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✪ A.P ✪ T.S ✪ Chandigarh ✪ Karnataka ✪ Tamilnadu ✪ Maharastra ✪ Delhi ✪ Ranchi

Jee Main 2020(Sep)

06-Sep-2020 (Morning Shift)

Question Paper, Key and Solutions

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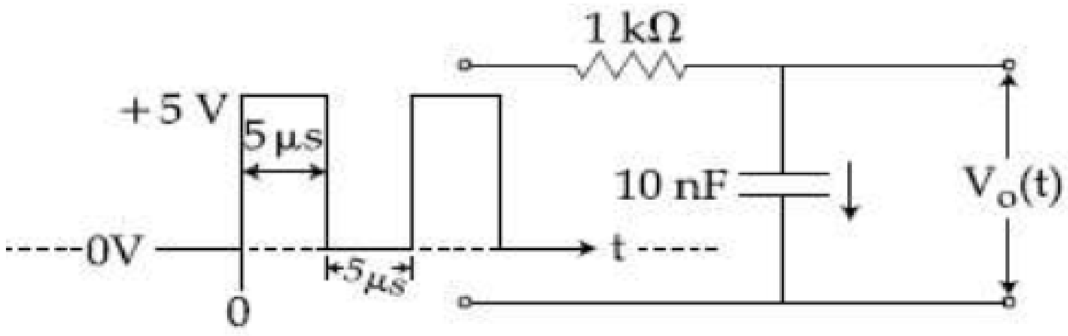
Physics

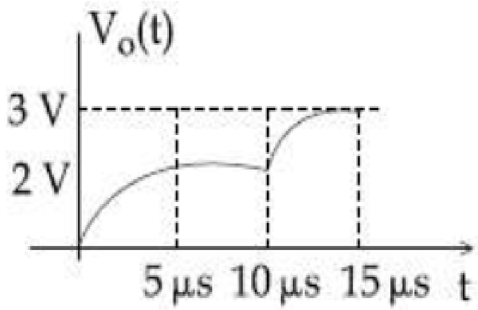
(SINGLE CORRECT ANSWER TYPE)

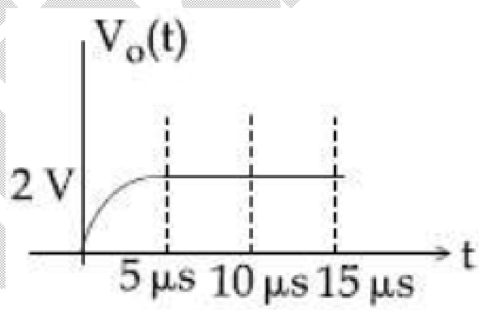
This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

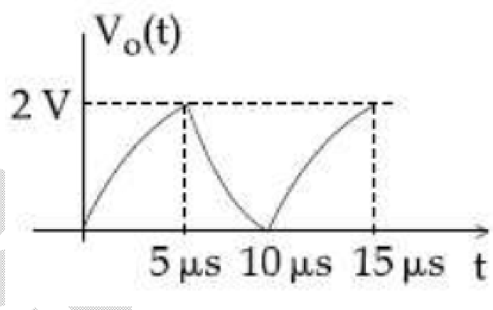
Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

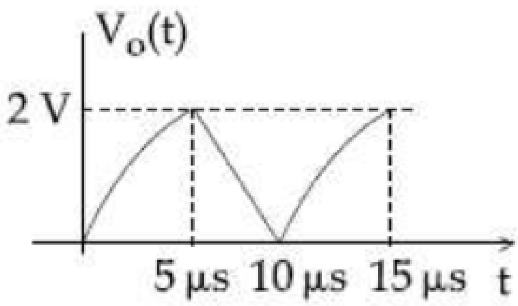
1. For the given input voltage waveform $V_{in}(t)$, the output waveform $V_0(t)$, across the capacitor is correctly depicted by :



1) 

2) 

3) 

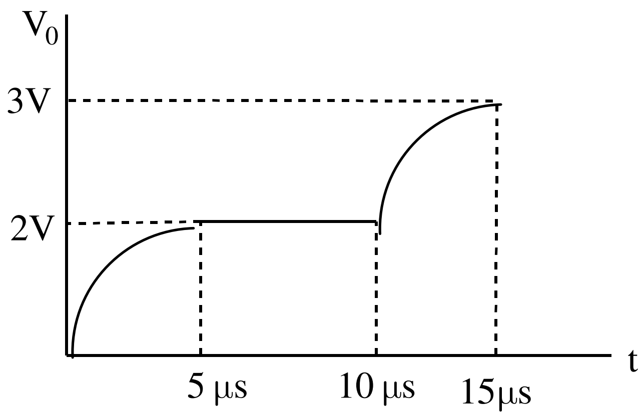
4) 

Key: **1**

Sol: Time constant $\tau = RC = 10^3 \times 10 \times 10^{-9} = 10^{-5} = 10\mu s$

$V_0 = \frac{q}{C} = v_{max}\left[1-e^{-t/\tau}\right] = 5\left(1-e^{-t/10}\right)$ where t is μs

At $t = 5\mu s,\ V_0 = 5\left(1-e^{-1/2}\right) = 2V$



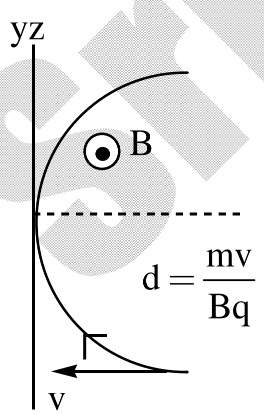
For $5\mu s < t < 10\mu s$, $V_0 = 2V$ (constant). Then again increases.

2. A particle of charge 'q' and mass 'm' is moving with a velocity $-v\hat{i}\,(v \neq 0)$ towards a large screen placed in Y-Z plane at a distance d. If there is a magnetic field $\vec{B} = B_0\hat{k}$, the minimum value of 'v' for which the particle will not hit the screen is :

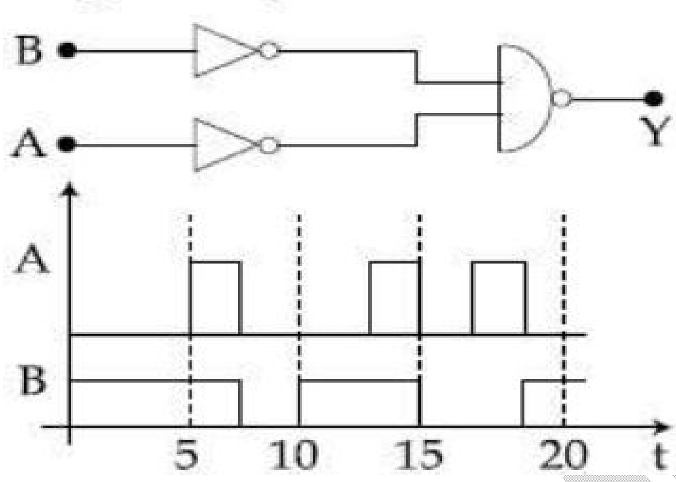
1) $\frac{qdB_0}{2m}$ 2) $\frac{2qdB_0}{m}$ 3) $\frac{qdB_0}{3m}$ 4) $\frac{qdB_0}{m}$

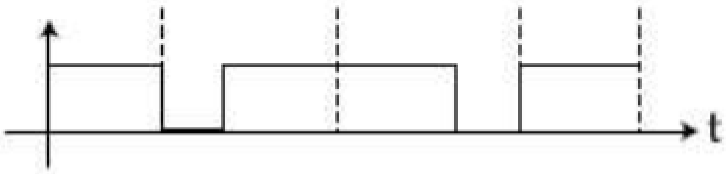
Key: **4**

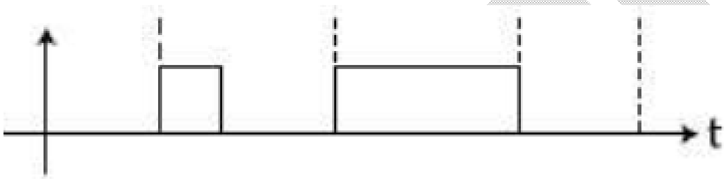
Sol : $\nu = \frac{qBd}{m}$

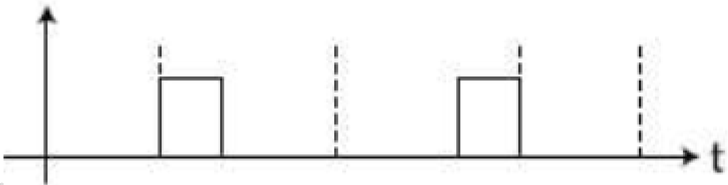


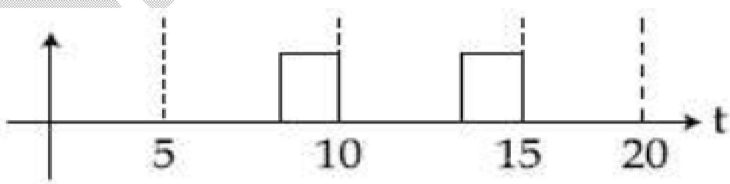
3. Identify the correct output signal 'Y' in the given combination of gates (as shown) for the given inputs A and B :



1) 

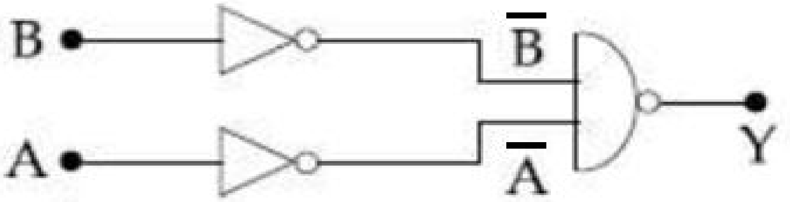
2) 

3) 

4) 

Key: **1**

Sol: $\overline{B}.\overline{A} = B + A$



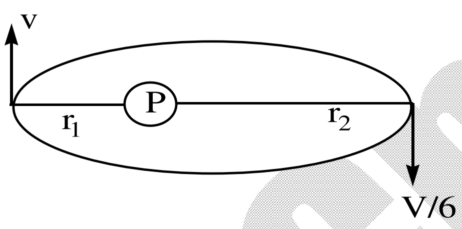
$Y = A + B$ OR gate

4. A satellite is in an elliptical orbit around a planet P. It is observed that the velocity of the satellite when it is farthest from the planet is 6 times less than that when it is closest to the planet. The ratio of distances between the satellite and the planet at closest and farthest point is :

1) 1 : 2 2) 3 : 4 3) 1 : 3 4) 1 : 6

Key: **4**

Sol: $(v)(r_1) = \left(\frac{v}{6}\right)(r_2)$



$\frac{r_1}{r_2} = \frac{1}{6}$

5. An AC circuit has $R = 100\,\Omega, C = 2\mu F$ and $L = 80\,mH$, connected in series. The quality factor of the circuit is :

