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A.P ✪ T.S ✪ Chandigarh ✪ Karnataka ✪ Tamilnadu ✪ Maharastra ✪ Delhi ✪ Ranchi

Jee Main 2020(Sep)

06-Sep-2020 (Evening Shift)

Question Paper, Key and Solutions

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Physics

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. A student measuring the diameter of a pencil of circular cross-section with the help of a vernier scale records the following four readings 5050 mm, 5.45 mm; 5.65 mm. The average of these four readings is 5.5375 mm and the standard deviation of the data is 0.073.95 mm. The deviation of the data is 0.7395 mm. The average diameter of the pencil should therefore be recorded as :

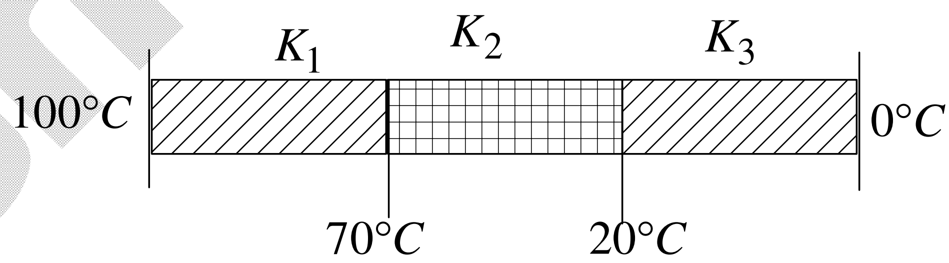
1) $(5.5375 \pm 0.0739)mm$ 2) $(5.5375 \pm 0.0740)mm$

3) $(5.54 \pm 0.07)mm$ 4) $(5.538 \pm 0.074)mm$

Key : 3

Sol : As significant figures in each measurement is 3, the diameter should be recorded in three significant figures.
$\therefore$ key is option (3).

2. Three rods of identical cross-section and lengths are made of three different materials of thermal conductivity K_1, K_2 and K_3 respectively. They are joined together at their ends to make a long rod (see figure). One end of the long rod is maintained at 100°C and the other at 0°C (see figure). If the joints of the rod are at 70°C and 20°C in steady state and there is no loss of energy from the surface of the rod, the correct relationship between K_1, K_2 and K_3



1) $K_1 > K_2 > K_3$ 2) $K_1:K_2=5:2,\ K_1:K_3=3:5$ 3) $K_1:K_3=2:3,\ K_2:K_3=2:5$ 4) $K_1 < K_2 < K_3$

Key : 3

Sol : Let cross-section area and length of each rod is A and l respectively. Then current (thermal) through eaqch of them are equal

So $\frac{(100-70)AK_1}{l}=\frac{(70-20)AK_2}{l}=\frac{(20-0)AK_3}{l}$

$\Rightarrow 30K_1=50K_2=20K_3 \qquad \Rightarrow \frac{K_1}{K_3}=\frac{2}{3}$ and $\frac{K_2}{K_3}=\frac{2}{5}$

3. When a car is at rest, its driver sees rain drops falling on it vertically. When driving the car with speed v, he sees that rain drops are coming at an angle 60° from the horizontal. On further increasing the speed of the car to $(1+\beta)v$, this angle changes to 45°. The value of β is close to :

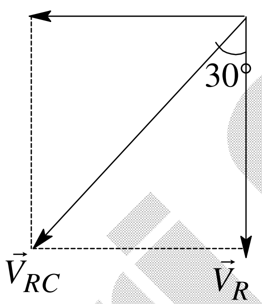
1) 0.41 2) 0.73 3) 0.37 4) 0.50

Key : 2

Sol : $\vec{V}_{RC}=\vec{V}_R-\vec{V}_C$

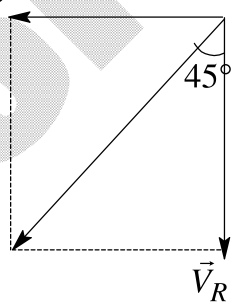
So $\tan 30°=\frac{\vartheta}{\vartheta_R}=\frac{1}{\sqrt{3}}$

$-\vec{V}_C=\upsilon$



Next, let speed of the car is $\vartheta^1=(1+\beta)\vartheta$

$V_c=\upsilon'$



So $V^1=V_R=\sqrt{3}\vartheta \qquad \Rightarrow (1+\beta)\vartheta=\sqrt{3}\vartheta$

$\Rightarrow \beta=\sqrt{3}-1=0.73$

4. For a plane electromagnetic wave, the magnetic field at a point x and time t is

$$\vec{B}(x,t)=\left[1.2\times10^{-7}\sin\left(0.5\times10^{3}x+1.5\times10^{11}t\right)\hat{k}\right]T$$

The instantaneous electric field $\vec{E}$ corresponding to $\vec{B}$ is (speed of light $c=3\times10^{8}ms^{-1}$)

1) $\vec{E}(x,t)=\left[36\sin\left(1\times10^{3}x+0.5\times10^{11}t\right)\hat{j}\right]\frac{V}{m}$

2) $\vec{E}(x,t)=\left[36\sin\left(0.5\times10^{3}x+1.5\times10^{11}t\right)\hat{i}\right]\frac{V}{m}$

3) $\vec{E}(x,t)=\left[-36\sin\left(0.5\times10^{3}x+1.5\times10^{11}t\right)\hat{j}\right]\frac{V}{m}$

4) $\vec{E}(x,t)=\left[36\sin\left(0.5\times10^{3}x+1.5\times10^{11}t\right)\hat{k}\right]\frac{V}{m}$

Key : 3

Sol : $E_o=cB_o=36\frac{V}{m}$

Also E-field and B-field should be mutually perpendicular and have the same frequency and wavelength as they both represents the same wave.

Therefore correct key is option : 3

5. Consider the force F on a charge 'q' due to a uniformly charged spherical shell of radius R carrying charge Q distributed uniformly over it. Which one of the following statements is true for F, if 'q' is placed at distance r from the centre of the shell ?

1) $F=\frac{1}{4\pi\in_0}\frac{Qq}{R^2}\ for\ r<R$

2) $F=\frac{1}{4\pi\in_0}\frac{Qq}{r^2}\ for\ r>R$

3) $\frac{1}{4\pi\in_0}\frac{qQ}{R^2}>F>0\ for\ r<R$

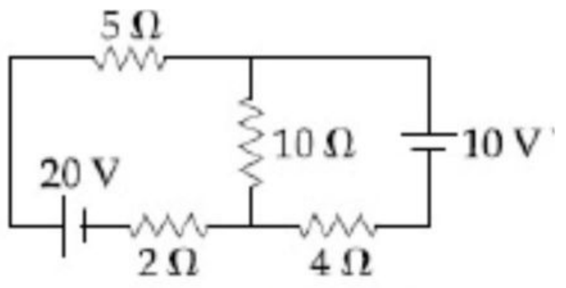
4) $F=\frac{1}{4\pi\in_0}\frac{Qq}{r^2}\ for\, all\ r$

Key : 2

Sol : Electric field inside the charged spherical shell is zero, so force should be zero for r < R.

For outside is $r>R,\ E=\frac{1}{4\pi\in_o}\frac{Q}{r^2}$ So, $F=\frac{1}{4\pi\in_o}\frac{Qq}{r^2}$

6.



In the figure shown, the current in the 10 V battery is close to :

1) 0.42 A from positive to negative terminal

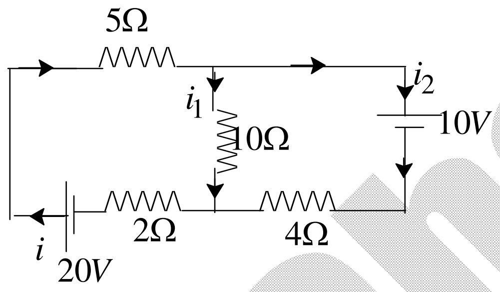
2) 0.21 A from positive to negative terminal

3) 0.36 A from negative positive terminal

4) 0.71 A from positive to negative terminal

Key : 2

Sol :



$i = i_1 + i_2$

$20 - 5i - 10i_1 - 2i = 0$

$10 - 10i_1 + 4i_2 = 0$

Solving $i_2 = 0.21A$

7. Two identical electric point dipoles have dipole moments $\vec{P}_1 = P\hat{i}$ and $\vec{P}_2 = P\hat{i}$ and are held on the x axis at distance $'a'$ from each other. When released, they move along the x-axis with the direction of their dipole moments remaining, unchanged. If the mass of each dipole is $'m'$, their speed when they are infinitely far apart is :

1) $\frac{P}{a}\sqrt{\frac{2}{\pi \in_0 ma}}$

2) $\frac{P}{a}\sqrt{\frac{1}{2\pi \in_0 ma}}$

3) $\frac{P}{a}\sqrt{\frac{1}{\pi \in_0 ma}}$

4) $\frac{P}{a}\sqrt{\frac{3}{2\pi \in_0 ma}}$

Key : 1

Sol : Electric field at the location of 2nd dipole due to 1st dipole is $\vec{E}_1 = \frac{1}{4\pi \in_o} \frac{2p}{a^3}\hat{i}$

