Sri Chaitanya

IIT Academy., India

JEE Main 2020

08 Jan 2020, Slot - 2

(2.30 PM - 5.30 PM)

Question Paper

Solutions

Corporate Office : Plot No-304, Kasetty Heights, Ayyappa Society Madhapur, Hyderabad-500081

www.srichaitanya.net

PHYSICS

(SINGLE CORRECT ANSWER TYPE)

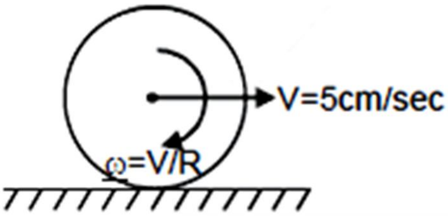
This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. A uniform sphere of mass 500 g rolls without slipping on a plane horizontal surface with its centre moving at a speed of 5.00 cm/s. Its kinetic energy is :

1) $1.13\text{x}10^{-3}$J 2) $6.25\text{x}10^{-4}$J 3) $8.75\text{x}10^{-3}$J 4) $8.75\text{x}10^{-4}$J

Ans: 4



Sol:

K.E of the sphere = Translational K.E + Rotational K.E.

$\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)$ K= Radius of gyration

$\frac{1}{2}\times\frac{1}{2}\times\left(\frac{5}{100}\right)^2\left(1+\frac{2}{5}\right)$ $\frac{35}{4}\times10^{-4}J$

2. A plane electromagnetic wave of frequency 25 GHz is propagating in vacuum along the z-direction. At a particular point in space and time, the magnetic field is given by $\vec{B}=5\text{x}10^{-8}\hat{j}\text{T}$ The corresponding electric field $\vec{E}$ is (speed of light $c=3\text{x}10^8 ms^{-1}$)

1) $-1.66\text{x}10^{-16}\hat{i}V/m$ 2) $1.66\text{x}10^{-16}\hat{i}V/m$

3) $-15\hat{i}V/m$ 4) $15\hat{i}V/m$

Ans: 4

Sol: $|\vec{B}|=\left|\frac{\vec{E}}{C}\right|\rightarrow|\vec{E}|=BC$ $=5\times10^{-8}\times3\times10^8$ $=15$

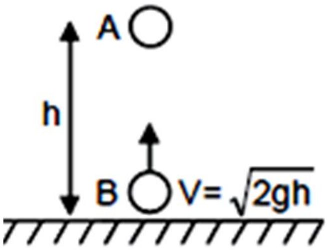
direction of propagation is parallel to

$\vec{E}\times\vec{B}$ $\therefore\vec{K}\parallel\left(\vec{E}\times\hat{j}\right)$ $\therefore\vec{E}\parallel\hat{i}$ $\therefore\vec{E}=15\hat{i}$

3. A particle of mass m is dropped from a height h above the ground. At the same time another particle of the same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2gh}$. If they collide head-on completely inelastically, the time taken for the combined mass to reach the ground, in units of $\sqrt{\frac{h}{g}}$ is :

1) $\sqrt{\frac{3}{2}}$ 2) $\frac{1}{2}$ 3) $\sqrt{\frac{1}{2}}$ 4) $\sqrt{\frac{3}{4}}$

Ans: 1

Sol: 

time for collision $t_1 = \frac{\text{relative distance}}{\text{relative velocity}} = \frac{h}{\sqrt{2gh}}$

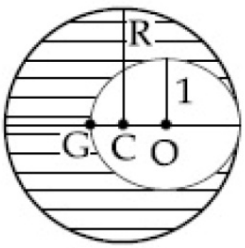
After t_1 $V_A = 0 - gt_{12} = -\sqrt{\frac{gh}{2}}$

and $V_B = \sqrt{2gh} - gt_1 = \sqrt{gh}\left[\sqrt{2} - \frac{1}{\sqrt{2}}\right] = \sqrt{\frac{gh}{2}}$

at the time of collision $\vec{P}_i = \vec{P}_f$ $\Rightarrow m\vec{V}_A + m\vec{V}_B = 2m\vec{V}_f$ $V_f = 0$

and height from ground $h - \frac{1}{2}gt_1^2 = h - \frac{h}{4} = \frac{3h}{4}$ so time $\sqrt{2 \times \frac{\left(\frac{3h}{4}\right)}{g}} = \sqrt{\frac{3h}{2g}}$

4. As shown in fig. when a spherical cavity (central at O) of radius 1 is cut out of a uniform sphere of radius R (centred at C), the centre of mass of remaining (shaded) part of sphere is at G, i.e on the surface of the cavity. R can be determined by the equation :



1) $(R^2 + R - 1)(2 - R) = 1$ 2) $(R^2 + R + 1)(2 - R) = 1$

3) $(R^2 - R + 1)(2 - R) = 1$ 4) $(R^2 - R - 1)(2 - R) = 1$

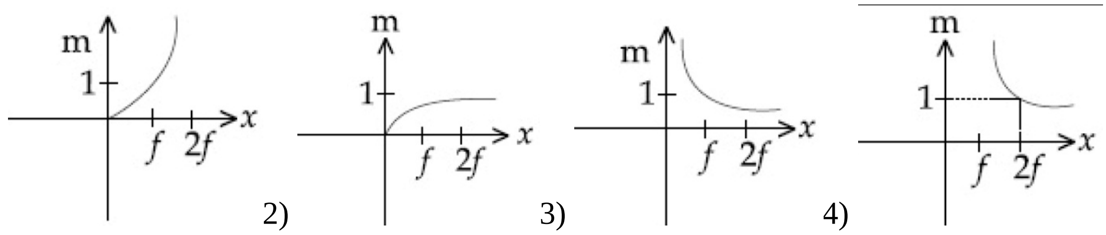
Ans: 2

Sol: Moment of masses about COM is zero

$\therefore M_{remaining}(2 - R = M_{cavity}(1 - R)$

$\Rightarrow (R^3 - 1^3)(2 - R) = 1^3[R - 1]$ $\Rightarrow (R^3 + R + 1)(2 - R) = 1$

5. An object is gradually moving away from the focal point of a concave mirror along the axis of the mirror. The graphical representation of the magnitude of linear magnification (m) versus distance of the object from the mirror(x) is correctly given by (Graphs are drawn schematically and are not to scale)



1) 2) 3) 4)

Ans: 4

Sol: As object moves from focus its magnification decreases from ∞

6. A transverse wave travels on a taut steel wire with a velocity of ν when tension in it is $2.06\text{x}10^4 N$. When the tension is changed to T, the velocity changed to $\nu/2$. The value of T is close to :

1) $5.15\text{x}10^3 N$ 2) $10.2\text{x}10^2 N$ 3) $2.50\text{x}10^4 N$ 4) $30.5\text{x}10^4 N$

Ans: 1

$v \propto \sqrt{T}$ $\quad \frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} \Rightarrow \frac{v}{(v/2)} = \sqrt{\frac{2.06\times10^4}{T}} \Rightarrow T = \frac{2.06\times10^4}{4} N = 0.515\times10^4 N$

7. A simple pendulum is being used to determine the value of gravitational acceleration g at a certain place. The length of the pendulum is 25.0 cm and a stop watch with 1s resolution measures the time taken for 40 oscillations to be 50s. The accuracy in g is :

1) 2.40% 2) 5.40% 3)3.40% 4) 4.40%

Ans: 4

Sol: $\frac{\Delta T}{T} = \frac{1}{2}\left(\frac{\Delta g}{g} + \frac{\Delta L}{L}\right) \quad \frac{\Delta g}{g} = \frac{2\Delta T}{T} + \frac{\Delta L}{L}; = 2\left(\frac{1}{50}\right) + \frac{0.1}{25.0}$ =4.4%

8. A galvanometer having a coil resistance 100Ω gives a full scale deflection when a current of 1 mA is passed through it. What is the value of the resistance which can convert this galvanometer into a voltmeter giving full scale deflection for a potential difference of 10V?

