36 Years of Excellence 1986-2021

Sri Chaitanya

JEE MAIN 2021

PHASE - IV

Key & Solutions

26-Aug-2021 | Shift - 1

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A right Choice for the Real Aspirant

ICON Central Office – Madhapur – Hyderabad

Jee-Main_Final_26-Aug-2021_Shift-01

PHYSICS **Max Marks: 100**

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. Two narrow bores of diameter $5.0\,mm$ and $8.0\,mm$ are joined together to form a U – shaped tube open at the both ends. If this U – tube contains water, what is the difference in the level of two limbs of the tube?

[Take surface tension of water $T = 7.3\times10^{-2}\,Nm^{-1}$, angle of constant = 0 , $g = 10\,ms^{-2}$ and density of water $= 1.0\times10^{3}\,kg\,m^{-3}$]

1) $2.19\,mm$ 2) $3.62\,mm$ 3) $4.97\,mm$ 4) $5.34\,mm$

Key: 1

Sol: $T = \dfrac{rh\rho g}{2\cos\theta}\left(\because \cos\theta = \cos 0 = 1\right)$

$$\therefore T = \frac{rh\rho g}{2} \text{ or } h = \frac{2T}{r\rho g}$$

$$\therefore \Delta h = h_1 - h_2 = \frac{2T}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\Delta h = \frac{2\times7.3\times10^{-2}}{1000\times10}\left(\frac{2}{5\times10^{-3}} - \frac{2}{8\times10^{-3}}\right)$$

$$h_1 - h_2 = \frac{14.6\times10^{-2}}{10^{4}}\times2000\left(\frac{1}{5} - \frac{1}{8}\right)$$

$$h_1 - h_2 = \frac{21.9}{10}\times10^{-3} = 2.19\times10^{-3}\,m = 2.19\,mm$$

2. The fractional change in the magnetic field intensity at a distance ‘r’ from the centre on the axis of current carrying coil of radius ‘a’ to the magnetic field intensity at the centre of the same coil is : (Take $r < a$)

1) $\dfrac{3}{2}\dfrac{a^2}{r^2}$ 2) $\dfrac{3}{2}\dfrac{r^2}{a^2}$ 3) $\dfrac{2}{3}\dfrac{r^2}{a^2}$ 4) $\dfrac{2}{3}\dfrac{a^2}{r^2}$

Key: 2

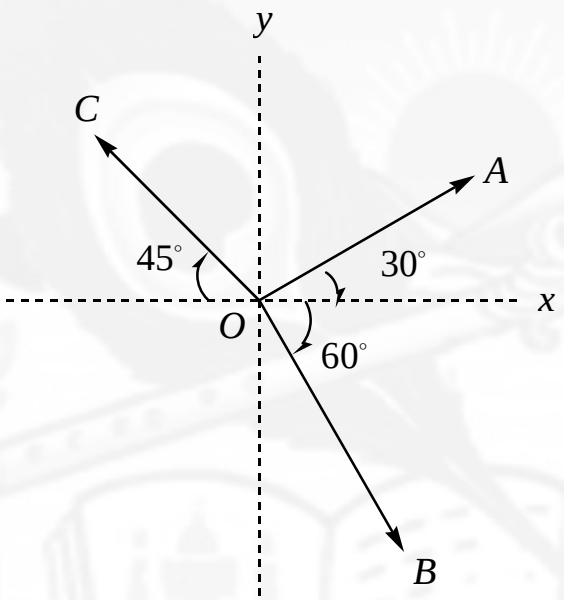
Sol: $\frac{B_c}{B_a}=\left(1+\frac{r^2}{a^2}\right)^{\frac{3}{2}}$

$\frac{B_c}{B_a}=1+\frac{3}{2}\frac{r^2}{a^2}$

$\frac{B_c}{B_a}-1=\frac{3}{2}\frac{r^2}{a^2}$

Fractional change = $\frac{3}{2}\frac{r^2}{a^2}$

3. The magnitudes of the vectors $\overrightarrow{OA},\overrightarrow{OB}$ and $\overrightarrow{OC}$ in the given figure are equal. The direction of $\overrightarrow{OA}+\overrightarrow{OB}-\overrightarrow{OC}$ with $x-$axis will be:



1) $\tan^{-1}\frac{\left(1+\sqrt{3}-\sqrt{2}\right)}{\left(1-\sqrt{3}-\sqrt{2}\right)}$

2) $\tan^{-1}\frac{\left(\sqrt{3}-1+\sqrt{2}\right)}{\left(1+\sqrt{3}-\sqrt{2}\right)}$

3) $\tan^{-1}\frac{\left(1-\sqrt{3}-\sqrt{2}\right)}{\left(1+\sqrt{3}+\sqrt{2}\right)}$

4) $\tan^{-1}\frac{\left(\sqrt{3}-1+\sqrt{2}\right)}{\left(1-\sqrt{3}+\sqrt{2}\right)}$

Key: 3

Sol: $\overrightarrow{OA}=P\cos 30\hat{i}+P\sin 30\hat{j}$

$\overrightarrow{OA}=\frac{\sqrt{3}P}{2}\hat{i}+\frac{P}{2}\hat{j}$

$\left|\overrightarrow{OA}\right|=\left|\overrightarrow{OB}\right|=\left|\overrightarrow{OC}\right|=p$

$$\overrightarrow{OB} = P\cos 60\hat{i} - P\sin 60\hat{j}$$

$$\overrightarrow{OB} = \frac{P}{2}\hat{i} - \frac{\sqrt{3}P}{2}\hat{j}$$

$$\overrightarrow{OC} = P\sin 45\left(-\hat{i}\right) + P\cos 45\hat{j}$$

$$\overrightarrow{OC} = \frac{P}{\sqrt{2}}\hat{i} + \frac{P}{\sqrt{2}}\hat{j}$$

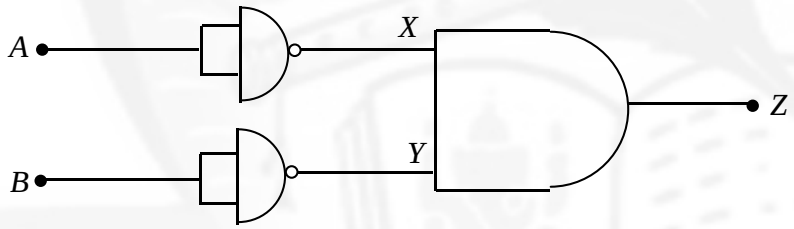
$$\overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC} = \hat{i}\left(\frac{\sqrt{3}}{2}P + \frac{P}{2} + \frac{P}{\sqrt{2}}\right) + \hat{j}\left(\frac{P}{2} - \frac{\sqrt{3}}{2}P - \frac{P}{\sqrt{2}}\right)$$

$$\overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC} = \hat{i}\left(\frac{\sqrt{2}\sqrt{3}P + \sqrt{2}P + 2P}{2\sqrt{2}}\right) + \hat{j}P\left(\frac{\sqrt{2} - \sqrt{2}\sqrt{3} - 2}{2\sqrt{2}}\right)$$

$$\tan\theta = \frac{P_y}{P_x} \Rightarrow \tan\alpha = \frac{\sqrt{2}\left[1 - \sqrt{3} - \sqrt{2}\right]}{\sqrt{2}\left[\sqrt{3} + 1 + \sqrt{2}\right]}$$

$$\tan\alpha = \frac{\left(1 - \sqrt{3} - \sqrt{2}\right)}{\left(1 + \sqrt{3} + \sqrt{2}\right)}$$

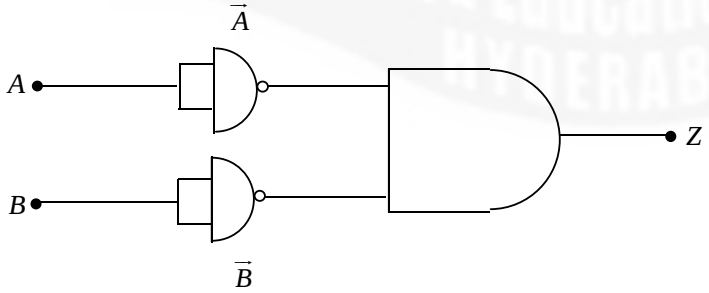
4. Identify the logic operation carried out by the given circuit:



1) NOR 2) NAND 3) OR 4) AND

Key: 1

Sol:



$$Z = \vec{A}.\vec{B} = \overrightarrow{A + B} = NOR$$

5. Inside a uniform spherical shell:

(a) The gravitational field is zero.

(b) The gravitational potential is zero.

(c) The gravitational field is same everywhere.

(d) The gravitational potential is same everywhere.

(e) All of the above

1) (b), (c) and (d) only 2) (a), (c) and (d) only

3) (e) only 4) (a), (b) and (c) only

Key: 2

Sol: Inside a uniform spherical shell

a) Gravitation field E = 0 true

b) Gravitational potential $v \neq 0$ false

c) E is same every where

d) V is same every where

6. In a Screw Gauge, fifth division of the circular scale coincides with the reference line when the ratchet is closed. There are 50 divisions on the circular scale and the main scale moves by $0.5mm$ on a complete rotation. For a particular observation the reading on the main scale is $5mm$ and the 20^{th} division of the circular scale coincides with reference line. Calculate the true reading.

1) $5.20mm$ 2) $5.25mm$ 3) $5.15mm$ 4) $5.00mm$

Key: 3

Sol: Zero error $= +5$

correction $= -5$

Least count $= \frac{0.5}{50} = \frac{1}{100} mm$

Reading=MSR+(Coincidence+Correction)LC

$= 5 + (20 - 5) \times \frac{1}{100}$

Reading=5.15 mm

7. The material field between the plates of a parallel plate capacitor has resistivity $200\Omega m$. The value of capacitance of the capacitor is $2\,pF$. If a potential difference of $40V$ is applied across the plates of the capacitor, then the value of leakage current flowing out of the capacitor is: (given the value of relative permittivity of material is 50)

1) $0.9mA$ 2) $0.9\mu A$ 3) $9.0mA$ 4) $9.0\mu A$

Key: 1

Sol: leakage current $i=\frac{V_0}{R}=\frac{Q_0}{CR}=\frac{Q_0}{\rho k\in_0}\left(\because V_0=\frac{Q_0}{C}\right)$

$$i=\frac{CV}{\rho k\in_0}=\frac{2\times10^{-12}\times40}{2W\times50\times8.8\times10^{-12}}$$

$$i=\frac{100}{11}\times10^{-4}=9\times10^{-4}A=\frac{9\times10^{-3}}{10}$$

$$i=0.9mA$$

8. A series LCR circuit driven by $300V$ at a frequency of 50 Hz contains a resistance $R=3k\Omega$, an inductor of inductive resistance $X_L=250\pi\Omega$ and an unknown capacitor. The value of capacitance to maximize the average power should be: (take $\pi^2=10$)

1) $4\mu F$ 2) $25\mu F$ 3) $40\mu F$ 4) $400\mu F$

Key: 1

Sol: maximize average power

$$X_L=X_C$$

$$X_L=\frac{1}{\omega C}$$

$$250\pi=\frac{1}{2\pi fC}=\frac{100}{2\pi}g$$

$$f=\frac{1}{2\left(\pi^2\right)(250)(50)}=4\mu F$$

9. The rms speed of the molecules of Hydrogen, Oxygen and Carbondioxide at the same temperature are V_H, V_O and V_C respectively then:

1) $V_H=V_O>V_C$ 2) $V_C>V_O>V_H$ 3) $V_H=V_O=V_C$ 4) $V_H>V_O>V_C$

Key: 4

Sol: $V_{rms} = \sqrt{\frac{3RT}{M}}$

$V_{rms} \alpha \frac{1}{\sqrt{M}}$

$\because M_H < M_O < M_C$

$\therefore V_H > V_O > V_C$

10. A particular hydrogen like ion emits radiation of frequency $2.92 \times 10^{15}\, Hz$ when it makes transition from $n = 3 \text{ to } n = 1$. The frequency in Hz of radiation emitted in transition from $n = 2$ to $n = 1$ will be:

1) 4.38×10^{15} 2) 2.46×10^{15} 3) 6.57×10^{15} 4) 0.44×10^{15}

Key: 2

Sol: $h\vartheta = 13.6\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$$\frac{2.92 \times 10^{15}}{\vartheta} = \frac{\left[\frac{1}{1^2} - \frac{1}{3^2}\right]}{\left[\frac{1}{1^2} - \frac{1}{2^2}\right]}$$

$$\frac{2.92 \times 10^{15}}{\vartheta} = \frac{\left[\frac{8}{9}\right]}{\left[\frac{3}{4}\right]} = \frac{8}{9} \times \frac{4}{3}$$

$\vartheta = \frac{27}{32} \times 2.92 \times 10^{15} = 2.46 \times 10^{15}\, Hz$

11. An electric appliance supplies $6000\, J/\min$ heat to the system delivers a power of $90 W$. How long it would take to increase the internal energy by $2.5 \times 10^3\, J$?