1) 2 2) 400 3) 0.5 4) 20

Key: **1**

Sol: $Q = \left(\sqrt{\frac{L}{C}}\right)\frac{1}{R}$

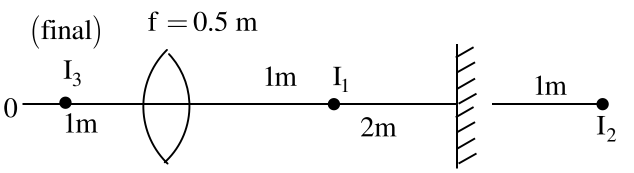
$= \sqrt{\frac{80 \times 10^{-3}}{2 \times 10^{-6}}} \times \frac{1}{100} = 2$

6. A point like object is placed at a distance of 1m in front of a convex lens of focal length 0.5 m. A plane mirror is placed at a distance of 2m behind the lens. The position and nature of the final image formed by the system is :

1) 1m from the mirror, virtual
2) 2.6 m from the mirror, real
3) 1m from the mirror, real
4) 2.6 m from the mirror, virtual

Key: **2**

Sol:

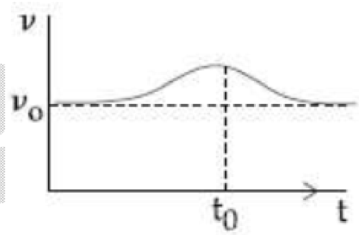


$\frac{1}{v_1} = \frac{1}{(-1)} = \frac{1}{0.5} \Rightarrow v_1 = 1\text{m}$ $\qquad$ $\frac{1}{v} - \frac{1}{(-3)} = \frac{1}{0.5} \Rightarrow v = \frac{3}{5}$

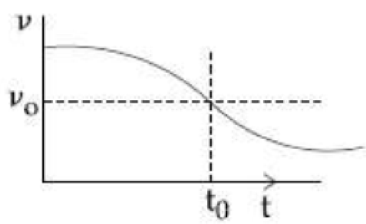
Final image is formed at 2.6m from mirror and is real

7. A sound source S is moving along a straight track with speed 'v', and is emitting sound of frequency v_0 (see fig.) An observer is standing at a finite distance, at the point 'O', from the track. The time variation of frequency heard by the observer is best represented by :

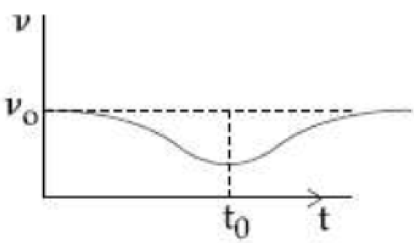
(t_0 represents the instant when the distance between the source and observer is minimum)



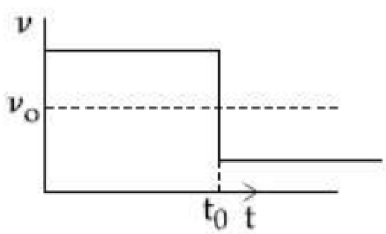
1)



2)



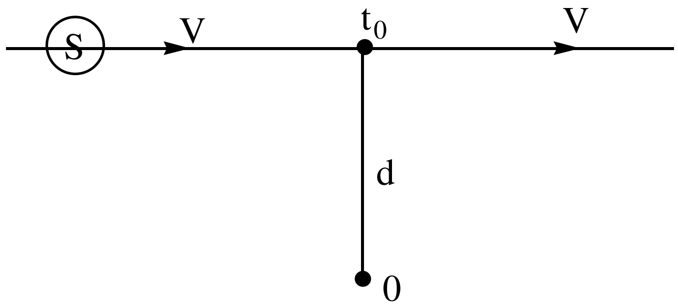
3)

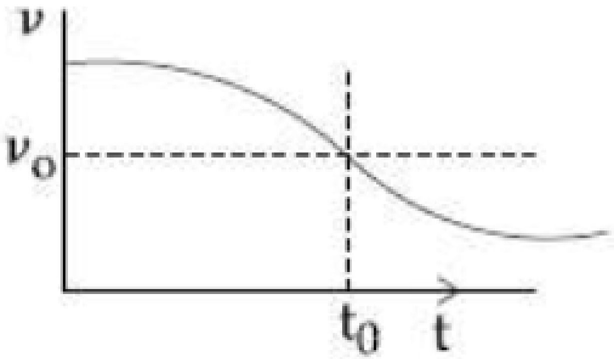


4)

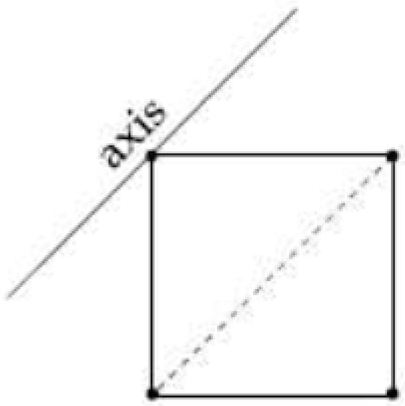
Key: **2**

Sol:





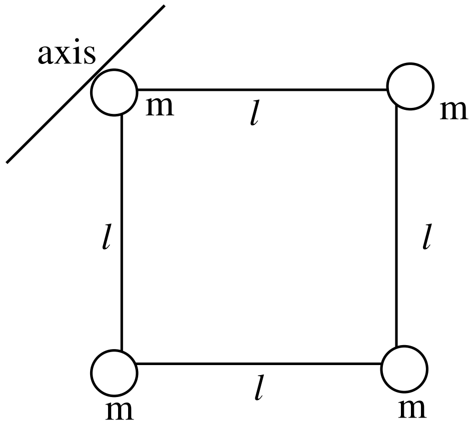
8. Four point masses, each of mass 'm', are fixed at the corners of a square of side 'l'. The square is rotating with angular frequency ω, about an axis passing through one of the corners of the square and parallel to its diagonal, as shown in the fig. The angular momentum of the square about this axis is :



1) $ml^2\omega$ 2) $2ml^2\omega$ 3) $3ml^2\omega$ 4) $4ml^2\omega$

Key: **3**

Sol: $I = 0 + 2m\frac{l^2}{2} + m\left(2l^2\right) = 3ml^2$



$L = I\omega = 3ml^2\omega$

9. Molecules of an ideal gas are known to have three translational degrees of freedom and two rotational degrees of freedom. The gas is maintained at a temperature of 'T'. The total internal energy, U of a mole of this gas, and the value of $\gamma\left(=\frac{C_p}{C_v}\right)$ are given, respectively, by :

1) $U = \frac{5}{2}RT$ and $\gamma = \frac{7}{5}$
2) $U = 5RT$ and $\gamma = \frac{7}{5}$
3) $U = 5RT$ and $\gamma = \frac{6}{5}$
4) $U = \frac{5}{2}RT$ and $\gamma = \frac{6}{5}$

Key: **1**

Sol: The gas is diatomic

dof = 5

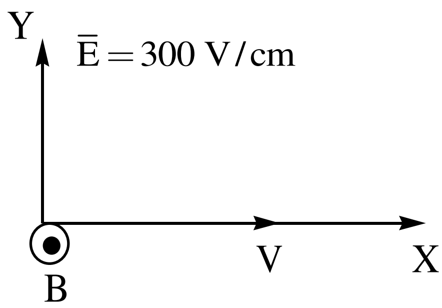
$\gamma = \frac{C_p}{C_V} = \frac{7}{5}$ and $U = nC_vT = \frac{5}{2}RT$

10. An electron is moving along +x direction with a velocity of $6\times10^{6}\ ms^{-1}$. It enters a region of uniform electric field of 300 V/cm pointing along +y direction. The magnitude and direction of the magnetic field set up in this region such that the electron keeps moving along the x-direction will be

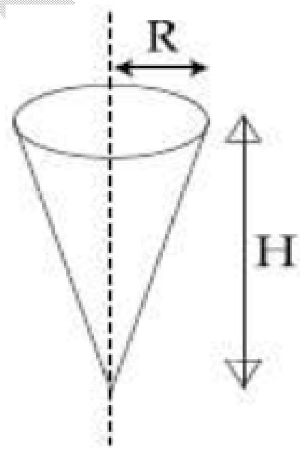
1) $3\times10^{-4}T$, along –z direction
2) $5\times10^{-3}T$, along –z direction
3) $3\times10^{-4}T$, along +z direction
4) $5\times10^{-3}T$, along +z direction

Key: **4**

Sol: $B=\frac{E}{V}=\frac{300\times10^{2}}{6\times10^{6}}=5\times10^{-3}T$ along +Z



11. Shown in the fig. is a hollow icecream cone (it is open at then top). If its mass is M, radius of its top, R and height, H, then its moment of inertia about its axis is :

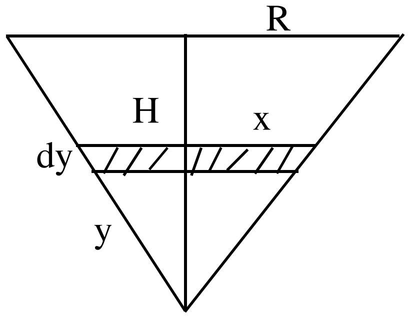


1) $\frac{MR^{2}}{3}$
2) $\frac{MR^{2}}{2}$
3) $\frac{M\left(R^{2}+H^{2}\right)}{4}$
4) $\frac{MH^{2}}{3}$

Key: **2**

Sol:

$$\frac{x}{y}=\frac{R}{\sqrt{R^2+H^2}}$$



$$I=\int dm\ x^2 \qquad =\frac{M}{\pi R\sqrt{R^2+H^2}}\int 2\pi x^3\,dy$$

$$I=\frac{2M}{R\sqrt{R^2+H^2}}\times\frac{R^3}{\left(R^2+H^2\right)^{\frac{3}{2}}}\times\left[\frac{y^4}{4}\right]_0^{\sqrt{R^2+H^2}}=\frac{MR^2}{2}$$

12. If the potential energy between two molecules is given by $U=-\frac{A}{r^6}+\frac{B}{r^{12}}$, then at equilibrium, separation between molecules, and the potential energy are :

1) $\left(\frac{B}{A}\right)^{\frac{1}{6}},0$
2) $\left(\frac{B}{2A}\right)^{\frac{1}{6}},-\frac{A^2}{2B}$
3) $\left(\frac{2B}{A}\right)^{\frac{1}{6}},-\frac{A^2}{2B}$
4) $\left(\frac{2B}{A}\right)^{\frac{1}{6}},-\frac{A^2}{4B}$

Key: **4**

Sol: $U=-\frac{A}{r^6}+\frac{B}{r^{12}}$ $\qquad \frac{du}{dr}=0\Rightarrow\frac{6A}{r^5}=\frac{12B}{r^{11}}$

$\therefore r=\left(\frac{2B}{A}\right)^{\frac{1}{6}}$ and $U=-\frac{A^2}{4B}$

13. A screw gauge has 50 divisions on its circular scale. The circular scale is 4 units ahead of

the pitch scale marking, prior to use. Upon one complete rotation of the circular scale, a displacement of 0.5 mm is noticed on the pitch scale. The nature of zero error involved, and the least count of the screw gauge, are respectively:

1) Positive, 0.1 mm
2) Positive, 10 μm
3) Positive, $0.1\,\mu$m
4) Negative, $2\,\mu$m

Key: **2**

Sol: Zero error = + 0.04 mm
Pitch = 0.5 mm
$LC = \frac{0.5}{50} = 0.01$ mm

14. You are given that mass ${}_3^7Li = 7.0160\,u$,
Mass of ${}_2^4He = 4.0026$ u,
And mass of ${}_1^1H = 1.0079$ u
When 20g of ${}_3^7Li$ is converted into ${}_2^4He$ by proton capture, the energy liberated, (in kWh), is :
$\left[\text{Mass of nucleon} = 1\,\text{GeV}/c^2\right]$

1) 6.82×10^5
2) 8×10^6
3) 1.33×10^6
4) 4.5×10^5

Key: **3**

Sol: ${}_3^7Li \quad + \quad P \rightarrow \quad 2\,{}_2^4He$
(7.0160 u) (1.0079 u) $(2\times4.0026u)$
$\Delta E = (7.0160 + 1.0079 - 2\times4.0026)u = 0.0187\,u$
for 20g of ${}_3^7Li$ $\quad \Delta E = 0.01874\times\frac{20}{7}\times6\times10^{23}$
$$= \frac{120}{7}\times10^{23}\times\frac{0.0187\times10^9\times1.6\times10^{-19}}{1000\times3600} = 1.33\times10^6\,\text{kWh}$$

15. An object of mass 'm' is suspended at the end of a massless wire of length L and area of cross-section, A. Young modulus of the material of the wire is Y. If the mass is pulled down slightly its frequency of oscillation along the vertical direction is :

1) $f = \frac{1}{2\pi}\sqrt{\frac{mA}{YL}}$ 2) $f = \frac{1}{2\pi}\sqrt{\frac{mL}{YA}}$

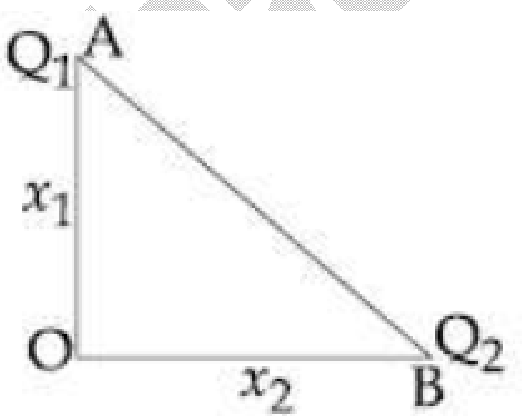
3) $f = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$ 4) $f = \frac{1}{2\pi}\sqrt{\frac{YL}{mA}}$.

Key: **3**

Sol: $K = \frac{yA}{L}$

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{yA}{mL}}$

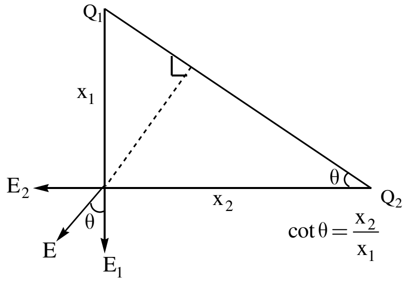
16. Charge Q_1 and Q_2 are at points A and B of a right angle triangle OAB (see fig). The resultant electric field at point O is perpendicular to the hypotenuse, then Q_1 / Q_2 is proportional to:



1) $\frac{x_1^3}{x_2^3}$ 2) $\frac{x_2^2}{x_1^2}$ 3) $\frac{x_2}{x_1}$ 4) $\frac{x_1}{x_2}$

Key: **4**

Sol:



$$E_1 \sin\theta = E_2 \cos\theta$$

$$\frac{kQ_1}{x_1^2}\sin\theta = \frac{kQ_2}{x_2^2}\cos\theta$$

$$\frac{Q_1}{Q_2} = \left(\frac{x_1}{x_2}\right)^2 \quad \cot\theta = \frac{x_1}{x_2}$$

17. An electron, a doubly ionized helium ion $\left(He^{++}\right)$ and a proton are having the same kinetic energy. The relation between their respective de-Broglie wavelengths $\lambda_e, \lambda_{He^{++}}$ and λ_p is:

1) $\lambda_e < \lambda_P < \lambda_{He^{++}}$

2) $\lambda_e > \lambda_{He^{++}} > \lambda_P$

3) $\lambda_e > \lambda_P > \lambda_{He^{++}}$

4) $\lambda_e < \lambda_{He^{++}} = \lambda_P$

Key: **3**

Sol: $\lambda = \frac{h}{\sqrt{2km}} \Rightarrow \lambda \propto \frac{1}{\sqrt{m}}$

$\lambda_e : \lambda_{He^{++}} : \lambda_p \; \frac{1}{\sqrt{m_e}} : \frac{1}{\sqrt{m_{He}}} : \frac{1}{\sqrt{m_p}} \qquad \lambda_e > \lambda_p > \lambda_{He^{++}}$

18. A clock has a continuously moving second's hand of 0.1 m length. The average acceleration of the tip of the hand (in units of ms^{-2}) is of the order of :

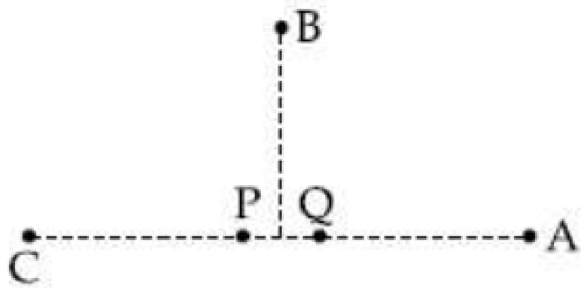
1) 10^{-2} 2) 10^{-1} 3) 10^{-4} 4) 10^{-3}

Key: **4**

Sol: $a = \omega^2 r = \frac{4\pi^2}{3600} \times 0.1 = 1.1 \times 10^{-3}$

Objection: Bonus
Reason: As it is in circular motion, average acceleration is zero

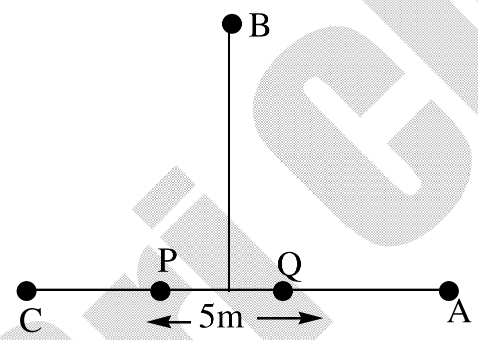
19. In the fig. below, P and Q are two equally intense coherent sources emitting radiation of wavelength 20 m. The separation between P and Q is 5m and the phase of P is ahead of that of Q by 90^0. A, B and C are three distinct points of observation, each equidistant from the midpoint of PQ. The intensities of radiation at A, B, C will be in the ratio :



1) 2 : 1 : 0 2) 0 : 1 : 4 3) 4 : 1 : 0 4) 0 : 1 : 2

Key: **1**

Sol: $\lambda = 20\ m$



$PQ = \frac{\lambda}{4}$ $\Delta\phi = 90^0$

$\Delta\phi$ at $A = 0 \Rightarrow$ intensity is I_{max}

$\Delta\phi$ at $B = 90^0 \Rightarrow$ intensity is $\frac{I_{max}}{2}$

$\Delta\phi$ at $C = 180^0 \Rightarrow$ intensity is 0

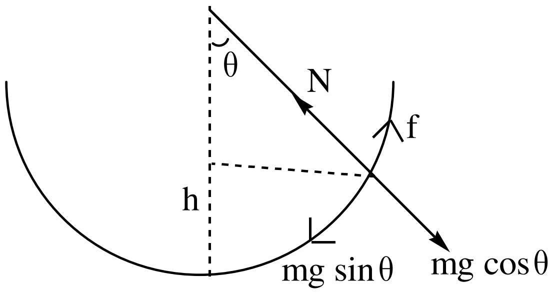
Intensity ratio = 2 : 1 : 0

20. An insect is at the bottom of a hemispherical ditch of radius 1m. It crawls up the ditch but starts slipping after it is at height 'h' from the bottom. If the coefficient of friction between the ground and the insect is 0.75, then 'h' is: $\left(g = 10\ ms^{-2}\right)$

1) 0.60 m 2) 0.45 m 3) 0.20 m 4) 0.80 m

Key: **3**

Sol:



$f = mg\sin\theta \qquad \mu mg\cos\theta = mg\sin\theta$

as $\mu = 0.75,\ \theta = 37^0 \qquad \therefore h = R(1-\cos\theta) = 0.20\ m$

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value(in decimal notation, truncated/ rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. Initially a gas of diatomic molecules is contained in a cylinder of volume V_1 at a pressure P_1 and temperature 250 K. As summing that 25% of the molecules get dissociated causing a change in number of moles. The pressure of the resulting gas at temperature 2000 K, when contained in a volume $2V_1$ is given by P_2. The ratio P_2/P_1 is ……

Key: **5**

Sol: Applying PV = nRT

$P_1V_1 = nR(250)$

$P_2(2V_1) = 1.25\ nR(2000)\dfrac{P_2}{P_1} = 5$

22. The density of a solid metal sphere is determined by measuring its mass and its diameter. The maximum error in the density of the sphere is $\left(\frac{x}{100}\right)\%$. If the relative errors in measuring the mass and the diameter are 6.0% and 1.5% respectively, the value of 'x' is............

Key: **1050**

Sol: $P = \frac{6m}{\pi d^3}$

$$\frac{\Delta\rho}{\rho}\times 100 = \frac{\Delta m}{m}\times 100 + 3\frac{\Delta d}{d}\times 100$$

$$= 6 + 3\times 1.5 = 10.5\%$$

23. Suppose that intensity of a laser is $\left(\frac{315}{\pi}\right)W/m^2$. The rms electric field, in units of V/m associated with this source is close to the nearest integer is

$\left(\varepsilon_0 = 8.86\times 10^{-12} C^2Nm^{-2}; c = 3\times 10^8 ms^{-1}\right)$

Key: **194 or 195**

Sol: $I = \varepsilon_0 E_{rms}^2 c$

$$\frac{315}{\pi} = \varepsilon_0 E^2\left(3\times 10^8\right) \quad \frac{105\times 10^{-8}}{\pi\varepsilon_0} = E^2$$

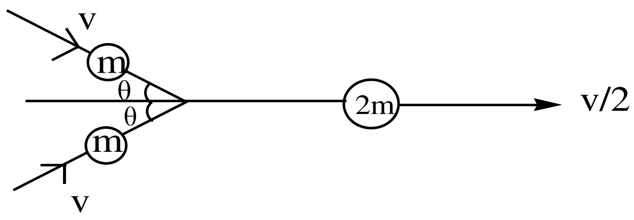
$$E^2 = 4\times 105\times 10^{-8}\times 9\times 10^9 = 36\times 105\times 10$$

$$E = \sqrt{36\times 1050} = 6\times 32.4 = 194.4 \ V/m$$

24. Two bodies of the same mass are moving with the same speed, but in different directions in a plane. They have a completely inelastic collision and move together thereafter with a final speed which is half of their initial speed. The angle between the initial velocities of the two bodies (in degree) is ....