1) $9.9k\Omega$ 2) $7.9k\Omega$ 3) $8.9k\Omega$ 4) $10k\Omega$

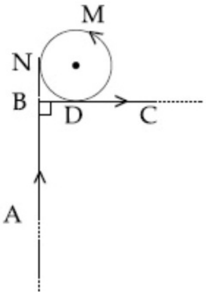
Ans: 1

Sol: $V = i_g\left(R_g + R\right)$

$10 - 10^{-3}(100 + R)$

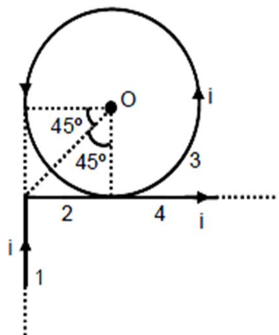
R= 9900 Ω

9. A very long wire ABDMNDC is shown in figure carrying current I. AB and BC parts are straight, long and at right angle. At D wire forms a circular turn DMND of radius R. AB, BC parts are tangential to circular turn at N and D. Magnetic field at the centre of circle is :



1) $\frac{\mu_0 I}{2\pi R}(\pi+1)$ 2) $\frac{\mu_0 I}{2\pi R}\left(\pi+\frac{1}{\sqrt{2}}\right)$ 3) $\frac{\mu_0 I}{2R}$ 4) $\frac{\mu_0 I}{2\pi R}\left(\pi-\frac{1}{\sqrt{2}}\right)$

Ans: 2



$$\vec{B}_0=\left(\vec{B}_0\right)_1+\left(\vec{B}_0\right)_2+\left(\vec{B}_0\right)_3+\left(\vec{B}_0\right)_4$$

$$\frac{-\mu_0 i}{4\pi R}\left[\sin 90^0-\sin 45^0\right]\otimes+\frac{\mu_0 i}{2R}\odot+\frac{\mu_0 i}{4\pi R}\left(\sin 45^0+\sin 90^0\right)\odot$$

$$=\frac{-\mu_0 i}{4\pi R}\left[1-\frac{1}{\sqrt{2}}\right]+\frac{\mu_0 i}{2R}+\frac{\mu_0 i}{4\pi R}\left[\frac{1}{\sqrt{2}}+1\right]\odot$$

$$\frac{-\mu_0 i}{4\pi R}\left[-1+\frac{1}{\sqrt{2}}+2\pi+\frac{1}{\sqrt{2}}+1\right]\odot=\frac{\mu_0 i}{4\pi R}\left[\sqrt{2}+2\pi\right]\odot\ \frac{\mu_0 i}{2\pi R}\left[\frac{1}{\sqrt{2}}+\pi\right]\odot$$

10. A particle moves such that its position vector $\vec{r}(t)=\cos\omega t\hat{i}+\sin\omega t\hat{j}$ where ω is a constant and t is time. Then which of the following statements is true for the velocity $\vec{\nu}(t)$ and acceleration $\vec{a}(t)$ of the particle :

1) $\vec{\nu}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed towards the origin

2) $\vec{\nu}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed away from the origin

3) $\vec{\nu}$ and $\vec{a}$ both are parallel to $\vec{r}$

4) $\vec{\nu}$ and $\vec{a}$ both are perpendicular to $\vec{r}$

Ans: 1

Sol: $\vec{r} = \cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = \omega\left(\sin\omega t\,\hat{i} + \cos\omega t\,\hat{j}\right)$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \omega^2\left(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}\right)$$

$\vec{a} = -\omega^2\vec{r}$

$\therefore \vec{a}$ is anti parallel to $\vec{r}$

$\vec{a}$, $\vec{r}$

$\vec{v}.\vec{r} = \omega\left(-\sin\omega t\cos\omega t + \cos\omega t\sin\omega t\right) = 0$

So $\vec{v} \perp \vec{r}$

11. In a double-slit experiment, at a certain point on the screen the path difference between the two interfering waves is $\frac{1}{8}$th of a wavelength. The ratio of the intensity of light at that point to that at the centre of a bright fringe is :

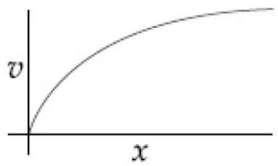
1) 0.568 2) 0.760 3)0.853 4) 0.672

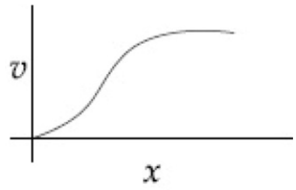
Ans: 3

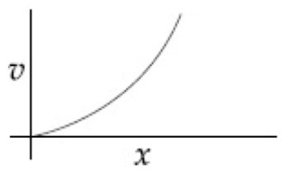
Sol: $I = \frac{I}{I_0}\cos^2\left(\frac{\Delta\phi}{2}\right)$

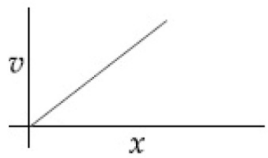
$$\frac{I}{I_0}\cos^2\left[\frac{\frac{2\pi}{\lambda}\times\Delta x}{2}\right] = \cos^2\left(\frac{\pi}{8}\right); \frac{I}{I_0} = 0.853$$

12. A particle of mass m and charge q is released from rest in a uniform electric field. If there is no other force on the particle, the dependence of its speed ν on the distance x travelled by it is correctly given by (graphs are schematic and not drawn to scale)

1) 

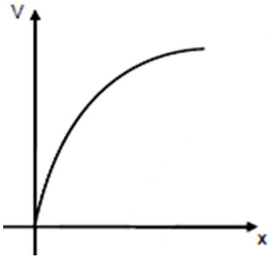
2) 

3) 

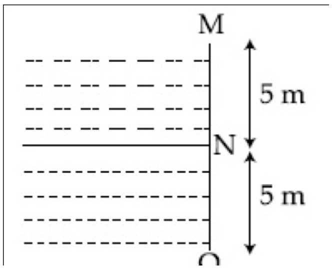
4) 

Ans: 1

Sol: $V^2 = \frac{2qE}{m}x$



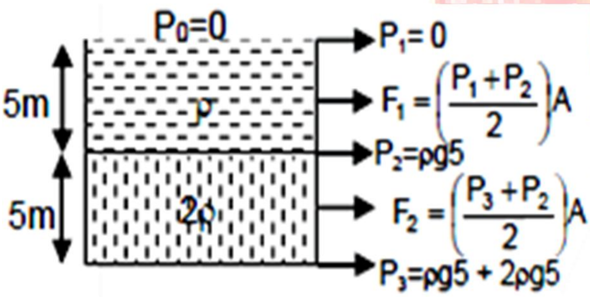
13.



Two liquids of densities ρ_1 and $\rho_2\left(\rho_2 = 2\rho_1\right)$ are filled up behind a square wall of side 10 m as shown in figure. Each liquid has a height of 5 m. The ratio of the forces due to these liquids exerted on upper part MN to that at the lower part NO is (Assume that the liquids are not mixing):

1) 1/4 2) 1/2 3)2/3 4) 1/3

Ans: 1



Force on upper halt = $P_{avg}A$

$= \frac{\rho g5}{5}A$

Force on lower halt = $P_{avg}A$ $= \frac{2(\rho g5) + 2\rho g5}{2}A$

$2\rho g5A$

$= \therefore \frac{F_1}{F_2} = \frac{\rho g5A}{2(2\rho g5)A} = \frac{1}{4}$

14. Consider a mixture of n moles of helium gas and 2n moles of oxygen gas (molecules taken to be rigid) as an ideal gas. Its C_P / C_V value will be :

1) 23/15 2) 67/45 3)19/13 4) 40/27

Ans: 3

sol: $\gamma_{mix} = \frac{n_1 c_{p_1} + n_2 c_{p_2}}{n_1 c_{v_1} + n_2 c_{v_2}} = \frac{n\left(\frac{5}{2}R\right) + 2n\left(\frac{7}{2}R\right)}{n\left(\frac{3}{2}R\right) + 2n\left(\frac{5}{2}R\right)} = \frac{5+14}{5+10} = \frac{19}{13}$

15. Consider two charged metallic sphere S_1 and S_2 of radii R_1 and R_2, respectively. The electric fields E_1(on S_1) and E_2(on S_2) on their surfaces are such that $E_1 / E_2 = R_1 / R_2$. Then the ratio $V_1(on\ S_1)/V_2(on\ S_2)$ of the electrostatic potentials on each sphere is :

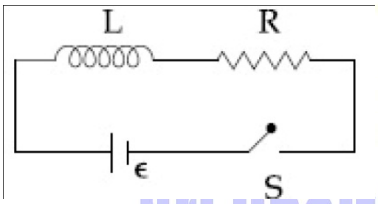
1) (R_2 / R_1) 2) $(R_1 / R_2)^2$ 3) $\left(\frac{R_1}{R_2}\right)^3$ 4) R_1 / R_2

Ans: 2

Sol: $\frac{E_1}{E_2} = \frac{r_1}{r_2}$

$\frac{V_1}{V_2} = \frac{E_1 r_1}{E_2 r_2} = \frac{r_1}{r_2} \times \frac{r_1}{r_2} = \left(\frac{r_1}{r_2}\right)^2$

16.



As shown in the figure, a battery of emf $\in$ is connected to an inductor L and resistance R in series. The switch is closed at t=0. The total charge that flows from the battery, between t = 0 and t = t_c(t_c is the time constant of the circuit) is :

1) $\frac{\in L}{R^2}\left(1 - \frac{1}{e}\right)$ 2) $\frac{\in R}{eL^2}$ 3) $\frac{\in L}{R^2}$ 4) $\frac{\in L}{eR^2}$

Ans: 4

Sol: $q = \int_0^{T_c} i dt$ $\quad i = \frac{\varepsilon}{R}\left(1 - e^{\frac{-t}{10}}\right)$

$\frac{E}{R}\left(T_c + T_c\left(e^{-1} - 1\right)\right)$ $\quad \frac{\varepsilon}{R} \times \frac{1}{e} \times \frac{L}{R} : \frac{\varepsilon L}{R^2 e}$

17. An electron (mass m) with initial velocity $\vec{\nu} = \nu_0 \hat{i} + \nu_0 \hat{j}$ is in an electric field $\vec{E} = -E_0 \hat{k}$. If λ_0 is initial de-Broglie wavelength of electron, its de-Broglie wavelength at time t is given by :