1) $2.5 \times 10^2\, S$ 2) $2.5 \times 10^1\, S$ 3) $2.4 \times 10^3\, S$ 4) $4.1 \times 10^1\, S$

Key: 1

Sol: $\frac{dQ}{dt} = 6000\, J/\min = 100\ J/s$

P = 90 W

$dU = 2.5 \times 10^3\, J$

$$\frac{dQ}{dt} = \frac{dU}{dt} + \frac{dW}{dt}$$

$$100 = \frac{dU}{dt} + 90$$

$$\frac{dU}{dt} = 10$$

Total time = $\frac{total\ internal\ energy}{\frac{dU}{dt}}$

$T = \frac{2.5\times10^3}{10} = 2.5\times10^2 \sec$

12. In a photo electric experiment ultraviolet light of wavelength 280 nm is used with lithium cathode having work function $\phi = 2.5eV$. If the wave length of incident light is switched to 400 nm, find out the change in the stopping potential.
($h = 6.63\times10^{-34} Js, c = 3\times10^8 ms^{-1}$)

1) 1.1 V 2) 1.9 V 3) 1.3 V 4) 0.6 V

Key: 1

Sol: $eV_{s1} - eV_{s2} = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2}$

$e(V_{s1} - V_{s2}) = \frac{12400}{2800} - \frac{12400}{400}\left(\because \frac{hc}{\lambda}\frac{eV}{(A^0)}\right)$

$e(V_{s1} - V_{s2}) = (4.4 - 3.1)eV\left(\because \frac{hc}{\lambda} = \frac{12400}{\lambda(A^0)}\right)$

$\Delta V = 1.3V$

13. If E, L, M and G denote the quantities as energy, angular momentum, mass and constant of gravitation respectively, then the dimensions of P in the formula $P = EL^2M^{-5}G^{-2}$ are :

1) $[M^0L^1T^0]$ 2) $[M^{-1}L^{-1}T^2]$ 3) $[M^1L^1T^{-2}]$ 4) $[M^0L^0T^0]$

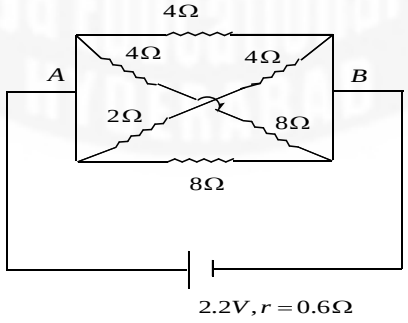
Key: 4

Sol: $[P] = \left[\left[ML^2T^{-2}\right]\left[ML^2T^{-1}\right]^2\left[M^{-5}\right]\left[\frac{MLT^{-2}L^2}{M^2}\right]^{-2}\right]$

$= \left[M^{1+2-5+2}L^{2+4-6}T^{-2-2+4}\right]$

$= \left[M^\circ L^\circ T^\circ\right]$

14. In the given figure, the emf of the cell is $2.2V$ and if internal resistance is $0.6\ \Omega$. Calculate the power dissipated in the whole circuit:



1) 0.65 W 2) 1.32 W 3) 4.4 W 4) 2.2 W

Key: 4

Sol: Reduce the network

$2\Omega \, senses \, with \, 4\Omega = 2 + 4 = 6\Omega$

$4\Omega \, senses \, with \, 8\Omega = 4 + 8 = 12\Omega$

$6\Omega, 12\ \Omega \, and \, 4\Omega \, and \, 8\Omega$ are in parallel

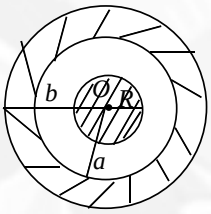
$$\therefore \frac{1}{R} = \frac{1}{6} + \frac{1}{12} + \frac{1}{4} + \frac{1}{8} = \frac{4+2+6+3}{24}$$

$$R = \frac{24}{15} = 1.6\Omega$$

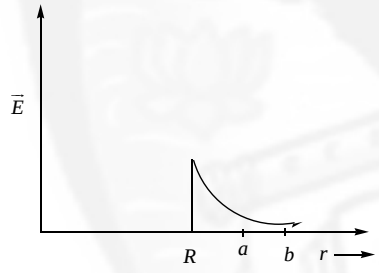
$$i = \frac{E}{R+r} = \frac{2.2}{1.6+0.6} = \frac{2.2}{2.2} = 1$$

$$\therefore P = i^2(R+r) = 1^2(1.6+0.6) = 2.2W$$

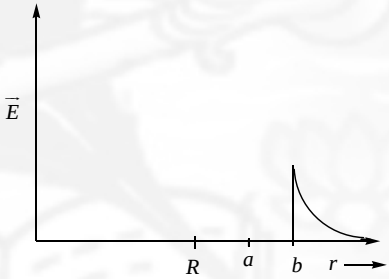
15. A solid metal sphere of radius R having charge q is enclosed inside the concentric spherical shell of inner radius a and the outer radius b as shown in figure. The approximate variation electric field $\vec{E}$ as function of distance r from centre O is given by:



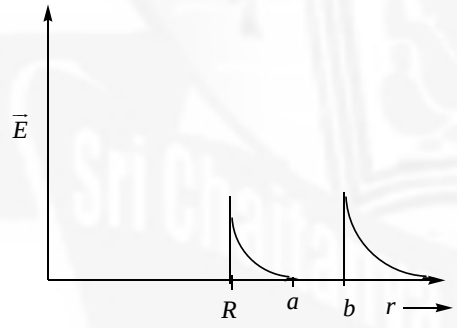
1)



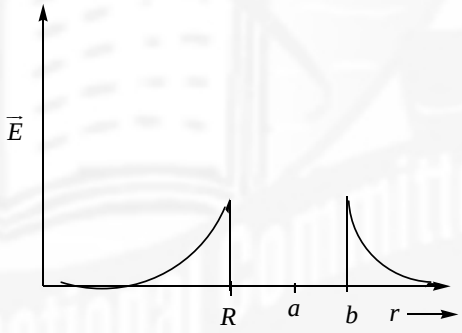
2)



3)



4)



Key: 3

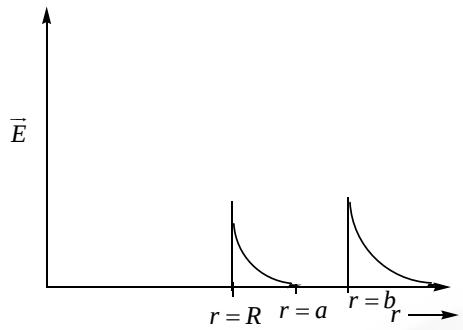
Sol: Consider outer spherical shell is conducting

$r < R, E = 0$

$$R \le r < a \;\; E = \frac{kQ}{r^2}$$

$a \le r < b \; E = 0$

$$r \ge b \; E = \frac{kQ}{r^2}$$



16. Statement I:
By doping silicon semiconductor with pentavalent material, the electrons density increases.
Statement II:
The n – type semiconductor has net negative charge.
In the light of the above statements, choose the most appropriate answer from the options given below
1) Statement I is false but Statement II is true
2) Both Statement I and Statement II are false
3) Statement I is true but Statement II is false
4) Both Statement I and Statement II are true

Key: 3

Sol: penta valent is majority of electrons.
Adding impurity may not give net charge because its atom

17. What equal length of an iron wire and a copper – nickel alloy wire, each of 2 mm diameter connected parallel to give an equivalent resistance of 3Ω ?
(Given resistivities of iron and copper – nickel alloy were are $12\,\mu\Omega cm$ and $51\,\mu\Omega cm$ respectively)
1) 82 m 2) 97 m 3) 110 m 4) 90 m

Key: 2

Sol: $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$

$$\frac{1}{3}=\frac{1}{\frac{\rho_1 \ell}{A}}+\frac{1}{\frac{\rho_2 \ell}{A}}$$

$$\frac{1}{3}=\left(\frac{1}{\frac{12\times10^{-6}}{10^{+2}}l}+\frac{1}{\frac{51\times10^{-6}}{10^{+2}}\times l}\right)\frac{22}{7}\times\left(\frac{2}{2}\times10^{-3}\right)^2$$

$$\frac{1}{3}=\frac{1}{l}\left[\frac{1}{10^{-8}}\right]\left[\frac{51+12}{12\times51}\right]\left[\frac{22}{7}\right]\left[10^{-6}\right]$$

$$l=3\times\frac{63}{12\times51}\times\frac{22}{7}\times10^{+2}=97\text{ m}$$

18. An inductor coil stores 64 J of magnetic field energy and dissipates energy at the rate of 640 W when a current of 8 A is passed through it. If this coil is joined across an ideal battery, find the time constant of the circuit in seconds:

1) 0.2 2) 0.4 3) 0.8 4) 0.125

Key: 1

Sol: $U = \frac{1}{2}Li^2 \Rightarrow 64J = \frac{1}{2}L(8^2)$

L = 2 H

II) $P = 640\ W$

$L^2R = 640$

$R = \frac{640}{i^2} = \frac{640}{8^2} = 10\Omega$

III) time constant $T = \frac{L}{R} = \frac{2}{10} = 0.2\sec$

19. Car B overtakes another car A at a relative speed of $40ms^{-1}$ .How fast will the image of car B appear to move in the mirror of focal length 10 cm fitted in car A, when the car B is 1.9 m away from the car A?

1) $4ms^{-1}$ 2) $40ms^{-1}$ 3) $0.2ms^{-1}$ 4) $0.1ms^{-1}$

Key: 4

Sol: $V_A - V_B = 40ms^{-1}$

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} + \frac{1}{-190} = \frac{1}{10}$

$\frac{1}{v} = \frac{1}{10} + \frac{1}{190} = \frac{190+10}{1900} = \frac{200}{1900} = \frac{2}{19}$

$v = \frac{19}{2}\ cm$

$\frac{d}{dt}\left[\frac{1}{v}\right] + \frac{d}{dt}\left(\frac{1}{u}\right) = \frac{d}{dt}\left(\frac{1}{f}\right)$

$\frac{1}{v^2}\frac{dv}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0$

$\frac{dv}{dt} = -\frac{v^2}{u^2}\left(\frac{du}{dt}\right)$

$\frac{dv}{dt} = -\left[\frac{19}{2} \times \frac{1}{190}\right]^2 40 = -\frac{1}{4 \times 100} \times 40 = 0.1ms^{-1}$

20. The initial mass of a rocket is 1000 kg. Calculate at what rate the fuel should be burnt so that the rocket is given an acceleration of $20 ms^{-2}$. The gases come out at a relative speed of $500 ms^{-1}$ with respect to the rocket:

[use $g = 10 ms^{-2}$]

1) $10 kg\, s^{-1}$ 2) $500 kg\, s^{-1}$ 3) $6.0 \times 10^2 kg\, s^{-1}$ 4) $60 kg\, s^{-1}$

Key: 4

Sol: Thrust - mg = ma

$$\vartheta \frac{dm}{dt} = m(g + a)$$

$$500 \frac{dm}{dt} = 1000(10 + 20)$$

$$\frac{dm}{dt} = \frac{1000}{500} \times 30 = 60 \text{ kg s}^{-1}$$

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. Two travelling waves produces a standing wave represented by equation. $y = 1.0 mm \cos\left(1.57 cm^{-1}\right) x \sin\left(78.5 s^{-1}\right) t$. The node closest to the origin in the region $x > 0$ will be at $x =$ _____cm.

Key: 1

Sol: Standing wave equation given is

$$y = 1.0 mm \cos\left(1.57 cm^{-1}\right) x \sin\left(78.5 s^{-1}\right) t$$

$$A(x) = 1.0 mm \cos\left(1.57 cm^{-1}\right) x$$

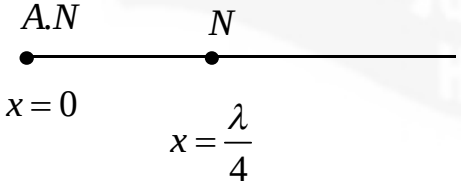
Amplitude is function f x

At $x = 0 \Rightarrow A(x)$ is maximum

So anti node formula

$$\frac{2\pi}{\lambda} = 1.57 = \frac{\pi}{2}$$

$$\lambda = 4 cm$$



$\therefore$ Distance of closest node from $x = 0$ is $x = \frac{\lambda}{4} = \frac{4}{4} = 1$

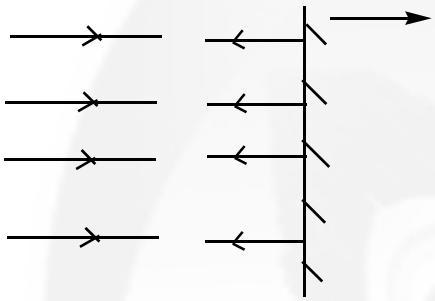
22. The electric field in a plane electromagnetic wave is given by

$$\vec{E} = 200\cos\left[\left(\frac{0.5\times10^3}{m}\right)x - \left(1.5\times10^{11}\frac{rad}{s}\times t\right)\right]\frac{V}{m}\hat{j}$$

If this wave falls normally on a perfectly reflecting surface having an area of $100\,cm^2$. If the radiation pressure exerted by the E.M wave on the surface during a 10 minute exposure is $\frac{x}{10^9}\frac{N}{m^2}$. Find the value of x.

Key: 354

Sol: $\vec{E} = 200\cos\left[\left(\frac{0.5\times10^3}{m}\right)x - \left(1.5\times10^{11}\frac{rad}{s}\times t\right)\right]\frac{V}{m}\hat{j}$



$E_0 = 200$

$I = \frac{1}{2}\in_0 E_0^2 C$

Radiation pressure

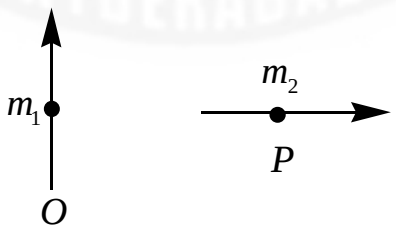
$$P = \frac{2I}{C} = \left(\frac{2}{C}\right)\left(\frac{1}{2}\in_0 E_0^2 C\right) = \in_0 E_0^2$$

$$P = \in_0 E_0^2$$

$$P = 8.85\times10^{-12}\times200^2 = 8.85\times10^{-8}\times4 = \frac{354}{10^9}$$

$x = 354$

23. Two short magnetic dipoles m_1 and m_2 each having magnetic moment of $1\,Am^2$ are placed at a point O and P respectively. The distance between OP is 1 meter. The torque experienced by the magnetic dipole m_2 due to the presence of m_1 is _____ $\times10^{-7}\,Nm$.



