Sri Chaitanya

JEE MAIN 2021

PHASE - IV

Key & Solutions

27-Aug-2021 | Shift - 1

Jee-Main_Final_27-Aug-2021_Shift-01

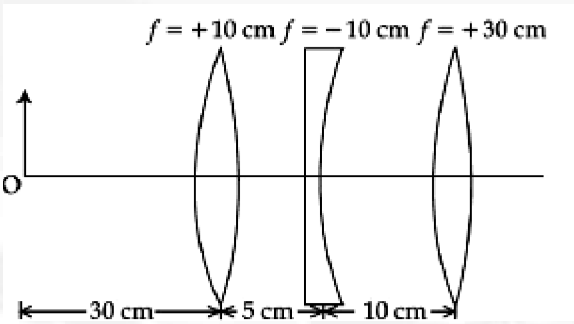
PHYSICS **Max Marks: 100**

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. Find the distance of image from object O, formed by the combination of lenses in the figure :



1) 20 cm **2)** 10 cm **3)** infinity **4)** 75 cm

Key: 4

Sol: 1st image

$$\frac{1}{v_1} = \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{-30} = \frac{1}{10}$$

$v = 15$

2nd image

$$\frac{1}{v} = \frac{-1}{10} = \frac{-1}{10}$$

$v = \infty$

Similarly $v_3 = 30cm$ from lense

So from object its 75 cm

2. For a transistor in CE mode to be used as an amplifier, it must be operated in :

1) The active region only **2)** Cut-off region only

3) Saturation region only **4)** Both Cut-off and Saturation

Key: 1

Sol: Conceptual

3. A uniformly charged disc of radius R having surface charge density σ is placed in the xy plane with its center at the origin. Find the electric field intensity along the z-axis at a distance Z from origin:

1) $E=\frac{\sigma}{2\varepsilon_0}\left(1+\frac{Z}{(Z^2+R^2)^{1/2}}\right)$ **2)** $E=\frac{2\varepsilon_0}{\sigma}\left(\frac{1}{(Z^2+R^2)^{1/2}}+Z\right)$

3) $E=\frac{\sigma}{2\varepsilon_0}\left(\frac{1}{(Z^2+R^2)}+\frac{1}{Z^2}\right)$ **4)** $E=\frac{\sigma}{2\varepsilon_0}\left(1-\frac{Z}{(Z^2+R^2)^{1/2}}\right)$

Key: 4

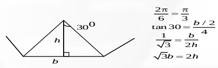
Sol: Electric field along axis of disc formula

$$E=\frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{R^2+x^2}}\right]$$

$$E=\frac{\sigma}{2\varepsilon_0}\left[1-\frac{Z}{\sqrt{R^2+x^2}}\right]$$

$$E=\frac{\sigma}{2\varepsilon_0}\left[1-\frac{Z}{\sqrt{R^2+Z^2}}\right]$$

4. In Millikan's oil drop experiment, what is viscous force acting on an uncharged drop of radius $2.0\times10^{-5}m$ and density $1.2\times10^3 kgm^{-3}$? Take viscosity of liquid = $1.8\times10^{-5} Nsm^{-2}$ (Neglect buoyancy due to air)

1) $1.8\times10^{-10}N$ **2)** $3.8\times10^{-11}N$ **3)** $3.9\times10^{-10}N$ **4)** 

Key: 3

Sol: $mg=F$

$\rho vg=F$

$1.2\times10^3\times\frac{4}{3}nR^3.g=F$

5. A balloon carries a total load of 185 kg at normal pressure of 27^0 C. What load will the balloon carry on rising to a height at which the barometric pressure is 45 cm of Hg and the temperature is -7^0 C. Assuming the volume constant?

1) 123.54 kg **2)** 214.15 kg **3)** 219.07 kg **4)** 181.46 kg

Key: 1

Sol: $mg=kvg$

$pv=nRT$

$$\therefore\ 185\left[\frac{45}{76}\times\frac{300}{266}\right]$$

$$185\times[0.59\times1.12]$$

$$185\times0.66\ = 123$$

6. Which of the following is not a dimensionless quantity?

1) Power factor **2)** Permeability of free space (μ_0)

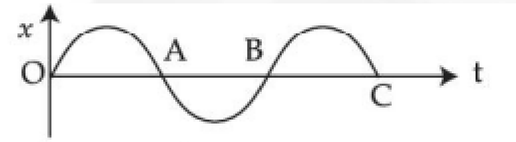
3) Quality factor **4)** Relative magnetic permeability (μ_r)