1) $\dfrac{\lambda_0}{\sqrt{1+\dfrac{e^2E^2t^2}{m^2\nu_0^2}}}$ 2) $\dfrac{\lambda_0\sqrt{2}}{\sqrt{1+\dfrac{e^2E^2t^2}{m^2\nu_0^2}}}$ 3) $\dfrac{\lambda_0}{\sqrt{2+\dfrac{e^2E^2t^2}{m^2\nu_0^2}}}$ 4) $\dfrac{\lambda_0}{\sqrt{1+\dfrac{e^2E^2t^2}{2m^2\nu_0^2}}}$

Ans:4

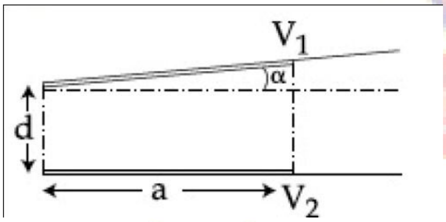
Sol: Initially $m\left(\sqrt{2}V_0\right)=\dfrac{h}{\lambda_0}$

Velocity as function of time $v_0\hat{i}+v_0\hat{j}+\dfrac{eE_0}{m}tk^-$

so wavelength $\lambda=\dfrac{h}{m\sqrt{2v_0^2+\dfrac{e^2E_0^2t^2}{m^2}}}$

$\lambda=\dfrac{\lambda_0}{\sqrt{1+\dfrac{e^2E_0^2}{m^2v_0^2}t^2}}$

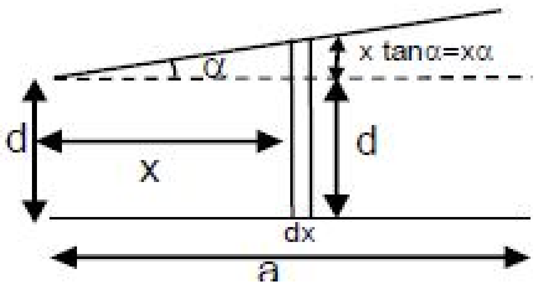
18. A capacitor is made of two square plates each of side ‘a’ making a very small angle α between them, as shown in figure. The capacitance will be close to :



1) $\dfrac{\in_0 a^2}{d}\left(1+\dfrac{\alpha a}{d}\right)$ 2) $\dfrac{\in_0 a^2}{d}\left(1-\dfrac{\alpha a}{4d}\right)$ 3) $\dfrac{\in_0 a^2}{d}\left(1-\dfrac{\alpha a}{2d}\right)$ 4) $\dfrac{\in_0 a^2}{d}\left(1-\dfrac{3\alpha a}{2d}\right)$

Ans:3

Sol:



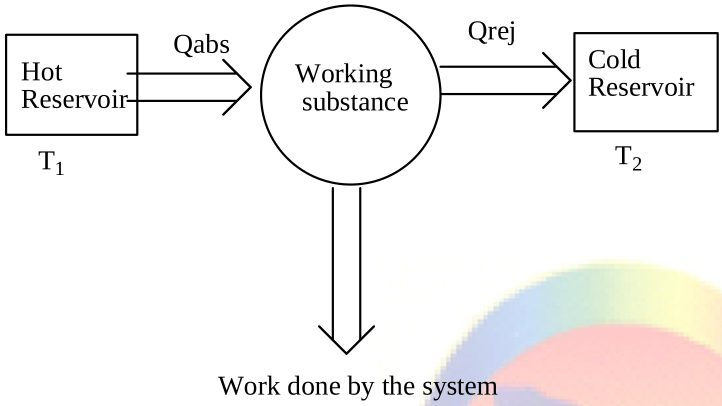
$dc=\dfrac{\varepsilon_0 a dx}{d+\alpha x}$ $\qquad \Rightarrow c=\dfrac{\varepsilon_0 a}{\alpha}\left[\ln(d+\alpha x)\right]_0^a$

$=\dfrac{\varepsilon_0 a}{\alpha}\ln\left(1+\dfrac{\alpha a}{d}\right)=\dfrac{\varepsilon_0 a^2}{d}\left(1-\dfrac{\alpha a}{2d}\right)$

19. A Carnot engine having an efficiency of $\frac{1}{10}$ is being used as a refrigerator. If the work done on the refrigerator is 10J, the amount of heat absorbed from the reservoir at lower temperature is :

1) 99 J 2) 100 J 3) 90 J 4) 1 J

Ans: 3



Sol:

In heat engine,

$$\eta = \frac{\text{work done}}{Q_{absorbed}} = \frac{1}{10} = \frac{10}{Q_{abs}}$$

$Q_{abs} = 100$

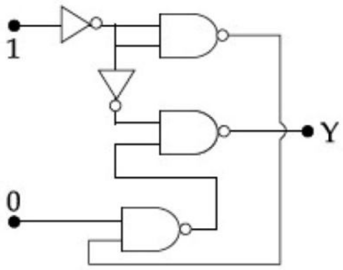
$Q_{rej} = Q_{abs} - w = 90$

In refrigerator,

$Q_{abs} = Q_{rej}$ in heat engine

$\therefore Q_{abs} = 90$

20. In the given circuit, value of Y is :



1) toggles between 0 and 1 2) 1

3) will not execute 4) 0

Ans: 4

Sol: $Y = \overline{\overline{AB}.A}$

$= \overline{\overline{AB}} + \overline{A}$

$= AB + \overline{A}$

= 0+0 = 1

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value(in decimal notation, truncated/rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer , 0 if not attempted and 0 in all other cases.

21. The first member of the Balmer series of hydrogen atom has a wavelength of 6561 $\overset{0}{A}$. The wavelength of the second member of the Balmer series (in nm) is ----

Ans: 486

Sol: $\frac{1}{\lambda}RZ^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$ $\frac{1}{\lambda_1}R(1)^2\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=\frac{5R}{36}$

$\frac{1}{\lambda_2}R(1)^2\left(\frac{1}{2^2}-\frac{1}{4^2}\right)=\frac{3R}{16}$ $\frac{\lambda_2}{\lambda_1}=\frac{20}{27}$

$\lambda_2=\frac{20}{27}\times 6561\overset{0}{A}=4860\overset{0}{A}$ =486nm

22. A ball is dropped from the top of a 100 m high tower on a planet. In the last $\frac{1}{2}$s before hitting the ground, it covers a distance of 19 m. Acceleration due to gravity (in ms^{-2}) near the surface on that planet is -----------

Ans: 8

Sol: $t=\sqrt{\frac{2(100)}{a}}=\sqrt{\frac{200}{a}}$ .....(1)

$t-\frac{1}{2}=\sqrt{\frac{2(81)}{a}}$ .....(2)

dividing 1 by 2 $\frac{t}{t-\frac{1}{2}}=\sqrt{\frac{100}{81}}$ 9t= 10t -5

5 =t t = 5

Substituting in 1) and then squaring it $25=\frac{200}{a}$ a=8

23. Three containers C_1,C_2 and C_3 have water at different temperatures. The table below shows the final temperature T when different amounts of water (given in liters) are taken from each container and mixed (assume no loss of heat during the process)

C_1	C_2	C_3	T
1l	2l	-	60^0C
-	1l	2l	30^0C
2l	-	1l	60^0C
1l	1l	1l	θ

The value of θ (in ^0C to the nearest integer) is -----------

Ans: 50

Sol: $1+\theta_1+2\theta_2=(1+2)60$

$\theta_1+2\theta_2=180$ .... (1)

$0\times\theta_1+1\times\theta_2+2\times\theta_3=(1+2)30$

$\theta_2+2\theta_3=90$ .... (2)

$2\times\theta_1+0\times\theta_2+1\times\theta_3=(2+1)60$

$2\theta_1+\theta_3=180$ .... (3)

Solving

$\theta_1=80, \theta_2=50$

$\theta_3=20$

$\therefore 20+80+50=3\theta R$

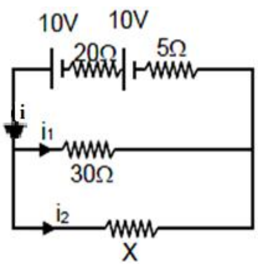
$\theta_{\frac{1}{2}}=50$

24. The series combination of two batteries, both of the same emf 10 V, but different internal resistance of 20 Ω and 5 Ω, is connected to the parallel combination of two resistors 30Ω and R Ω. The voltage difference across the battery of internal resistance 20 Ω is zero, the value of R(in Ω) is ------

Ans: 30

Sol: $V_1=\varepsilon_1-i.r_1$ $\quad 0-10-i\times 20$ $\quad i=0.5A$ $\quad V_2=\varepsilon_2-ir_2 \;=10-0.5\times 5$

$V_2=7.5V$ $\quad \mathrm{i}=\mathrm{i}_1+\mathrm{i}_2$ $\quad 0.5=\dfrac{7.5}{30}+\dfrac{7.5}{x}$ $\quad = 0.5=0.25+\dfrac{7.5}{x}\;\;\dfrac{7.5}{x}=0.25$



25. An asteroid is moving directly towards the centre of the earth. When at a distance of 10R(R is the radius of the earth) from the earth's centre, it has a speed of 12 km/s. Neglecting the effect of earth's atmosphere, what will be the speed of the asteroid when it hits the surface of the earth (escape velocity from the earth is 11.2 km/s)? Give your answer to the nearest integer in kilometer/s-------

Ans: 16

sol: $-\dfrac{GMm}{10R}+\dfrac{1}{2}m\left(12\times10^3\right)^2=\dfrac{-GMm}{R}+\dfrac{1}{2}m\upsilon_+^2$

$\sqrt{\dfrac{18gR}{10}+\left(12\times10^3\right)^2}=\upsilon_f$ $\qquad \Rightarrow \upsilon_f=\sqrt{\dfrac{9\left(11\times10^3\right)^2}{10}+\left(12\times10^3\right)^2}\,\text{m / sec}$

$\upsilon_f=\sqrt{108.9+144}\times10^3\,\text{m / sec}$ $\qquad \upsilon_f=16\,\text{km / sec}$

CHEMISTRY

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. The increasing order of the atomic radii of the following elements is:

(a) C (b) O (c) F (d) Cl (e) Br

(1) b < c < d < a < e (2) a < b < c < d < e

(3) d < c < b < a < e (4) c < b < a < d < e

Ans. 4

Sol: F < O < C < Cl < Br

2. For the following Assertion and Reason, the correct option is:

Assertion: The pH of water increases with increase in temperature.

