 2)}}{x - 2} = \lim_{x \rightarrow 2^+} \frac{\sqrt{2} \left| \sin\left(\frac{x^2 - x - 2}{2}\right) \right|}{x - 2} = \lim_{x \rightarrow 2^+} \frac{\sqrt{2}(x+1)}{2} = \frac{3}{\sqrt{2}}$$

So, option (3) is the correct Keywer

20. If four complex numbers $z, \bar{z}, \bar{z} - 2\operatorname{Re}(\bar{z})$ and $z - 2\operatorname{Re}(z)$ represent the vertices of the square of the side 4 units in the Argand plane, then $|z|$ is equal to:

- 1) 2 2) 4 3) $4\sqrt{2}$ 4) $2\sqrt{2}$

Key: 4

Sol: Let $z = x + iy \Rightarrow \bar{z} = x - iy$

So, the vertices will be $(x, y), (x, -y), (-x, -y), (-x, y)$

So, sides of squares will be $2x, 2y$ which meKey $x = y = 2$

So, $|z| = 2\sqrt{2}$

So, option (4) is the correct Keywer

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value (in decimal notation, truncated/ rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct Keywer, 0 if not attempted and 0 in all other cases.

21. If the line $2x - y + 3 = 0$ is at a distance $\frac{1}{\sqrt{5}}$ and $\frac{2}{\sqrt{5}}$ from the line $4x - 2y + \alpha = 0$ and $6x - 3y + \beta = 0$, respectively, then the sum of all possible values of α and β is _____

Key: 30

Sol: Distance between $4x - 2y + \alpha = 0$ and $4x - 2y + 6 = 0$ is $\frac{1}{\sqrt{5}} \Rightarrow \frac{|\alpha - 6|}{2\sqrt{5}} = \frac{1}{\sqrt{5}} \Rightarrow \alpha = 6 \pm 2$

Distance between $6x - 3y + \beta = 0$ and $6x - 3y + 9 = 0$ is $\frac{2}{\sqrt{5}} \Rightarrow \frac{|\beta - 9|}{3\sqrt{5}} = \frac{1}{\sqrt{5}} \Rightarrow \beta = 9 \pm 3$

So, sum of all values will be $2(9 + 6) = 30$

22. The natural number m , for which the coefficient of x in binomial expansion of

$\left(x^m + \frac{1}{x^2}\right)^{22}$ is 1540, is _____.

Key: 13

Sol: General term $T_{r+1} = {}^{22}C_r (x^m)^{22-r} \left(\frac{1}{x^2}\right)^r = {}^{22}C_r x^{22m - rm - 2r}$

Given, ${}^{22}C_r = 1540$ and $22m - rm - 2r = 1$

$$1540 = 22 \times 70 = \frac{22 \times 21 \times 20}{3 \times 2} = {}^{22}C_3 = {}^{22}C_{19}$$

$$\text{So, } r = 3 \text{ or } r = 19 \quad 22m - rm - 2r = 1 \Rightarrow m = \frac{1 + 2r}{22 - r}$$

We have $m \in \mathbb{N}$, so we get $r = 19$ and $m = 13$

23. Let $f(x) = x \left[\frac{x}{2} \right]$, for $-10 < x < 10$, where $[t]$ denotes greatest integer function.

Then the number of points of discontinuity of f is equal to _____.

Key: 8

Sol:

$$f(x) = x \left\lfloor \frac{x}{2} \right\rfloor = \begin{cases} -5x, & x \in (-10, -8) \\ -4x, & x \in [-8, -6) \\ -3x, & x \in [-6, -4) \\ -2x, & x \in [-4, -2) \\ -x, & x \in [-2, 0) \\ 0, & x \in [0, 2) \\ x, & x \in [2, 4) \\ 2x, & x \in [4, 6) \\ 3x, & x \in [6, 8) \\ 4x, & x \in [8, 10) \end{cases}$$

So, we can see that $f(x)$ is discontinuous at $x = -8, -6, -4, -2, 2, 4, 6, 8$

Number of points of discontinuity will be 8

24. Four fair dice are thrown independently 27 times. Then the expected number of times, at least two dice show up three or a five, is _____.

Key: 11

Sol: Expected number = $27 \left[{}^4C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^2 + {}^4C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right) + {}^4C_4 \left(\frac{1}{3}\right)^4 \right]$

$$= \frac{6(2)^2 + 4(2) + 1}{3} = \frac{24 + 8 + 1}{3} = 11$$

25. The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is _____.

Key: 240

Sol: We have pairs of S, L and Y, A, B, U are single

Required number will be ${}^2C_1 {}^5C_2 \frac{4!}{2!} = 240$

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