So potential energy is $U = \vec{p}_2 . \vec{E}_1 \quad = \frac{1}{4\pi \in_o} \times \frac{2p^2}{a^3} = \frac{p^2}{2\pi \in_o a^3}$

When they will be infinitely far apart U= 0 and if speed of each dipole is ϑ, then kinetic energy is $\frac{1}{2}m\vartheta^2 + \frac{1}{2}mv^2 = m\vartheta^2$

So $m\vartheta^2 = \frac{p^2}{2\pi \in_o a^3} \Rightarrow \vartheta = \frac{p}{a} \times \sqrt{\frac{1}{2\pi \in_o ma}}$

8. A circuit to verify Ohm's law uses ammeter and voltmeter in series or parallel connected correctly to the resistor. In the circuit:

1) Both, ammeter any voltmeter must be connected in series

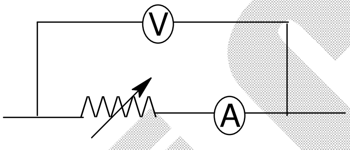
2) Both, ammeter any voltmeter must be connected in parallel

3) ammeter is always connected in series and voltmeter in parallel

4) ammeter is always used in parallel and voltmeter is series

Key : 3

Sol :



9. In a dilute gas at pressure P and temperature T, the mean time between successive collisions of a molecule varies with T as :

1) $\sqrt{T}$ 2) T 3) $\frac{1}{T}$ 4) $\frac{1}{\sqrt{T}}$

Key : 1

Sol : mean free path $\lambda = \frac{RT}{\sqrt{2}\pi d^2 N_A p}$

So mean time between successive collisions is

$\tau = \frac{\lambda}{\bar{\vartheta}_{rel}} = \frac{RT}{\sqrt{2}\pi d^2 N_A P\sqrt{2}\bar{\vartheta}}$ But $\bar{\vartheta} = \sqrt{\frac{8RT}{\pi M}}$ So $\tau \alpha \sqrt{T}$

10. When a particle of mass m is attached to a vertical spring of spring constant k and released, its motion is described by $y(t)=y_0\sin^2\omega t$, where $'y'$ is measured from the lower end of unstretched spring. Then ω is :

1) $\frac{1}{2}\sqrt{\frac{g}{y_0}}$ 2) $\sqrt{\frac{g}{y_0}}$ 3) $\sqrt{\frac{g}{2y_0}}$ 4) $\sqrt{\frac{2g}{y_0}}$

Key : 3

Sol : $y(t)=y_o\sin^2(\omega t)=\frac{y_o}{2}(1-\cos 2\omega t)$

So elongation in the spring in equilibrium is $\frac{y_o}{2}$.

Therefore $mg=\frac{ky_o}{2}$

$$\Rightarrow \sqrt{\frac{k}{m}}=\sqrt{\frac{2g}{y_o}}=2\omega$$

$$\Rightarrow \omega=\sqrt{\frac{g}{2y_o}}$$

11. A fluid is flowing through a horizontal pipe of varying cross-section, with speed vms^{-1} at a point where the pressure is P Pascal. At another point where pressure is $\frac{P}{2}$ Pascal its speed is Vms^{-1}. If the density of the fluid is $\rho\ kg\,m^{-3}$ and the flow is streamline, then V is equal to :

1) $\sqrt{\frac{P}{\rho}+v^2}$ 2) $\sqrt{\frac{P}{2\rho}+v^2}$ 3) $\sqrt{\frac{P}{\rho}+v}$ 4) $\sqrt{\frac{2P}{\rho}+v^2}$

Key : 1

Sol : $P+\frac{1}{2}\rho\vartheta^2=\frac{P}{2}+\frac{1}{2}\rho V^2$

$$\Rightarrow V=\sqrt{\frac{P}{\rho}+\vartheta^2}$$

12. A square loop of side 2a and carrying current I is kept in xz plane with its centre at origin. A long wire carrying the same current I is placed parallel to z-axis and passing through point $(0,b,0),\left(b>>a\right)$. The magnitude of torque on the loop about z-axis will be :

1) $\frac{\mu_0 I^2 a^2 b}{2\pi\left(a^2+b^2\right)}$ 2) $\frac{\mu_0 I^2 a^2}{2\pi b}$ 3) $\frac{2\mu_0 I^2 a^2}{\pi b}$ 4) $\frac{2\mu_0 I^2 a^2 b}{\pi\left(a^2+b^2\right)}$

Key : 3

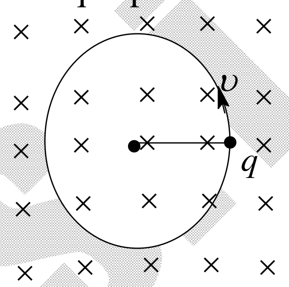
Sol : Magnetic moment of loop is $4a^2 I\,\hat{j}$ and magnetic filed induction at origin due to long wire $\vec{B}=\frac{\mu_0 I}{2\pi b}\hat{i}$. So as b>>a, $|\vec{\tau}|=|\vec{\mu}\times\vec{B}|=\frac{2\mu_0 I^2 a^2}{\pi b}$

13. A charged particle going around in a circle can be considered to be a current loop. A particle of mass m carrying charge q is moving in a plane with speed v under the influence of magnetic field $\vec{B}$. The magnetic moment of this moving particle :

1) $-\frac{mv^2\vec{B}}{B^2}$ 2) $-\frac{mv^2\vec{B}}{2B^2}$ 3) $-\frac{mv^2\vec{B}}{2\pi B^2}$ 4) $\frac{mv^2\vec{B}}{2B^2}$

Key : 2

Sol : Let q is positive and $\vec{B}$ is into the plane



So charge particle is moving in anticlockwise sense on a circle of radius $r=\frac{m\vartheta}{qB}$

Hence magnetic moment $\mu=i\times\pi r^2=\frac{m\vartheta^2}{2B}$

Clearly magnetic moment is out of the plane so $\vec{\mu}=-\frac{m\vartheta^2}{2B^2}\times\vec{B}$

14. A particle moving in the xy plane experiences a velocity dependent force $\vec{F} = k\left(v_y\hat{i} + v_x\hat{j}\right)$, where v_x and v_y are the x and y components of its velocity $\vec{v}$. If $\vec{a}$ is the acceleration of the particle, then which of the following statements is true for the particle?

1) Kinetic energy of particle is constant in time

2) Quantity $\vec{v}.\vec{a}$ is constant in time

3) Quantity $\vec{v}\times\vec{a}$ is constant in time

4) $\vec{F}$ arises due to a magnetic field

Key : 3

Sol :

let mass of the partcile is m then $\vec{a} = \frac{k}{m}\left(\vartheta_y\hat{i} + \vartheta_x\hat{j}\right)$ and $\vec{\vartheta} = \vartheta_x\hat{i} + \vartheta_y\hat{j}$

Let $\frac{k}{m} = \alpha$ (constant) So $\alpha V_y = \frac{d\vartheta_x}{dl}$ and $\alpha\vartheta_x = \frac{d\vartheta_y}{dt}$

$\Rightarrow \alpha V_y = \frac{1}{\alpha}\frac{d^2V_y}{dt^2}$ and $\alpha V_x = \frac{1}{\alpha}\frac{d^2V_x}{dt^2}$ $\Rightarrow \frac{d^2V_y}{dt^2} = \alpha^2V_y$ and $\frac{d^2V_x}{dt^2} = \alpha^2V_x$

Now, $\frac{d^2V_y}{dt^2} = \alpha^2V_y \Rightarrow V_y = V_{oy}e^{\alpha t}$

And $\frac{d^2V_x}{dt^2} = \alpha^2V_x \Rightarrow V_x = V_{ox}e^{\alpha t}$

Where V_{oy} and V_{ox} are constants and represent initial y=component and x-component of velocity

Also $V_{oy} = V_{ox} = V_0(say)\, as\, \frac{dVy}{dt} = \alpha V_x\, and\, \frac{dVx}{dt} = \alpha V_y$

So speed ϑ of the particle is $V = \left(\sqrt{V_{ox}^2 + V_{oy}^2}\right)e^{\alpha t} = \sqrt{2}V_oe^{\alpha t}$

Hence speed is not constant and therefore kinetic energy is not constant.

Also $\vec{V}.\vec{a} = 2\alpha V_yV_x = 2\alpha V_o^2e^{2\alpha t}$

which is not constant in time

$\vec{V}\times\vec{a} = \alpha\left(V_x^2 - V_y^2\right)\hat{k}$ $= \alpha\left(V_{ox}^2 - V_{oy}^2\right)e^{2\alpha t}\,\hat{k} = 0\; as\; V_{0x} = V_{oy}$

Which is constant in time

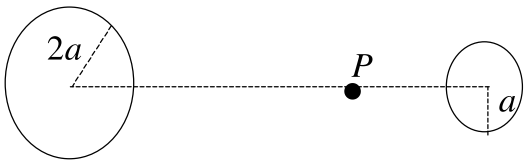
Also as v is not constant, the force can not be due to magnetic field.