Key: **120**

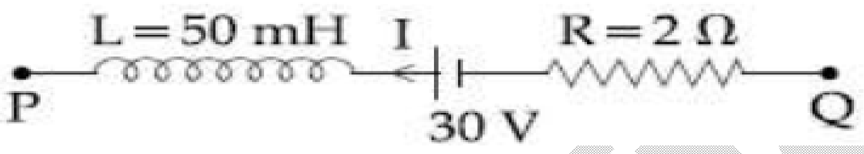
Sol: By COLM



$2mv\cos\theta = 2m \times \frac{v}{2}$ $\theta = 60^0$

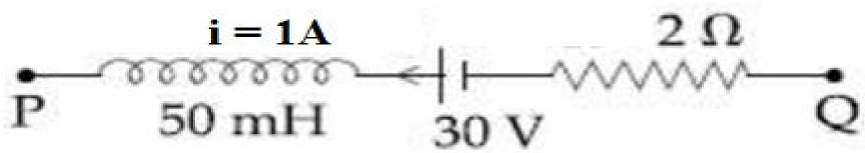
$2\theta = 120^0$

25. A part of a complete circuit is shown in the fig. At some instant, the value of current I is 1 A and it is decreasing at a rate of 10^2 A s^{-1}. The value of the potential difference $V_P - V_{Q'}$ (in volts) at that instant, is ……..



Key: **33**

Sol: $\frac{di}{dt} = -10^2$ A/s



$V_P - V_Q = -L\frac{di}{dt} + 30 - iR$

$= -\frac{50}{1000}\left(-10^2\right) + 30 - 1\times 2 = 33V$

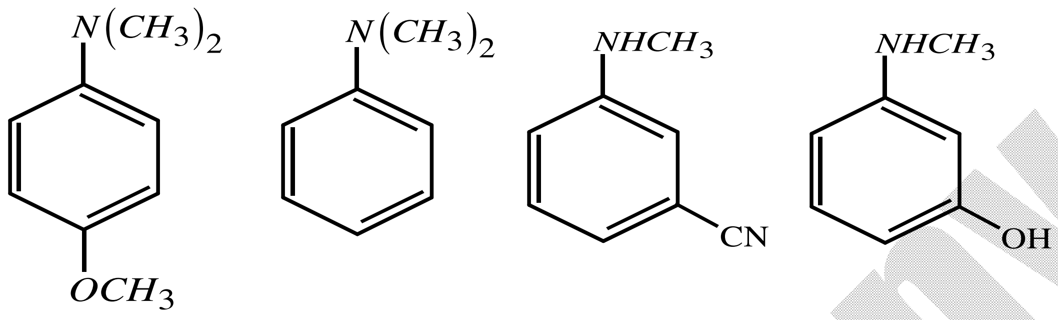
CHEMISTRY

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

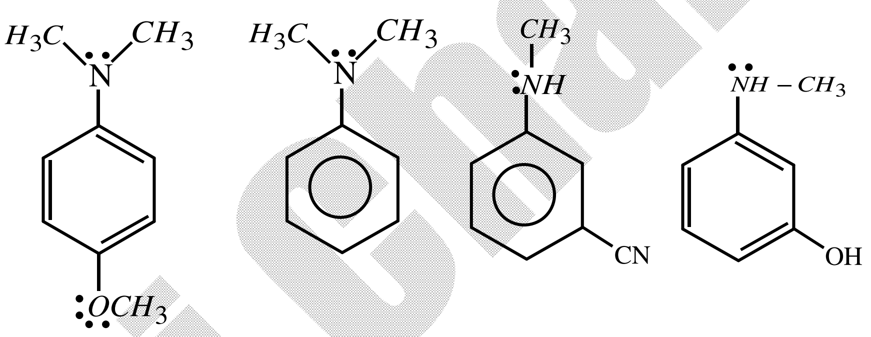
1. The increasing order of pK_b values of the following compounds is:



1) I < II < IV < III
2) II < IV < III < I
3) II < I < III < IV
4) I < II < III < IV

Key: **1**

Sol:



I +M | II | III -I max | IV -I min

Electron donating groups α basic strength

$K_b\ order \Rightarrow I > II > IV > III$

$_pK_b\ order \Rightarrow III > IV > II > I$

$K_b\ \alpha\ \frac{1}{_pK_b}$

2. The species that has a spin-only magnetic moment of 5.9 BM, is: (T.H = tetrahedral)

1) $[Ni(CN)_4]^{2-}$ (square planar) 2) $[MnBr_4]^{2-}$ (T.H)

3) $[NiCl_4]^{2-}$ (T.H) 4) $[Ni(CO)_4]$(T.H)

Key: **2**

Sol:

$i)\ [Ni(CN)_4]^{2-}$

No. of 'd' electrons $= z - x - 18$

$= 28 - 2 - 18$

$= d^8$

$CN^- \rightarrow strong\ ligand$

$dx^2 - y^2 > d_{xy}^2 > d_{z^2}^2 > d_{yz}^2 = d_{zx}^2$

No. of unpaired $e^- = 0$

dia magnetic species

$ii)\ (M_nBr_4)^{2-}$

No. of $electrons\ d = 25 - 2 - 18 = d^5$

$\mu = \sqrt{n(n+2)}B.M$

$Br^- \rightarrow$ weak ligand and obeys Hund's rule of multiplicity and contains 5 unpaired e^-

$iii)\ (NiCl_4)^{2-}\ \ No.of\ d\ e^- = 28 - 2 - 18 = d^8$

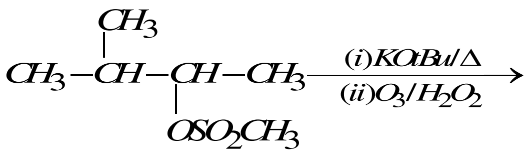
No.of unpaired $e^- = 2$

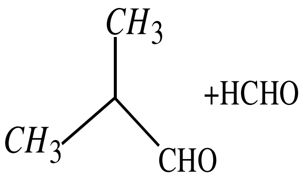
$iv)\ (Ni(CO)_4)$

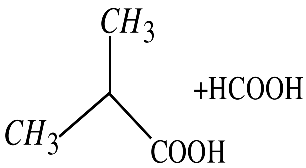
No.of $d\ \ e^- = 28 - 0 - 18 = d^{10}$

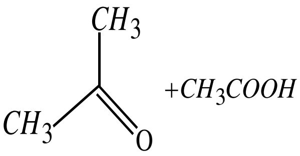
Dia magnetic species answer is $[MnBr_4]^{2-}$ (T.H).

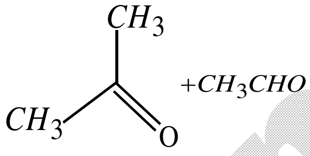
3. The major products of the following reaction are



1) 

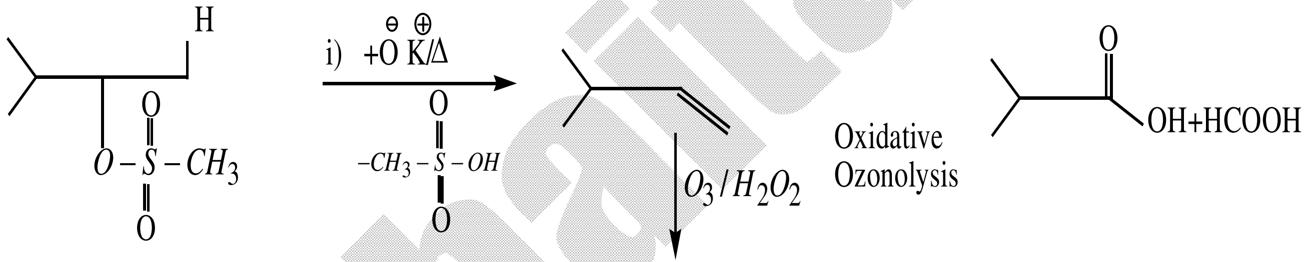
2) 

3) 

4) 

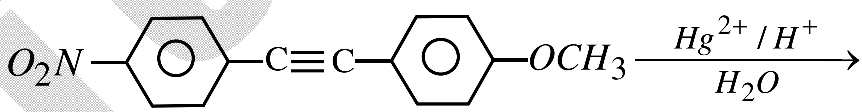
Key: **2**

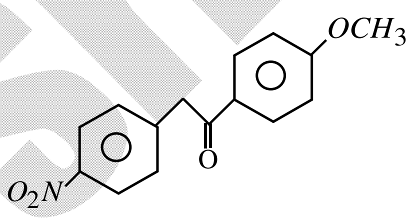
Sol:

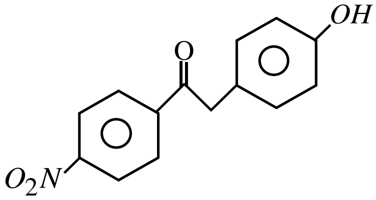


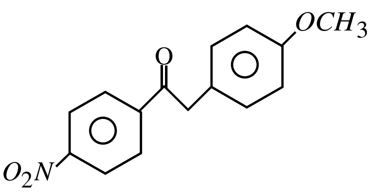
$\Rightarrow t_{Bu}O^{\odot}K^{\oplus}$ is bulky base less substituted alkene is major.