Key: 1

Sol: $M_1 = M_2 = 1Am^2$

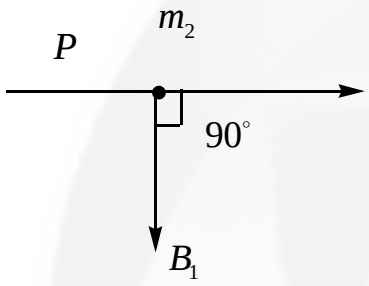
$OP = 1m$

"P" is the equal point for M_1

"B" due to M_1 at $P \Rightarrow B_1 = \frac{\mu_0}{4\pi}\frac{M_1}{d^3}\left(d = OP = 1m\right)$

$B_1 = 10^{-7} \times \frac{1}{1} = 10^{-7}T\left(opposite\,to\,m_1\right)$

Torque experienced by M_2 due to M_1



$\tau = M_2 B_1 \sin 90^\circ = (1)\left(10^{-7}\right)(1)$

$\tau = 10^{-7}Nm$

$\therefore \tau = 1 \times 10^{-7}Nm$

Answer = 1

24. An amplitude modulated wave is represented by $C_m(t) = 10\left(1 + 0.2\cos 12560t\right)\sin\left(111 \times 10^4 t\right)volts$. The modulating frequency in kHz will be_____

Key: 2

Sol: Amplitude modulated wave is represented by

$C_m(t) = 10\left(1 + 0.2\cos 12560t\right)\sin\left(111 \times 10^4 t\right)volts$

Modulations frequency _____ kHz

$C_m(t) = A_c\left(1 + \frac{A_m}{A_c}\sin 2\pi\ f_m\right)$

$2\pi f_m = 12560$

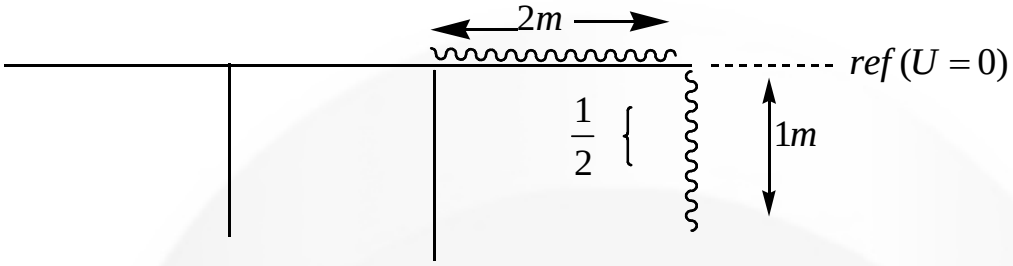
$2 \times 3.14 f_m = 12560$

$f_m = \frac{12560}{6.28} = 2000Hz = 2kHz$

25. A uniform chain of length 3 meter and mass 3 kg overhangs a smooth table with 2 meter lying on the table. If k is the kinetic energy of the chain in joule as it completely slips off the table, then the value of k is _____(take $g = 10 ms^{-2}$)

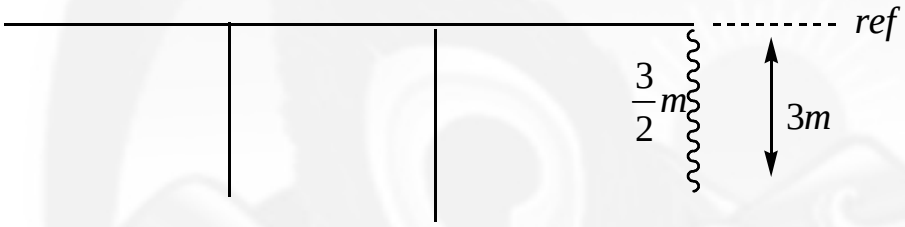
Key: 40

Sol: L =3m, m = 3 kg



$$U_i = (1)(10)\left(-\frac{1}{2}\right) = -5J$$

$$K_i = 0$$



$$U_f = (3)(10)\left(-\frac{3}{2}\right) = -45J$$

$$K_f = ?$$

From the law of conservation of mechanical energy

$$\Rightarrow U_i + K_i = U_f + K_f$$

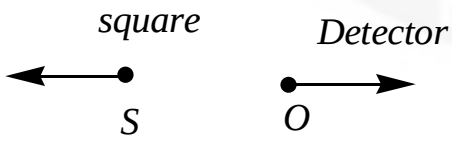
$$-5 + 0 = -45 + K_f$$

$$K_f = 40J$$

26. A source and a detector move away from each other in absence of wind with a speed of $20 m/s$ with respect to the ground. If the detector detects a frequency of 1800 Hz of the sound coming from the source, then the original frequency of source considering speed of sound in air $340 m/s$ will be _____ Hz.

Key: 2025

Sol:



$$V_s = 20 m/s \qquad V_0 = 20 m/s$$

$$f_0 = ?$$

Apparent frequency f = 1800Hz

$$f = f_0\left(\frac{V - V_0}{V + V_s}\right)$$

$$1800 = f_0\left(\frac{340 - 20}{340 + 20}\right)$$

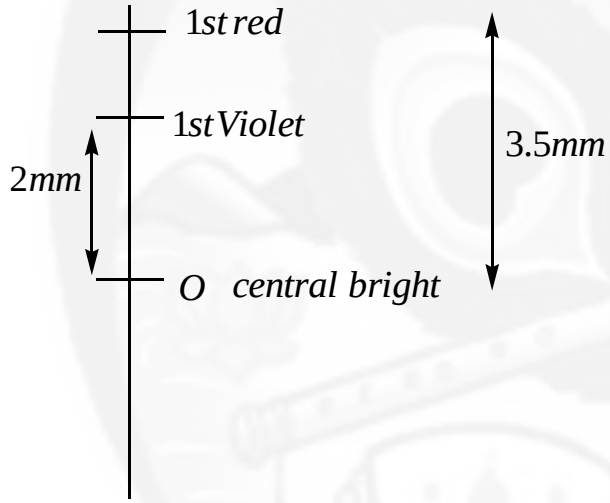
$$f_0 = \frac{1800 \times 36}{32} = 2025Hz$$

27. White light is passed through a double slit and interference is observed on a screen 1.5 m away. The separation between the slits is 0.33 mm. The first violet and red fringes are formed 2.0 mm and 3.5 mm away from the central white fringes. The difference in wavelengths of red and violet light is ___ nm.

Key: 300

Sol: D = 1.5 m

d = 0.33 mm = $0.3 \times 10^{-3} m$



$$\beta = \frac{\lambda D}{d}$$

$$\lambda = \frac{\beta d}{D}$$

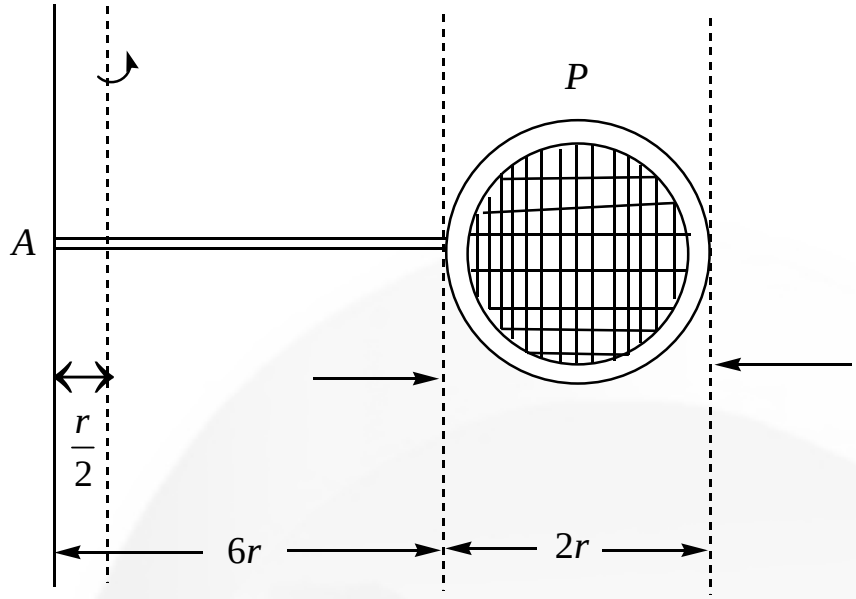
$$\lambda_V = \frac{\beta_V d}{D}, \lambda_R = \frac{\beta_R d}{D}$$

$$\Delta\lambda = \lambda_R - \lambda_V = \frac{(\beta_R - \beta_V)d}{D}$$

$$\Delta\lambda = \frac{(3.5 - 2)10^{-3} \times 0.3 \times 10^{-3}}{1.5} = 0.3 \times 10^{-6} m$$

$$\Delta\lambda = 3 \times 10^{-7} m = 300 \times 10^{-9} m = 300nm$$

28. Consider a badminton racket with length scales as shown in the figure.



If the mass of the linear and circular portions of the badminton racket are same (M) and the mass of the threads are negligible, the moment of inertia of the racket about an axis perpendicular to the handle and in the plane of the ring at , $\frac{r}{2}$ distance from the end A of the handle will be _____ Mr^2.

Key: 52.5

Sol: I = M.I of linear portion + M.I of circular portion

$$I = \left[\frac{M(6r)^2}{12} + M\left(3r - \frac{r}{2}\right)^2\right] + \left[Mr^2 + M\left(7r - \frac{r}{2}\right)^2\right]$$

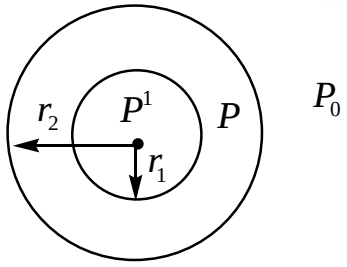
$$I = \left[M3r^2 + M\left(\frac{25r^2}{4}\right)\right] + \left[Mr^2 + \frac{169r^2}{4}\right]$$

$$I = \frac{37Mr^2}{4} + \frac{173Mr^2}{4} = \frac{210Mr^2}{4} = 52.5Mr^2$$

29. A soap bubble of radius 3 cm is formed inside the another soap bubble of radius 6 cm. The radius of an equivalent soap bubble which has the same excess pressure as inside the smaller bubble with respect to the atmospheric pressure is _____ cm.

Key: 2

Sol:



Excess pressure inside the smaller soap bubble

$$\Delta P = \frac{4S}{r_1} + \frac{4S}{r_2} ----1$$

The excess pressure inside the equivalent soap bubble

$$\Delta P = \frac{4S}{R} ----2$$

From 1 & 2

$$\frac{4S}{R} = \frac{4S}{r_1} + \frac{4S}{r_2}$$

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{6} + \frac{1}{3}$$

$$R = 2\ cm$$

30. Two spherical balls having equal masses with radius of 5 cm each are thrown upwards along the same vertical direction at an interval of 3s with the same initial velocity of $35 m/s$, then these balls collide at a height of _____ m.

Key: 50

Sol: 1st ball thrown at t = 0

2nd ball thrown at t = 35

Both collide at t = 8

For 1st

$$h = ut - \frac{1}{2}gt^2$$

For 2nd

$$h = u(t-3) - \frac{1}{2}g(t-3)^2$$

$$ut - \frac{1}{2}gt^2 = u(t-3) - \frac{1}{2}g(t-3)^2$$

$$3u + \frac{9g}{2} = 3gt$$

$$3 \times 35 + \frac{90}{2} = 30t$$

$$105 + 45 = 30t \Rightarrow t = \frac{150}{30} = 5s$$

$$h = 35(5) - \frac{1}{2}(10)(5)^2 = 175 - 125 = 50m$$

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

31. The incorrect statement is:

1) F_2 is a stronger oxidizing agent than $C\ell_2$ in aqueous solution

2) On hydrolysis $C\ell F$ forms $HOC\ell$ and HF

3) F_2 is more reactive than $C\ell F$

4) $C\ell_2$ is more reactive than $C\ell F$

Key: 4

Sol: In $C\ell F_3$ the $C\ell - F$ bond is weaker than $C\ell - C\ell$ in $C\ell_2$ thus $C\ell F_3$ is more reactive than $C\ell_2$

32. What are the products formed in sequence when excess of CO_2 is passed in slaked lime?

1) $CaCO_3, Ca(HCO_3)_2$ 2) $CaO, Ca(HCO_3)_2$

3) $Ca(HCO_3)_2, CaCO_3$ 4) $CaO, CaCO_3$

Key: 1

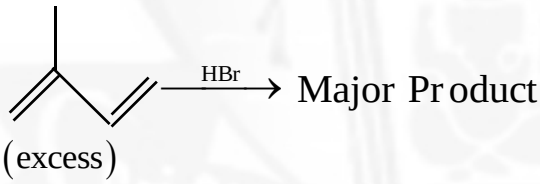
Sol: $Ca(OH)_2 + CO_2 \rightarrow \underset{Insoluble}{CaCO_3} + H_2O$

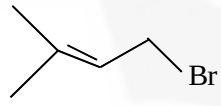
When CO_2 passed through slaked lime it turns milky due to formation of $CaCO_3$ (Insoluble in water)

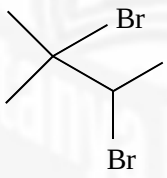
When excess of CO_2 is passed then,

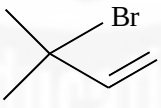
$CaCO_3 + H_2O + \underset{excess}{CO_2} \rightarrow \underset{water\ soluble}{Ca(HCO_3)_2}$

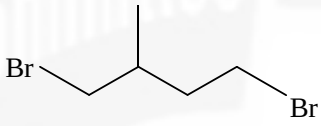
33. The major product formed in the following reaction is:



1) 

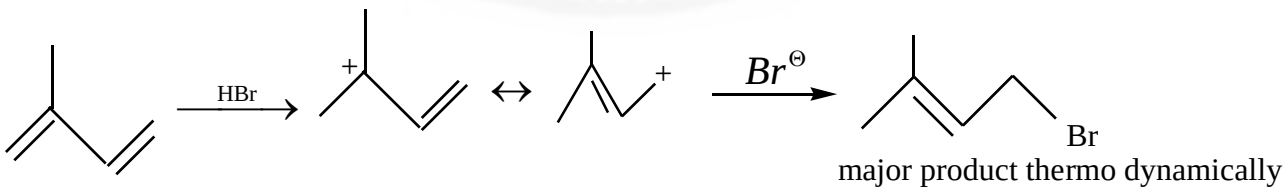
2) 

3) 

4) 

Key: 1

Sol:



major product thermo dynamically controlled product

34. Which one of the following when dissolved in water gives coloured solution in nitrogen atmosphere?

1) $ZnC\ell_2$ 2) $AgC\ell$ 3) $CuC\ell_2$ 4) $Cu_2C\ell_2$

Key: 3

Sol: $CuC\ell_2$ is green solid but when dissolved in water it gives $\left(Cu\left(H_2O\right)_4\right)^{+2}$ and turns to blue colour.