15. Two planets have masses M and 16 M and their radii are a and 2a, respectively. The separation between the centres of the planets is 10a. A body of mass m is fired from the surface of the larger planet towards the smaller planet along the line joining their centres. For the body to be able to reach at the surface of smaller planer, the minimum firing speed needed is :

1) $\sqrt{\frac{GM^2}{ma}}$ 2) $\sqrt{\frac{GM}{a}}$ 3) $\sqrt{\frac{5GM}{a}}$ 4) $\sqrt{\frac{GM}{a}}$

Key : 3

Sol :



Let gravitational field intansity at P is zero. Then if distance of P from the center of bigger sphere is r,

$$\frac{-G16M}{r^2} = \frac{-G\times M}{(10a-r)^2} \Rightarrow r = 8a$$

The partciel is projected from surface of the bigger sphere, and if it just manages to cross P then it will surely land on the smaller sphere.

So applying energy conservation.

$$-\frac{G16Mm}{2a} - \frac{GMm}{8a} + \frac{1}{2}m\vartheta^2$$

$$= -\frac{G16Mm}{8a} - \frac{GMm}{2a}$$

$$\Rightarrow \vartheta = \frac{3}{2}\times\sqrt{\frac{5Gm}{a}}$$

16. The linear mass density of a thin rod AB of length L varies from A to B as $\lambda(x) = \lambda_0\left(1+\frac{x}{L}\right)$, where x is the distance from A. If M is the mass of the rod then its moment of inertia about an axis passing through A and perpendicular to the rod is:

1) $\frac{3}{7}ML^2$ 2) $\frac{7}{18}ML^2$ 3) $\frac{5}{12}ML^2$ 4) $\frac{2}{5}ML^2$

Key : 2

Sol : $M = \int_o^L \lambda_o \left(1+\frac{x}{L}\right) dx = \frac{3}{2}\lambda_o L$

$L = \int_o^L \lambda_o \left(1+\frac{x}{L}\right) dx \times x^2 \qquad = \frac{7}{18} ML^2$

17. A double convex lens has power P and same radii of curvature R of both the surfaces. The radius of curvature of a surface of a plano-convex lens made of the same material with powr 1.5 P is

1) $\frac{3R}{2}$ 2) 2R 3) $\frac{R}{2}$ 4) $\frac{R}{3}$

Key : 4

Sol : $\left.\begin{array}{l} P=(\mu-1)\left(\frac{2}{R}\right) \\ 1.5P=(\mu-1)\times\frac{1}{R'} \end{array}\right\} \Rightarrow R' = \frac{R}{3}$

18. Given the masses of various atomic particles $m_p = 1.0072u$, $m_n = 1.0087u$, $m_e = 0.000548u, m_{\bar{v}} = 0, m_d = 2.0141u$, where P = portion, n = neutron, e = electron, $\bar{v}$ = antineutrino and d = deuteron. Which of the following process is allowed by momentum and energy conservation?

1) $n+n \rightarrow$ deuterium atom (electron bound to the nucleus)

2) $p \rightarrow n+e^{+}+\bar{v}$

3) $e^{+}+e^{-} \rightarrow \gamma$

4) $n+p \rightarrow d+\gamma$

Key : 4

Sol : $n+n \rightarrow$ deutrium atom is not energetically feasible as mass on product side is more than mass on reactant side.

$p \rightarrow n+e^{+}+v^{-}$ is also not energetically feasible.

$e^{+}+e^{-} \rightarrow r$ is not possible as momentum conservation does not hold.

rather $e^{+}+e^{-} \rightarrow 2r$ would be correct.

As $n+p \rightarrow d+r$ is energetically feasible, it could be possible.

19. Assuming the nitrogen molecule is moving with r.m.s velocity at 400 K, the de-Broglie wavelength of nitrogen molecule is close to :

(Given : nitrogen molecule weight : $4.64\times10^{-26}\,kg$, Boltzman constant : $1.38\times10^{-23}\,J/K$, Planck constant : $6.63\times10^{-34}\,J.s$)

1) $0.20\mathring{A}$ 2) $0.34\mathring{A}$ 3) $0.44\mathring{A}$ 4) $0.24\mathring{A}$

Key : 4

Sol : $\lambda = \frac{h}{m\vartheta} = \frac{h}{m\times\sqrt{\frac{3KT}{m}}} = \frac{h}{\sqrt{3mKT}} \simeq 0.24A^\circ$

20. Particle A of mass m_1 moving with velocity $\left(\sqrt{3}\hat{i}+\hat{j}\right)ms^{-1}$ collides with another particle B of mass m_2 which is at rest initially. Let $\vec{V_1}\,and\,\vec{V_2}$ be the velocities of particles A and B after collision respectively. If $m_1 = 2m_2$ and after collision $\vec{V_1} = \left(\hat{i}+\sqrt{3}\hat{j}\right)ms^{-1}$, the angle between $\vec{V_1}$ and $\vec{V_2}$ is

1) 15° 2) –45° 3) 60° 4) 105°

Key : 4

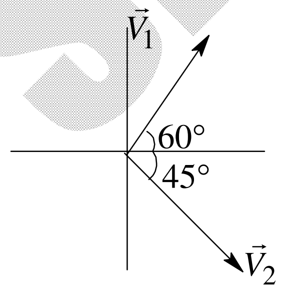
Sol : Let $\vec{V_2} = V_x\hat{i} + V_y\hat{j}$

Applying momentum conservation along x and y direction,

$m_1\times\sqrt{3} = m_1 + m_2 \Rightarrow V_x = 2\left(\sqrt{3}-1\right)$

And $m_1 = \sqrt{3}m_1 + m_2V_y \Rightarrow V_y = -2\left(\sqrt{3}-1\right)$

So $\vec{V_2} = 2\left(\sqrt{3}-1\right)\hat{i} - 2\left(\sqrt{3}-1\right)\hat{j}$, which is making angle 45° with x=axis as shown



So angle between $\vec{V_1}\,and\,\vec{V_2}$ is 105°

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value (in decimal notation, truncated/ rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer , 0 if not attempted and 0 in all other cases.

21. A Young's double- slit experiment is performed using monochromatic light of wavelength λ. The intensity of light at a point on the screen, where the path difference is λ, is K units. The intensity of light at a point where the path difference is $\frac{\lambda}{6}$ is given by $\frac{nK}{12}$, where n is an integer. The value of n is

Key : [9.00]

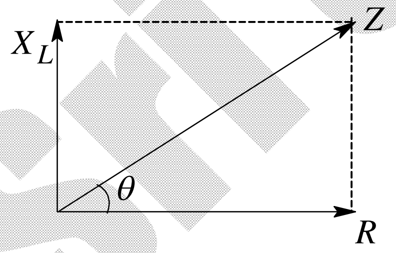
Sol : $4I_O = k$ so if path difference $\Delta x = \frac{\lambda}{6}$, phase difference $\phi = \frac{\pi}{3}$

So $I = 3I_O = \frac{3k}{4} = \frac{9k}{12}$

22. In a series LR circuit, power of 400 W is dissipated from a source of 250 V, 50 Hz. The power factor of the circuit is 0.8. In order to bring the power factor to unity, a capacitor of value C is added in series to the L and R. Taking the value of C as $\left(\frac{n}{3\pi}\right)\mu F$, then value of n is _____

Key : (400.00)

Sol :

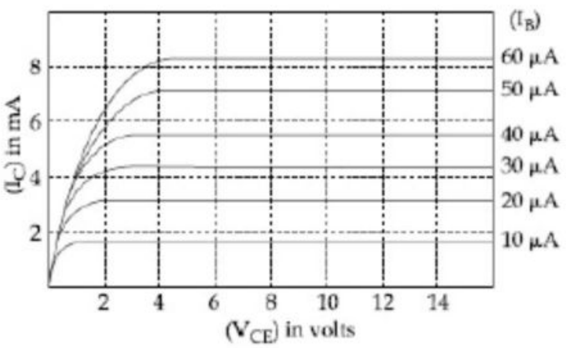


$Cos\theta = 0.8 \quad \Rightarrow Tan\theta = \frac{3}{4} = \frac{X_L}{R} \quad \Rightarrow X_L = \frac{3R}{4}$

And $Z = \frac{5R}{4} \quad P = \frac{V^2}{Z^2} \times R \Rightarrow \frac{P \times Z^2}{V^2} \Rightarrow R = 100\Omega$ So $X_L = 75\Omega$

For power fractor to be *1*, $X_C = X_L$ So $\frac{1}{2\pi fC} = 75 \Rightarrow C = \frac{400}{3\pi}\mu F$

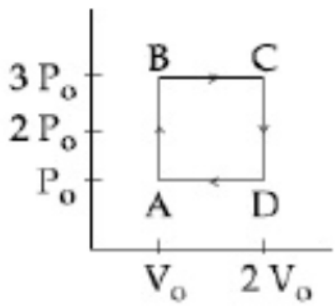
23. The output characteristics of a transistor is shown in the figure. When V_{CE} is 10V and $I_C = 4.0mA$, then value of β_{ac} is ____



Key : [133.34]

Sol : $\beta_{ac} = \frac{I_C}{I_B} = \frac{4mA}{30\mu A} = \frac{4000}{30} = 133.34$

24. An engine operates by taking a monoatomic ideal gas through the cycle shown in the figure. The percentage efficiency of the engine is close to _____



Key : 19.00

Sol : $W_{AB} = 0, \quad Q_{AB} = 3P_O V_O$ $\qquad W_{AC} = 3P_O V_O, \quad Q_{BC} = \frac{15}{2} P_O V_O$

$W_{CD} = 0, \quad Q_{CD} = -6P_O V_O$ $\qquad W_{DA} = -P_O V_O, \quad Q_{DA} = -\frac{5}{2} P_O V_O$

$\%\eta = \frac{W}{Q} \times 100 = \frac{W_{AB} + W_{BC} + W_{CD} + W_{DA}}{Q_{AB} + Q_{BC}} \times 100 \qquad = 19.05 \approx 19.00$

25. The centre of mass of a solid hemisphere of radius 8 cm is x cm from the centre of the flat surface. Then value of x is _______

Key : 3.00

Sol : $\frac{3R}{8} = x \Rightarrow x = 3cm$

CHEMISTRY

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

01. A crystal is made up of metal 'M_1' and 'M_2' and oxide ions. Oxide ions form a ccp lattice structure. The cation 'M_1' occupies 50% of octahedral voids and the cation 'M_2' occupies 12.5% of tetrahedral voids of oxide lattice. The oxidation numbers of 'M_1' and 'M_2' are, respectively.