Reason: The dissociation of water into H^+ and OH^- is an exothermic reaction.

1) Both assertion and reason are true, But the reason is not the correct explanation for the assertion

2) Assertion is not true, but reason is true

3) Both assertion and reason are false

4) Both assertion and reason are true, and the reason is the correct explanation for the assertion

Ans: 3

Sol: $H_2O + H_2O \rightleftharpoons H_3O^+ + OH^-$ (Endothermic Reaction)

Increase in temperature increases K_w and pH decreases.

3. Hydrogen has three isotopes (A), (B) and (C). If the number of neutron(s) in (A), (B) and (C) respectively, are (x), (y) and (z), the sum of (x), (y) and (z) is:

1) 2 2) 4 3) 1 4) 3

Ans: 4

Sol: $\overset{1}{H}, \overset{2}{H}, \overset{3}{H}$ are three isotopes of hydrogen.

$n(n) = 0 + 1 + 2 = 3$

4. Among the reactions (a) – (d), the reaction(s) that does/do not occur in the blast furnace during the extraction of iron is/are:

(a) $CaO + SiO_2 \rightarrow CaSiO_3$ (b) $3Fe_2O_3 + CO \rightarrow 2Fe_3O_4 + CO_2$

(c) $FeO + SiO_2 \rightarrow FeSiO_3$ (d) $FeO \rightarrow Fe + \frac{1}{2} O_2$

1) d 2) a 3) a & b 4) c and d

Ans: 4

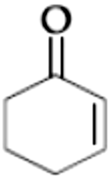
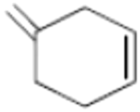
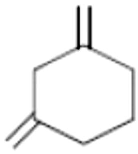
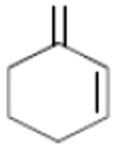
Sol: Both reactions c and d do not occur ion blast furnace.

5. An unsaturated hydrocarbon X absorbs two hydrogen molecules on catalytic hydrogenation, and also gives following reaction:

$$X \xrightarrow[Zn/H_2O]{O_3} A \xrightarrow{[Ag(NH_3)_2]^+}$$

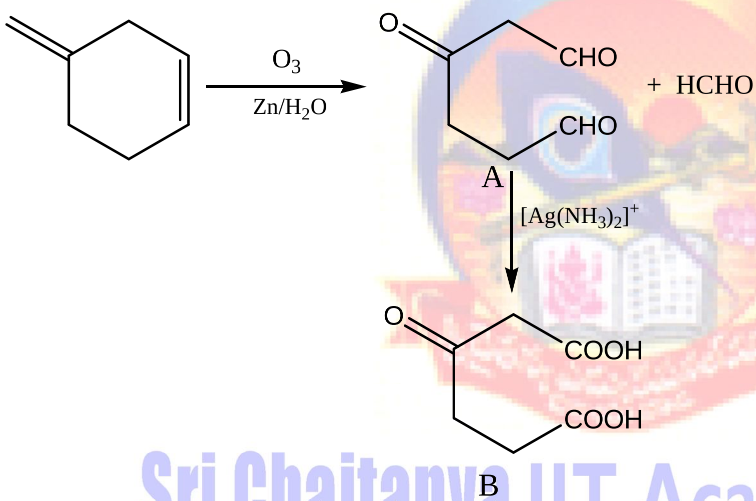
B (3-oxo-hexanedicarboxylic acid)

X will be:

1)  2)  3)  4) 

Ans: 2

Sol:



6. A metal (A) on heating in nitrogen gas gives compound B. B on treatment with H_2O gives a colourless gas which when passed through $CuSO_4$ solution gives a dark blue-violet coloured solution. A and B respectively, are:

1) Na & Na_3N 2) Na & $NaNO_3$ 3) Mg & Mg_3N_2 4) Mg & $Mg(NO_3)_2$

Ans: 3

Sol:
$$\underset{(A)}{3Mg} + N_2 \rightarrow \underset{(B)}{Mg_3N_2}$$

$$\underset{(B)}{Mg_3N_2} + 6H_2O \rightarrow 3Mg(OH)_2 + 2NH_3$$

$$CuSO_4 + 4NH_3 \rightarrow \underset{\text{Dark blue}}{\left[Cu(NH_3)_4\right]SO_4}$$

7. Which of the following compounds is likely to show both Frenkel and Schottky defects in its crystalline form?

1) KBr 2) AgBr 3) CsCl 4) ZnS

Ans: 2

Sol: Silver bromide crystals show both Frenkel and shortly defects.

8. The radius of the second Bohr orbit, in terms of the Bohr radius, a_0, in Li^{2+} is:

1) $\frac{2a_0}{3}$ 2) $\frac{2a_0}{9}$ 3) $\frac{4a_0}{9}$ 4) $\frac{4a_0}{3}$

Ans: 4

Sol: $r_{2,Li^{+2}} = \frac{a_0 \times 4}{z} = \frac{a_0 \times 4}{3} = \frac{4a_0}{3}$

9. Two monomers in maltose are:

1) α–D-glucose & α-D-Fructose 2) α–D-glucose & β-D- glucose

3) α–D-glucose & α-D-galactose 4) α–D-glucose & α-D-glucose

Ans: 4

Sol: Maltose is a disaccharide with two α–D-glucose units.

10. For the following Assertion and Reason, the correct option is:

Assertion: For hydrogenation reactions, the catalytic activity increases from Group 5 to Group 11 metals with maximum activity shown by Group 7 – 9 elements.

Reason: The reactants are most strongly adsorbed on group 7 – 9 elements.

1) Both assertion and reason are true, the reason is not the correct explanation for the assertion

2) Both assertion and reason are true, the reason is the correct explanation for the assertion

3) Both assertion and reason are false

4) The Assertion is true, but reason is false

Ans: 2

Sol: Reactants should be chemisorted reasonable strongly but not most strongly to linden the reaction progress

11. The correct order of the calculated spin only magnetic moments of complexes (A) to (D) is:

(A) $Ni(CO)_4$ (B) $[Ni(H_2O)_6]Cl_2$ (C) $Na_2[Ni(CN)_4]$ (D) $PdCl_2(PPh_3)_2$

1) (A) $\approx$ (C) $\approx$ (D)<(B) 2) (C) $\approx$ (D)<(B)<(A)

3) (C) < (D) < (B) < (A) 4) (A) $\approx$ (C)<(B) $\approx$ (D)

Ans: 1

Sol: $Ni(CO)_4, Na_2[Ni(CN)_4]$ and $Pd[Cl_2(PPh_3)_2]$ are diamagnetic with zero magnetic moment. A= C = D < B

12. Kjeldahl's method cannot be used to estimate nitrogen for which of the following compounds?

1) $C_6H_5NH_2$ 2) $NH_2-\overset{O}{\overset{\|}{C}}-NH_2$ 3) $CH_3CH_2-C\equiv N$ 4) $C_6H_5NO_2$

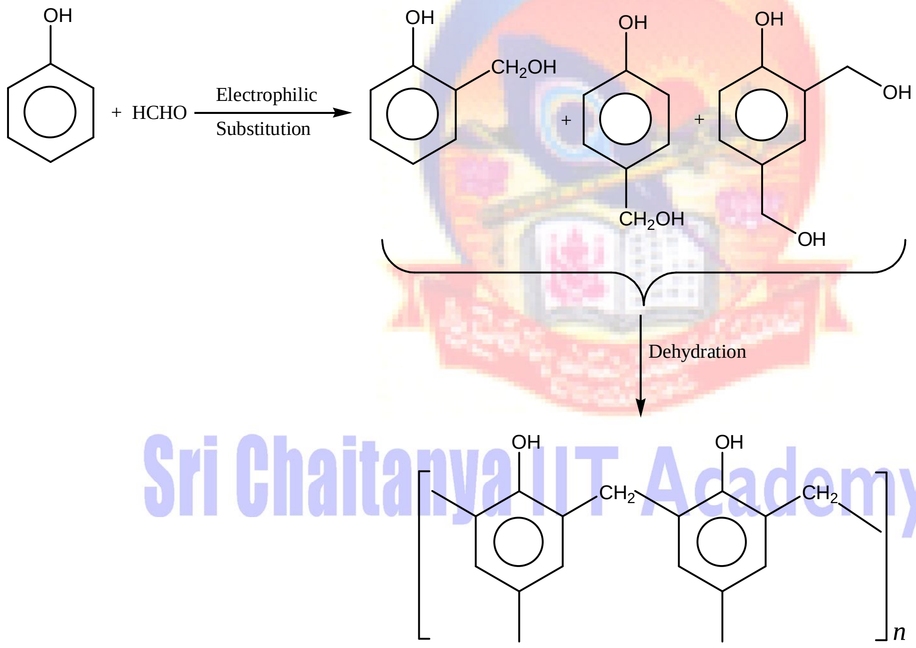
Ans: 4

Sol: $C_6H_5NO_2$ does not give NH_3 on reaction with H_2SO_4, hence Kjeldahl's method cannot be used to estimate nitrogen in $C_6H_5NO_2$.