35. Which one of the following complexes is violet in colour?

1) $\left[Fe\left(SCN\right)_6\right]^{4-}$ 2) $\left[Fe\left(CN\right)_6\right]^{4-}$

3) $Fe_4\left[Fe\left(CN\right)_6\right]_3.H_2O$ 4) $\left[Fe\left(CN\right)_5 NOS\right]^{4-}$

Key: 4

Sol: $Na_2S + \underset{\text{Sodium nitro prusside}}{\left[Na_2Fe\left(CN\right)_5 NO\right]} \rightarrow \underset{\text{Violet coloured complex}}{Na_4\left[Fe\left(CN\right)_5 NOS\right]}$

When Na_2S reacts with sodium nitro prusside forming violet coloured complex

36. Given below are two statements:

Statement I: The limiting molar conductivity of $KC\ell$ (strong electrolyte) is higher compared to that of CH_3COOH (weak electrolyte)

Statement II: Molar conductivity decreases with decrease in concentration of electrolyte.

In the light of the above statements choose the most appropriate answer from the options given below

1) Both statement I and Statement II are false
2) Statement I is false but Statement II is true
3) Both statement I and Statement II are true
4) Statement I is true but Statement II is false

Key: 4

Sol: Limiting molar conductivity of $KC\ell$ is higher compared to that of CH_3COOH because $KC\ell$ is strong electrolyte and can be extrapolated in the graph to calculate the limiting molar conductivity. Whereas CH_3COOH is a weak electrolyte and it cannot be ealenlated using the graphical method.

37. Which one of the following is correct for the adsorption of a gas at a given temperature on a solid surface ?

1) $\Delta H > 0, \Delta S > 0$ 2) $\Delta H < 0, \Delta S > 0$

3) $\Delta H < 0, \Delta S < 0$ 4) $\Delta H > 0, \Delta S < 0$

Key: 3

Sol: when a gas adsorbs on a solid, as the reaction is exothermic $\Delta H < 0$ and entropy decreases $\Delta S < 0$

38. The conversion of hydroxyapatite occurs due to presence of F^- ions in water. The correct formula of hydroxyapatite is:

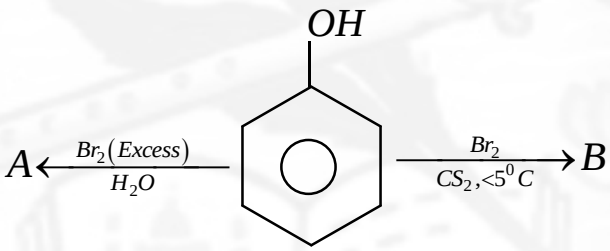
1) $\left[3Ca_3\left(PO_4\right)_2.Ca\left(OH\right)_2\right]$ 2) $\left[Ca_3\left(PO_4\right)_2.CaF_2\right]$

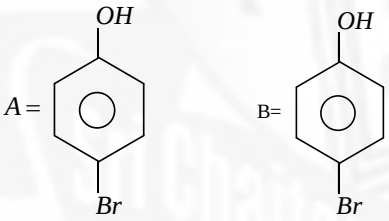
3) $\left[3Ca\left(OH\right)_2.CaF_2\right]$ 4) $\left[3Ca_3\left(PO_4\right)_2.CaF_2\right]$

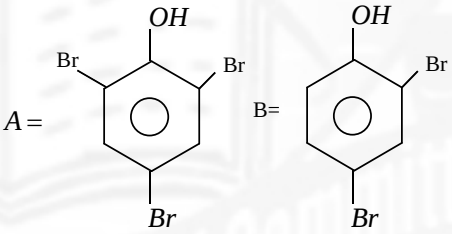
Key: 1

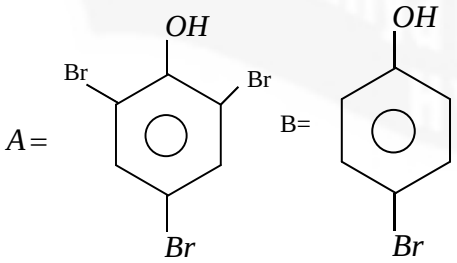
Sol: Hydroxyapatite is $\left[3Ca_3\left(PO_4\right)_2.Ca\left(OH\right)_2\right]$

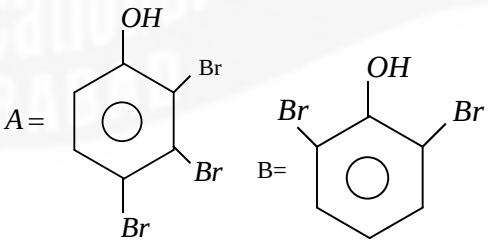
39. The correct options for the products A and B of the following reactions are:



1) 

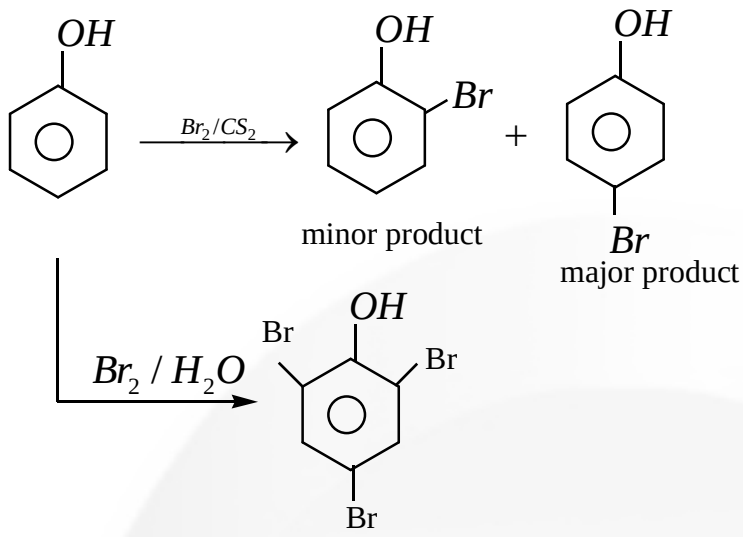
2) 

3) 

4) 

Key: 3

Sol:



2,4,6-tri bromo phenol

40. Given below are two statements:

Statement I: In the titration between strong acid and weak base methyl orange is suitable as an indicator.

Statement II: For titration of acetic acid with NaOH phenolphthalein is not a suitable indicator.

In the light of the above statements, choose the most appropriate answer from the options given below:

1) Statement I is false but Statement II is true
2) Both statement I and Statement II are false
3) Both statement I and Statement II are True
4) Statement I is true but Statement II is false

Key: 2

Sol: P^H range of strong acid weak base reaction is form 4 to 7 and methyl range P^H range is 3.2 to 4.5 hence in strong acid weak base reaction methyl orange can be used. P^H range of weak acid strong base reaction is form 7 to 11 and P^H range of phenolphthalein is 8.7 to 10.5. Hence phenolphthalein can be used. Statement I is correct II is false.

41. Given below are two statements:

Statement I: Frenkel defects are vacancy as well as interstitial defects.

Statement II: Frenkel defect leads to colour in ionic solids due to presence of F-centres.

Choose the most appropriate answer for the statements from the options given below:

1) Statement I is false but Statement II is true

2) Both statement I and Statement II are true

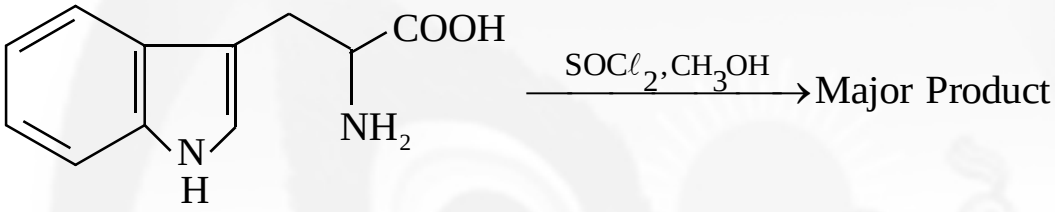
3) Both statement I and Statement II are false

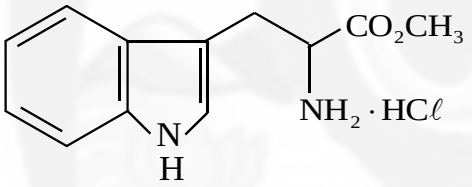
4) Statement I is true but Statement II is false

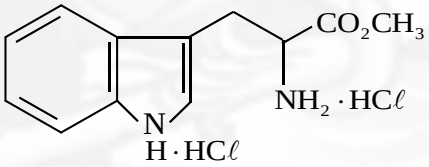
Key: 4

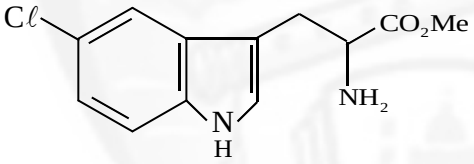
Sol: 'F' – centers are due to metal excess defects.

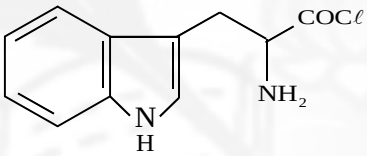
42. The major product formed in the following reaction is:



1) 

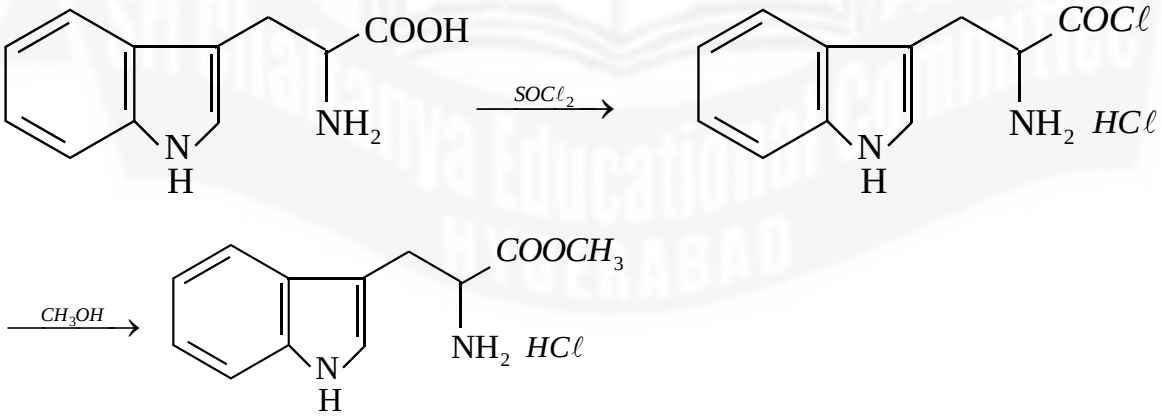
2) 

3) 

4) 

Key: 1

Sol:



43. Given below are two statements:

Statement I; The choice of reducing agents for metals extraction can be made by using: Ellingham diagram a plot of ΔG vs temperature.

Statement II: The value of ΔS increases from left to right in Ellingham diagram. In the light of the above statements, choose the most appropriate answer from the options given below:

1) Statement I is false but Statement II is true

2) Both statement I and Statement II are true

3) Statement I is true but Statement II is false

4) Both statement I and Statement II are false

Key: 3

Sol: Ellingham diagram helps in selecting suitable reducing agent at a particular temperature.

$\therefore$ statement I is true.

'ΔS' value change depends on type of chemical reaction involved in the process and no of gaseous moles involved in process.

$\therefore$ Statement II is false.

44. Which one of the following methods is most suiable for preparing deionized water?

1) Permutit method 2) Calgon's method

3) Synthetic resin method 4) Clark's method

Key: 3

Sol: Synthetic resin method is most suitable for preparing deionized water.

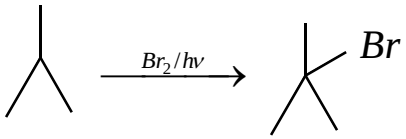
45. Excess of isobutane on reaction with Br_2 in presence of light at $125^0 C$ gives which one of the following as the major product?