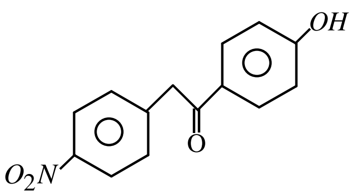
4. The major product obtained from the following reaction is



1) 

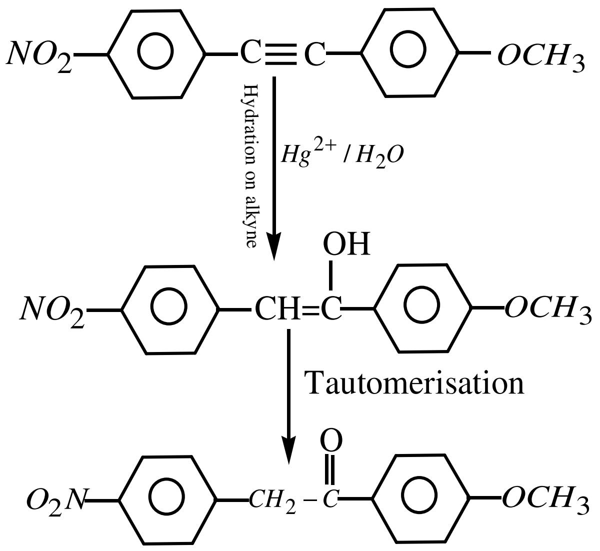
2) 

3) 

4) 

Key: **1**

Sol:



(Final product)

5. For the reaction $Fe_2N(s)+\frac{3}{2}H_2(g) \rightleftharpoons 2Fe(s)+NH_3(g)$

1) $K_c = K_p(RT)^{3/2}$
2) $K_c = K_p(RT)$
3) $K_c = K_p(RT)^{-1/2}$
4) $K_c = K_p(RT)^{1/2}$

Key: **4**

Sol:

$$Fe_2N_{(s)}+\frac{3}{2}H_{2(g)} \rightarrow 2Fe_{(s)}+NH_{3(g)}$$

$\Delta ng =$ no. of moles of gaseous products - no. of moles of gaseous reactants

$$=1-\frac{3}{2}$$

$$=-1/2$$

$$K_p = K_c(RT)^{\Delta n_g}$$

$$\Rightarrow K_p = K_c(RT)^{-1/2}$$

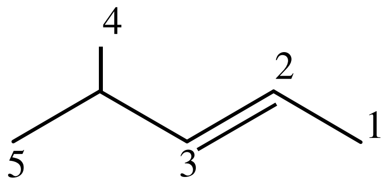
$$\Rightarrow K_c = K_p(RT)^{1/2}$$

6. Which of the following compounds shows geometrical isomerism ?

1) 2 – methylpent – 1 – ene
2) 2 – methylpent – 2 – ene
3) 4 – methylpent – 1 – ene
4) 4 – methylpent – 2 – ene

Key: **4**

Sol:



4-methyl pent-2ene

This compound shows geometrical isomerism

7. The correct statement with respect to dinitrogen is

1) It can combine with dioxygen at 25^0C
2) N_2 is paramagnetic in nature
3) Liquid dinitrogen is not used in cryosurgery
4) It can be used as an inert diluent for reactive chemicals

Key: **4**

Sol:

$$i) N_2 + O_2 \xrightarrow{25^0 C} No\ reaction$$

$$ii) N_2 \rightarrow dia\ magnetic\ gas$$

$$iii)\ liq\ N_2 is\ used\ in\ cryosurgery$$

$$iv)\ statement\ is\ correct$$

$$S + F_2 \xrightarrow[with\ N_2]{diluted} SF_4$$

i) Ist reaction is possible only at high temp

ii) N_2 contains completely filled bonding molecular orbitals.

8. The variation of equilibrium constant with temperature is given below

Temperature	Equilibrium Constant
$T_1 = 25^0 C$	$K_1 = 10$
$T_2 = 100^0 C$	$K_2 = 100$

The values of $\Delta H^0, \Delta G^0$ at T_1 and ΔG^0 at T_2 $\left(\text{in kJ mol}^{-1}\right)$ respectively, are close to

$\left[\text{use } R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}\right]$

1) $0.64, -5.71$ and -14.29
2) $0.64, -7.14$ and -5.71
3) $28.4, -5.71$ and -14.29
4) $28.4, -7.14$ and -5.71

Key: **3**

Sol: $$\log_{10}\left(\frac{K_2}{K_1}\right) = \frac{\Delta H^0}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\Rightarrow \log(10) = \frac{\Delta H^0}{2.303R}\left[\frac{1}{298} - \frac{1}{373}\right]$$

$$\Rightarrow \frac{\Delta H^0}{2.303 \times 8.314} = \left[\frac{298 \times 373}{75}\right]$$

$$\Rightarrow \Delta H^0 = 28377.1 \, J/mol$$

$$= 28.4 \, kJ/mol$$

At T_1

$$\Delta G^0 = -RT_1 \times 2.303 \, \log_{10} K_1$$

$$= -2.303 \times 8.314 \times 298 \log_{10}^{10}$$

$$= -5.71 \, kJ/mol$$

At T_2

$$\Delta G^0 = -2.303RT_2 \log K_2$$

$$= -2.303 \times 8.314 \times 373 \times 2$$

$$= -14.29 \, kJ/mol.$$

9. A solution of two components containing n_1 moles of the 1st component and n_2 moles of the 2nd component is prepared. M_1 and M_2 are the molecular weights of component 1 and 2 respectively. If d is the density of the solution in g mL^{-1}, C_2 is the molarity and x_2 is the mole fraction of the 2nd component, then C_2 can be expressed as

1) $C_2 = \frac{dx_1}{M_2 + x_2(M_2 - M_1)}$ 2) $C_2 = \frac{dx_2}{M_2 + x_2(M_2 - M_1)}$

3) $C_2 = \frac{1000dx_2}{M_1 + x_2(M_2 - M_1)}$ 4) $C_2 = \frac{1000x_2}{M_1 + x_2(M_2 - M_1)}$

Key: **3**

Sol: $C_2 = \frac{n_2}{V_{(ml)}} \times 1000 \quad \rightarrow 1$

$d = \frac{m}{V_{(ml)}}$

$\Rightarrow V = \frac{m}{d}$

$m = n_1M_1 + n_2M_2$

$V = \frac{n_1M_1 + n_2M_2}{d} \quad \rightarrow 2$

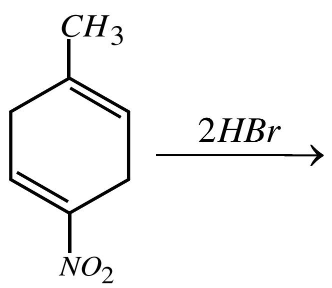
$x_2 = \frac{n_2}{n_1 + n_2} \quad \Rightarrow \quad n_2 = x_2(n_1 + n_2) \quad \rightarrow 3$

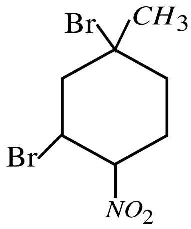
Substitute V, n_2 values in equation 1

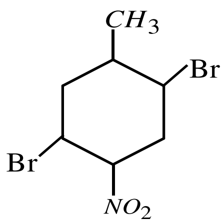
$$\Rightarrow \quad C_2 = \frac{x_2(n_1 + n_2)1000d}{n_1M_1 + n_2M_2}$$

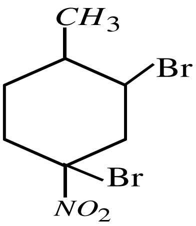
$$= \frac{x_2 1000d}{x_1M_1 + x_2M_2} = \frac{x_2 1000d}{(1 - x_2)M_1 + x_2M_2} = \frac{1000dx_2}{M_1 + [M_2 - M_1]x_2}.$$

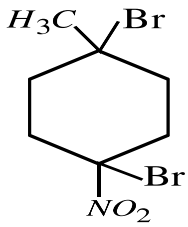
10. The major product of the following reaction is



1) 

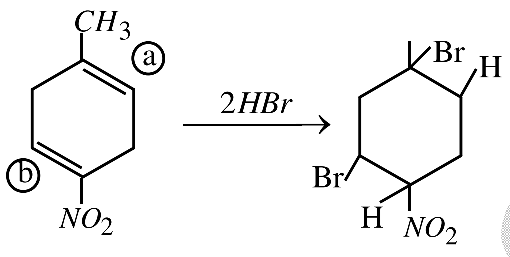
2) 

3) 

4) 

Key: **1**

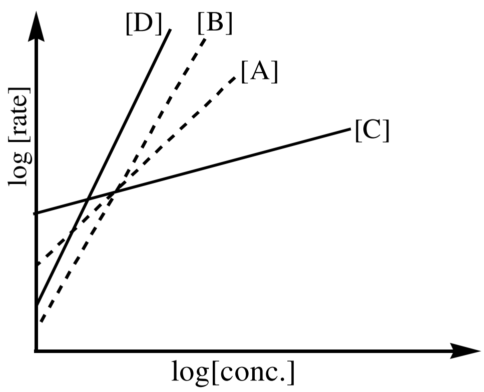
Sol:



'a' position markovnikov addition takes place

'b' position anti markovnikov addition takes place

11. Consider the following reactions $A \rightarrow P1; B \rightarrow P2; C \rightarrow P3; D \rightarrow P4$, The order of the above reactions are a, b, c and d, respectively. The following graph is obtained when log [rate] vs. log [conc.] are plotted



Among the following the correct sequence for the order of the reactions is

1) d > a > b > c　　2) a > b > c > d

3) d > b > a > c　　4) c > a > b > d

Key: **3**

Sol: $rate = K[conc]^n$

$where\ n = order$

$\Rightarrow \log rate = \log K + n \log[conc]$

$y = \quad C + mx$

order(n) is more, slope is more $d > b > a > c$.

12. Consider the Assertion and Reason given below.

Assertion (A): Ethene polymerized in the presence of Ziegler Natta Catalyst at high temperature and pressure is used to make buckets and dustbins

Reason (R): High density polymers are closely packed and are chemically inert.

Choose the correct answer from the following:

1) Bothe (A) and (R) are correct and (R) is the correct explanation of (A)

2) Bothe (A) and (R) are correct but (R) is not correct explanation of (A)

3) (A) is correct but (R) is wrong

4) (A) and (R) both are wrong

Key: **1**

Sol: Both state ments correct. As per NCERT.
Ziegler-Natta catalyst at a temperature of 3333 K to 343 K and under a pressure of 6-7 atm HDP $\in$ produced. Consists of linear molecules and has a high density due to close packing. Chemically Inert.

13. The set that contains atomic numbers of only transition elements is

1) 9,17,34,38　　2) 37,42,50,64

3) 21,32,53,64　　4) 21,25,42,72

Key: **4**

Sol: $i)\ 9 \rightarrow VIIA, 17 \rightarrow VIIA, 34 \rightarrow VIIA, 38 \rightarrow IIA$

$ii)\ 37 \rightarrow IA, 42 \rightarrow VIB, 50 \rightarrow IVA, 64 \rightarrow 4f.series\ element$

$iii)\ 21 \rightarrow IIIB, 32 \rightarrow IVA, 53 \rightarrow VIIA, 64 \rightarrow Lanthanide$

$iv)\ 21 \rightarrow IIIB, 25 \rightarrow VIIB, 42 \rightarrow VIB, 72 \rightarrow IB - Transitional\ elements$

$3d \rightarrow 21-29$

$4d \rightarrow 39-47$

$5d \rightarrow 57, 72-79$

Transitional elements

14. Kraft temperature is the temperature

1) Above which the formation of micelles takes place

2) Below which the formation of micelles takes place

3) Below which the aqueous solution of detergents starts freezing

4) Above which the aqueous solution of detergents starts boiling

Key: **1**

Sol: kraft temperature is the temperature above which formation of micelle takes place.

15. Among the sulphates of alkaline earth metals, the solubilities of $BeSO_4$ and $MgSO_4$ in water, respectively are

1) high and high
2) poor and poor
3) poor and high
4) high and poor

Key: **1**

Sol: $SO_4^{2-}(sulphates)$ Solubility top to bottom decreases due to decreasing of Hydrational energy is more on comparison with decreasing of Lattice energy

$BeSO_4 > MgSO_4 > CaSO_4 > SrSO_4 > BaSO_4$.