Key: 2

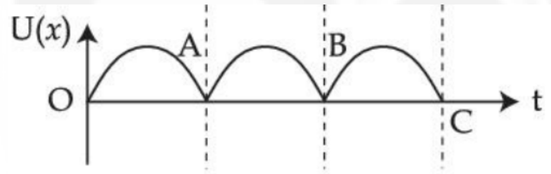
Sol: Magnetic permeability

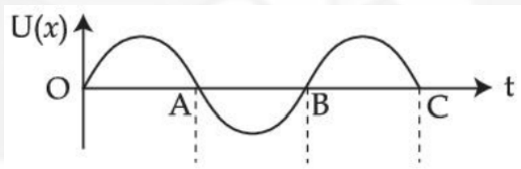
Dimension in $(mLT^{-2}A^{-2})$

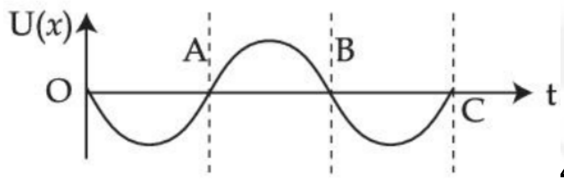
7. The variation of displacement with time of a particle executing free simple harmonic motion is shown in the figure.

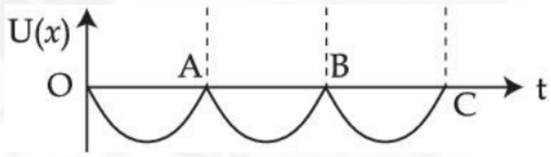


The potential energy U(x) versus time (t) plot of the particle is correctly shown in figure:

1) 

2) 

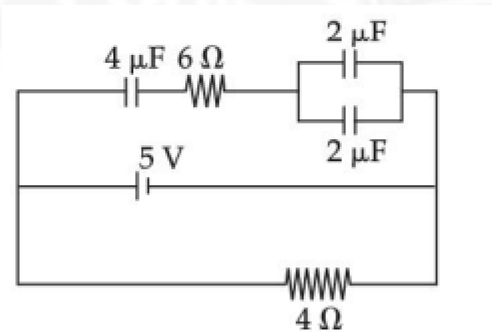
3) 

4) 

Key: 1

Sol: Conceptual

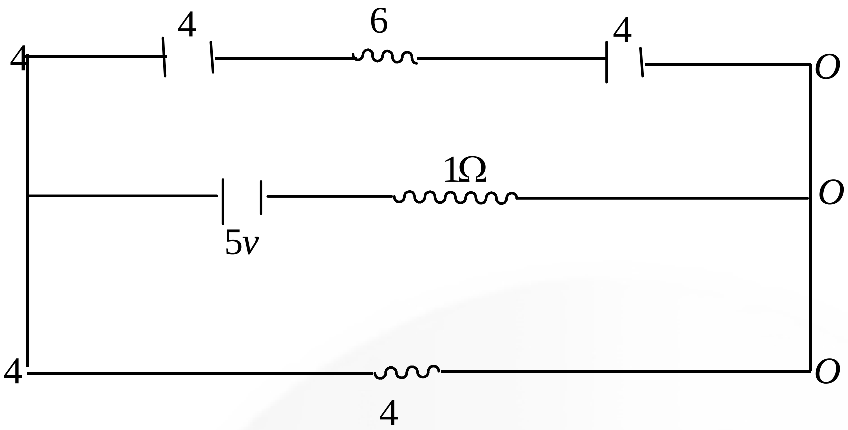
8. Calculate the amount of change on capacitor of 4 μF. The internal resistance of battery is 1Ω:



1) Zero **2)** $4\mu C$ **3)** $8\mu C$ **4)** $16\mu C$

Key: 3

Sol:



Voltage speed a cross 1Ω is $1v$

$i=\frac{5}{4+1}=1$

$\therefore$ P. D across each capacitor is $2v$

$\therefore\ 9=4\times2=8$

9. Electric field in a plane electromagnetic wave is given by $E=50\sin\left(500x-10\times10^{10}t\right)$ V/m The velocity of electromagnetic wave in this medium is :

(Given C= speed of light in vacuum)