1) $CH_3 - \underset{\underset{CH_3}{|}}{\overset{\overset{Br}{|}}{C}} - CH_2 - Br$ 2) $CH_3 - \underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{C}} - Br$

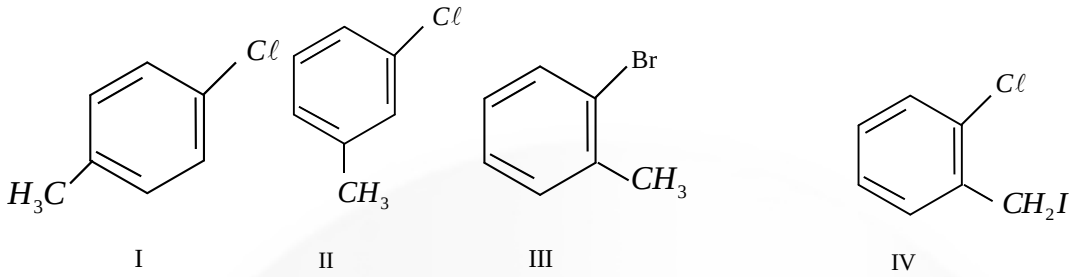
3) $CH_3 - \underset{\underset{CH_3}{|}}{CH} - CH_2Br$ 4) $CH_3 - \underset{\underset{CH_2Br}{|}}{CH} - CH_2Br$

Key: 2

Sol:



46. Among the following compounds I-IV, which one forms a yellow preceipitate on reacting sequentially with (i) $NaOH$ (ii) $dil.HNO_3$ (iii) $AgNO_3$?

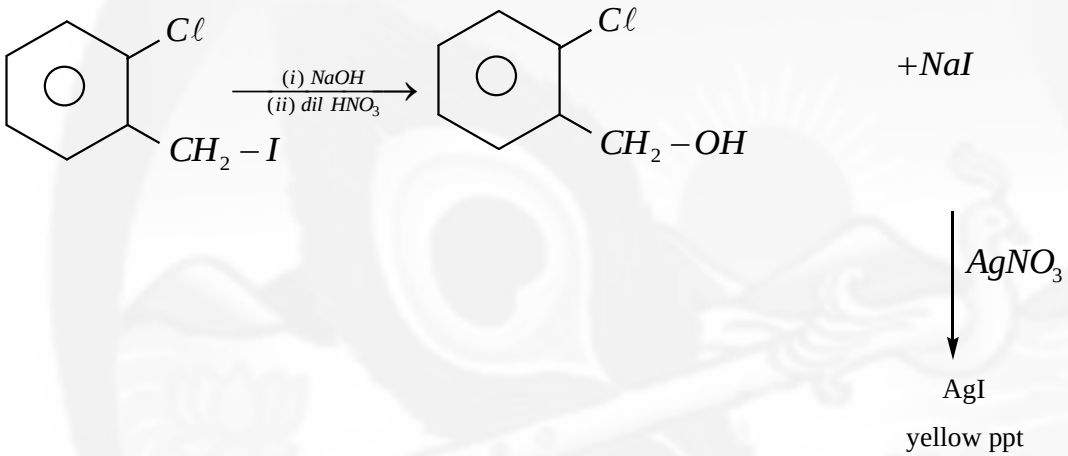


I II III IV

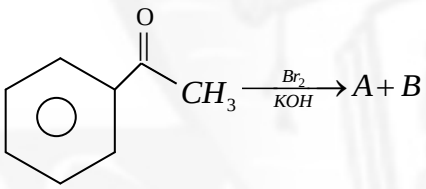
1) I 2) IV 3) II 4) III

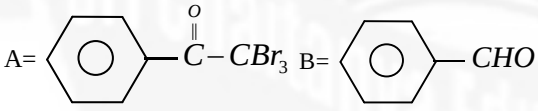
Key: 2

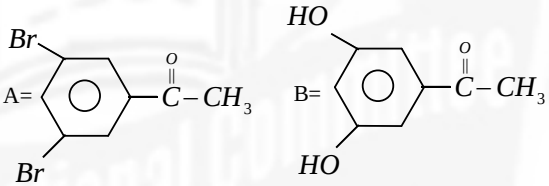
Sol:

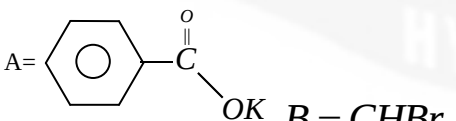


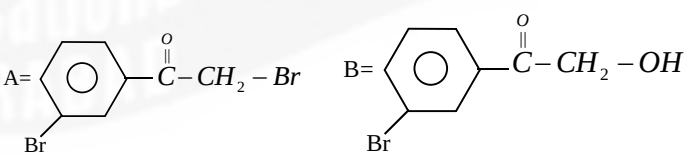
47. The major products formed in the following reaction sequence A and B are:



1) 

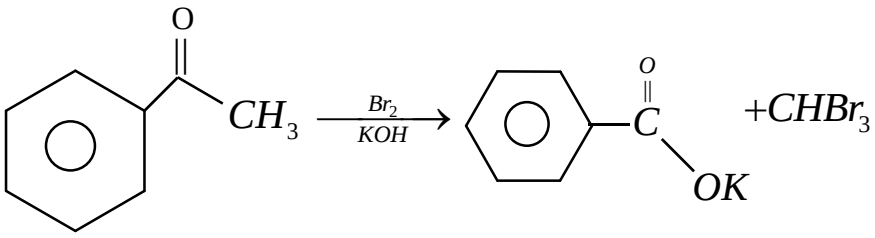
2) 

3) 

4) 

Key: 3

Sol:



48. The correct sequential addition of reagents in the preparation of 3-nitrobenzoic acid from benzene is :

1) $HNO_3 / H_2SO_4, Br_2 / A\ell Br_3, Mg / ether, CO_2, H_3O^+$

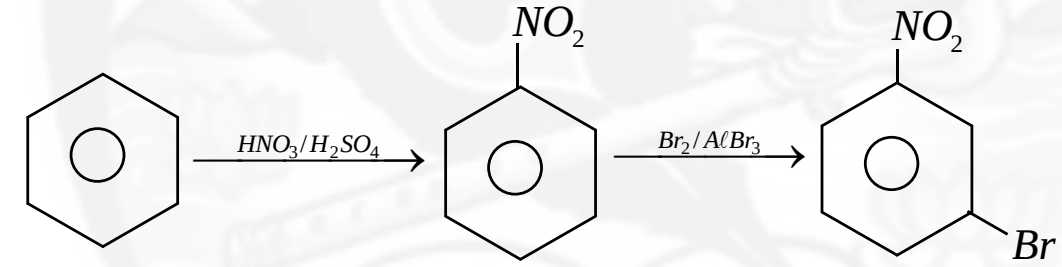
2) $Br_2 / A\ell Br_3, NaCN, H_3O^+, HNO_3 / H_2SO_4$

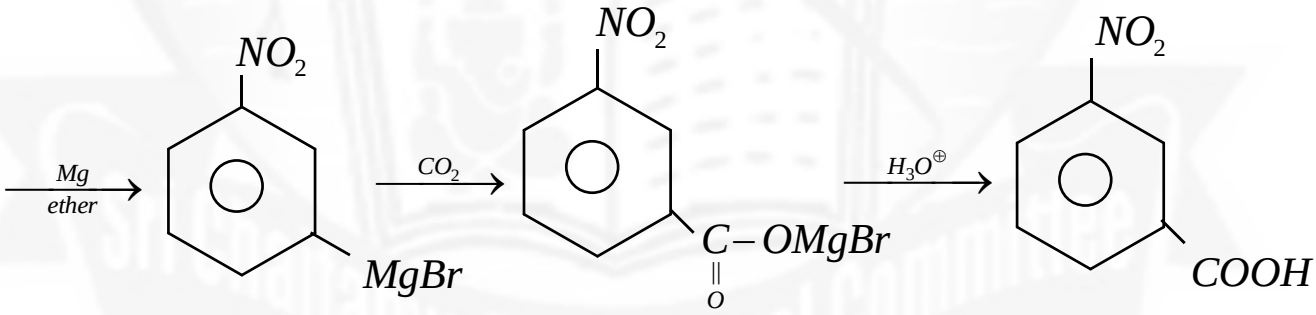
3) $Br_2 / A\ell Br_3, HNO_3 / H_2SO_4, Mg / ether, CO_2, H_3O^+$

4) $Br_2 / A\ell Br_3, HNO_3 / H_2SO_4, NaCN, H_3O^+$

Key: 1

Sol:





49. The polymer formed on heating, Novolac with formaldehyde is:

1) Polyster 2) Nylon 6,6 3) Melamine 4) Bakelite

Key: 4

Sol: $Novolac \xrightarrow[\Delta]{CH_2O/H^\oplus} Bakelite$

50. Given below are two statements:

Statement I: According to Bohr's model of an atom, qualitatively the magnitude of velocity of electron increases with decrease in positive charges on the nucleus as there is no strong hold on the electron by the nucleus.

Statement II: According to Bohr's model of an atom, qualitatively the magnitude of velocity of electron increases with decrease in principal quantum number. In the light of the above statements, choose the most appropriate answer from the options given below:

1) Statement I is true but Statement II is false
2) Statement I is false but Statement II is true
3) Both Statement I and Statement II are false
4) Both Statement I and Statement II are true

Key: 2

Sol: The velocity of electron is an orbit can be calculated by using the formula.

$$v = \frac{2.18 \times 10^6 \times z}{n} m/\sec$$

$\therefore$ 'v' is directly proportional to 'z' and is inversely proportional to 'n'

'n' $\rightarrow$ principal quantum number.

$\therefore$ statement I false statement II true.

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

51. The number of 4f electrons in the ground state electronic configuration of Gd^{2+} is [Atomic number of $Gd = 64$]

Key: 7

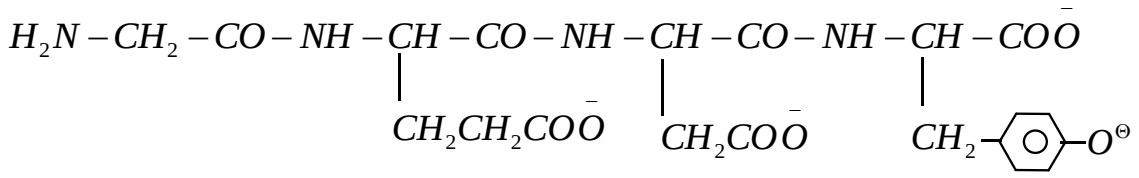
Sol: $Gd : 4f^7 5d^1 6s^2$

$Gd^{+2} : 4f^7 5d^1$

52. The total number of negative charge in the tetrapeptide, Gly-Glu-Asp-Tyr, at pH 12.5 will be _____. (Integer answer)

Key: 4

Sol: Gly-Glu-Asp-Tyr



Total negative charge = 4

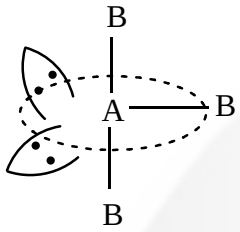
53. AB_3 is an interhalogen T-shaped molecule. The number of lone pairs of electrons on A is _____. (Integer answer)

Key: 2

Sol: In AB_3 molecule the contain atom 'A' under goes sp^3d hybridization. It has 'T'-shape

2-lone pairs

3-bond pairs



54. The ratio of number of water molecules in Mohr's salt and potash alum is ___$\times 10^{-1}$. (Integer answer)

Key: 5

Sol: Mobr's salt: $\left(NH_4\right)_2 SO_4.FeSO_4.6H_2O$

$KA\ell\left(SO_4\right)_2\ 12H_2O$

The ratio of number of water molecules

$$\frac{6}{12}=\frac{1}{2}=0.5(or)5\times10^{-1}$$

55. The OH^- concentration in a mixture of $5.0\,mL$ of $0.0504M$ $NH_4C\ell$ and 2 mL of $0.0210\,M$ NH_3 solution is $x\times10^{-6}M$. The value of x is ____(Nearest integer)

Key: 3

Sol: $V_{salt}=5\ m\ell; M_{salt}=0.0504M$

$V_{base}=2m\ell\ M_{base}=0.0210M$

It is a basic buffer solution

$$PoH=pk_b+\log\frac{[salt]}{[Base]}$$

$$=4.74+\log\frac{[0.0504\times5]}{[0.0210\times2]}$$

$$=4.74+\log\frac{[0.252]}{[0.042]}$$

$$=4.74+\log60$$

$$=4.74+1.778$$

$$PoH=6.5181$$

$$\therefore\left[OH^-\right]=anti\log\ 6.5181$$

$\left[OH^-\right] = 3\times10^{-6}$

56. Of the following four aqueous solutions, total number of those solutions whose freezing point is lower than that of $0.10M\ C_2H_5OH$ is ____.(Integer answer)

(i) $0.10\ M\ Ba_3\left(PO_4\right)_2$

(ii) $0.10\ M\ Na_2SO_4$

(iii) $0.10\ M\ KC\ell$

(iv) $0.10\ M\ Li_3PO_4$

Key: 4

Sol: freezing point of a solution depends on number of solute particles. More the number of solute particles less is the freezing point

All given options dissociates

$\therefore$ They have less Freezing point

57. The following data was obtained for chemical reaction given below at 975 K.

$2NO_{(g)} + 2H_{2(g)} \rightarrow N_{2(g)} + 2H_2O_{(g)}$

	$[NO]$	$[H_2]$	Rate
	$molL^{-1}$	$molL^{-1}$	$molL^{-1}s^{-1}$
(A) 8×10^{-5}	8×10^{-5}	7×10^{-9}	
(B) 24×10^{-5}	8×10^{-5}	2.1×10^{-8}	
(C) 24×10^{-5}	32×10^{-5}	8.4×10^{-8}	

The order of the reaction with respect to NO is _____.(Integer answer)

Key: 1

Sol: $rate = k\left[NO\right]^x\left[H_2\right]^y$

$7\times10^{-9} = k\left(8\times10^{-5}\right)^x\left(8\times10^{-5}\right)^4$ .....(1)

$2.1\times10^{-8} = k\left(24\times10^{-5}\right)^x\left(8\times10^{-5}\right)^y$ .....(2)

$$\frac{2.1\times\cancel{10^{-8}}}{7\times\cancel{10^{-9}}} = \frac{\cancel{k}\left[\overset{3}{\cancel{24}}\times10^{-5}\right]^x\left[8\times\cancel{10^{-5}}\right]^y}{\cancel{k}\left[\cancel{8}\times10^{-5}\right]^x\left[8\times\cancel{10^{-5}}\right]^y}$$

$3 = 3^x$

$\therefore x = 1$

$\therefore$w.r.t 'NO' it is a first order reaction.