13. Preperation of Bakelite proceeds via reactions:

1) Electrophilic addition and dehydration
2) Condensation and elimination
3) Electrophilic substitution and dehydration
4) Nucleophilic addition and dehydration

Ans: 3



Sol:

Bakelite

14. Arrange the following bonds according to their average bond energies in descending order:

C – Cl, C – Br, C – F, C - I

1) C – Cl > C – Br > C – I > C – F 2) C – I > C – Br > C – Cl > C – F
3) C – F > C – Cl > C – Br > C – I 4) C – Br > C – I > C – Cl > C – F

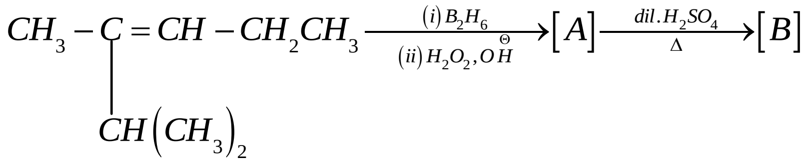
Ans: 3

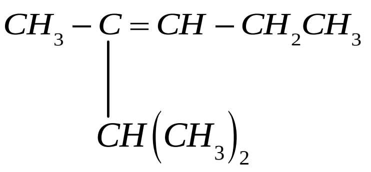
Sol: C – F < C – Cl < C – Br < C – I [order of bond length]

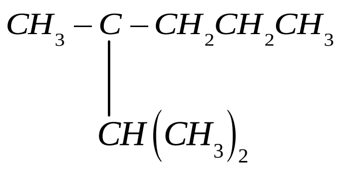
In this case bond energy $\propto \dfrac{1}{\text{Bond length}}$.

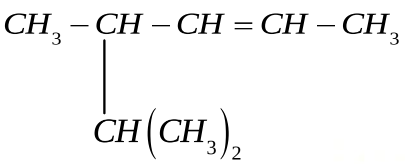
Hence bond energy order is C – F > C – Cl > C – Br > C – I.

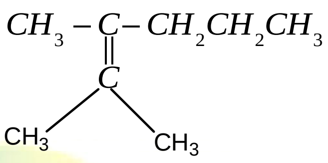
15. The major product [B] in the following sequence of reactions is:



1)


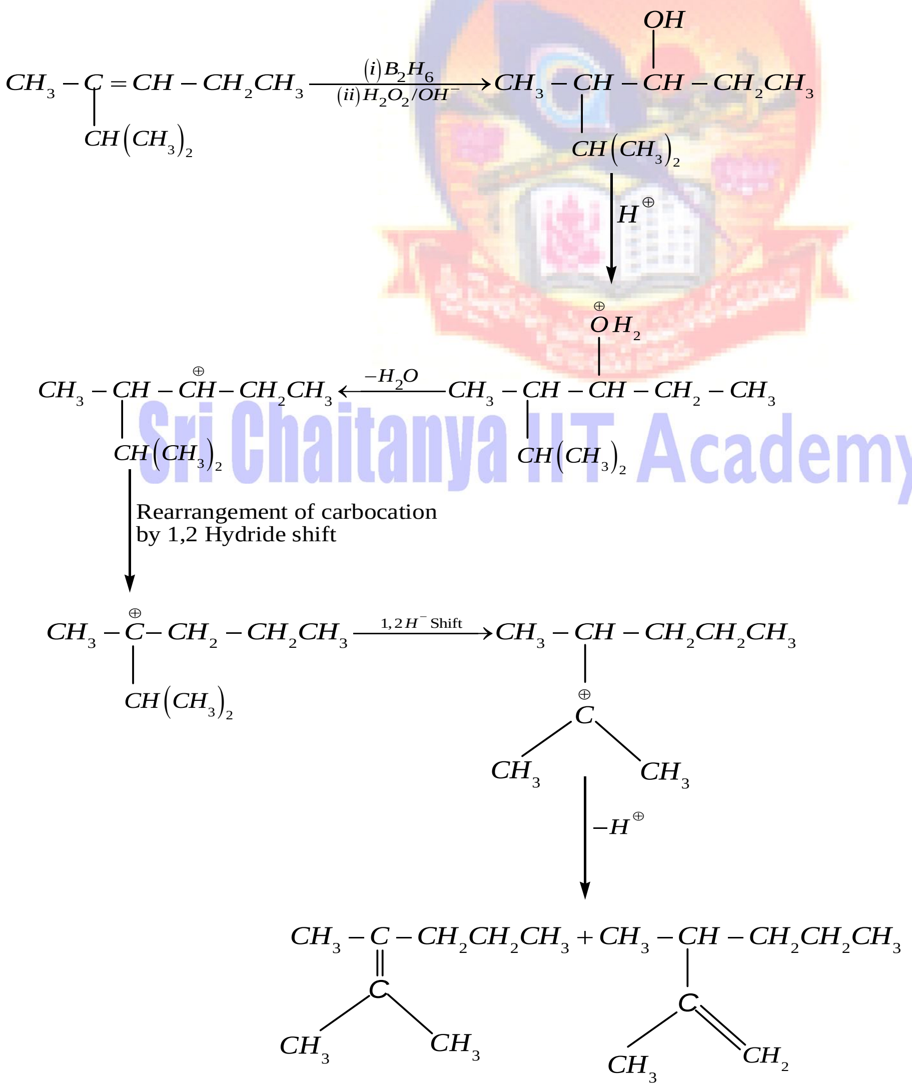
2)


3)


4)


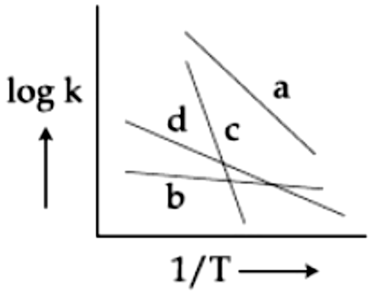
Ans: 4

Sol:



Major Product

16. Consider the following plots of rate constant versus $\frac{1}{T}$ for four different reactions. Which of the following orders is correct for the activation energies of these reactions?



1) $E_a > E_c > E_d > E_b$ 2) $E_c > E_a > E_d > E_b$

3) $E_b > E_a > E_d > E_c$ 4) $E_b > E_d > E_c > E_a$

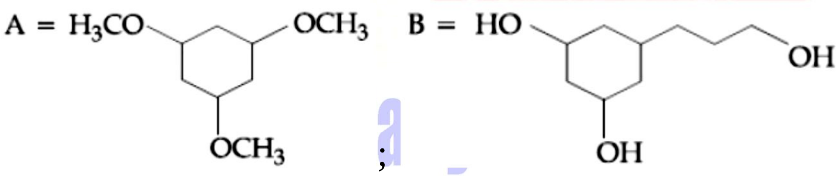
Ans: 2

Sol: $k = Ae^{-Ea/RT}$ Arheneous equation

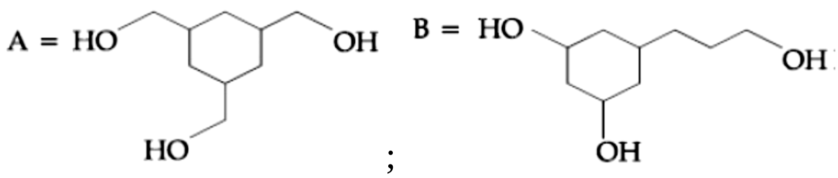
$$\log k = \log A - \frac{E_a}{RT}$$

17. Among the compounds A and B with molecular formula $C_9H_{18}O_3$, A is having higher boiling point than B. The possible structures of A and B are:

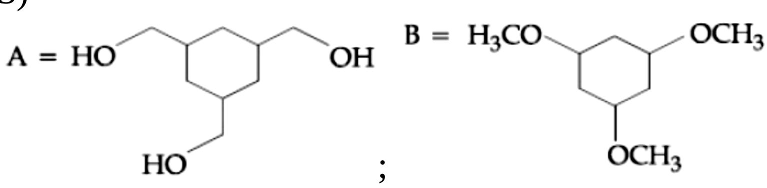
1)



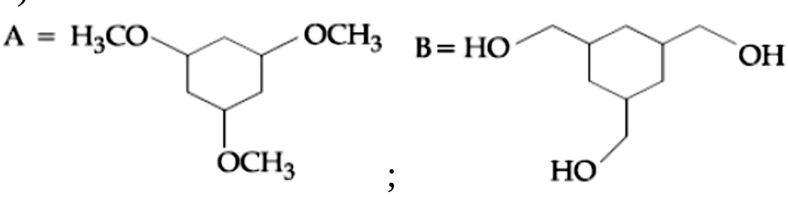
2)



3)



4)



Ans: 3

Sol: A(alcohol) have higher boiling point due to H-bonding which is absent in B(ether).