1) +2, +4 2) +1, +3 3) +4, +2 4) +3, +1

Key: 4

Sol: If O^{2-} ions form ccp that gives 4.0v's and 8 T.v's

50% of O.V i.e, 2.0 Vs are occupiiied by M_1

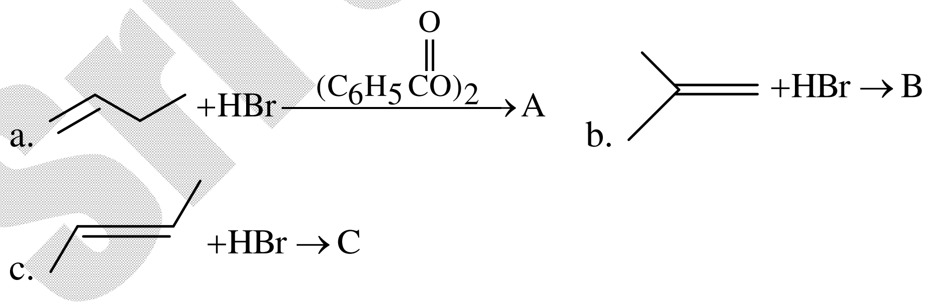
12.5% of T.V i.e 7 T.V is occupied by M_2

So in one formule unit there is $2M_1\ 1M_2$ and $4O^{-2}$

So M_1 must be M_1^{+3} and M_2 must be M_2^{2+}

$\Rightarrow M_2(M_1)_2O_4$

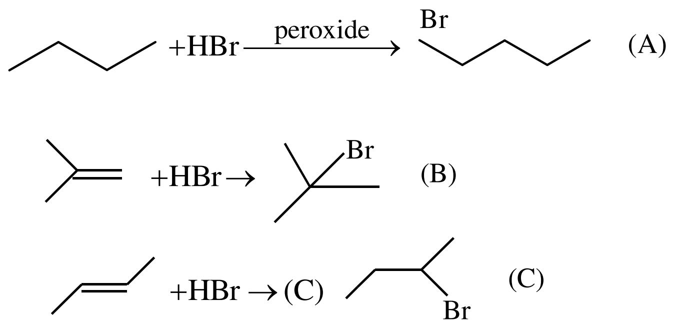
02. The increasing order of the boiling points of the major products A, B and C of the following reactions will be:



1) B < C < A 2) A < B < C 3) C < A < B 4) A < C < B

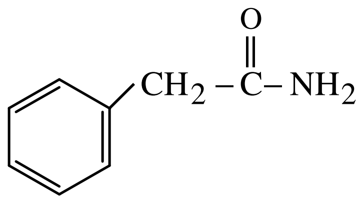
Key: 1

Sol:

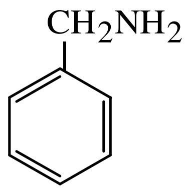


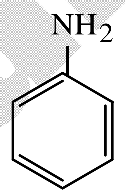
Intermolecular attractions increases if branching decreases among isomeric alkyl halides

03. Which of the following compounds can be prepared in good yield by Gabriel phthalimide synthesis?

1) 

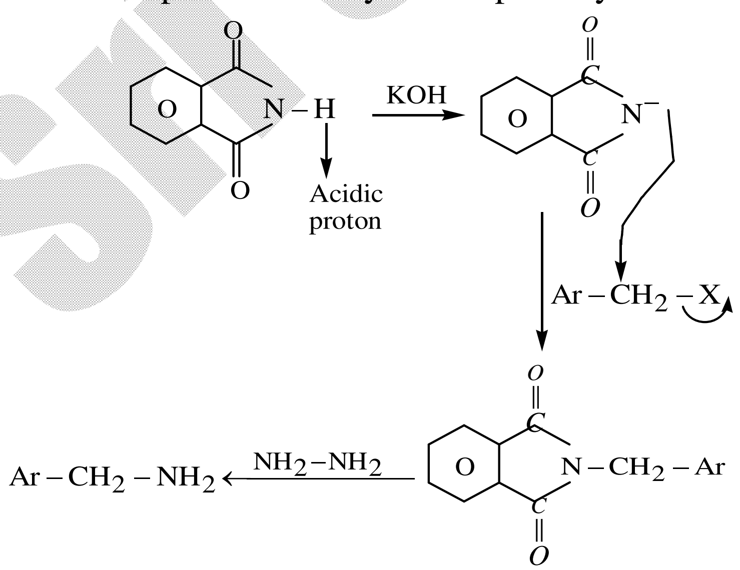
2) $CH_3-CH_2-NHCH_3$

3) 

4) 

Key: 3

Sol: In Gabriel phthalimide synthesis primary amine can be prepared in good yield



04. The correct match between Item -1 (starting material) and Item II (reagent) for the

preparation of benzaldehyde is:

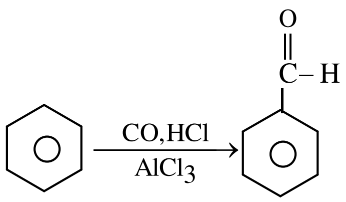
Item I	Item II
I. Benzene	P. HCl and $SnCl_2, H_3O^+$
II. Benzonitrile	Q. $H_2, Pd-BaSO_4$, S and quinoline
III. Benzoyl chloride	R. CO, HCl and $AlCl_3$

1) I – Q, II – R and III – P   2) I – R, II – P and III – Q

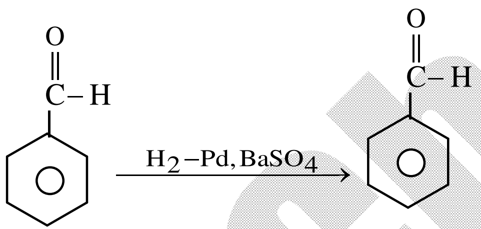
3) I – R, II – Q and III – P   4) I – P, II – Q and III – R

Key: 2

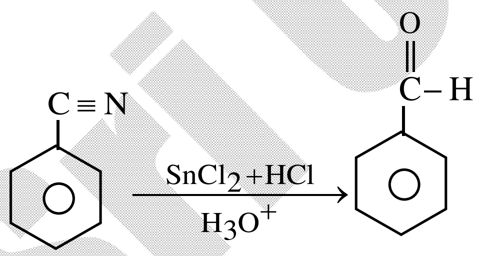
Sol:



Gattermann koch



Rosenmund reduction



Stephen reduction

05. For the following cell; $Cu(s)|Cu^{2+}(C_1M)||Cu^{2+}(C_2M)|Cu(s)$ change in Gibbs energy (ΔG) is negative if:

1) $C_2 = C_1/\sqrt{2}$ 2) $C_1 = 2C_2$ 3) $C_1 = C_2$ 4) $C_2 = \sqrt{2}C_1$

Key: 4

Sol: Anode: $Cu_{(s)} \rightleftharpoons Cu^{2+}(C_1) + 2e^-$

Cathode: $Cu^{2+}(C_2) + 2e^- \rightleftharpoons Cu_{(s)}$

$Cu^{2+}(C_2) \rightleftharpoons Cu^{2+}(C_1)$

For which equilibrium constant will be K_{eq}

From thermodynamics $\Delta G = -nFE$ for the overall reaction to be spontaneous ΔG has to be negative so E must be positive know

$$E = E^\circ - \frac{0.059}{2}\log\frac{C_1}{C_2}$$

$E^\circ = O$ for concentration cell

For 'E' to be positive $C_2 > C_1$

06. The reaction of NO with N_2O_4 at 250 K gives

1) NO_2 2) N_2O 3) N_2O_3 4) N_2O_5

Key: 3

Sol: $2NO + N_2O_4 \xrightarrow{250K} 2N_2O_3$ blue colour solid

NCERT table 7.3

Oxides of nitrogen

07. The element that can be refined by distillation is:

1) zinc 2) tin 3) nickel 4) gallium

Key: 1

Sol: $\left.\begin{matrix} Zn \\ Cd \\ Hg \end{matrix}\right\}$ Refined by distillation