58. The Born-Haber cycle for $KC\ell$ is evaluated with the following data:

$\Delta_f H^\Theta$ for $KC\ell = -436.7\ kjmol^{-1}; \Delta_{sub} H^\Theta$ for $K = 89.2\ kJ\ mol^{-1}$;

$\Delta_{ionization} H^\Theta\ for\ K = 419.0\,kj\,mol^{-1}\ \Delta_{electrongain} H^\Theta\ for\ C\ell_{(g)} = 348.6\,kj\,mol^{-1}$

$\Delta_{bond} H^\Theta\ for\ C\ell_2 = 243.0\,kj\,mol^{-1}$

The magnitude of lattice enthalpy of $KC\ell$ in $kj\ mol^{-1}$ is _____. (Nearest integer)

Key: 718

Sol: $-U = \Delta H_f - \left\{ +SE + IE + \frac{1}{2}(BE) - EA \right\}$

$= -436.7 - \left(89.2 + 419.0 + \frac{1}{2}(243.0) - 348.6 \right)$

$= -717.8\ kj / mol$

59. An aqueous $KC\ell$ solution of density $1.20g\ mL^{-1}$ has a molality of 3.30 mol kg^{-1} the molarity of the solution in $mol\ L^{-1}$ is _____ (Nearest integer) [Molar mass of $KC\ell = 74.5$]

Key: 3

Sol: $d = 1.20g / ml\ ;\ m = 3.30;\ M = ?$

$(mol\ wt\ KC\ell = 74.3)$

$M = \frac{wt\ of\ solute}{mwt\ of\ solute} \times \frac{1000}{vol\ of\ solution\ in\ m\ell}$

3.30 m means 3.30 moles in "1000 gm" solvent

$\therefore Wt\ of solute(KC\ell) = 3.30 \times 74.3 = 245.19$

Total wt of solution $= 1000 + 245.19 = 1245.19$

Vol of solution can be calculated by using density

$d_{solution} = \frac{mass\ of\ solution}{vol\ of\ solution}$

$vol\ of\ solution = \frac{1245.19}{1.20} = 1037.65$

$\therefore M = 3.30 \times \frac{1000}{1037.65} = 3.18$

60. These are physical properties of an element

(A) Sublimation enthalpy

(B) Ionisation enthalpy

(C) Hydration enthalpy

(D) Electron gain enthalpy

Total number of above properties that affect the reduction potential is ____. (Integer answer)

Key: 3

Sol: Reduction potential it is a tendency to gain electron.

$\therefore$ It is influenced by (i) sublimation enthalpy

(ii) Hydration enthalpy

(iii) electron gain enthalpy

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

61. The sum of solutions of the equation $\frac{\cos x}{1+\sin x} = |\tan 2x|,\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$ is:

1) $\frac{\pi}{10}$ 2) $-\frac{\pi}{15}$ 3) $-\frac{7\pi}{30}$ 4) $-\frac{11\pi}{30}$

Key: 4

Sol: $\frac{\cos x}{1+\sin x} = \pm\left(\frac{2\tan x}{1-\tan^2 x}\right)$

$\Rightarrow \frac{\cos x}{1+\sin x} = \pm\left(\frac{2\sin x}{\cos x\left(\frac{\cos^2 x - \sin^2 x}{\cos^2 x}\right)}\right)$

$\Rightarrow \cos^2 x - \sin^2 x = \pm\left(2\sin x + 2\sin^2 x\right)\ \ (\because \cos x \neq 0)$

$\Rightarrow 1 - 2\sin^2 x = 2\sin x + 2\sin^2 x$

$4\sin^2 x + 2\sin x - 1 = 0$

$\sin x = \frac{\sqrt{5}-1}{4}$ and $\frac{-\left(\sqrt{5}+1\right)}{4}$

$1 - 2\sin^2 x = -2\sin x - 2\sin^2 x$

$\sin x = -\frac{1}{2}$

$x = -\frac{\pi}{6}$

$x = \frac{\pi}{10},\ \frac{-3\pi}{10}$

Sum of roots $= \frac{\pi}{10} - \frac{3\pi}{10} - \frac{\pi}{6} = \frac{-\pi}{5} - \frac{\pi}{6} = \frac{-11\pi}{30}$

62. If a line along a chord of the circle $4x^2 + 4y^2 + 120x + 675 = 0$, passes through the point $(-30, 0)$ and is tangent to the parabola $y^2 = 30x$, then the length of this chord is:

1) 5 2) $3\sqrt{5}$ 3) $5\sqrt{3}$ 4) 7

Key: 2

Sol: d = perpendicular distance from $c(-15, 0)$ to $x - 2y + 30 = 0$

$$= \left|\frac{-15+30}{\sqrt{5}}\right| = \frac{15}{\sqrt{5}} = 3\sqrt{5}$$

$$2\sqrt{r^2 - d^2} = 2\sqrt{\left(\frac{15}{2}\right)^2 - \left(3\sqrt{5}\right)^2}$$

$$= 2\sqrt{\frac{225}{4} - 45}$$

$$= 2\sqrt{\frac{225-180}{4}}$$

$$= 2\sqrt{\frac{45}{4}} = 2.\frac{\sqrt{45}}{2}$$

$$= \sqrt{45}$$

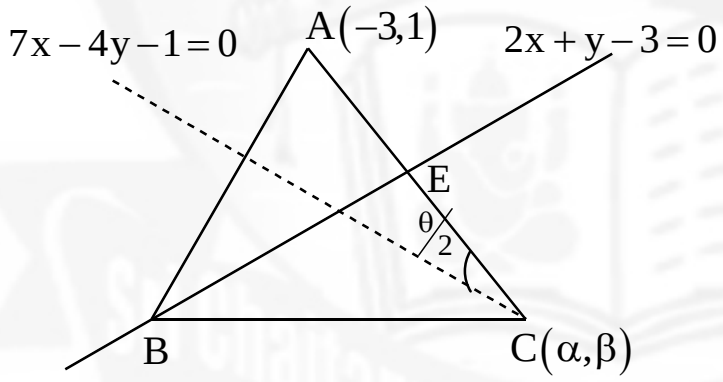
$$= 3\sqrt{5}$$

63. Let ABC be a triangle with $A(-3,1)$ and $\angle ACB = \theta, 0 < \theta < \frac{\pi}{2}$. If the equation of the median through B is $2x + y - 3 = 0$ and the equation of angle bisector of C is $7x - 4y - 1 = 0$, then $\tan\theta$ is equal to:

1) $\frac{1}{2}$ 2) $\frac{3}{4}$ 3) 2 4) $\frac{4}{3}$

Key: 4

Sol:



$E\left(\frac{\alpha-3}{2}, \frac{\beta+1}{2}\right)$ lies on $2x + y - 3 = 0$. So $2\alpha + \beta - 11 = 0 \text{ ---} (1)$

(α,β) lies on $7x - 4y - 1 = 0$ so $7\alpha - 4\beta - 1 = 0 \text{ ----} (2)$

by solving (1) & (2)

$\alpha = 3, \beta = 5$

equation of AC is $2x - 3y + 9 = 0$ $\quad m_2 = \frac{2}{3}$

solve of $7x-4y-1=0$ is $m_2=\frac{7}{4}$

$$\tan\frac{\theta}{2}=\left|\frac{\frac{2}{3}-\frac{7}{4}}{1+\frac{2}{3}\cdot\frac{7}{4}}\right|=\frac{1}{2}$$

$$\tan\theta=\frac{4}{3}$$

64. On the ellipse $\frac{x^2}{8}+\frac{y^2}{4}=1$ let P be a point in the second quadrant such that the tangent at P to the ellipse is perpendicular to the line $x+2y=0$. Let S and S' be the foci of the ellipse and e be its eccentricity. If A is the area of the triangle SPS' then, the value of $\left(5-e^2\right)$. A is:

1) 12 2) 6 3) 24 4) 14

Key: 2

Sol: $\frac{x^2}{8}+\frac{y^2}{4}=1$

$a^2=8\Rightarrow b^2=4$

$$e=\sqrt{\frac{a^2-b^2}{a^2}}=\sqrt{\frac{8-4}{8}}=\sqrt{\frac{4}{8}}=\frac{1}{\sqrt{2}}$$

$$e=\frac{1}{\sqrt{2}}$$

$S(ae,0)=(2,0)$

$S^1(-ae,0)=(-2,0)$

$y=mx+c$ $\qquad c=\sqrt{a^2m^2+b^2}$

$y=2x+6$ $\qquad c=\sqrt{8.4+4}=2.3=6$

$2x-y+6=0$ .....(1) equation of tangent at $P(x_1,y_1)$

$\frac{xx_1}{8}+\frac{yy_1}{4}-1=0$ ....(2)

(1), (2) represent same line

$$\frac{x_1}{8.2}=\frac{\frac{y_1}{4}}{-1}=\frac{-1}{6}$$

$$x_1=\frac{-8}{3}\quad y_1=\frac{2}{3}$$

$$\text{Area}=\frac{1}{2}.\frac{2}{3}.4=\frac{4}{3}$$

$$\left(5-e^2\right)A=\left(5-\frac{1}{2}\right)\left(\frac{4}{3}\right)=6$$

65. If the truth value of the Boolean expression $\left((p\vee q)\wedge(q\to r)\wedge(\sim r)\right)\to(p\wedge q)$ is false, then the truth values of the statements p,q,r respectively can be:

1) TFT 2) FTF 3) FFT 4) TFF

Key: 4

Sol: By Verification

$$\left((p\vee q)\wedge(q\to r)\wedge(-r)\right)\to(p\wedge q)=\text{False}$$

$$p=T,\ q=F,\ r=F$$

$$\left((T\vee F)\wedge(F\to F)\wedge(T)\right)\to(T\wedge F)$$

$$(T\wedge T\wedge T)\to F$$

$$T\to F\ =\text{False}$$

66. Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If $K\%$ of them are suffering from both ailments, then K can not belong to the set:

1) $\{80, 83, 86, 89\}$ 2) $\{84, 87, 90, 93\}$

3) $\{79, 81, 83, 85\}$ 4) $\{84, 86, 88, 90\}$

Key: 3

Sol: $\max\left(n(A\cup B)\right)=\max\left\{n(A),n(B)\right\}$

$$\max(A\cup B)=100 \text{ of } n(A)+n(B)\geq 100$$

$$n(A\cap B)=n(A)+n(B)-n(A\cap B)$$

$$\max(A\cap B)=n(A)=89\%$$

$$\max\ n(A\cap B)=n(A)+n(B)-100$$

$$=89+98-100$$

$$=87\%$$

$$n(A\cap B)=\{87, 88, 89\}$$

67. The value of $\int_{-1/\sqrt{2}}^{1/\sqrt{2}} \left(\left(\frac{x+1}{x-1} \right)^2 + \left(\frac{x-1}{x+1} \right)^2 - 2 \right)^{\frac{1}{2}} dx$ is:

1) $2\log_e 16$ 2) $\log_e 4$ 3) $\log_e 16$ 4) $4\log_e \left(3+2\sqrt{2}\right)$

Key: 3

Sol: $\int_{-1/\sqrt{2}}^{1/\sqrt{2}} \left(\left(\frac{x+1}{x-1} \right)^2 + \left(\frac{x-1}{x+1} \right)^2 - 2 \right)^{\frac{1}{2}} dx$

$= \left(\left(\frac{x+1}{x-1} - \frac{x-1}{x+1} \right)^2 \right)^{\frac{1}{2}} = \left(\frac{x+1}{x-1} - \frac{x-1}{x+1} \right)$

$f(x) = \text{even}$

$f(x) = \int_{-1/\sqrt{2}}^{1/\sqrt{2}} \left| \frac{4x}{x^2-1} \right| dx = 2 \int_{0}^{1/\sqrt{2}} \frac{4x}{1-x^2} dx$

$= -4 \left[\log\left(1-x^2\right) \right]_0^{\frac{1}{\sqrt{2}}}$

$= -4 \log\left(\frac{1}{2}\right)$

$= \log_e 16$

68. The sum of the series $\frac{1}{x+1} + \frac{2}{x^2+1} + \frac{2^2}{x^4+1} + ... + \frac{2^{100}}{x^{2^{100}}+1}$ when $x = 2$ is:

1) $1 - \frac{2^{100}}{4^{100}-1}$ 2) $1 + \frac{2^{101}}{4^{101}-1}$ 3) $1 + \frac{2^{100}}{4^{101}-1}$ 4) $1 - \frac{2^{101}}{4^{101}-1}$

Key: 4

Sol: $\frac{-1}{x-1} + \frac{1}{x+1} + \frac{2}{x^2+1} + \frac{2^2}{x^4+1} + ... + \frac{2^{100}}{x^{2^{100}}} + \frac{1}{x-1}$

$-\left[\frac{2}{x^2-1} - \frac{2}{x^2+1} - \frac{2^2}{x^4+1} - .... - \frac{2^{100}}{x^{2^{100}}+1} \right] + \frac{1}{x-1}$

$= -\left[\frac{2^2}{x^4-1} - \frac{2^2}{x^4+1} - .... - \frac{2^{100}}{x^{2^{100}}+1} \right] + \frac{1}{x-1}$