16. Arrange the following solutions in the decreasing order of pOH

A) 0.01 M HCl B) 0.01 M NaOH C) 0.01 M CH_3COONa D) 0.01 M NaCl

1) $B > D > C > A$
2) $A > C > D > B$
3) $A > D > C > B$
4) $B > C > D > A$

Key: 3

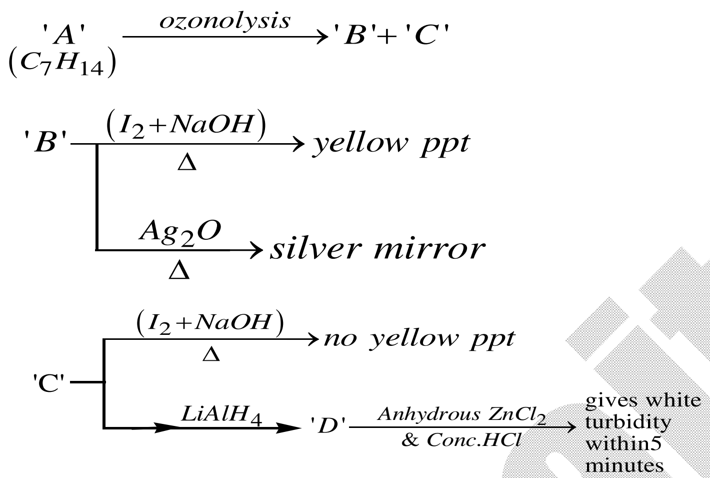
Sol: since *conc* for all solutions same

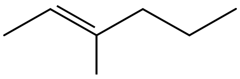
$\left[OH^-\right]$ order:- $NaOH > CH_3COONa > NaCl > HCl$

If $\left[OH^-\right]$ more, pOH is less

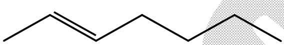
pOH order:- $HCl > NaCl > CH_3COONa > NaOH$.

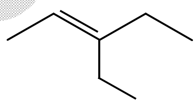
17. Consider the following reactions



1) 

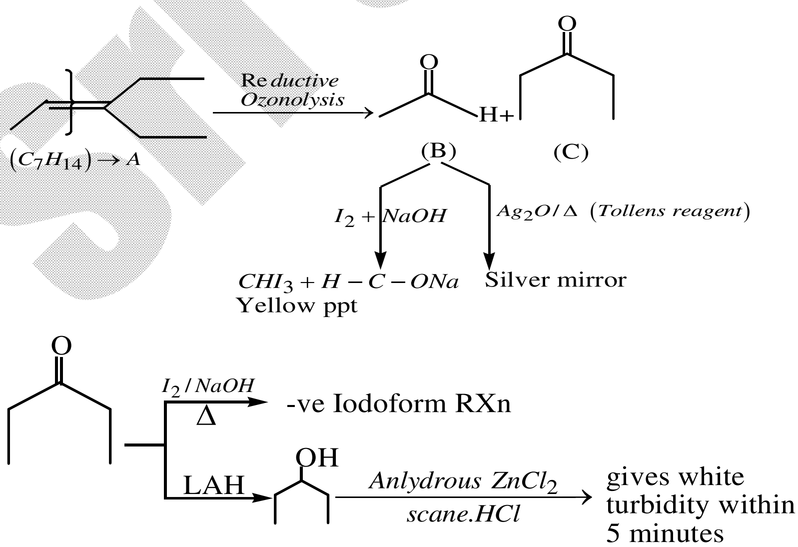
2) 

3) 

4) 

Key: 4

Sol:



2^0 – alcohols gives turbidity within '5' minutes.

18. The presence of soluble fluoride ion upto 1 ppm concentration in drinking water is

1) harmful to skin
2) safe for teeth
3) harmful for teeth
4) harmful to bones

Key: **2**

Sol: Fluoride ion conc upto 1ppm safe for teeth
Above 2ppm shows brown mottling of teeth
Above 3ppm harmful for bones.

19. The INCORRECT statement is

1) brass is an alloy of copper and nickel
2) german silver is an alloy of zinc copper and nickel
3) bronze is an alloy of copper and tin
4) cast iron is used to manufacture of wrought iron

Key: **1**

Sol: $i) Brass \rightarrow Cu + Zn$
$ii)$ $German\ silver \rightarrow Cu + Zn + Ni$
$iii)$ $Bronze \rightarrow Cu + Sn$
iv) Cast iron is used to manufacture of wrought Iron (purest form of Fe)
In correct statement is 1.

20. The lanthanoid that does not shows +4 oxidation state is

1) Tb
2) Dy
3) Eu
4) Ce

Key: **3**

Sol: i) $Ce, \text{Pr}, Nd, Tb\ and\ Dy\ shows + 4\ oxidation\ state$
ii) $Sm, Eu, Yb \rightarrow shows + 2\ state$
iii) Lanthanides common oxidation state is +3
iv) Eu shows only +2 or +3 states

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value (in decimal notation, truncated/ rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer , 0 if not attempted and 0 in all other cases.

21. The number of Cl=O bonds in perchloric acid is,"________________"

Key: **3**

Sol: $HClO_4$ $Structureis$

$$H-O-\overset{\overset{O}{\|}}{\underset{\underset{O}{\|}}{Cl}}=O$$

No. of $Cl=0$ $bonds \rightarrow 3$

22. The elevation of boiling point of 0.10 m aqueous $CrCl_3.xNH_3$ solution is two times that of 0.05 m aqueous $CaCl_2$ solution. The value of x is ______________.

[Assume 100% ionization of the complex and $CaCl_2$, coordination number of Cr as 6, and that all NH_3 molecules are present inside the coordination sphere]

Key: **5**

Sol: $\Delta T_{b_{complex}} = 2\Delta T_{b_{CaCl_2}}$

$iK_b \times 0.1 = 2 \times i \times K_b \times 0.05$

$CaCl_2 \rightarrow Ca^{2+} + 2Cl^-$ $\qquad i_{CaCl_2} = 3 \qquad \Rightarrow i = 3$

i.e no. of Ions after dissociation of complex '3'

$\left[Cr(NH_3)_5Cl\right]Cl_2 \rightarrow \left[Cr(NH_3)_5Cl\right]^- + 2Cl^- \qquad \therefore x = 5$

23. Potassium chlorate is prepared by the electrolysis of KCl in basic solution $6OH^- + Cl^- \rightarrow ClO_3^- + 3H_2O + 6e^-$. If only 60% of the current is utilized in the reaction, the time (rounded to the nearest hour) required to produce 10g of $KClO_3$ using a current of 2 A is______

(Given : F=96,500 C mol^{-1}; molar mass of $KClO_3 = 122$ g mol^{-1})

Key: **11**

Sol: $6OH^- + Cl^- \rightarrow ClO_3^- + 3H_2O + 6e^-$

For $6\times 96500\,C \to 122g\ \ KClO_3$

$Q \qquad\qquad \to 10g\ \ KClO_3$

$$Q=\frac{579\times 10^4}{122}=47459$$

Actural current used (i) $=0.6\times 2=1.2$

$Q=i\times t$

$$\Rightarrow \frac{47459}{1.2}=t \qquad \Rightarrow t=39549.2\sec$$

$t=11\ \ hours$

24. A spherical balloon of radius 3cm containing helium gas has a pressure of 48×10^{-3} bar. At the same temperature, the pressure, of a spherical balloon of radius 12cm containing the same amount of gas will be __________$\times 10^{-6}$ bar.

Key: **750**

Sol: $P_1V_1=P_2V_2$

$$\Rightarrow 48\times 10^{-3}\times r_1^3=P_2\times r_2^3 \quad \because V=\frac{4}{3}\pi r^3$$

$$\Rightarrow P_2=48\times 10^{-3}\times\left(\frac{3}{12}\right)^3 \qquad =0.75\times 10^{-3} \qquad =750\times 10^{-6}\ bar.$$

25. In an estimation of bromine by Carius method, 1.6 g of an organic compound gave 1.88 g of AgBr. The mass percentage of bromine in the compound is _________

(Atomic mass, Ag=108, Br =80g mol^{-1}).

Key: **50**

Sol: $$\because\ \%\ \ of\ \ halogen=\frac{(Atomic\ mass\ of\ 'X')\times(Mass\ of\ \ AgX)}{(108+Atomic\ mass\ of\ 'X')(Mass\ of\ organic\ compound)}\times 100$$

$$=\frac{80\times 1.88}{(108+80)\times 1.6}\times 100 \qquad =\frac{15040}{300.8}$$

$\%\ of\ 'Br'=50\%$.

MATHEMATICS

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated?

1) $(3!)^3.(4!)$ 2) $3!.(4!)^3$ 3) $2!3!4!$ 4) $(3!)^2(4!)$

Key: **1**

Sol: Let A_1, A_2, A_3 be members of one family
Let B_1, B_2, B_3 be members of another family
Let C_1, C_2, C_3, C_4 be members of another family

$A_1\ A_2\ A_3$	$B_1\ B_2\ B_3$	$C_1\ C_2\ C_3\ C_4$
1 unit	1 unit	1 unit

Number of ways of arranging 3 units $=(3P_3)=\lfloor 3$
Number of internal arrangements of $A_1, A_2, A_3 =(3P_3)=\lfloor 3$
Number of internal arrangements of $B_1, B_2, B_3 =(3P_3)=\lfloor 3$
Number of internal arrangements of $C_1, C_2, C_3, C_4 =(4P_4)=\lfloor 4$
$\therefore$ Total Number of arrangements, so that members of same family are not separated
$=(\lfloor 3\times\lfloor 3\times\lfloor 3\times\lfloor 4)=(\lfloor 3)^3.\lfloor 4$

2. Which of the following points lies on the locus of the foot of perpendicular drawn upon any tangent to the ellipse $\frac{x^2}{4}+\frac{y^2}{2}=1$ from any of its foci?