1) $\frac{2}{3}C$ **2)** $\frac{4}{T}$ **3)** $\frac{1}{T}$ **4)** $\frac{3}{T}$

Key: 1

Sol: $E=50\sin(500x-10\times10^{10}t)$

$E=A\sin(kx-wt)$

$v=\frac{w}{k}=\frac{10\times10^{10}}{500}$

$v=2\times10^{8}$

So $\frac{2}{3}C$

10. An ideal gas is expanding such that $PT^3=\text{constant}$. The coefficient of volume expansion of the gas is :

1) $\frac{2}{T}$ **2)** $\frac{4}{T}$ **3)** $\frac{1}{T}$ **4)** $\frac{3}{T}$

Key: 2

Sol: $v=v_0(1+\gamma.\Delta T$

$v=v_0+v_0\gamma.\Delta T$

$v-v_0=v_0\gamma.\Delta T$

$\frac{\Delta v}{\Delta v_0}=\gamma.\Delta T$

$$\gamma = \frac{1}{v} \times \frac{dv}{dt}$$

$PT^3 =$ constant

$\frac{T}{v} T^3 =$ constant

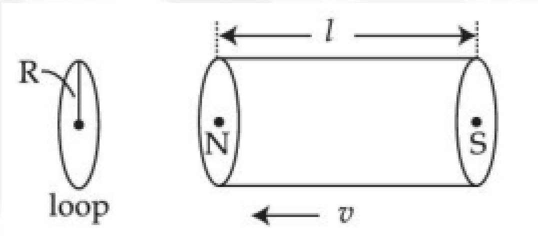
$T^4 v^{-1} =$ constant

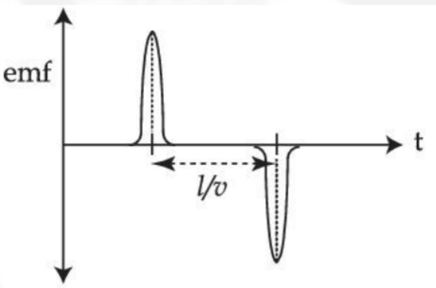
$$v^{-1}.4T^3 dt + T^4 \times \frac{-1}{v^2}.dv = 0$$

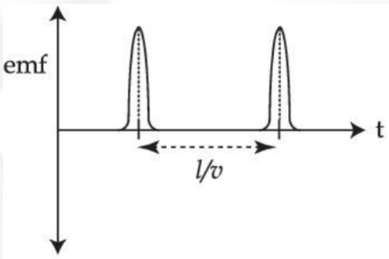
$$\therefore \frac{dv}{dT} = \frac{4T^3 / v}{T^4 v^2} = \frac{4v}{T}$$

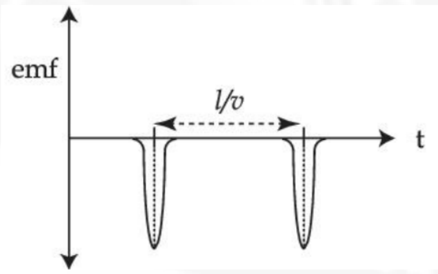
$\therefore \gamma = 4/T$

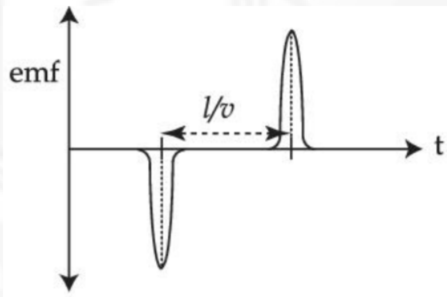
11. A bar magnet is passing through a conducting loop of radius R with velocity υ. The radius of the bar magnet is such that it just passes through the loop. The induced e.m.f. in the loop can be represented by the approximate curve:



1) 

2) 

3) 

4) 

Key: 4

Sol: 1) AC magnet approaches e.m.f will be maximum (negative)

2) When middle feen zero

3) When going ant again maximum but in opposite direction (positive)

12. If E and H represents the intensity of electric field and magnetizing field respectively, then the unit of E/H will be :

1) Newton **2)** Joule **3)** Ohm **4)** Mho

Key: 3

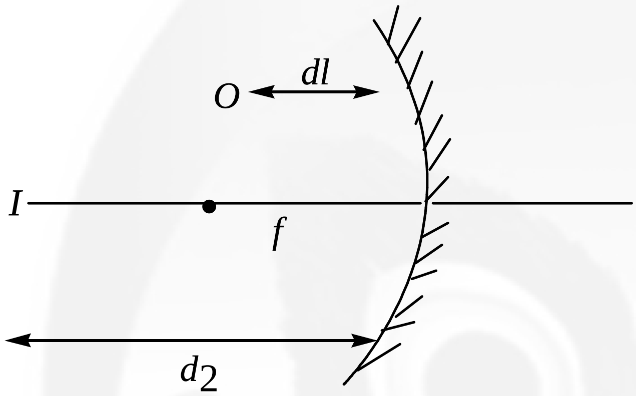
Sol: $\frac{E}{H} = \frac{N/C}{N/A-m}$

13. An object is placed beyond the centre of curvature C of the given concave mirror. If the distance of the object is d_1 from C and the distance of the image formed is d_2 from C, the radius of curvature of this mirror is :