$= -\frac{2^{101}}{x^{2^{101}}-1} + \frac{1}{x-1} = put \;\; x = 2$

$$=1-\frac{2^{101}}{2^{2^{101}}-1}=1-\frac{2^{101}}{4^{101}-1}$$

69. The value of $\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{2n-1}\frac{n^2}{n^2+4r^2}$ is:

1) $\frac{1}{2}\tan^{-1}(2)$ 2) $\frac{1}{4}\tan^{-1}(4)$ 3) $\frac{1}{2}\tan^{-1}(4)$ 4) $\tan^{-1}(4)$

Key: 3

Sol: $\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{2n-1}\frac{n^2}{n^2+4r^2}$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{2n-1}\frac{1}{1+4\left(\frac{r}{n}\right)^2}$$

$$=\int_0^2\frac{1}{1+4x^2}dx=\frac{1}{2}\tan^{-1}(2x)$$

$$=\frac{1}{2}\tan^{-1}(4)-0$$

$$=\frac{1}{2}\tan^{-1}(4)$$

70. If ${}^{20}C_r$ is the co-efficient of x^r in the expansion of $(1+x)^{20}$, then the value of $\sum_{r=0}^{20}r^2\,{}^{20}C_r$ is equal to:

1) 420×2^{18} 2) 380×2^{18} 3) 380×2^{19} 4) 420×2^{19}

Key: 1

Sol: $\sum_{r=0}^{20}r^2\,{}^{20}C_r=n(n+1)2^{n-2}$

$$=20(20+1)\cdot 2^{18}$$

$$=420\cdot(2^{18})$$

71. Let $f(x)=\cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right)$, $0<x<1$. Then:

1) $(1-x)^2 f'(x)+2(f(x))^2=0$ 2) $(1+x)^2 f'(x)+2(f(x))^2=0$

3) $(1-x)^2 f'(x)-2(f(x))^2=0$ 4) $(1+x)^2 f'(x)-2(f(x))^2=0$

Key: 1

Sol: $f(x)=\cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right)\quad(0<x<1)$

$$=\cos\left(2\tan^{-1}\left(\sin\left(\sin^{-1}\frac{1}{\sqrt{1+\frac{1-x}{x}}}\right)\right)\right)$$

$$=\cos\left(2\tan^{-1}\left(\sqrt{x}\right)\right)=\cos\left(\cos^{-1}\frac{1-x}{1+x}\right)=\frac{1-x}{1+x}$$

$$f(x)=\frac{1-x}{1+x}\Rightarrow\log f(x)=\log(1-x)-\log(1+x)$$

$$\frac{f'(x)}{f(x)}=\frac{-1-x-1+x}{1-x^2}$$

$$\Rightarrow\frac{f'(x)}{f(x)}=\frac{-2}{(1-x)^2}$$

$$\Rightarrow\frac{f^1(x)}{f(x)}=\frac{-2}{1-x^2}\times\frac{1-x}{1+x}\times\frac{1+x}{1-x}=\frac{-2f(x)}{(1-x)^2}$$

$$\Rightarrow(1-x)^2\,f^1(x)+2\left(f(x)\right)^2=0$$

72. A plane P contains the line $x+2y+3z+1=0=x-y-z-6$, and is perpendicular to the plane $-2x+y+z+8=0$. Then which of the following points lies on P?

1) $(-1,1,2)$ 2) $(0,1,1)$ 3) $(1,0,1)$ 4) $(2,-1,1)$

Key: 2

Sol: Required plane is $\pi_1+\lambda\pi_2=0$

$$(x+2y+3z+1)+\lambda(x-y-z-6)=0$$

$$(1+\lambda)x+(2-\lambda)y+(3-\lambda)z+(1-6\lambda)=0$$

which is perpendicular to $\pi_3=-2x+y+z+8=0$

Then $-2(1+\lambda)+1(2-\lambda)+1(3-\lambda)=0$

$$-2-2\lambda+2-\lambda+3-\lambda=0$$

$$3=4\lambda\Rightarrow\lambda=\frac{3}{4}$$

sub $\lambda=\frac{3}{4}$ in (1)

$$(1)\Rightarrow 4x+8y+12z+4+3x-3y-3z-18=0$$

$$7x+5y+9z-14=0$$

the point $p(0,1,1)$ lines on plane $0+5+9-14=0$

$\therefore (0,1,1)$

73. If $A=\begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$, $B=\begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}$, $i=\sqrt{-1}$, and $Q=A^TBA$, then the inverse of the matrix $AQ^{2021}A^T$ is equal to:

1) $\begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$

2) $\begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$

3) $\begin{pmatrix} \frac{1}{\sqrt{5}} & -2021 \\ 2021 & \frac{1}{\sqrt{5}} \end{pmatrix}$

4) $\begin{pmatrix} 1 & -2021i \\ 0 & 1 \end{pmatrix}$

Key: 2

Sol: $A=\begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}$ $B=\begin{bmatrix} 1 & 0 \\ i & 1 \end{bmatrix}$

$AA^T=I$

$Q=A^TBA$

$Q^2=(A^TBA)(A^tBA)$

$=A^TBAA^TBA$

$Q^2=A^TB^2A$

$Q^{2021}=A^TB^{2021}.A$

$B^2=\begin{bmatrix} 1 & 0 \\ i & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ i & 1 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 2i & 1 \end{bmatrix}$

$AQ^{2021}A^T=\begin{bmatrix} 1 & 0 \\ 2021i & 1 \end{bmatrix}$

Its inverse $=\frac{1}{1-0}\begin{bmatrix} 1 & -0 \\ -2021i & 1 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ -2021i & 1 \end{bmatrix}$

74. Let A and B be independent events such that $P(A)=p$, $P(B)=2p$. The largest value of p, for which P (exactly one of A, B occurs) $=\frac{5}{9}$, is:

1) $\frac{5}{12}$ 2) $\frac{4}{9}$ 3) $\frac{2}{9}$ 4) $\frac{1}{3}$

Key: 1

Sol: Let A & B are independent events such that

$P(A)=p,\ P(B)=2p$

$p(\text{exactly one of } A,B \text{ occurs})=\frac{5}{9}$

$p(A)+p(B)-2p(A\cap B)=\frac{5}{9}$

$p+2p-2.p.2p=\frac{5}{9}$

$3p-4p^2=\frac{5}{9} \Rightarrow 27p-36p^2=5$

$36p^2-27p+5=0$

$12p(3p-1)-5(3p-1)=0,\ p=\frac{1}{3},\ p=\frac{5}{12}$

Largest value of $p=\frac{5}{12}$

75. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$. If $\vec{c}$ is a vector such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}\cdot\vec{c}=3$, then $\vec{a}\cdot(\vec{b}\times\vec{c})$ is equal to:

1) -2 2) -6 3) 2 4) 6

Key: 1

Sol: Let $\bar{a}=\bar{i}+\bar{j}+\bar{k};\ \bar{b}=\bar{j}-\bar{k},\ \bar{c}$ is a vector

such that $\bar{a}\times\bar{c}=\bar{b}$ and $\bar{a}\cdot\bar{c}=3$ then $\bar{a}\cdot(\bar{b}\times\bar{c})$

$\bar{a}\times\bar{c}=\bar{b} \Rightarrow \bar{c}\times\bar{a}=-\bar{b}$

$\Rightarrow(\bar{c}\times\bar{a})\cdot\bar{b}=-\bar{b}.\bar{b}=-|\bar{b}|^2=-2$

$\Rightarrow[\bar{c}\ \bar{a}\ \bar{b}]=[\bar{a}\ \bar{b}\ \bar{c}]=-2$

76. Let $\theta \in \left(0, \frac{\pi}{2}\right)$. If the system of linear equations.

$\left(1+\cos^2\theta\right) x + \sin^2\theta\, y + 4\sin 3\theta\, z = 0$

$\cos^2\theta\, x + \left(1+\sin^2\theta\right) y + 4\sin 3\theta\, z = 0$

$\cos^2\theta\, x + \sin^2\theta\, y + \left(1+4\sin 3\theta\right) z = 0$

has a non-trivial solution, then the value of θ is:

1) $\frac{7\pi}{18}$ 2) $\frac{5\pi}{18}$ 3) $\frac{4\pi}{9}$ 4) $\frac{\pi}{18}$

Key: 1

Sol: $: \begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$

$C_1 + C_2 \quad \begin{vmatrix} 2 & \sin^2\theta & 4\sin 3\theta \\ 2 & 1+\sin^2\theta & 4\sin 3\theta \\ 1 & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$

$R_1 - R_2, R_2 - 2R_3$

$\begin{vmatrix} 0 & -1 & 0 \\ 0 & 1-\sin^2\theta & -2-4\sin 3\theta \\ 1 & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$

(1) $2 + 4\sin 3\theta = 0 \Rightarrow \sin 3\theta = -\frac{1}{2}$

$3\theta = -\frac{\pi}{6}$

$3\theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$

$= n\pi + (-1)^{n+1}\frac{\pi}{6}$

$\theta = \frac{x\pi}{3} + (-1)^{n+1}\frac{\pi}{18}$

$n = 0; \qquad \theta = \frac{-\pi}{18}$

$n = 1; \ \frac{\pi}{3} + \frac{\pi}{18} = \frac{7\pi}{18}$

77. The mean and standard deviation of 20 observations were calculated as 10 and 2.5 respectively. It was found that by mistake one data value was taken as 25 instead of 35. If α and $\sqrt{\beta}$ are the mean and standard deviation respectively for correct data, then (α,β) is:

1) $(10.5,25)$ 2) $(10.5,26)$ 3) $(11,26)$ 4) $(11,25)$

Key: 2

Sol: $n = 20$

$\bar{x} = 10,\ \sigma_1 = 2.5$

$$\frac{X_1 + X_2 + ----+X_{20}}{20} = 10$$

$$\text{Correct sum} = X_1 + X_2 + ---+ X_{20} = 200 - 25 + 35$$

$$= 210$$

$$\text{new mean} \Rightarrow \frac{210}{20} = \frac{21}{2} = 10.5$$

$$\text{Variance} = \frac{\sum x_i^{\,2}}{n} - \left(\bar{x}\right)^2$$

$$\sigma = \sqrt{\frac{\sum x_i^{\,2}}{n} - \left(\bar{x}\right)^2} = 26$$

$(10.5,26)$

78. The equation $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$ represents a circle with:

1) centre at $(0,1)$ and radius 2

2) centre at $(0,1)$ and radius $\sqrt{2}$

3) centre at $(0,-1)$ and radius $\sqrt{2}$

4) centre at $(0,0)$ and radius $\sqrt{2}$

Key: 2

Sol: $$\frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1} = \frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy}$$

$$= \frac{(x+1)(x-1)-iy(x-1)+iy(x+1)-i^2y^2}{(x+1)^2+y^2}$$

$$= \frac{x^2-1+y^2+i[xy+y-xy+y]}{(x+1)^2+y^2}$$

$$= \frac{\left(x^2+y^2-1\right)+i(2y)}{(x+1)^2+y^2}$$

$$\text{Arg}\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$$

$$\text{So } \frac{2y}{x^2+y^2-1}=1$$

$$x^2+y^2-1=2y$$

$$x^2+y^2-2y-1=0$$

centre $(0,1)$, $r=\sqrt{2}$

79. If the sum of an infinite GP $a, ar, ar^2, ar^3, \ldots$ is 15 and the sum of the squares of its each term is 150, then the sum of $ar^2, ar^4, ar^6, \ldots$ is:

1) $\frac{5}{2}$ 2) $\frac{25}{2}$ 3) $\frac{1}{2}$ 4) $\frac{9}{2}$

Key: 3

Sol: $a+ar+a\vec{r}\ldots=\frac{a}{1-r}=15$ .....(1)

$$a^2+a^2r^2+\ldots..$$

$$\frac{a^2}{1-r^2}=150 \ldots.(2)$$

$$\frac{1}{2} \Rightarrow \frac{a}{1+r}=10 \quad \ldots.(3)$$

$$\frac{1}{3} \Rightarrow \frac{a}{1-r}\times\frac{1+r}{a}=\frac{15}{10}$$

$$\frac{1+r}{1-r}=\frac{3}{2}$$

$$2+2r=3-3r \Rightarrow 5r=2 \Rightarrow r=\frac{1}{5}$$

From (1) $\frac{a}{1-\frac{1}{5}}=15 \Rightarrow 5a=60$

$$\Rightarrow a=12$$

Sum of required series $\frac{ar^2}{1-r^2}=\frac{12\times\frac{1}{25}}{1-\frac{1}{25}}=\frac{12}{24}=\frac{1}{2}$

80. Let $y=y(x)$ be a solution curve of the differential equation $(y+1)\tan^2 x\,dx+\tan x\,dy+y\,dx=0$, $x\in\left(0,\frac{\pi}{2}\right)$. If $\lim\limits_{x\to 0+} xy(x)=1$, then the value of $y\left(\frac{\pi}{4}\right)$ is:

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{4}-1$ 3) $\frac{\pi}{4}+1$ 4) $-\frac{\pi}{4}$

Key: 1

Sol: $(y+1)\tan^2 x\,dx+\tan x\,dy+y\,dx=0$

$y\tan^2 x\,dx+\tan^2 x\,dx+\tan x\,dy+y\,dx=0$

$y\,dx(1+\tan^2 x)+\tan^2 x\,dx+\tan x\,dy=0$

$\tan x\frac{dy}{dx}+y\sin^2 x=-\tan^2 x$

$\frac{dy}{dx}+y\frac{1}{\sin x\cos x}=-\tan x$

I.F $=e^{\int\frac{1}{\sin x\cos x}dx}$

$=e^{\int(\tan x+\cot x)dx}$

$=e^{\log\sec x+\log\sin x}$

$=e^{\log\sec x\sin x}$

$=\tan x$

$y(I.F)=\int Q(x)(I.F)\,dx$

$y\tan x=\int-\tan x\tan x d$

$y\tan x=-\int(\sec^2 x-1)\,dx$

$y\tan x=-\tan x+x+c$

$y=-1+x\cot x+c\cot x$

$$\lim_{x\to 0} xy = 1$$

$$\lim_{x\to 0} x\left(-1 + x\cot x + c\cot x\right) = 1$$

$$c = 1$$

$$y = -1 + x\cot x + \cot x$$

$$y\left(\frac{\pi}{4}\right) = -1 + \frac{\pi}{4} + 1 = \frac{\pi}{4}$$

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

81. If ${}^{1}P_1 + 2\cdot {}^{2}P_2 + 3\cdot {}^{3}P_3 + ... + 15\cdot {}^{15}P_{15} = {}^{q}P_r - s$, $0 \le s \le 1$, then ${}^{q+s}C_{r-s}$ is equal to

Key: 136

Sol: Given $1.1! + 2.2! + 3.3! ..... + 15.15! = {}^{q}P_r - s$

$1.1! + 2.2! + ... + n.n! = (n+1)! - 1$

$1.1! + 2.2! + ... + 15.15! = 16! - 1 = {}^{q}P_r - s$

$\therefore s = 1,\ q = r = 16$

${}^{q+s}C_{r-s} = {}^{16+1}C_{16-1} = {}^{17}C_{15} = {}^{17}C_2 = \frac{17\times 16}{2} = 136$

82. The sum of all integral values of $k\left(k \ne 0\right)$ for which the equation $\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$ in x has no real roots, is _____.