18. Among (a) – (d), the complexes that can display geometrical isomerism are:

(a) $\left[Pt(NH_3)_3Cl\right]^{+}$ (b) $\left[Pt(NH_3)Cl_5\right]^{-}$

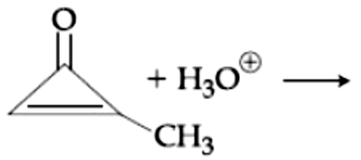
(c) $\left[Pt(NH_3)_2Cl(NO_2)\right]$ (d) $\left[Pt(NH_3)_4ClBr\right]^{2+}$

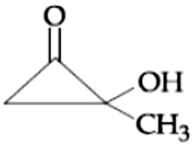
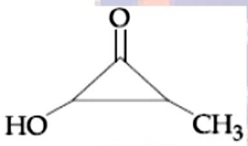
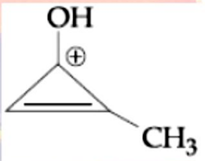
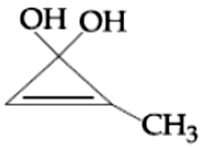
1) a & b 2) a & d 3) c & d 4) b & c

Ans: 3

Sol: c and d exhibit Cis – trans isomers

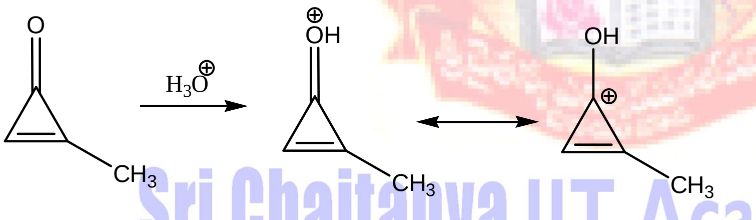
19. The major product in the following reaction is:



1)  2)  3)  4) 

Ans: 3

Sol:



Aromatic cation

20. White phosphorus on reaction with concentrated NaOH solution in an inert atmosphere of CO_2 gives phosphine and compound (X). (X) on acidification with HCl gives compound (Y). The basicity of compound (Y) is:

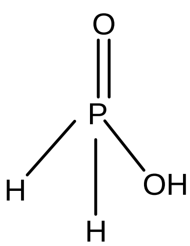
1) 4 2) 2 3) 1 4) 3

Ans: 3

Sol: $P_4 + 3NaOH + 3H_2O \rightarrow 3NaH_2PO_2 + PH_3$

$NaH_2PO_2 + HCl \rightarrow NaCl + H_3PO_2$

H_3PO_2 is mono basic and as it contain only one – OH group.



(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value(in decimal notation, truncated/rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

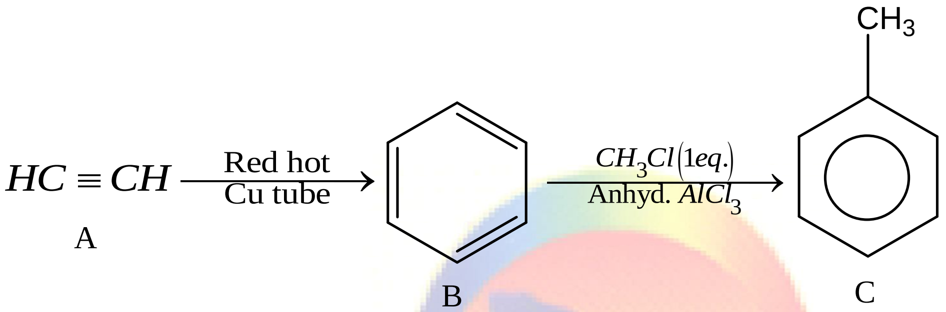
Marking scheme: +4 for correct answer , 0 if not attempted and 0 in all other cases.

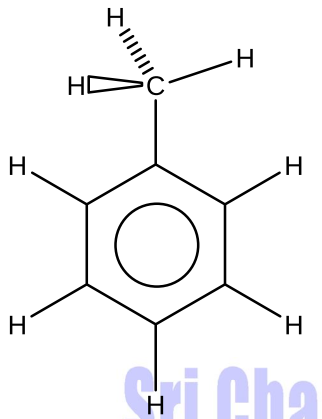
21. In the following sequence of reactions the maximum number of atoms present in molecule 'C' in one plane is____

$$A \xrightarrow[\text{Cu tube}]{\text{Red hot}} B \xrightarrow[\text{Anhydrous } AlCl_3]{CH_3Cl(1eq.)} C$$

(A is a lowest molecular weight alkyne).

Ans: 13





Total 13 atoms in same plane.

Sol:

22. At constant volume, 4 mol of an ideal gas when heated from 300 K to 500 K changes its internal energy by 5000 J. The molar heat capacity at constant volume is_____.

Ans: 6.25

Sol: $\Delta U = nC_{v,m}\Delta T$

$5000 = 4 \times C_{v,m} \times 200$

$$C_{v,m} = \frac{5000}{4 \times 200} = 6.25$$

23. For an electrochemical cell

$Sn(s) | Sn^{2+}(aq,1M) || Pb^{2+}(aq,1M) | Pb(s)$

The ratio $\frac{[Sn^{2+}]}{[Pb^{2+}]}$ when this cell attains equilibrium is______

(Given: $E^0_{Sn^{2+}|Sn} = -0.14\,V, E^0_{Pb^{2+}|Pb} = -0.13\,V, \frac{2.303RT}{F} = 0.06$)

Ans: 2.15

Sol: $E^0 = \frac{0.06}{2}\log\frac{[Sn^{+2}]}{[Pb^{+2}]}$

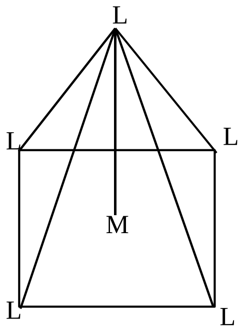
$0.14 - 0.13 = 0.03\log\frac{[Sn^{+2}]}{[Pb^{+2}]}$

$\frac{[Sn^{+2}]}{[Pb^{+2}]} = 10^{\frac{1}{3}} = 2.15$

24. Complexes (ML_5) of metals Ni and Fe have ideal square pyramidal and trigonal bipyramidal geometries, respectively. The sum of the 90^0, 120^0 and 180^0 L-M-L angles in the two complexes is____

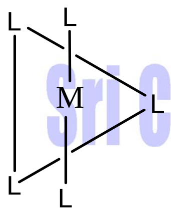
Ans: 20

Sol: ML_5 with Square pyramid.



Contains eight 90^0 angle and two diagonal 180^0 angles (total=10).

ML_5 with TBP structure.



Contains six 90^0 and three 120^0 and one 180^0. So total = 10.

Sum in both = 10 + 10 = 20.

25. $NaClO_3$ is used, even in spacecrafts, to produce O_2. The daily consumption of pure O_2 by a person is 492 L at 1 atm, 300 K. How much amount of $NaClO_3$, in grams, is required to produce O_2 for the daily consumption of a person at 1 atm, 300 K?

$NaClO_3$ (s) + Fe(s) → O_2 (g) + NaCl (s) + FeO (s)

R = 0.082 L atm mol^{-1} K^{-1}

Ans: 2130

Sol: $NaClO_3 + Fe \rightarrow O_2 + NaCl + FeO$

$\frac{x}{106.5} = \frac{1 \times 492}{300 \times 0.082}$

x=2130gms

MATHEMATICS

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. The mirror image of the point $(1,2,3)$ in a plane is $\left(\frac{-7}{3},\frac{-4}{3},-\frac{1}{3}\right)$. Which of the following lies on this plane?

   1) $(-1,-,1,-1)$ 2) $(1,1,1)$ 3) $(-1,-1,1)$ 4) $(1,-1,1)$

Ans: 4

Sol: d.r of normal to the plane are $\left\langle\frac{10}{3},\frac{10}{3},\frac{10}{3}\right\rangle$ i.e. $\langle 1,1,1\rangle$

Midpoint of given point is $\left(\frac{-2}{3},\frac{1}{3},\frac{4}{3}\right)$

$\therefore$ Equation of plane is x + y + z = 1

2. If α and β be the coefficients of x^4 and x^2 respectively in the expansion of $\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6$ then:

   1) $\alpha-\beta=60$ 2) $\alpha+\beta=60$ 3) $\alpha-\beta=-132$ 4) $\alpha+\beta=-30$

Ans: 3

Sol: $2[\,{}^6C_0x^6+{}^6C_2x^4\left(x^2-1\right)^2+{}^6C_4x^2\left(x^2-1\right)^2+{}^6C_6\left(x^2-1\right)^3\,]$

$=2[x^6+15\left(x^6-x^4\right)+15x^2\left(x^4-2x^2+1\right)+\left(-1+3x^2-3x^4+x^6\right)]$

$=2\left(32x^6-48x^4+18x^2-1\right)$ $\alpha=-96$ and $\beta=36$ $\therefore$ $\alpha-\beta=-132$

3. If a line, $y=mx+c$ is a tangent to the circle, $(x-3)^2+y^2=1$ and it is perpendicular to a line L_1, where L_1 is the tangent to the circle, $x^2+y^2=1$ at the point $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$; then:

   1) $c^2-7c+6=0$ 2) $c^2-6c+7=0$ 3) $c^2+6c+7=0$ 4) $c^2+7c+6=0$

Ans: 3

Sol: Slope of tangent to $x^2 + y^2 = 1$ at $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ is 1.