08. Mischmetal is an alloy consisting mainly of:

1) Lanthanoid metals

2) Actinoid and transition metals

3) actinoid metals

4) Lanthanoid and actinoid metals

Key: 1

Sol: Misch metal is an alloy consisting of 95% lanthanoids 5% Iron and Ca, Al, C,S in traces

09. The value of K_c is 64 at 800K for the reaction $N_2(g)+3H_2(g) \rightleftharpoons 2NH_3(g)$. The value of K_c for the following reaction is: $NH_3(g) \rightleftharpoons \frac{1}{2}N_2(g)+\frac{3}{2}H_2(g)$

1) 1/64 2) 8 3) 1/4 4) 1/8

Key: 4

Sol: Given $N_{2(g)}+3H_{2(g)} \rightleftharpoons 2NH_{3(g)}$ $K_C=64$

If we reverse the reaction then equilibrium constant becomes $\frac{1}{K_c}$

$\Rightarrow 2NH_{3(g)} \rightleftharpoons N_{2(g)}+3H_{2(g)}$ $K_C^1=\frac{1}{64}$

If the entire reaction is multiplied by 'X' then the equilibrium constant becomes $\left(\frac{1}{K^x}\right)$

$NH_{3(g)} \rightleftharpoons \frac{1}{2}N_{2(g)}+\frac{3}{2}H_{2(g)}$

$K_{eq}=\frac{1}{\sqrt{64}}=\frac{1}{8}$

10. Dihydrogen of high purity (> 99.95%) is obtained through

1) the electrolysis of warm $Ba(OH)_2$ solution using Ni electrodes

2) the reaction of Zn with dilute HCl

3) the electrolysis of brine solution

4) the electrolysis of acidified water using Pt electrodes

Key: 1

Sol: Dehydrogen of high purity (>99.95%) is obtained by the electrolysis of warm $Ba(OH)_2$ solution using Ni electrons

11. Match the followigng compounds (column I) with their uses (column II)

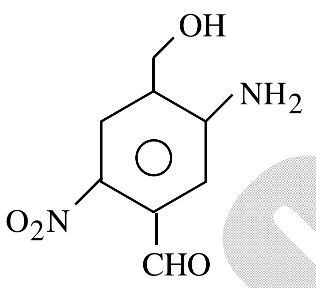
S.No	column I	S.No	Column II
I.	$Ca(OH)_2$	A.	casts of statues
II.	NaCl	B.	white wash
III.	$CaSO_4.\frac{1}{2}H_2O$	C.	antacid
IV.	$CaCO_3$	D.	washing soda preparation

1) $I-B, II-C, III-D, IV-A$
2) $I-C, II-D, III-B, IV-A$
3) $I-D, II-A, III-C, IV-B$
4) $I-B, II-D, III-A, IV-C$

Key: 4

Sol: $CaCO_3$ is used as an antacid to relieve acid indigation, heart burn.
$Ca(OH)_2 \rightarrow$ white wash
$NaCl \rightarrow$ In the preparation of of Na_2CO_3
$CaSO_4\frac{1}{2}H_2O \rightarrow$ plaster of pairs casts of statues

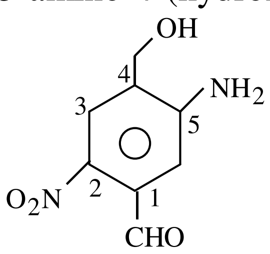
12. The IUPAC name of the following compound is



1) 3-amino-4-hydroxymethyl-5-nitrobenzaldehyde
2) 4-amino-2-formyl-5-hydroxymethyl nitrobenzene
3) 2-nitro-4-hydroxymethyl-5-aminobenzaldehyde
4) 5-amino-4-hydroxymethyl1-2-nitrobenzaldehyde

Key: 4

Sol: 5-amino-4-(hydroxymethyl)-2-nitrobenzenzaldehyde



13. For a reaction $4M(s)+nO_2(g)\rightarrow 2M_2O_n(s)$, the free energy change is plotted as a function of temperature. The temperature below which the oxide is stable could be inferred fromt the plot as the point at which:

1) the free energy change shows a change from negative to positive value
2) the slope change from negative to positive
3) the slope change from positive to zero
4) the slope change from positive to negative

Key: 4

Sol: If the ΔG for the formation of oxide is negative then the oxide is stable more negative the ΔG more is stability of oxide

14. Match the following:

Test/Method	Reagent
i. Lucas Test	a. $C_6H_5SO_2Cl/aq.KOH$
ii. Dumas method	b. $HNO_3/AgNO_3$
iii. Kjeldahl's method	c. CuO/CO_2
iv. Hinsberg test	d. conc. HCl and $ZnCl_2$
	e. H_2SO_4

1) $i-d, ii-c, iii-b, iv-e$
2) $i-b, ii-a, iii-c, iv-d$
3) $i-b, ii-d, iii-e, iv-a$
4) $i-d, ii-c, iii-e, iv-a$

Key: 4

Sol: Lucas reagent $\rightarrow ZnCl_2/\text{conc HCl}$
$CuO/\Delta \rightarrow$ Dumas method
conc $H_2SO_4 \rightarrow$ Kjeldhal method
$C_6H_5SO_2Cl/\text{aq KOH} \rightarrow$ Hinsberg reagent

15. Which one of the following statements is not true?

1) Lactose is a reducing sugar and it gives Fehling's test

2) Lactose $(C_{11}H_{22}O_{11})$ is a disaccharide and it contains 8 hydroxyl groups

3) On acid hydrolysis, lactose gives one molecule of D(+) –glucose and one molecule of D (+)-galactose

4) Lactose contains $\alpha-$glycosidic linkage between C_1 of galactose and C_4 of glucos

Key: 3

Sol: Lactose is a disaccharides of $\beta-D-$galactose and $\alpha-or-\beta-D-$glucose

16. Reaction of an inorganic sulphite X with dilute H_2SO_4 generates compound Y. Reaction of Y with NaOH gives X. Further, the reaction of X with Y and water affords compound Z. Y and Z, respectively, are:

1) S and Na_2SO_3
2) SO_3 and $NaHSO_3$
3) SO_2 and $NaHSO_3$
4) SO_2 and Na_2SO_3

Key: 4

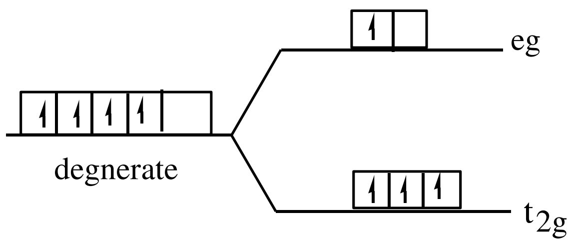
Sol: $S+\text{conc}\,H_2SO_4 \rightarrow \underset{X}{SO_{4(g)}} \xrightarrow{NaOH} \underset{Y}{Na_2SO_3}$

17. For a d^4 metal ion in an octahedral field, the correct electronic configuration is:

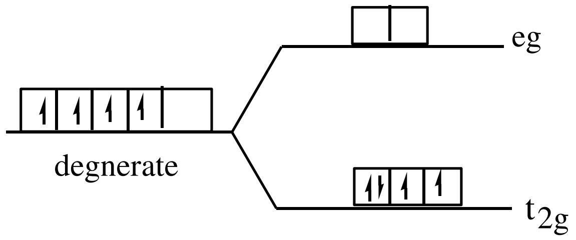
1) $t_{2g}^3\,e_g^1$ when $\Delta_0 < P$
2) $e_g^2\,t_{2g}^2$ when $\Delta_0 < P$
3) $t_{2g}^4 e_g^0$ when $\Delta_0 < P$
4) $t_{2g}^3 e_g^1$ when $\Delta_0 > P$

Key: 1

Sol:



if $\Delta_0 < P$



if $\Delta_0 > P$

18. The correct match between item I and Item II is:

Item I	Item II
a. Natural rubber	I. 1, 3 – butadiene + syrene
b. Neoprene	II. 1, 3-butadiene +acrylonitrile
c. Buna – N	III. Chloroprene
d. Buna – S	IV. Isoprene

1) $a-IV, b-III, c-II, d-I$ 2) $a-III, b-IV, c-I, d-II$

3) $a-III, b-IV, c-II, d-I$ 4) $a-IV, b-III, c-I, d-II$

Key: 1

Sol: The monomer of natural rubber is isoprene
In Buna-S one of the monomer is acrylonitrile
In Buna N one of the monomer is styrene

19. The average molar mass of chloride is $35.5\,g\,mol^{-1}$. The ratio of ^{35}Cl to ^{37}Cl in naturally occuring chloride is close to:

1) 4:1 2) 3:1 3) 1:1 4) 2:1

Key:2

Sol:

	Cl^{35}	Cl^{37}
mole fraction	x	$1-x$

$M_{avg} = x \times 35 + (1-x)(37) = 35.5$

$\Rightarrow 35x + 37 - 37x = 35.5$

$2x = \frac{3}{2}$

$x = \frac{3}{4}$

$\frac{Cl^{35}}{Cl^{37}} = \frac{x}{(1-x)} = \frac{3/4}{1/4} = 3:1$

20. A set of solutions is prepared using 180g of water as a solvent and 10g of different non –volatile solutes A, B and C. The relative lowering of vapour pressure in the presence of these solutes are in the order [Given molar mass of A = 100g mol^{-1}; B = 200 g mol^{-1}; C = 10,000g mol^{-1};]

1) C > B > A
2) A > C > B
3) B > C > A
4) A > B > C

Key: 1

Sol: RLVP = colligative property which depends up on number of moles of solute (particles)We know more is the molar mass less is the number of particles and less is the value of colligative property

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value (in decimal notation, truncated/ rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. The atomic number of Unnilunium is..............