1) $\frac{2d_1d_2}{d_1-d_2}$ **2)** $\frac{d_1d_2}{d_1-d_2}$ **3)** $\frac{d_1d_2}{d_1+d_2}$ **4)** $\frac{2d_1d_2}{d_1+d_2}$

Key: 1

Sol:



$(f+d_1)(f-d_2) = f^2$

$f = \frac{d_1.d_2}{d_1-d_2}$

$\therefore C = \frac{2d_1.d_2}{d_1-d_2}$

14. There are 10^{10} radioactive nuclei in a given radioactive element. Its half-life time is 1 minute. How many nuclei will remain after 30 seconds? $\left(\sqrt{2}=1.414\right)$

1) 4×10^{10} **2)** 2×10^{10} **3)** 7×10^{9} **4)** 10^{5}

Key: 3

Sol: $N = N_0e^{-\lambda T}$

$T = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda}$

$N = N_0\left(\frac{1}{2}\right)^{1/2}$

$N = \frac{N_0}{\sqrt{2}} = \frac{10^{10}}{1.414}$

7.07×10^{9}

15. A huge circular are of length 4.4 ly subtends an angle '4s' at the centre of the circle. How long it would take for a body to complete 4 revolution if its speed is 8 AU per second?

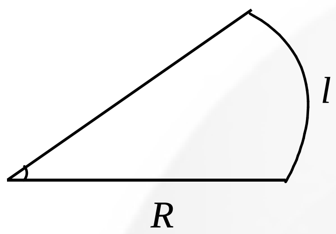
Given : $1\ \text{ly} = 9.46\times10^{15}\ \text{m}$

$1\ \text{AU} = 1.5\times10^{11}\ \text{m}$

1) $7.2\times10^{8}\ \text{s}$ **2)** $3.5\times10^{6}\ \text{s}$ **3)** $4.5\times10^{10}\ \text{s}$ **4)** $4.1\times10^{8}\ \text{s}$

Key: 3

Sol:



Speed = 8AV

$l = 4.4ly = 4.4\times9.46\times10^{15}$

$\therefore 8\times1.5\times10^{11}$

$12\times10^{11} m/s$

Time = $\frac{d}{dv} = \frac{4\times2\mu R}{12\times10^{11}}$

$t = 4.5\times10^{10}$

Arc length = $R\times\theta$

$\theta = 4\times5 = 4\times4.83\times10^{-6}$

$\theta = 1.94\times10^{-5}$ radians

$4.4\times9.46\times10^{15} = R\times1.94\times10^{-5}$

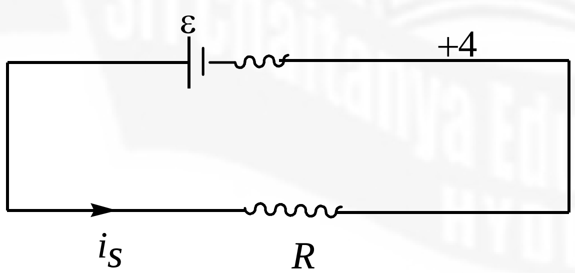
$R + 2.145\times10^{2}$

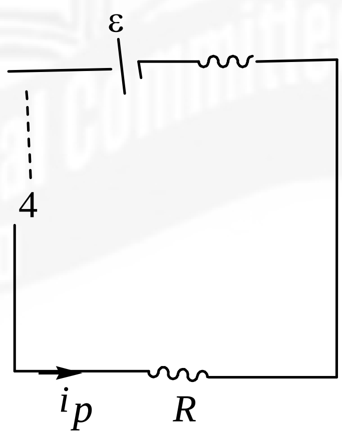
16. Five identical cells each of internal resistance 1Ω and emf 5V are connected in series and in parallel with an external resistance 'R'. For what value of 'R', current in series and parallel combination will remain the same?

1) 10Ω **2)** 5Ω **3)** 1Ω **4)** 25Ω

Key: 3

Sol:





$i_s = \frac{5\varepsilon}{R+5r}$

$i_p = \frac{\varepsilon}{R+r/5}$

$i_S = i_P$

$$\frac{5\varepsilon}{R+5r} = \frac{\varepsilon}{R+r/5}$$

$\therefore\ R = 1\Omega$

17. Moment of inertia of a square plate of side l about the axis passing through one of the corner and perpendicular to the plane of square plate is given by:

1) $\frac{Ml^2}{12}$ **2)** $\frac{Ml^2}{6}$ **3)** Ml^2 **4)** $\frac{2}{3}Ml^2$