1) $(1, 2)$ 2) $(-2, \sqrt{3})$ 3) $(-1, \sqrt{2})$ 4) $(-1, \sqrt{3})$

Key: **4**

Sol: Auxiliary circle of $\frac{x^2}{4}+\frac{y^2}{2}=1$ is $x^2+y^2=4$ satisfies $(-1,\sqrt{3})$

3. $\lim_{x\to 1}\left(\dfrac{\int_0^{(x-1)^2} t\cos\left(t^2\right)dt}{(x-1)\sin(x-1)}\right)$

1) is equal to $\frac{1}{2}$
2) is equal to $-\frac{1}{2}$
3) does not exist
4) is equal to 1

Key: **BONUS**

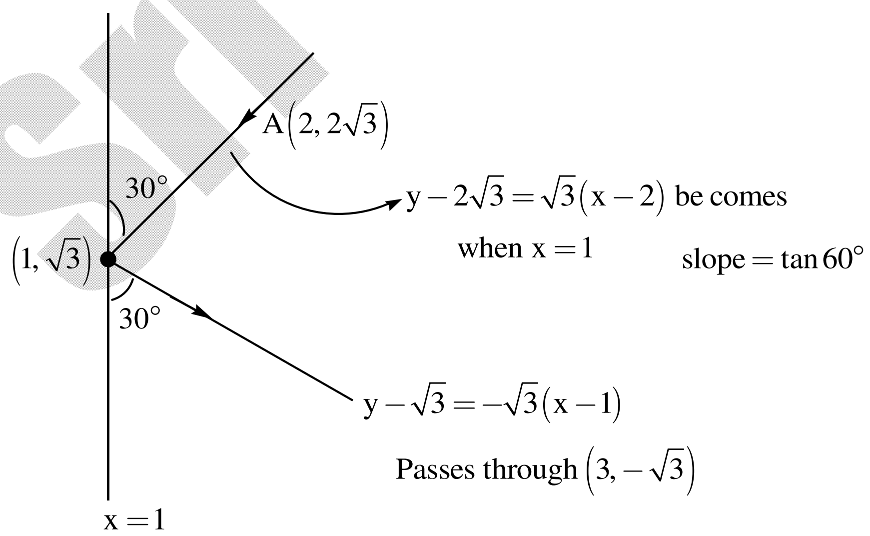
Sol: $\mathop{Lt}_{x\to 1}\dfrac{\int_0^{(x-1)^2} t.\cos(t)^2\,dt}{(x-1)\sin(x-1)}$ $1\mathop{Lt}_{x\to 1}\dfrac{\int_0^{(x-1)^2} t.\cos(t)^2\,dt}{(x-1)^2}$

$\mathop{Lt}_{x\to 1}\dfrac{(x-1)^2\cos(x-1)^4\,2(x-1)}{2(x-1)}$ $= 0$

4. A ray of light coming from the point $\left(2,2\sqrt{3}\right)$ is incident at an angle 30° on the line $x=1$ at the point A. The ray gets reflected on the line $x=1$ and meets x-axis at the point B. Then, the line AB passes through the point:

1) $\left(3,-\frac{1}{\sqrt{3}}\right)$ 2) $\left(4,-\sqrt{3}\right)$ 3) $\left(3,-\sqrt{3}\right)$ 4) $\left(4,-\frac{\sqrt{3}}{2}\right)$

Key: **3**
Sol:



5. Let L_1 be a tangent to the parabola $y^2 = 4(x+1)$ and L_2 be a tangent to the parabola $y^2 = 8(x+2)$ such that L_1 and L_2 intersect at right angles. Then L_1 and L_2 meet on the straight line

1) $x+3=0$ 2) $x+2=0$ 3) $2x+1=0$ 4) $x+2y=0$

Key: **1**

Sol: $y^2 = 4(x+1)$ (1)

$y^2 = 8(x+2)$ (2)

$y = m(x+1) + \frac{1}{m}$ is tangent to (1)

$y = -\frac{1}{m}(x+2) - 2m$ is tangent to (2)

$m(x+1) + \frac{1}{m} = \frac{-1}{m}(x+2) - 2m.$

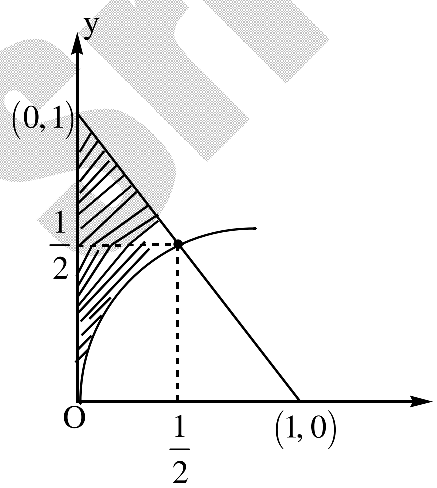
$x\left(m + \frac{1}{m}\right) = -m\frac{1}{m} - \frac{2}{m} - 2m \quad = \left(m + \frac{1}{m}\right)(-3) \quad x+3=0$

6. The area (in sq. units) of the region $A = \{(x,y): |x|+|y| \le 1,\ 2y^2 \ge |x|\}$ is :

1) $\frac{1}{3}$ 2) $\frac{7}{6}$ 3) $\frac{1}{6}$ 4) $\frac{5}{6}$

Key: **4**

Sol:



$|x|+|y| \le 1$ represent region of inside a square

$2y^2 \geq |x| \quad \Rightarrow \begin{cases} 2y^2 \geq x & \text{if } x \geq 0 \\ \geq -x & \text{if } x < 0 \end{cases}$

which represents region outside/on the two parabolas $y^2 = x/2$ and $y^2 = -x/2$

By symmetry,

the required area $= 4 \times [\text{Area bounded in } Q_1]$

$$= 4 \times \left| \text{Area bounded by } x + y = 1,\ y^2 = \frac{x}{2} \text{ and } y - \text{axis} \right|$$

$$= 4 \times \left[\int_0^{\frac{1}{2}} x\,dy + \text{Area of } \Delta^{le} \text{ formed by } x + y = 1, y = \frac{1}{2},\ y - \text{axis} \right]$$

$$= 4 \times \left[\int_0^{\frac{1}{2}} (2y^2)\,dy + \frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \right]$$

$$= 4 \times \left[\left[\frac{2y^3}{3}\right]_0^{\frac{1}{2}} + \frac{1}{8} \right] = 4 \times \left[\frac{2}{3} \times \frac{1}{8} + \frac{1}{8}\right] = \frac{1}{3} + \frac{1}{2} = \frac{5}{6} \text{ sq.units}$$

7. Let m and M be respectively the minimum and maximum values of

$$\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$$

Then the ordered pair (m, M) is equal to:

1) $(-3,3)$ 2) $(1,3)$ 3) $(-3,-1)$ 4) $(-4,-1)$

Key: **3**

Sol: $\Delta = \begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$

by $C_1 \to C_1 + C_2$

$$\Delta = \begin{vmatrix} 2 & 1+\sin^2 x & \sin 2x \\ 2 & \sin^2 x & \sin 2x \\ 1 & \sin^2 x & 1+\sin 2x \end{vmatrix}$$

by $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$\Delta = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & \sin^2 x & 1+\sin 2x \end{vmatrix}$$

$$\Rightarrow \Delta = (-1)[(1+\sin 2x)-(-1)] = -(2+\sin 2x)$$

(by expanding along R_1)

Now $\begin{cases} M = \max(\Delta) = -1 \\ m = \min(\Delta) = -3 \end{cases}$

$\therefore (m, M) = (-3, -1)$

8. If $f(x+y) = f(x)f(y)$ and $\sum_{x=1}^{\infty} f(x) = 2$, $x, y \in N$, where N is the set of all natural numbers, then the value of $\frac{f(4)}{f(2)}$ is:

1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) $\frac{1}{9}$ 4) $\frac{4}{9}$

Key: **4**

Sol: $f(x+y) = f(x).f(y); x, y \in N \to (1)$ and $\sum_{x=1}^{\infty} f(x) = 2 \to (2)$

Let $f(x) = a^x$

from (2) $\sum_{x=1}^{\infty}(a^x) = 2 \Rightarrow \left(\frac{a}{1-a}\right) = 2 \therefore a = \frac{2}{3}$

$\therefore f(x) = \left(\frac{2}{3}\right)^x \qquad \frac{f(4)}{f(2)} = \frac{(2/3)^4}{(2/3)^2} = (2/3)^2 = 4/9$

9. The shortest distance between the lines

$$\frac{x-1}{0}=\frac{y+1}{-1}=\frac{z}{1} \text{ and } x+y+z+1=0,\ 2x-y+z+3=0 \text{ is:}$$

1) 1 2) $\frac{1}{\sqrt{3}}$ 3) $\frac{1}{\sqrt{2}}$ 4) $\frac{1}{2}$

Key: **2**

Sol: $L_1:\frac{x-1}{0}=\frac{y+1}{-1}=\frac{z}{1}$

$L_2: x+y+z+1=0, 2x-y+z+3=0$

Equation of plane $(\pi=0)$ through L_2 and parallel to L_1 is

$x+y+z+1+\lambda(2x-y+z+3)=0$

$0(1+2\lambda)-1(1-\lambda)+1(1+\lambda)=0$

$-1+\lambda+1+\lambda=0$

$\lambda=0$

$x+y+z+1=0$

Shortest distance =distance of $(1,-1,0)$ from $x+y+z+1=0$ is $\frac{|\cancel{1}-\cancel{1}+0+1|}{\sqrt{1+1+1}}=\frac{1}{\sqrt{3}}$

10. If α and β be two roots of the equation $x^2-64x+256=0$. Then the value of

$$\left(\frac{\alpha^3}{\beta^5}\right)^{1/8}+\left(\frac{\beta^3}{\alpha^5}\right)^{1/8} \text{ is:}$$

1) 3 2) 2 3) 4 4) 1

Key: **2**

Sol: α,β are roots of $x^2-64x+256=0$

$\Rightarrow \alpha+\beta=64$ and $\alpha\beta=256=2^8$

$$\left(\frac{\alpha^3}{\beta^5}\right)^{1/8}+\left(\frac{\beta^3}{\alpha^5}\right)^{1/8}=\frac{\alpha+\beta}{(\alpha\beta)^{5/8}}=\frac{64}{32}=2$$

11. The position of a moving car at time t is given by $f(t) = at^2 + bt + c, t > 0$, where a, b and c are real numbers greater than1. Then the average speed of the car over the time interval $[t_1, t_2]$ is attained at the point

1) $a(t_2 - t_1) + b$
2) $(t_1 + t_2)/2$
3) $2a(t_1 + t_2) + b$
4) $(t_2 - t_1)/2$

Key: **2**

Sol: $f(t) = at^2 + bt + c, \ t > 0$

$f'(t) = 2at + b$

$$\text{Average} = \frac{2at_1 + b + 2at_2 + b}{2} = a(t_1 + t_2) + b \ \text{ at point } \frac{t_1 + t_2}{2}$$

12. Out of 11 consecutive natural numbers if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference, is:

1) $\frac{5}{33}$
2) $\frac{10}{99}$
3) $\frac{15}{101}$
4) $\frac{5}{101}$

Key: **1**

Sol:

Consider 11 consecutive natural numbers 1, 2, 3, ...,11

3 numbers are chosen (without repetition)

$\Rightarrow n(S) = \left({}^{11}C_3\right)$

E be the event that the selected numbers are in A.P.