Key: 101

Sol: $\underset{1}{\underset{\downarrow}{\text{un}}}\ \underset{0}{\underset{\downarrow}{\text{nil}}}\ \underset{1}{\underset{\downarrow}{\text{un}}}\ \underset{\text{suffix}}{\underset{\downarrow}{\text{ium}}}$

22. If the solubility product of AB_2 is $3.20\times10^{-11}M^3$, then the solubility of AB_2 in pure water is.......$\times10^{-4}$ $mol\,L^{-1}$. [Assuming that neither kind of ion reacts with water]

Key: 0.0002

Sol: $AB_2 \rightleftharpoons \underset{s}{A^{+2}} + \underset{2s}{2B^-}$

$K_{sp} = s\times(2s)^2 = 4s^3$

$4s^3 = 3.2\times10^{-11}$

$s^3 = 8\times10^{-12}$

$\Rightarrow s = \sqrt[3]{8\times10^{-12}} = 2\times10^{-4}M$

23. A solution of phenol in chloroform when treated with aqueous NaOH gives compound P as a major product. The mass percentage of carbon in P is............. (to the nearest integer) (Atomic mass: C =12 ; H =1; O = 16)

Key: 68.85%

Sol: Reimer Tiemann reaction

$C_7H_6O_2$

$$\text{molar mass} = 7\times 12g + 6g + 32g = 122g$$

$$\%\text{of 'C'} = \frac{84}{122}\times 100 \quad = 68.85\%$$

24. The rate of a reaction decreased by 3.555 times when the temperature was changed from 40°C to 30°C. The activation energy (in kJ mol^{-1}) of the reaction is.............Take R=8.314 J $mol^{-1}K^{-1}$ ln 3.555 = 1.268.

Key: 99.98

Sol: $\ln\left(\frac{k_1}{k_2}\right) = \frac{E_a}{R}\left[\frac{1}{T_1} = \frac{1}{T_2}\right]$

$$\ln\left(\frac{1}{3.555}\right) = \frac{E_a}{R}\left[\frac{1}{313} = \frac{1}{303}\right]$$

$$\ln\left(\frac{1}{3.555}\right) = \frac{E_a}{R}\left[\frac{1}{313} = \frac{1}{303}\right] E_a = \frac{1.268\times 8.314\times 94839}{10} = 99980.7J$$

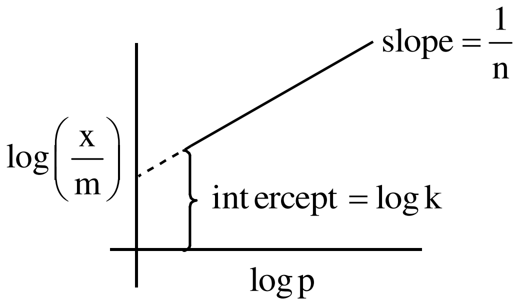
$$E_{a(KJ)} = 99.98KJ$$

25. For Freundlich adsorption isotherm, a plot of log (x/m) (y-axis) and log P (x-axis) gives a straight line. The intercept and slope for the line is 0.4771 and 2, respectively. The mass of gas, adsorbed per gram of adsorbent if the initial pressure is 0.04 atm, is...........$\times 10^{-4}$g. ($\log 3 = 0.4771$)

Key: 2

Sol: Freundlich isotherm $\frac{x}{m} = kP^{1/n} k \to 1$

Table $\log_{10}$ on both sides



$$\log\left(\frac{x}{m}\right) = \log k + \frac{1}{n}\log P$$

If intercept = 0.4771 = log K

K = 3

If slope = 1/n = 2 $\frac{x}{m} = 3 \times (0.04)^2 = 0.0048$

MATHEMATICS

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

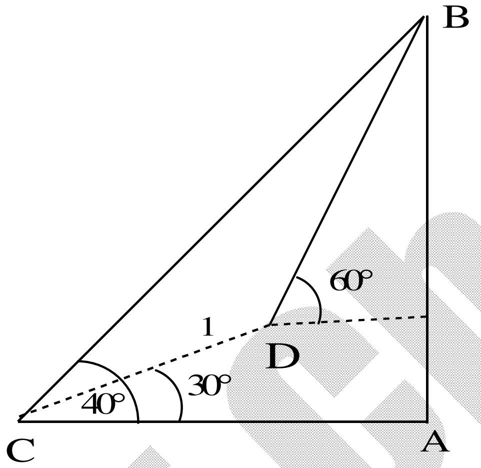
Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

01. The angle of elevation of the summit of a mountain from a point on the ground is 45°. After climbing up one km towards the summit at an inclination of 30° from the ground, the angle of elevation of the summit is found to be 60°. Then the height (in km) of the summit from the ground is:

1) $\frac{1}{\sqrt{3}-1}$ 2) $\frac{1}{\sqrt{3}+1}$ 3) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ 4) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

Key: 1

Sol: In ΔBCD,



$\frac{CD}{\sin B} = \frac{BC}{\sin D}$

$\Rightarrow \frac{1}{\sin 15°} = \frac{BC}{\sin 150°}$ $\Rightarrow BC = \frac{\sqrt{2}}{\sqrt{3}-1}$

In ΔABC, $\sin 45° = \frac{AB}{BC}$ $\Rightarrow AB = \frac{\sqrt{2}}{\sqrt{3}-1}.\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{3}-1}$

02. Let L denote the line in the $xy-$ plane with x and y intercepts as 3 and 1 respectively. Then the image of the point $(-1,-4)$ in this line is :

1) $\left(\frac{29}{5},\frac{11}{5}\right)$ 2) $\left(\frac{29}{5},\frac{8}{5}\right)$ 3) $\left(\frac{11}{5},\frac{28}{5}\right)$ 4) $\left(\frac{8}{5},\frac{29}{5}\right)$

Key: 3

Sol: Equation of L is $\frac{x}{3}+\frac{y}{1}=1$

$\Rightarrow x+3y-3=0$

$\frac{h+1}{1}=\frac{k+4}{3}=\frac{-2(-1+3(-4)-3)}{1^2+3^2}$

$\Rightarrow \frac{h+1}{1}=\frac{k+4}{3}=\frac{16}{5}$

Image = $(h,k)=\left(\frac{11}{5},\frac{28}{5}\right)$

03. If the constant term in the binomial expansion of $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ is 405, then $|k|$ equals

1) 3 2) 9 3) 1 4) 2

Key: 1

Sol: $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$

$T_{r+1}={}^{10}C_r\left(\sqrt{x}\right)^{10-r}.\left(\frac{-k}{x^2}\right)^r$

$=(-1)^r.{}^{10}C_r k^r x^{\frac{10-r}{2}-2r}$

T_{r+1} is a constant $\Rightarrow \frac{10-r}{2}-2r=0$

$\Rightarrow r=2$

Now $T_3=405$

$\Rightarrow {}^{10}C_2 k^2=45 \Rightarrow k^2=9$ $\Rightarrow |k|=3$

04. The common difference of the A.P. $b_1,b_2,....,b_m$ is 2 more than the common difference of A.P. $a_1,a_2....,a_n$. If $a_{40}=-159$, $a_{100}=-399$ and $b_{100}=a_{70}$, then b_1 is equal to :

1) 81 2) -127 3) -81 4) 127

Key: 3

$a_{40} = -159 \Rightarrow a_1 + 39d = -159$

Sol: $a_{100} = -399 \Rightarrow \underline{a_1 + 99d = -399}$

$\quad Solving \quad a_1 = -3,\ d = -4$

$b_{100} = a_{70} = a_1 + 69d$

$\Rightarrow b_1 + 99(2+d) = -3 - 276$

$\Rightarrow b_1 = -81$

05. The probabilities of three events A, B and C are given by $P(A) = 0.6, P(B) = 0.4$ and $P(C) = 0.5$. If $P(A \cup B) = 0.8, P(A \cap C) = 0.3, P(A \cap B \cap C) = 0.2$, $P(B \cap C) = \beta$ and $P(A \cup B \cup C) = \alpha$ where $0.85 \le \alpha \le 0.95$, then β lies in the interval :

1) $[0.20, 0.25]$ 2) $[0.25, 0.35]$ 3) $[0.35, 0.36]$ 4) $[0.36, 0.40]$

Key: 2

Sol: $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$

$-P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$\Rightarrow \alpha = 1.2 - \beta$

$0.85 \le \alpha \le 0.95$

$\Rightarrow 0.85 \le 1.2 - \beta \le 0.95$ $\Rightarrow 0.25 \le \beta \le 0.35$

06. If $y = \left(\frac{2}{\pi}x - 1\right)\operatorname{cosec} x$ is the solution of the differential equation,

$\frac{dy}{dx} + p(x)y = \frac{2}{\pi}\operatorname{cosec} x, 0 < x < \frac{\pi}{2}$, then the function $p(x)$ is equal to :