Key: 4

Sol: $I = I_{cm} + md^2$

$$= \frac{ml^2}{6} + m\left(\frac{l}{\sqrt{2}}\right)^2$$

$$I = \frac{ml^2}{6} + \frac{ml^2}{2} = \frac{4ml^2}{6}$$

$$I = \frac{2}{3}ml^2$$

18. In a photoelectric experiment, increasing the intensity of incident light:

1) Increases the frequency of photons incident and the K.E. of the ejected electron remains unchanged

2) Increases the number of photons incident and the K.E. of the ejected electrons remains unchanged

3) Increase the number of photons incident and also increases the K.E. of the ejected electrons

4) Increase the frequency of photons incident and increases the K.E. of the ejected electrons

Key: 2

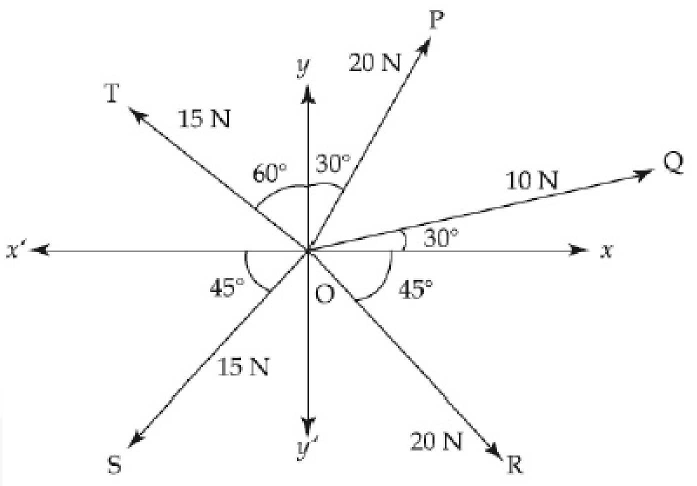
Sol: $\varepsilon = \frac{hc}{\lambda}$

$P = \frac{h}{\lambda}$

No of photons will increase

19. The resultant of these forces $\overrightarrow{OP}, \overrightarrow{OQ}, \overrightarrow{OR}, \overrightarrow{OS}$, and $\overrightarrow{OT}$ is approximately _______N.

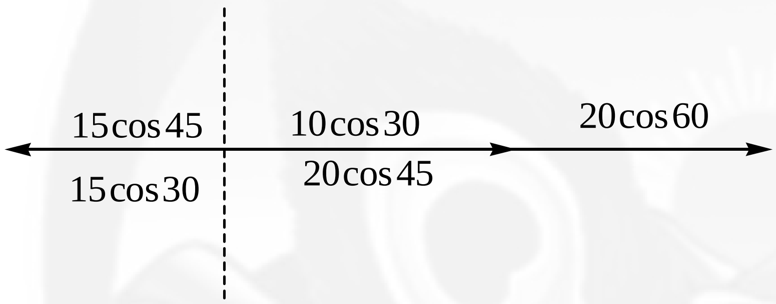
[Take $\sqrt{3}=1.7$, $\sqrt{2}=1.4$ Given $\hat{i}$ and $\hat{j}$ unit vectors along x, y axis)



1) $2.5\hat{i}-14.5\hat{j}$ **2)** $9.25\hat{i}+5\hat{j}$ **3)** $3\hat{i}-15\hat{j}$ **4)** $-1.5\hat{i}-15.5\hat{j}$

Key: 2

Sol:



$$10\times\frac{\sqrt{3}}{2}+20\times\frac{1}{\sqrt{2}}+20\times\frac{1}{2}-15\times\frac{1}{\sqrt{2}}-15\times\frac{\sqrt{3}}{2}$$

$$10\times\frac{1.7}{2}+\frac{20}{1.4}+10-\frac{15}{1.4}-\frac{15\times1.7}{2}$$

$$8.5+14.2+10-10.7-12.75$$

$$8.5+14.2+10-23.45$$

$$9.25$$

20. Two ions of masses 4 amu and 16 amu have charges +2e and +3e respectively. These ions pass through the region of constant perpendicular magnetic field. The kinetic energy of both ions is same.Then:

1) Both ions will be deflected equally

2) Lighter ion will be deflected less than heavier ion

3) Lighter ion will be deflected more than heavier ion

4) No ion will be deflected

Key: 3

Sol: $r=\frac{p}{qB}=\frac{\sqrt{2mk}}{qB}$

$$r\alpha\frac{\sqrt{m}}{q}$$

$$\frac{r_1}{r_2} = \frac{\sqrt{4}}{2} \times \frac{3}{\sqrt{16}} = 3/4$$

$$\sin\theta = \frac{d}{R}$$

$$\theta \alpha \frac{1}{R}$$

$$R_2 > R_1 \ \& \ \theta_2 < \theta_1$$

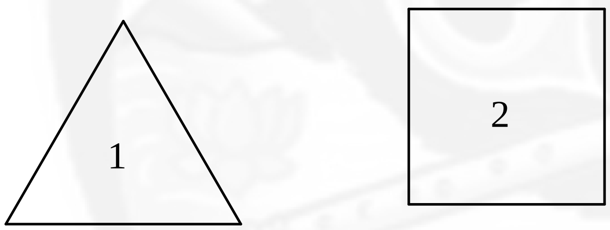
(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. A uniform conducting wire of length is 24a, and resistance R is wound up as a current carrying coil in the shape of an equilateral triangle of side 'a' and then in the form of a square of side 'a'. The coil is connected to a voltage source V_0. The ratio of magnetic moment of the coils in case of equilateral triangle to that for square is $1:\sqrt{y}$ where y is __________.

Key: 3

Sol:



$3l = d4a$ $\quad$ $4l = 24a$

$\therefore$ no of terms = 8 $\quad$ no = 6

$$\frac{m_1}{m_2} = \frac{n_i A_1}{n_i A_2} = \frac{8 \times \sqrt{3}/4a^2}{6 \times a^2}$$

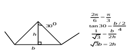
$$\frac{\sqrt{3} \times 8a^2}{24a^2} = \frac{\sqrt{3}}{3} a^2 = \frac{1}{\sqrt{3}}$$

$y = 3$

22. First, a set of n equal resistors of 10Ω each are connected in series to a battery of emf 20 V and internal resistance 10Ω. A current I is observed to flow. Then, the n resistors are connected in parallel to the same battery. It is observed that the current is increased 20 times, then the value of n is _______.

Key: 20

Sol: $R = 10\Omega$



$$v=20$$
$$i_p=20i_S$$
$$\frac{\varepsilon}{R+r}=\frac{20t}{R+r}$$
$$\frac{20}{\left(\frac{10}{n}+10\right)}=\frac{20\times 20}{(n10+10)}$$
$$(10n+10)=20\times 10/n+200$$
$$10n-\frac{200}{n}=200-10$$
$$\frac{10n^2-200}{n}=190$$
$$10n^2-200=190n$$
$$10n^2-190n-200=0$$
$$n^2-19n-20=0$$
$$n=20$$

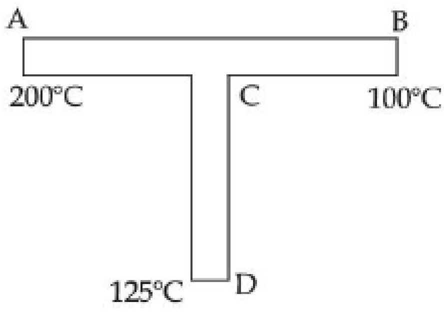
23. If the velocity of a body related to displacement x is given by $\upsilon=\sqrt{5000+24x}\ m/s,$ then the acceleration of the body is ________ m/s^2

Key: 12

Sol: $v=\sqrt{5000+24x}$

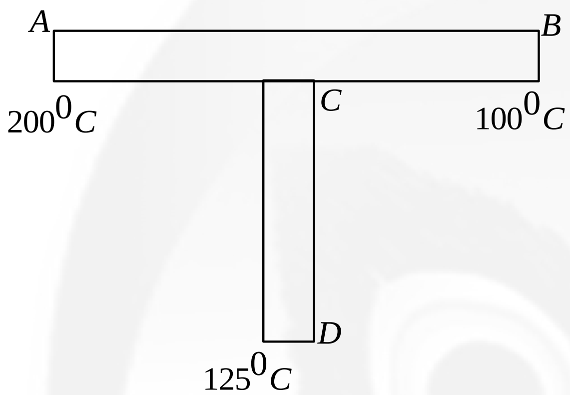
$$a=\frac{dv}{dm}v$$
$$\therefore a=\sqrt{5000+24x}\times\frac{d}{dx}\sqrt{5000+24x}$$
$$a=\sqrt{5000+24x}\times\frac{1}{2\sqrt{5000+24x}}\times\frac{d}{dx}24x$$
$$\frac{1}{2}\times 24=12$$

24. A rod CD of thermal resistance $10.0\,KW^{-1}$ is joined at the middle of an identical rod AB as shown in figure. The ends A,B and D are maintained at $200^0C, 100^0C$ *and* 125^0C respectively. The heat current in CD is P watt. The value of P is ______