$x^2 + y^2 = 1$

$2x + 2yy' = 0$

$y'=-\frac{x}{y}=-1$

$\therefore y=x+c$ is tangent of $(x-3)^2 + y^2 = 1$

Now distance of (3, 0) from y = x + c is equal to radius

$\left|\frac{c+3}{\sqrt{2}}\right|=1 \quad \Rightarrow c^2 + 6c + 9 = 2 \Rightarrow c^2 + 6c + 7 = 0$

4. $\lim_{x\to 0}\frac{\int_0^x t\sin(10t)\,dt}{x}$ is equal to:

1) $\frac{1}{10}$ 2) $-\frac{1}{10}$ 3) $-\frac{1}{5}$ 4) 0

Ans: 4

Sol: Using L'Hospital Rule, we get

$$\lim_{x\to 0}\frac{x\sin(10x)}{1}=0$$

5. Let $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b}\times\vec{c}=\vec{b}\times\vec{a}$ and $\vec{c}.\vec{a}=0$, then $\vec{c}.\vec{b}$ is equal to:

1) $-\frac{3}{2}$ 2) $\frac{1}{2}$ 3) -1 4) $-\frac{1}{2}$

Ans: 4

Sol: $\vec{a}\times(\vec{b}\times\vec{c})=\vec{a}\times(\vec{b}\times\vec{a}) \qquad -(\vec{a}.\vec{b})\vec{c}=(\vec{a}.\vec{a})\vec{b}-(\vec{a}.\vec{b})\vec{a}$

$-4\vec{c}=6(\hat{i}-\hat{j}+\hat{k})-4(\hat{i}-2\hat{j}+\hat{k}) \qquad \vec{c}=-\frac{1}{2}(\hat{i}+\hat{j}+\hat{k}) \qquad \vec{b}.\vec{c}=-\frac{1}{2}$

6. If $A=\begin{pmatrix}2 & 2\\ 9 & 4\end{pmatrix}$ and $I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, then $10A^{-1}$ is equal to:

1) 6I – A 2) 4I – A 3) A – 4I 4) A – 6I

Ans: 4

Sol: Characteristics equation of matrix 'A' is

$\begin{vmatrix}2-x & 2\\ 9 & 4-x\end{vmatrix}=0 \quad \Rightarrow \quad x^2-6x-10=0$

$\therefore \quad A^2-6A-10I=0 \quad \Rightarrow \quad 10A^{-1}=A-6I$

7. The differential equation of the family of curves, $x^2=4b(y+b), b\in R$ is:

1) $x(y')^2=x-2yy'$ 2) $xy''=y'$

3) $x(y')^2=2yy'-x$ 4) $x(y')^2=x+2yy'$

Ans: 4

Sol: $2x=4by' \quad \Rightarrow \quad b=\frac{x}{2y'}$

So. differential equation is $x^2=\frac{2x}{y'}.y+\left(\frac{x}{y'}\right)^2$

8. The system of linear equations

$\lambda x+2y+2z=5$

$2\lambda x+3y+5z=8$

$4x+\lambda y+6z=10$ has :

1) infinitely many solutions when $\lambda=2$

2) no solution when $\lambda=2$

3) a unique solution when $\lambda=-8$

4) no solution when $\lambda=8$

Ans: 2

Sol: $D=\begin{vmatrix}\lambda & 2 & 2\\ 2\lambda & 3 & 5\\ 4 & \lambda & 6\end{vmatrix}$

$D=(\lambda+8)(2-\lambda)$

for $\lambda=2$

$D_1=\begin{vmatrix}5 & 2 & 2\\ 8 & 3 & 5\\ 10 & 2 & 6\end{vmatrix}$

= 5[18 – 10] – 2 [48 – 50] + 2 (16 – 30]

= 40 + 4 – 28 $\neq 0$

No solutions for $\lambda=2$

9. Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability that A or B occurs is $\frac{1}{2}$, then the probability of both of them occur together is:

1) 0.20 2) 0.01 3) 0.02 4) 0.10

Ans: 4

Sol: P(exactly one) = $\frac{2}{5}$ $\Rightarrow P(A)+P(B)-2P(A\cap B)=\frac{2}{5}$

$P(A\cup B)=\frac{1}{2}$ $\Rightarrow P(A)+P(B)-2P(A\cap B)=\frac{1}{2}$

$\therefore\ P(A\cap B)=\frac{1}{2}-\frac{2}{5}=\frac{5-4}{10}=\frac{1}{10}$

10. Let $\alpha=\frac{-1+i\sqrt{3}}{2}$. If $a=(1+\alpha)\sum_{k=0}^{100}\alpha^{2k}$ and $b=\sum_{k=0}^{100}\alpha^{3k}$, then a and b are the roots of the quadratic equation:

1) $x^2-102x+101=0$ 2) $x^2+101x+100=0$ 3) $x^2+102x+101=0$ 4) $x^2-101x+100=0$

Ans: 1

Sol: $\alpha=\omega,\quad b=1+\omega^3+\omega^6+......=101$

$a=(1+\omega)\left(1+\omega^2+\omega^4+......\omega^{198}+\omega^{200}\right)$

$=(1+\omega)\dfrac{\left(1-(\omega^2)^{101}\right)}{1-\omega^2}=\dfrac{(1+\omega)(1-\omega)}{1-\omega^2}=1$

Equation : $x^2-(101+1)x+(101)\times1=0 \qquad \Rightarrow x^2-102x+101=0$

11. The mean and variance of 20 observations are found to be 10 and 4, respectively. On rechecking, it was found that an observation 9 was incorrect and the correct observation was 11. Then the correct variance is:

1)4.02 2) 4.01 3) 3.99 4) 3.98

Ans: 3

Sol: $\dfrac{\sum x_i}{20}=10 \qquad ....(i)$

$\dfrac{\sum x_i^2}{20}-100=4 \qquad ....(ii)$

$\sum x_i^2=104\times20=2080$

Actual mean $=\dfrac{200-9+11}{20}=\dfrac{202}{20}$

Actual Variance $=\dfrac{2080-81+121}{20}-\left(\dfrac{202}{20}\right)^2$

$=\dfrac{2120}{20}-(10.1)^2=106-102.01=3.99$

12. Let $f:(1,3)\to R$ be a function defined by $f(x)=\dfrac{x[x]}{1+x^2}$, where $[x]$ denotes the greatest integer $\le x$. Then the range of f is:

1) $\left(\frac{2}{5},\frac{4}{5}\right]$ 2) $\left(\frac{2}{5},\frac{3}{5}\right]\cup\left(\frac{3}{4},\frac{4}{5}\right)$ 3) $\left(\frac{2}{5},\frac{1}{2}\right)\cup\left(\frac{3}{5},\frac{4}{5}\right]$ 4) $\left(\frac{3}{5},\frac{4}{5}\right)$

Ans: 3

Sol: $f(x)=\begin{cases}\dfrac{x}{x^2+1}; & x\in(1,2)\\ \dfrac{2x}{x^2+1}; & x\in[2,3)\end{cases} \qquad \therefore \qquad f(x)$ is a decreasing function

If $x\in(1,2)$ then $f(x)\in\left(\frac{2}{5},\frac{1}{2}\right)$

If $x\in[2,3)$ then $f(x)\in\left(\frac{3}{5},\frac{4}{5}\right] \qquad \Rightarrow \qquad y\in\left(\frac{2}{5},\frac{1}{2}\right)\cup\left(\frac{3}{5},\frac{4}{5}\right]$

13. The length of the perpendicular from the origin, on the normal to the curve, $x^2+2xy-3y^2=0$ at the point $(2,2)$ is:

1) $4\sqrt{2}$ 2) 2 3) $2\sqrt{2}$ 4) $\sqrt{2}$

Ans: 3

Sol: $x^2+2xy-3y^2=0 \Rightarrow \frac{dy}{dx}=\frac{-(x+y)}{x-3y} \Rightarrow \left.\frac{dy}{dx}\right|_{(2,2)}=1$

$\therefore$ Equation of normal is x + y = 4

Perpendicular distance from origin $\left|\frac{0+0-4}{\sqrt{2}}\right|=2\sqrt{2}$

14. Which of the following statements is a tautology?

1) $\sim(p\vee\sim q)\rightarrow p\wedge q$ 2) $\sim(p\wedge\sim q)\rightarrow p\vee q$

3) $\sim(p\vee\sim q)\rightarrow p\vee q$ 4) $p\vee(\sim q)\rightarrow p\wedge q$