Number of A.P's with common difference $1 = 9$

Number of A.P's with common difference $2 = 7$

Number of A.P's with common difference $3 = 5$

Number of A.P's with common difference $4 = 3$

Number of A.P's with common difference $5 = 1$

$\therefore n(E) = 9 + 7 + 5 + 3 + 1 = 25$

$$\therefore P(E) = \frac{25}{{}^{11}C_3} = \left(\frac{25 \times 6}{11 \times 10 \times 9}\right) = \frac{5}{33}$$

13. Let a, b, c, d and p be any non zero distinct real numbers such that

$$\left(a^2+b^2+c^2\right)p^2-2(ab+bc+cd)p+\left(b^2+c^2+d^2\right)=0.\text{ Then :}$$

1) a, b, c, d are in G.P
2) a, c, p are in G.P
3) a, c, p are in A.P
4) a, b, c, d are in A.P

Key: **1**

Sol: a, b, c, d $\in R^{\circ}$ are all different

Given $\left(a^2+b^2+c^2\right)p^2-2(ab+bc+cd)p+\left(b^2+c^2+d^2\right)=0$

$\Rightarrow (ap-b)^2+(bp-c)^2+(cp-d)^2=0$

$\Rightarrow ap-b=0$ and $bp-c=0$ and $cp-d=0$

$\Rightarrow p=\frac{b}{a}=\frac{c}{b}=\frac{d}{c}$

$\Rightarrow a,b,c,d$ are in G.P

14. The general solution of the differential equation $\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$ is:

(where C is a constant of integration)

1) $\sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right)+C$

2) $\sqrt{1+y^2}-\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right)+C$

3) $\sqrt{1+y^2}-\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right)+C$

4) $\sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right)+C$

Key: **4**

Sol: $\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$

$$\sqrt{1+x^2}\sqrt{1+y^2}+xy\frac{dy}{dx}=0$$

$$\frac{\sqrt{1+x^2}}{x}dx+\frac{y}{\sqrt{1+y^2}}dy=0$$

Integrate $\frac{\sqrt{1+x^2}}{x}dx+\sqrt{1+y^2}=c$

$2tdt = 2+dt$

$1+x^2=t^2$ $dx=\frac{tdt}{x}$

$$\int\frac{t^2}{t^2-1}dt+\sqrt{1+y^2}=c$$

$$\int\left(\frac{t^2-1}{t^2-1}+\frac{1}{t^2-1}\right)+\sqrt{1+y^2}=c$$

$$t+\frac{1}{2}\log\left|\frac{t-1}{t+1}\right|+\sqrt{1+y^2}=c$$

$$\sqrt{1+x^2}+\frac{1}{2}\log\left|\frac{t-1}{t+1}\right|+\sqrt{1+y^2}=c$$

15. If $\sum_{i=1}^{n}(x_i-a)=n$ and $\sum_{i=1}^{n}(x_i-a)^2=na,\ (n,a>1)$ then the standard deviation of n observations $x_1, x_2,....,x_n$ is:

1) $a-1$ 2) $n\sqrt{a-1}$ 3) $\sqrt{n(a-1)}$ 4) $\sqrt{a-1}$

Key: **4**

Sol: Let $(x_i-a)=d$

Given $\sum_{i=1}^{n}(x_i-a)=n$ & $\sum_{i=1}^{n}(x_i-a)^2=na$

$$\Rightarrow\sum_{i=1}^{n}d_i=n \text{ and } \sum_{i=1}^{n}(d_i)^2=na$$

$$S.D=\sqrt{\frac{\sum d_i^2}{n}-\left(\frac{\sum d_i}{n}\right)^2}=\sqrt{\frac{na}{n}-\left(\frac{n}{n}\right)^2}=\sqrt{a-1}$$

16. The values of λ and μ for which the system of linear equations

$$x+y+z=2$$
$$x+2y+3z=5$$
$$x+3y+\lambda z=\mu$$

has infinitely many solutions are, respectively :

1) 4 and 9 2) 5 and 7 3) 6 and 8 4) 5 and 8

Key: **4**

Sol:

$$\left.\begin{array}{ll} x+y+z=2 & (1) \\ x+2y+3z=5 & (2) \\ x+3y+\lambda z=m & (3) \end{array}\right\} \text{has infinitely} - \text{many solutions}$$

$$\left.\begin{array}{ll} (2)-(1) \text{ gives } y+2z=3 & \rightarrow (4) \\ (3)-(2) \text{ gives } y+(\lambda-3)z=\mu-5 & \rightarrow (5) \end{array}\right\} \text{has infinitely} - \text{many solutions}$$

$$\Rightarrow \frac{1}{1}=\frac{\lambda-3}{2}=\frac{\mu-5}{3}$$

$$\Rightarrow \lambda=5 \text{ and } \mu=8$$

17. The negation of the Boolean expression $p \vee (\sim p \wedge q)$ is equivalent to :

1) $\sim p \wedge \sim q$ 2) $p \wedge \sim q$ 3) $\sim p \vee \sim q$ 4) $\sim p \vee q$

Key: **1**

Sol:

$$\sim (p \vee (\sim p \wedge q))$$

$$\equiv \sim ((p \vee \sim p)) \wedge (p \vee q) \text{ by distributive law}$$

$$\equiv \sim (t \wedge (p \vee q)) \quad [\because p \vee \sim p \equiv t]$$

$$\equiv \sim (p \vee q) \quad [\because t \wedge s \equiv s]$$

$$\equiv (\sim p \wedge \sim q) \text{ by De Morgan law.}$$

18. The region represented by $\{z = x+iy \in C : |z| - \text{Re}(z) \leq 1\}$ is also given by the inequality:

1) $y^2 \geq x+1$ 2) $y^2 \leq x+\frac{1}{2}$ 3) $y^2 \leq 2\left(x+\frac{1}{2}\right)$ 4) $y^2 \geq 2(x+1)$

Key: **3**

Sol: $|z| - \text{Re}\, z \le 1$

$z = x + iy$

$\sqrt{x^2 + y^2} - x \le 1$

$\sqrt{x^2 + y^2} \le 1 + x$

$\cancel{x^2} + y^2 \le 1 + \cancel{x^2} + 2x$

$y^2 \le 1 + 2x$

19. If $I_1 = \int_0^1 \left(1 - x^{50}\right)^{100} dx$ and $I_2 = \int_0^1 \left(1 - x^{50}\right)^{101} dx$ such that $I_2 = \alpha I_1$ then α equals to :

1) $\frac{5050}{5051}$ 2) $\frac{5051}{5050}$ 3) $\frac{5049}{5050}$ 4) $\frac{5050}{5049}$

Key: **1**

Sol: $I_2 = \int_0^1 \left(1 - x^{50}\right)^{100} dx$

$= \left(\left(1 - x^{50}\right)^{101} x\right)_0^1 - 0\int_0^1 101(1 - x50)^{100} \left(-50x^{49}\right) x dx$

$I_2 = 0 - 5050\int_0^1 \left(\left(-x^{50}\right)^{100} \left(-x^{50} + 1 - 1\right)\right) dx$

$I_2 = -5050 I_2 + 5050 I_1 \qquad \Rightarrow I_2 = \frac{5050}{5051} I_1$

20. If $\{p\}$ denotes the fractional part of the number p then $\left\{\frac{3^{200}}{8}\right\}$ is equal to:

1) $\frac{7}{8}$ 2) $\frac{1}{8}$ 3) $\frac{3}{8}$ 4) $\frac{5}{8}$

Key: **2**

Sol: $\left\{\frac{3^{200}}{8}\right\}=$ _____ Where $\{.\}$ denotes fractional part

We have $\frac{3^{200}}{8}=\frac{(9^{100})}{8}=\frac{(1+8)^{100}}{8}$

$=\frac{1}{8}\left[{}^{100}C_0+{}^{100}C_1(8)+{}^{100}C_2(8^2)+.....+{}^{100}C_{100}(8^{100})\right]$

$=\frac{1}{8}[1+8k]$ for some $k\in N$

$\therefore\left\{\frac{3^{200}}{8}\right\}=\left\{k+\frac{1}{8}\right\}=\frac{1}{8}$

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value (in decimal notation, truncated/ rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. Set A has m elements and Set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of m.n is

Key: **28**

Sol: Given $n(A)=m$ and $n(B)=n$

(Number of subsets of A) –(Number of subsets of B) =112

$\Rightarrow(2^m)-(2^n)=112=(2^7)-(2^4)$

$\because m=7$ and $n=4\Rightarrow m.n=28$

22. Let $f:R\to R$ be defined as

$$f(x)=\begin{cases}x^5\sin\left(\frac{1}{x}\right)+5x^2, & x<0\\ 0, & x=0\\ x^5\cos\left(\frac{1}{x}\right)+\lambda x^2, & x>0\end{cases}$$

The value of λ for which $f''(0)$ exists, is

Key: **5**

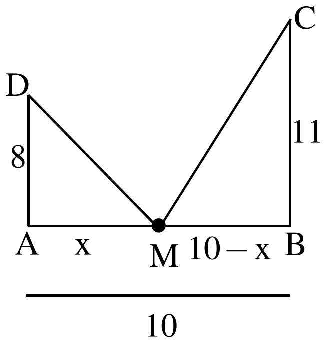
Sol: $f''(0)$ exist if $\lambda=5$

$L(f''(0))=10=2\lambda=R(f''(0))$

23. Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If AD = 8 m ,BC =11 m and AB =10 m; then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimum is ………

Key: **5**

Sol:



$$f(x) = DM^2 + MC^2 = x^2 + 64 + 11^2 + (10-x)^2$$
$$f'(x) = 2x - 2(10-x) = 0 \Rightarrow x = 5$$
$$f''(x) > 0$$

24. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}\left|\vec{a}+\vec{b}\right| + \left|\vec{a}-\vec{b}\right|$ is

Key: **4**

Sol: $\left|\bar{a}+\bar{b}\right|^2 = |a|^2 + \left|\bar{b}\right|^2 + 2\bar{a}.\bar{b}$

$= 2 + 2\cos\theta$ where $\theta = (\bar{a}, \bar{b})$

$= 4\cos^2\theta/2$

$\left|\bar{a}-\bar{b}\right| = 2 - 2\cos\theta$

$= 4\sin^2\theta/2$

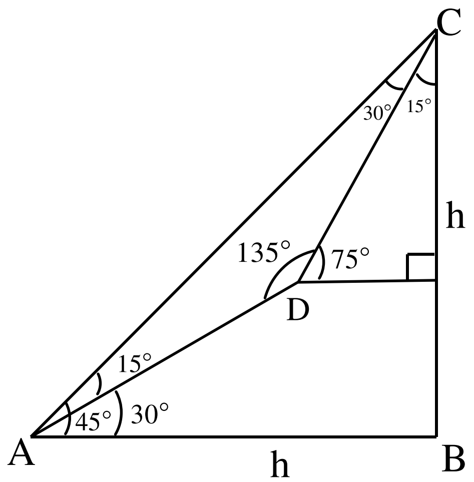
$\sqrt{3}\left|\bar{a}+\bar{b}\right| + \left|\bar{a}-\bar{b}\right| = 2\sqrt{3}\cos\theta/2 + 2\sin\theta/2$

$\le \sqrt{12+4} = 16 = 4$

25. The angle of elevation of the top of a hill from a point on the horizontal plane Passing through the foot of the hill is found to be $45°$. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of $30°$ to the horizontal plane, the angle of elevation of the top of the hill becomes $75°$. Then the height of the hill (in meters) is………………..

Key: **80**

Sol:



Δ^{le}ACD, by sine rule

$$\frac{80}{\sin 30°} = \frac{AC}{\sin 135°}$$

$$80 \times 2 = AC\sqrt{2}$$

$$\Delta^{le}ABC,\ x^2 + x^2 = \left(80\sqrt{2}\right)^2$$

$$2x^2 = (80)^2(2)$$

$$x^2 = (80)^2$$

$$x = 80$$

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