Key: 2

Sol:



$$Q_{AC} + Q_{BC} = Q_{CD}$$

Let temperature at C is T

$$\frac{KA\Delta T}{l/2} + \frac{KA\Delta T}{l/2} = \frac{KA\Delta T}{l}$$

$$\frac{2KA\Delta T}{l} + \frac{KA\Delta T}{l} = \frac{KA\Delta T}{l}$$

$$2(200-T) + 2(100-T) = (T-125)$$

$$400 - 2T + 200 - 2T = T - 125$$

$$600.4T = T - 125$$

$$600 - 5T = 125$$

$$725 = 5T$$

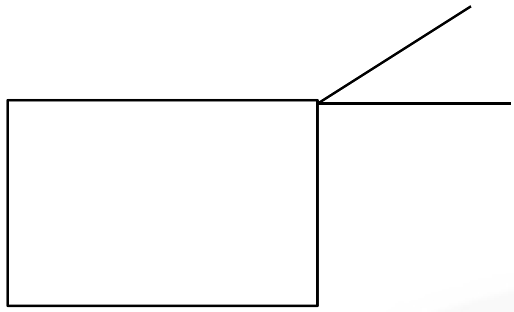
T =145

$$I = \frac{145-125}{10} = \frac{20}{10}w = 2w$$

25. Two persons A and B perform same amount of work in moving a body through a certain distance d with application of forces acting at angles 45° *and* 60° with the direction of displacement respectively. The ratio of force applied by person A to the force applied by person B is $\frac{1}{\sqrt{x}}$. The value of x is ___________.

Key: 2

Sol:



$w = F\cos\theta \times d$

$\frac{w}{d\cos\theta} = F_1$

$F_2 = \frac{w}{d\cos\theta_2}$

$\therefore \frac{F_1}{F_2} = \frac{w/d\cos\theta_1}{w/d\cos\theta_2}$

$\frac{F_1}{F_2} = \frac{d\cos\theta_2}{d\cos\theta_1} = \frac{\cos 60}{\cos 45} = \frac{1/2}{1/\sqrt{2}}$

$\frac{1}{2} \times \frac{\sqrt{2}}{1} = \frac{1}{\sqrt{2}}$

$x = 2$

26. Two cars X and Y are approaching each other with velocities 36 km/h and 72 km/h respectively. The frequency of a whistle sound as emitted by a passenger in car X, heard by the passenger in car Y is 1320 Hz. If the velocity of sound in air is 340 m/s, the actual frequency of the whistle sound produced is _______Hz

Key: 1210

Sol: $1320 = f_0\left[\frac{340+20}{340-10}\right]$

$\therefore\ f_0 = 1210$

27. The alternating current is given by $i = \left\{\sqrt{42}\ \sin\left(\frac{2\pi}{T}t\right) + 10\right\}A$ The r.m.s. value of this current is ___________A

Key: 11

Sol: $\left\langle i^2 \right\rangle = \frac{42}{2} + 100$

$i_{ms} = \sqrt{i^2}$

$\sqrt{121} = 11$

28. A transmitting antenna has a height of 320 m and that of receiving antenna is 2000 m. The maximum distance between them for satisfactory communication in line of sight mode is ‘d’. The value of ‘d’ is _______km

Key: 224km

Sol: $64\times10\times\sqrt{10}+16\times10^{2}\times\sqrt{10}$

$\sqrt{10}(640+1600)$

$\sqrt{2Rh_1}+\sqrt{2Rh_2}$

$\sqrt{2\times6400\times320\times10^{-3}}+\sqrt{26400\times320\times10^{-3}}$

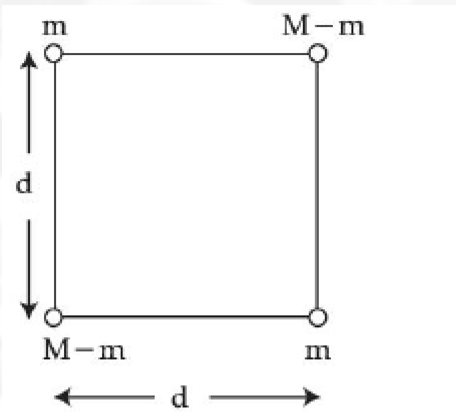
$\sqrt{64\times2\times32}+\sqrt{6400\times2\times2}$