Ans: 3

Sol: $(\sim p\wedge q)\rightarrow(p\vee q)$

$\sim\{(\sim p\wedge q)\wedge(\sim p\wedge\sim q)\}$

$\sim\{\sim p\wedge f\}$

15. Let S be the set of all real roots of the equation, $3^x(3^x-1)+z|3^x-1|+|3^x-2|$. Then S:

1) is an empty set 2) is a singleton

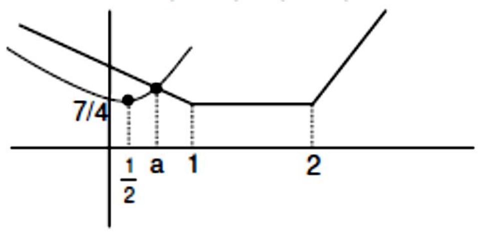
3) contains at least four elements 4) contains exactly two elements

Ans: 2

Sol: Let 3^x = t, then t > 0

$t(t-1)+2=|t-1|+|t-2|$

$t^2-t+2=|t-1|+|t-2|$



On + ve side of x – axis, there is only one point of intersection

$\therefore t=a\Rightarrow x=log_3 a$

16. The area (in sq. units) of the region $\{(x,y)\in R^2 : x^2\le y\le 3-2x\}$ is:

1) $\frac{29}{3}$ 2) $\frac{32}{3}$ 3) $\frac{31}{3}$ 4) $\frac{34}{3}$

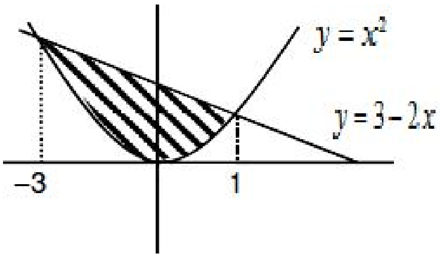
Ans: 2

Sol: $x^2 \le y \Rightarrow (x,y)$ lies above y = x^2 and $y \le 3-2x$ means (x, y) lies below 2x+y=3

Point of intersection of y = x^2 & y = - 2x + 3 is obtained by

$x^2 + 2x - 3 = 0 \Rightarrow x = -3, 1$

So, Area $= \int_{-3}^{1}(3-2x-x^2)dx = 3(4) - 2\left(\frac{1^2-3^2}{2}\right) - \left(\frac{1^3+3^3}{3}\right) = 12+8-\frac{28}{3} = \frac{32}{3}$



17. Let S be the set of all functions $f:[0,1]\to R$, which are continuous on $[0,1]$ and differentiable on $(0,1)$. Then for every f in S, there exists $a c \in (0,1)$ depending on f, such that:

1) $|f(c)+f(1)| < (1+c)|f'(c)|$ 2) $|f(c)-f(1)| < (1-c)|f'(c)|$

3) $|f(c)-f(1)| < |f'(c)|$ 4) $\frac{|f(1)-f(c)|}{1-c} = f'(c)$

Ans: Given by NTA 3 Our answer (BOUNS)

Sol: Options (1), (2), (3) are wrong, if we take f (x) as a constant function

Option (4) is incorrect if for f (x) = x^2

18. If $I = \int_1^2 \frac{dx}{\sqrt{2x^3-9x^2+12x+4}}$, then:

1) $\frac{1}{8} < I^2 < \frac{1}{4}$ 2) $\frac{1}{16} < I^2 < \frac{1}{9}$ 3) $\frac{1}{6} < I^2 < \frac{1}{2}$ 4) $\frac{1}{9} < I^2 < \frac{1}{8}$

Ans: 4

Sol: Let $f(x) = \frac{1}{\sqrt{2x^3-9x^2+12x+4}}$

$$f'(x) = \frac{-1}{2}\frac{(6x^2-18x+12)}{(2x^3-9x^2+12x+4)^{\frac{3}{2}}}$$

$$= \frac{-6(x-1)(x-2)}{2(2x^3-9x^2+12x+4)^{\frac{3}{2}}}$$

$f(1) = \frac{1}{3}$, $f(2) = \frac{1}{\sqrt{8}}$

$\frac{1}{3} < I < \frac{1}{\sqrt{8}}$

19. If the 10th term of an A.P. is $\frac{1}{20}$ and its 20th term is $\frac{1}{10}$, then the sum of its first 200 terms is:

1) 50 2) 100 3) $100\frac{1}{2}$ 4) $50\frac{1}{4}$

Ans: 3

Sol: $T_{10}=\frac{1}{20}=a+9d$ ....(i)

$T_{10}=\frac{1}{10}=a+19d$ ....(ii)

$\Rightarrow \quad a=\frac{1}{200}, d=\frac{1}{200} \quad \Rightarrow \quad S_{200}=\frac{200}{2}\left[\frac{2}{200}+\frac{199}{200}\right]=\frac{201}{2}=100\frac{1}{2}$

20. If a hyperbola passes through the point P(10, 16) and it has vertices at $(\pm 6, 0)$, then the equation of the normal to it at P is:

1) $3x+4y=94$ 2) $x+2y=42$ 3) $2x+5y=100$ 4) $x+3y=58$

Ans: 3

Sol: Vertex is at $(\pm 6,0)$

$\therefore \quad a=6$

Let the hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Putting point P(10, 16) on the hyperbola

$\frac{100}{36}-\frac{256}{b^2}=1 \quad \Rightarrow \quad b^2=144$

$\therefore$ Hyperbola is $\frac{x^2}{36}-\frac{y^2}{144}=1$

$\therefore$ Equation of normal is $\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2$

$\therefore$ Putting $(x_1, y_1)=(10,16)$, we get 2x + 5y = 100

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value(in decimal notation, truncated/rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}}=\frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}}=\frac{1}{\sqrt{10}}$

$\alpha, \beta, \in \left(0, \frac{\pi}{2}\right)$, then $\tan(\alpha+2\beta)$ is equal to

Ans: 1

Sol: $\frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha}=\frac{1}{7}$ and $\frac{\sqrt{2}\sin\beta}{\sqrt{2}}=\frac{1}{\sqrt{10}}$

$\tan\alpha=\frac{1}{7}$ $\quad\sin\beta=\frac{1}{\sqrt{10}}$ $\quad\tan\beta=\frac{1}{3}$

$$\tan 2\beta=\frac{2.\frac{1}{3}}{1-\frac{1}{9}}=\frac{\frac{2}{3}}{\frac{8}{9}}=\frac{3}{4}$$

$$\tan(\alpha+2\beta)=\frac{\tan\alpha+\tan 2\beta}{1-\tan\alpha\tan 2\beta}=\frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}.\frac{3}{4}}=\frac{\frac{4+21}{28}}{\frac{25}{28}}=1$$

22. Let $f(x)$ be a polynomial of degree 3 such that $f(-1)=10, f(1)=-6, f(x)$ has a critical point at x=-1 and $f'(x)$ has a critical point at x=1. Then $f(x)$ has a local minima at x=

Ans: 3

Sol: $f'(-1)=0\ \&\ f(-1)=10$

$\Rightarrow$ Local maxima at x = -1

Let $f''(x)=k(x-1)\left[\because f''(1)=0\right]$

$$\Rightarrow f'(x)=\frac{k}{2}\left(x^2-2x\right)+c$$

$$f'(-1)=0\Rightarrow c=\frac{-3k}{2}$$

$$\therefore f'(x)=\frac{k}{2}(x+1)(x-3)$$

$\Rightarrow$ Local minima at x = 3

23. The sum , $\sum_{n=1}^{7}\frac{n(n+1)(2n+1)}{4}$ is equal to

Ans: 504

Sol: $\frac{1}{4}\left[\sum_{n=1}^{7}\left(2n^3+3n^2+n\right)\right]$

$$\frac{1}{4}\left[2\left(\frac{7.8}{2}\right)^2+3\left(\frac{7.8.15}{6}\right)+\frac{7.8}{2}\right]$$

$$\frac{1}{4}[2\times 49\times 16+28\times 15+28]$$

$$\frac{1}{4}[1568+420+28]=504$$

24. The number of 4 letter words (with or without meaning) that can be formed from the eleven letters of the word "EXAMINATION" is __________

Ans: 2454

Sol: EXAMINATION

2N, 2A, 2I, E, X, M, T, O

Case I All are different so $^8P_4 = \frac{8!}{4!} = 8.7.6.5 = 1680$

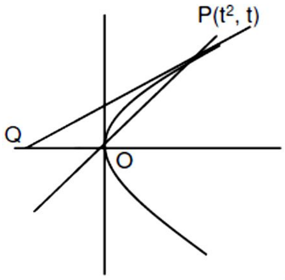
Case II 2 same and 2 different so $^3C_1.^7C_2.\frac{4!}{2!} = 3.21.12 = 756$

Case III 2 same and 2 same so $^3C_2.\frac{4!}{2!.2!} = 3.6 = 18$

Total = 1680 + 756 + 18 = 2454

25. Let a line $y = mx(m > 0)$ intersect the parabola, $y^2 = x$ at a point P, other than the origin. Let the tangent to it at P meet the x-axis at the point Q. If area $(\Delta OPQ) = 4$ sq. units, then m is equal to __________

Ans: 0.50



Sol:

2ty = x + t^2

Q($-t^2$, 0)

$$\frac{1}{2}\left\|\begin{matrix} 0 & 0 & 1 \\ t^2 & t & 1 \\ -t^2 & 0 & 1 \end{matrix}\right\| = 4$$

$$|t|^3 = 8$$

$$t = \pm 2 \quad (t > 0)$$

$$m = \frac{1}{2}$$

Prepared by

Sri Chaitanya Faculty