1) $\cot x$ 2) $\sec x$ 3) $\operatorname{cosec} x$ 4) $\tan x$

Key: 1

Sol: $y = \left(\frac{2x}{\pi} - 1\right)\operatorname{cosec} x$

$\frac{dy}{dx} = \left(\frac{2x}{\pi} - 1\right)$ cosecx cotx + $\frac{2}{\pi}\operatorname{cosec} x$

$= -y\cot x + \frac{2}{\pi}\operatorname{cosec} x \Rightarrow \frac{dy}{dx} + \cot x . y = \frac{2}{\pi}\operatorname{cosec} x$

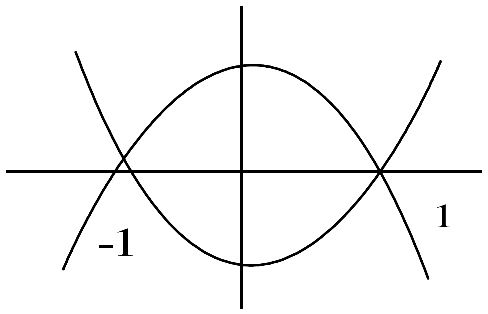
$\therefore p(x) = \cot x$

07. The area (in sq.units) of the region enclosed by the curves $y = x^2 - 1$ and $y = 1 - x^2$ is equal to :

1) $\frac{16}{3}$ 2) $\frac{8}{3}$ 3) $\frac{7}{2}$ 4) $\frac{4}{3}$

Key: 2

Sol:



$y = 1 - x^2,\ y = x^2 - 1$

Solving points of intersection are $(\pm 1, 0)$

Area = $\int_{-1}^{1}\left[\left(1-x^2\right)-\left(x^2-1\right)\right]dx$

$= \int_{-1}^{1}\left(2-2x^2\right)dx = \frac{8}{3}$

08. For all twice differentiable functions $f : R \to R$, with $f(0) = f(1) = f'(0) = 0$,

1) $f''(x) = 0$, for some $x \in (0,1)$ 2) $f''(x) = 0$, at every point $x \in (0,1)$

3) $f''(x) \neq 0$ at every point $x \in (0,1)$ 4) $f''(0) = 0$

Key: 1

Sol: $f(0) = f(1) = 0$

$\Rightarrow f^1(c) = 0$ for some $c \in (0,1)$

Now $f^1(0) = f^1(c) = 0$

$\Rightarrow f^{11}(x) = 0$ for some $x \in (0,c)$

09. For a suitably chosen real constant a, let a function, $f : R-\{-a\} \to R$ be defined by

$f(x)=\frac{a-x}{a+x}$. Further suppose that for any real number $x \neq -a$ and

$f(x) \neq -a, (fof)(x) = x$. Then $f\left(-\frac{1}{2}\right)$ is equal to :

1) $\frac{1}{3}$ 2) $-\frac{1}{3}$ 3) -3 4) 3

Key: 4

Sol: $(fof)(x) = x$

$$\Rightarrow \frac{a-f(x)}{a+f(x)} = x$$

$$\Rightarrow \frac{a-\frac{a-x}{a+x}}{a+\frac{a-x}{a+x}} = x \Rightarrow a = 1$$

$$f(x) = \frac{1-x}{1+x}$$

$$\therefore f\left(-\frac{1}{2}\right) = 3$$

10. Let $z = x+iy$ be a non-zero complex number such that $z^2 = i|z|^2$, where $i = \sqrt{-1}$, then z lies on the:

1) line, $y = -x$ 2) imaginary axis

3) real axis 4) line, y = x

Key: 4

Sol: $z^2 = i|z|^2$

$$\Rightarrow x^2 - y^2 + i.2xy = i\left(x^2 + y^2\right)$$

$$\Rightarrow x^2 - y^2 = 0,\ 2xy = x^2 + y^2$$

$$\Rightarrow x^2 = y^2,\ 2xy = x^2 + y^2 \qquad \Rightarrow 2xy = 2x^2$$

$$\Rightarrow x = 0 \ or\ x = y \qquad \Rightarrow x = y \ (\because z \neq 0)$$

11. A plane P meets the coordinate axes at A, B and C respectively. The centroid of ΔABC is given to be (1, 1,2). Then the equation of the line through this centroid and perpendicular to the plane P is

1) $\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-2}{2}$ 2) $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$

3) $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$ 4) $\frac{x-1}{2}=\frac{y-1}{1}=\frac{z-2}{1}$

Key:3

Sol: Plane be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

$A=(a,0,0)$, $B=(0,b,0)$, $C=(0,0,c)$

Centroid, $G=(1,1,2)$

$\Rightarrow\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)=(1,1,2)\Rightarrow a=3,\ b=3,\ c=6$

Plane is $\frac{x}{3}+\frac{y}{3}+\frac{z}{6}=1$ $\Rightarrow 2x+2y+z=6$

Dr's of the line perpendicular to the plane are (2, 2, 1)

$\therefore$ equation of the line are $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$

12. Let $\theta=\frac{\pi}{5}$ and $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$. If $B=A+A^4$, Then det (B) :

1) lies in (1, 2) 2) is zero 3) is one 4) lies in (2, 3)

Key: 1

Sol: $A=\begin{pmatrix}Cos\theta & Sin\theta\\ -Sin\theta & Cos\theta\end{pmatrix}$ $\Rightarrow A^4=\begin{pmatrix}Cos4\theta & Sin4\theta\\ -Sin4\theta & Cos4\theta\end{pmatrix}$

$=\begin{pmatrix}-Cos\theta & Sin\theta\\ -Sin\theta & -Cos\theta\end{pmatrix}(\because 5\theta=\pi)$

$B=A+A^4=\begin{pmatrix}0 & 2Sin\theta\\ -2Sin\theta & 0\end{pmatrix}$

$|B|=4\sin^2\theta$ $=4\left(\frac{\sqrt{10-2\sqrt{5}}}{4}\right)^2$ $=\frac{10-2\sqrt{5}}{4}$

13. Consider the statement: "For an integer n, if $n^3 - 1$ is even, then n is odd." The contrapositive statement of this statement is:

1) For an integer n, if n is odd, then $n^3 - 1$ is even

2) For an integer n, if n is even, then $n^3 - 1$ is odd

3) For an integer n, if $n^3 - 1$ is not even, then n is not odd

4) For an integer n, if n is even, then $n^3 - 1$ is even

Key: 2

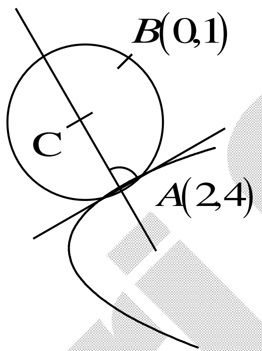
Sol: The contra positive of $p \to q$ is $\sim q \to \sim p$

14. The centre of the circle passing through the point (0,1) and touching the parabola $y = x^2$ at the point (2,4) is

1) $\left(\frac{-53}{10}, \frac{16}{5}\right)$ 2) $\left(\frac{6}{5}, \frac{53}{10}\right)$ 3) $\left(\frac{3}{10}, \frac{16}{5}\right)$ 4) $\left(\frac{-16}{5}, \frac{53}{10}\right)$

Key: 4

Sol:



Tangent to the parabola $y = x^2$ at $(2,4)$ is

$S_1 = 0$ $\Rightarrow \frac{y+4}{2} = x.2$ $\Rightarrow 4x - y - 4 = 0$

Normal at (2, 4) is $x + 4y = 18$

Centre of the circle be $c(\alpha, \beta)$

Normal passes through C $\Rightarrow \alpha + 4\beta = 18 ----(1)$

Also, CA = CB $\Rightarrow 2\alpha + 3\beta = \frac{19}{2} ----(2)$

Solving (1) & (2) Centre = $\left(-\frac{16}{5}, \frac{53}{10}\right)$

15. If α and β are the roots of the equation $2x(2x+1)=1$, then β is equal to

1) $-2\alpha(\alpha+1)$ 2) $2\alpha^2$ 3) $2\alpha(\alpha+1)$ 4) $2\alpha(\alpha-1)$

Key: 1

Sol: $2x(2x+1)=1$

$\Rightarrow 4x^2+2x-1=0$

Roots are $\dfrac{-2\pm\sqrt{4+16}}{8}=\dfrac{-1\pm\sqrt{5}}{4}$

Let $\alpha=\dfrac{\sqrt{5}-1}{4}, \beta=\dfrac{-1-\sqrt{5}}{4}$

Clearly $\alpha=Sin18°, \beta=-Cos36°$

$\beta=-(1-2Sin^2 18°)$

$=-1+2\alpha^2$ But $4\alpha^2+2\alpha-1=0$

$=-2\alpha-2\alpha^2=-2\alpha(\alpha+1)$

16. The set of all real values of λ for which the function

$f(x)=(1-\cos^2 x).(\lambda+\sin x), x\in\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$, has exactly one maxima and exactly one minima, is:

1) $\left(-\dfrac{3}{2},\dfrac{3}{2}\right)-\{0\}$ 2) $\left(-\dfrac{1}{2},\dfrac{1}{2}\right)-\{0\}$ 3) $\left(-\dfrac{3}{2},\dfrac{3}{2}\right)$ 4) $\left(-\dfrac{1}{2},\dfrac{1}{2}\right)$

Key: 1

Sol: $f(x)=(1-Cos^2x)(\lambda+Sinx)$ $=Sin^2x(\lambda+\sin x)$

$f^1(x)=Sin^2xCosx+(\lambda+\sin x)2\sin x\cos x$

$=\sin x\cos x(2\lambda+3\sin x)$

For maximum or minimum,

$f^1(x)=0$ $\Rightarrow \sin x=0$ *or* $\sin x=\dfrac{-2\lambda}{3}$

As there is exactly one max. and exactly one min, $f'(x)=0$ has exactly 2 roots

$-1<\dfrac{2\lambda}{3}<1$ *and* $\dfrac{-2\lambda}{3}\neq 0$ $\Rightarrow \lambda\in\left(\dfrac{-3}{2},\dfrac{3}{2}\right)-\{0\}$

17. If the normal at an end of a latus rectum of an ellipse passes through an extremity of the minor axis, then the eccentricity e of the ellipse satisfies:

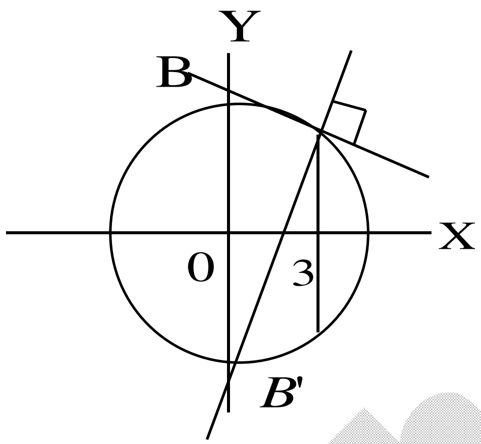
1) $e^2+e-1=0$ 2) $e^2+2e-1=0$ 3) $e^4+e^2-1=0$ 4) $e^4+2e^2-1=0$

Key: 3

Sol: Let ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

An end of latusrectum is $L\left(ae,\frac{b^2}{a}\right)$

Normal at L is



$$\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$$

$$\Rightarrow \frac{a^2x}{ae}-\frac{b^2y}{\left(b^2/a\right)}=a^2e^2----(1)$$

(1)passes through (0, -b)

$$\Rightarrow 0+ab=a^2e^2$$

$$\Rightarrow \frac{b}{a}=e^2$$

$$\Rightarrow \frac{b^2}{a^2}=e^4 \Rightarrow 1-e^2=e^4$$

$$\Rightarrow e^4+e^2-1=0$$

18. The integral $\int_1^2 e^x.x^x(2+\log_e x)\,dx$ equal

1) $4e^2-1$ 2) $e(4e+1)$ 3) $e(2e-1)$ 4) $e(4e-1)$

Key: 4

Sol: $I = \int_1^2 e^x x^x (2+\log_e x)\,dx \qquad = \int_1^2 (xe)^x (2+\log_e x)\,dx$

$Put\,(xe)^x = t \qquad \Rightarrow x(\log x + 1) = \log t$

$\Rightarrow \left(x\frac{1}{x} + (\log x + 1)1 \right) dx = \frac{1}{t} dt$

$\Rightarrow \ (xe)^x (2+\log x)dx = dt$

$x = 1 \Rightarrow t = e$

$x = 2 \Rightarrow t = (2e)^2 \qquad \therefore I = \int_e^{4e^2} dt = 4e^2 - e \qquad = e(4e-1)$

19. Let $f : R \to R$ be a function defined by $f(x) = \max\{x, x^2\}$. Let S denote the set of all points in R, where f is not differentiable. Then

1) {0,1} 2) {1} 3) {0} 4) ϕ (an empty set)

Key: 1

Sol: $f(x) = \max\{x, x^2\} \quad \Rightarrow f(x) = \begin{cases} x^2, & if\ \ x<0 \\ x, & if\ \ 0 \le x < 1 \\ x^2, & if\ \ x \ge 1 \end{cases}$

$f^1(x) = \begin{cases} 2x, & if\ \ x<0 \\ x, & if\ \ 0 < x < 1 \\ 2x, & if\ \ x > 1 \end{cases}$

Clearly $f^1(0-) \ne f^1(0+)$

and $f^1(1-) \ne f^1(1+)$

$\therefore$ f is not differentiable at 0, 1

20. If the tangent to the curve, $y = f(x) = x\log_e x, (x>0)$ at a point (c, f(c)) is parallel to the line-segment to the line – segment joining the points (1,0) and (e,e) then c is equal to

1) $e^{\left(\frac{1}{e-1}\right)}$ 2) $e^{\left(\frac{1}{1-e}\right)}$ 3) $\frac{1}{e-1}$ 4) $\frac{e-1}{e}$

Key: 1

Sol: $f(x) = x\log_e x$

$f^1(x) = 1 + \log_e x$

Now $f^1(c) = \frac{e-0}{e-1}$ $\Rightarrow 1 + \log c = \frac{e}{e-1}$

$\Rightarrow \log c = \frac{1}{e-1}$ $\Rightarrow c = e^{\left(\frac{1}{e-1}\right)}$

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value (in decimal notation, truncated/ rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is............

Key: 120

Sol: First, we arrange the letters LTTR in a row in $\frac{4!}{2!} = 12\,ways$

Now we arrange two vowels E, E in 2 of 5 gaps in ${}^5C_2$ ways

$\therefore$ No. of words = 12 x 10 = 120

22. Suppose that a function $f : R \to R$ satifies $f(x+y) = f(x)f(y)$ for all $x, y \in R$ and $f(1) = 3$. If $\sum_{i=1}^{n} f(i) = 363$, then n is equal to.........

Key: 5

Sol: $f(x+y) = f(x)f(y)$

$\Rightarrow f(x) = a^x$

$f(x) = 3 \Rightarrow a = 3$

$\therefore f(x) = 3^x$

Now $\sum_{i=1}^{n} f(i) = 363$

$\Rightarrow 3 + 3^2 + 3^3 + ...n\,terms = 363$ $\Rightarrow n = 5$

23. The sum of distinct values of λ for which the system of equations

$$(\lambda-1)x+(3\lambda+1)y+2\lambda z=0$$

$$(\lambda-1)x+(4\lambda-2)y+(\lambda+3)z=0$$

$$2x+(3\lambda+1)y+3(\lambda-1)z=0$$

has non zero solutions, is................

Key: 3

Sol: $\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3(\lambda-1) \end{vmatrix}=0$

$$\Rightarrow \begin{vmatrix} 2\lambda & 10\lambda & 6\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix}=0\ (R_1+R_2+R_3)$$

$$\Rightarrow 2\lambda\begin{vmatrix} 1 & 5 & 3 \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix}=0$$

$$\Rightarrow 2\lambda\begin{vmatrix} 1 & 0 & 0 \\ \lambda-1 & -\lambda+3 & -2\lambda+6 \\ 2 & 3\lambda-9 & 3\lambda-9 \end{vmatrix}=0$$

$$C_2-5C_1 \quad C_3-3C_1$$

$$\Rightarrow \lambda(\lambda-3)^2\begin{vmatrix} 1 & 0 & 0 \\ \lambda-1 & -1 & -2 \\ 2 & 3 & 3 \end{vmatrix}=0$$

$$\Rightarrow \lambda(\lambda-3)^2=0$$

$$\Rightarrow \lambda=0 \ or \ 3$$

Sum = 3

24. Consider the data on x taking the values of 0, 2, 4, 8,...... 2^n with frequencies $^nC_0, ^nC_1, ^nC_2, ..., ^nC_n$ respectively. If the mean of this data is $\frac{728}{2^n}$, then n is equal to....

Key: 6

Sol: $mean = \frac{728}{2^n}$

$\Rightarrow \frac{0.C_0 + 2.C_1 + 4.C_2 + ... + 2^n C_n}{C_0 + C_1 + C_2 + ... + C_n} = \frac{728}{2^n}$

$\Rightarrow \frac{(1+2)^n - 1}{2^n} = \frac{728}{2^n} \Rightarrow n = 6$

25. If $\vec{x}$ and $\vec{y}$ be two non – zero vectors such that $|\vec{x}+\vec{y}| = |\vec{x}|$ and $2\vec{x}+\lambda\vec{y}$ is perpendicular to $\vec{y}$, then the value of λ is..................

Key: 1

Sol: $|\bar{x} + \bar{y}| = |\bar{x}|$

$\Rightarrow |\bar{x} + \bar{y}|^2 = |\bar{x}|^2$

$\Rightarrow |\bar{x}|^2 + |\bar{y}|^2 + 2\bar{x}.\bar{y} = |\bar{x}|^2$

$\Rightarrow \bar{y}.\bar{y} + 2\bar{x}.\bar{y} = 0$

$\Rightarrow (2\bar{x} + \bar{y}).\bar{y} = 0$

$\Rightarrow (2\bar{x} + \bar{y}) \perp \bar{y}$

$\therefore \lambda = 1$

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