Key: 66

Sol: Given $\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$

$\left[2(x-2) - (x-1)\right]k = 2(x-1)(x-2)$

$(2x - 4 - x + 1)k = 2(x-1)(x-2)$

$(x-3)k = 2\left(x^2 - 3x + 2\right)$

$2x^2 - 6x + 4 - kx + 3k = 0$

$2x^2 - x(6+k) + 4 + 3k = 0$

x has no real. so $\Delta<0$

$(6+k)^2-4\cdot2\cdot(4+3k)<0$

$k^2-12k+4<0$

$k=\dfrac{12\pm\sqrt{144-16}}{2}$

$k=\dfrac{12\pm\sqrt{128}}{2}$

$k=6\pm\sqrt{32}$

$k\in\left(6-\sqrt{32},6+\sqrt{32}\right)$ integer between it are

$k=\{1,2,3,....11\}$

sum of 11 natural no's $\Rightarrow \dfrac{11\times12}{2}=11\times6=66$

83. The locus of a point, which moves such that the sum of squares of its distances from the points $(0,0),(1,0),(0,1),(1,1)$ is 18 units, is a circle of diameter d. Then d^2 is equal to ___.

Key: 16

Sol: Let point $P(x,y)$

Given points $A(0,0)\ B(1,0)\ C(0,1)\ D(1,1)$

Given $(AP)^2+(BP)^2+(CP)^2+(DP)^2=18$

$x^2+y^2+(x-1)^2+y^2+x^2+(y-1)^2+(x-1)^2+(y-1)^2=18$

$4x^2+4y^2-4x-4y-14=0$

$x^2+y^2-x-y-\dfrac{7}{2}=0$

$r=\sqrt{\dfrac{1}{4}+\dfrac{1}{4}+\dfrac{7}{2}}=\sqrt{\dfrac{1}{2}+\dfrac{7}{2}}=\sqrt{\dfrac{8}{2}}=2$

$d=2r=4$

$d^2=16$

84. Let $a, b \in R$, $b \neq 0$. Define a function

$$f(x)=\begin{cases} a\sin\frac{\pi}{2}(x-1), \text{ for } x \le 0 \\ \frac{\tan 2x-\sin 2x}{bx^3}, \text{for } x>0. \end{cases}$$

If f is continuous at $x=0$, then $10-ab$ is equal to _____.

Key: 14

Sol: $$f(x)=\begin{cases} a\sin\frac{\pi}{2}(x-1) & x \le 0 \\ \frac{\tan 2x-\sin 2x}{bx^3} & x>0. \end{cases}$$

$$\lim_{x\to 0^-} a\sin\frac{\pi}{2}(x-1)=\lim_{x\to 0^+}\frac{\frac{\sin 2x}{\cos 2x}(1-\cos 2x)}{bx^3}$$

$$a\cdot\sin\frac{\pi}{2}(0-1)=\frac{1}{b}\lim_{x\to 0}\frac{\tan 2x}{x}.\frac{1-\cos 2x}{x^2}$$

$$a(-1)=\frac{1}{b}(2)\left(\frac{(2)^2}{2}\right)$$

$$-ab=4$$

$$\therefore 10-ab=10+4=14$$

85. If $y=y(x)$ is an implicit functions of x such that $\log_e(x+y)=4xy$, then $\frac{d^2y}{dx^2}$ at $x=0$ is equal to ____.

Key: 40

Sol: Given $\log(x+y)=4xy$

$$x+y=e^{4xy}$$

Diff

$$1+y^1=e^{4xy}\,4\left(xy^1+y(1)\right)$$

put $x=0,\ y=1$ Then $y^1=3$

diff

$$y^{11} = 4e^{4xy}\left(xy^{11} + y^1 + y^1\right) + 4\left(xy^1 + y\right)e^{4xy}\left(4xy^1 + 4y(1)\right)$$

put $x = 0,\ y = 1,\ y^1 = 3$

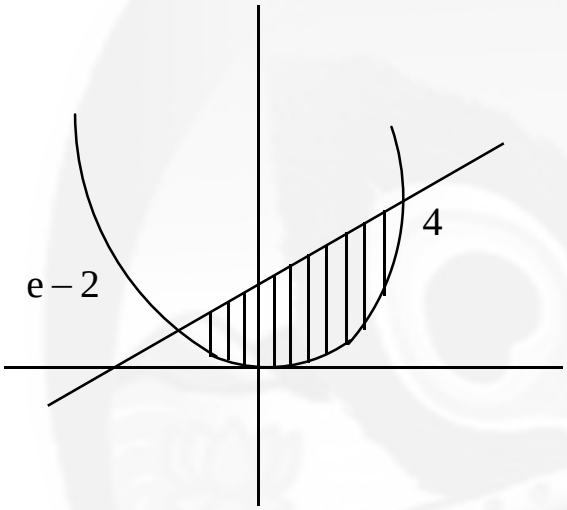
$$y^{11} = 4(1)(0+3+3) + 4(0+1)(1)(0+4)$$

$$y^{11} = 24 + 16 = 40$$

86. The area of the $S = \left\{(x, y): 3x^2 \le 4y \le 6x + 24\right\}$ regions is ____.

Key: 27 sq units

Sol: Given $3x^2 \le 44 \le 6x + 24$



p.o.i of $4y = 3x^2,\ 4y = 6x + 24$

$$3x^2 = 6x + 24$$

$$x = -2, 4$$

req Area $\int_{-2}^{4}\left(\frac{x+24}{4} - \frac{3x^2}{4}\right)dx$

$$= \frac{3}{4}\int_{-2}^{4}\left(2x + 8 - x^2\right)dx$$

$$= \frac{3}{4}(36) = 27 \text{ sq units}$$

87. A wire of length $36\,m$ is cut into two pieces, one of the pieces is bent to form a square and the other is bent to form a circle. if the sum of the areas of the two figures is minimum, and the circumference of the circle is k (meter), then $\left(\frac{4}{\pi}+1\right)k$ is equal to

Key: 36

Sol: $4x + 2\pi r = 36$

$2x + \pi r = 18$

$\pi r = 18 - 2x$

$r = \frac{18-2x}{\pi}$

$A = x^2 + \pi r^2$

$= x^2 + \pi\left(\frac{18-2x}{\pi}\right)^2$

$\frac{dA}{dx} = 0$

$2x + \frac{\pi}{\pi^2}.2(18-2x)(0-2) = 0$

$\therefore\ x = \frac{36}{\pi+4}$

$2\pi r = 36 - 4x$

$= 36 - 4\left(\frac{36}{\pi+4}\right)$

$2\pi r = 36\left(1 - \frac{4}{\pi+4}\right) = 36\left(\frac{\pi}{4+4}\right)$

Given circumference of circle $= k$

$\frac{36\pi}{\pi+4} = k$

$36 = k\frac{(\pi+4)}{\pi}$

88. Let $z=\frac{1-i\sqrt{3}}{2}$, $i=\sqrt{-1}$. Then the value of

$$21+\left(z+\frac{1}{z}\right)^3+\left(z^2+\frac{1}{z^2}\right)^3+\left(z^3+\frac{1}{z^3}\right)^3+.....+\left(z^{21}+\frac{1}{z^{21}}\right)^3$$

is ____.

Key: 13

Sol: $-z=\frac{-1+i\sqrt{3}}{2}$

Let $-z=\omega$

$z=-\omega$

$$z+\frac{1}{z}=-\omega-\frac{1}{\omega}=\frac{-\omega^2-1}{\omega}=\frac{\omega}{\omega}=1$$

$$z^2+\frac{1}{z^2}=(-\omega)^2+\frac{1}{(-\omega)^2}=\omega^2+\frac{1}{\omega^2}=\frac{\omega^4+1}{\omega^2}=\frac{\omega+1}{\omega^2}=\frac{-\omega^2}{\omega^2}=-1$$

$$z^3+\frac{1}{z^3}=(-\omega)^3+\frac{1}{(-\omega)^3}=-1-1=-2$$

$$\left(z+\frac{1}{z}\right)^3+\left(z^2+\frac{1}{z^2}\right)^3+\left(z^3+\frac{1}{z^3}\right)^3=1-1-8=-8$$

$$z^6+\frac{1}{z^6}=(-\omega)^6+\frac{1}{(-\omega)^6}=(1+1)^3=8$$

$$\left(z^3+\frac{1}{z^3}\right)^3+\left(z^6+\frac{1}{z^6}\right)^3=0$$

$$\left(z^9+\frac{1}{z^9}\right)^3+\left(z^{12}+\frac{1}{z^{12}}\right)^3=0$$

$$\left(z^{15}+\frac{1}{z^{15}}\right)^3+\left(z^{18}+\frac{1}{z^{18}}\right)^3=0$$

$$z^{21}+\frac{1}{z^{21}}=-8$$

$$\Rightarrow 21-8=13$$

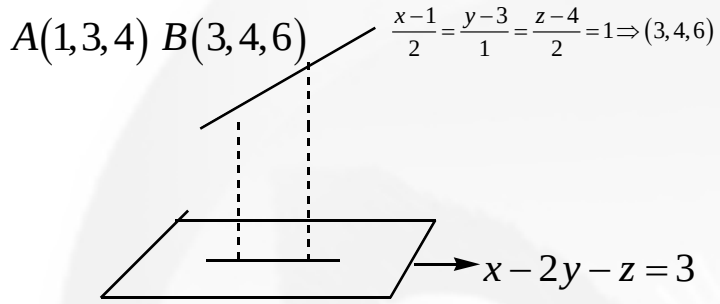
89. Let the line L be the projection of the line

$$\frac{x-1}{2}=\frac{y-3}{1}=\frac{z-4}{2}$$

in the plane $x-2y-z=3$. If d is the distance of the point $(0,0,6)$ from L, then d^2 is equal to ____.

Key: 26

Sol:



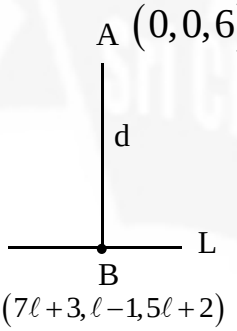
$A_1, B_1 \Rightarrow$ *foot of* $\perp$ A, B

$$\frac{\alpha-1}{1}-\frac{\beta-3}{-2}=\frac{\gamma-4}{-1}=\frac{-(1-6-4-3)}{6}=2\Rightarrow A_1=(3,-1,2)$$

$$\frac{\alpha-3}{1}-\frac{\beta-4}{-2}=\frac{\gamma-6}{-1}=\frac{-(3-8-6-3)}{6}=\frac{7}{3}\Rightarrow B_1=\left(\frac{16}{3},\frac{-2}{3},\frac{11}{3}\right)$$

Drs of $A_1B_1: \frac{16}{3}-3, \frac{-2}{3}+1, \frac{11}{3}-2 \Rightarrow 7,1,5$

$$\therefore L: \frac{x-3}{7}=\frac{y+1}{1}=\frac{z-2}{5}=\ell$$



$(7\ell+3, \ell-1, 5\ell+2)=B$

Dr,s of AB: $7\ell+3, \ell-1, 5\ell-4$

$\overline{AB} \perp L \Rightarrow 7(7\ell+3)+(\ell-1)+5(5\ell-4)=0$

$(49+1+25)kl+21-1-20=0$

$75\ell=0\Rightarrow \ell=0$

$B=(3,-1,2)$

$\therefore$ Req distance, $d=\sqrt{9+1+16}=\sqrt{26}$

$\Rightarrow d^2=26$

90. The number of three-digit even numbers, formed by the digits $0,1,3,4,6,7$ if the repetition of digits is not allowed, is ____.

Key: 52

Sol: Given digits $\{0,1,3,4,6,7\}$

$\_\ \_\ \underline{0}={}^5p_2=\frac{5!}{3!}=\frac{120}{6}=20$

$\overline{4}\ \overline{4}\ \frac{4}{1}\Rightarrow 16$

$\overline{4}\ \overline{4}\ \frac{6}{1}\Rightarrow 16$

Req $\Rightarrow 20+16+16=52$

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