$\sqrt{4096}+\sqrt{25600}$

$64+160$

224 km

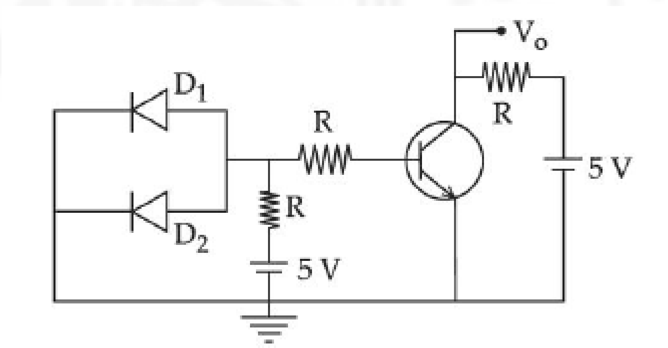
29. A body of mass (2M) splits into four masses {m,M-m,m, M-m}, which are rearranged to form a square as shown in the figure. The ratio of $\frac{M}{m}$ for which, the gravitational potential energy of the system becomes maximum is x:1. The value of x is ___________.



Key: 2

Sol: PE will be maximum when masses will be divided equally. i.e., $\frac{m}{M}=2$

30. A circuit is arranged as shown in figure. The output voltage V_0 is equal to _______V



Key: 5

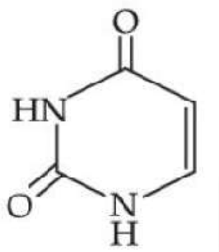
Sol: Both the diodes are forward bias transistor will go in steady state input current will be 0. Out put current will be zero so answer is 5

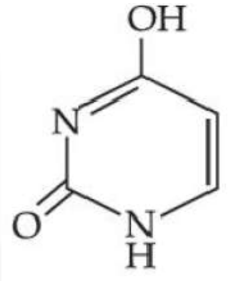
(SINGLE CORRECT ANSWER TYPE)

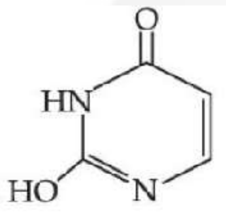
This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

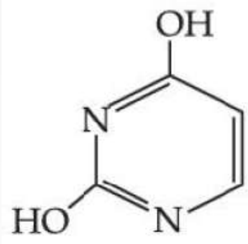
Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

31. Out of following isomeric forms of uracil, which one is present in RNA ?

1) 

2) 

3) 

4) 

Key: 1

Sol: NCERT

32. The number of water molecules in gypsum, dead burnt plaster and plaster of Paris, respectively are:

1) 2,0 and 1 **2)** 2,0 and 0.5 **3)** 0.5,0 and 2 **4)** 5,0 and 0.5

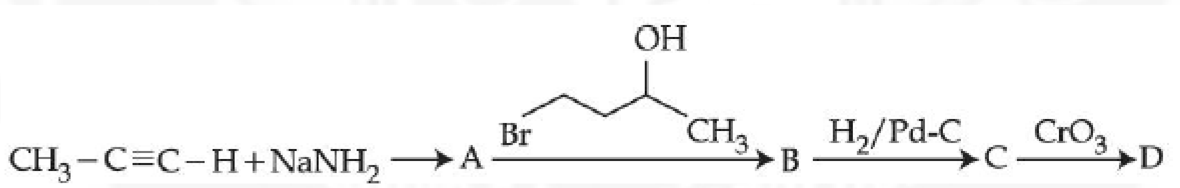
Key: 2

Sol: Gypsam has 2 moles of water molecules

dead burnt plaster has zero water molecules

Plaster of Paris has 0.5 moles of water molecules

33. In the following sequence of reactions, the final product D is:



1) $CH_3-CH=CH-CH_2-CH_2-CH_2-COOH$

2) $H_3C-CH_2-CH_2-CH_2-CH_2-\overset{\overset{O}{\|}}{C}-H$

3) $CH_3-CH_2-CH_2-CH_2-CH_2-\overset{\overset{O}{\|}}{C}-CH_3$

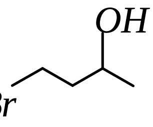
4) $H_3C-CH=CH-CH(OH)-CH_2-CH_2-CH_3$

Key: 3

Sol:

$$CH_3-C\equiv C-H+NaNH_2\rightarrow H_3C-C\equiv \overset{\ominus}{C}-\overset{\oplus}{Na}$$

$A \downarrow$



$$H_3C-C\equiv C-CH_2-CH_2-\underset{}{\overset{OH}{\overset{|}{CH}}}-CH_3$$

$B \downarrow H_2 / Pd / C$

$$H_3C-CH_2-CH_2-CH_2-CH_2-\overset{OH}{\overset{|}{CH}}-CH_3$$

$C \downarrow CrO_3$

$$CH_3-CH_2-CH_2-CH_2-CH_2-\overset{O}{\overset{||}{C}}-CH_3$$

D

34. Deuterium resembles hydrogen in properties but:

1) Reacts vigorously than hydrogen

2) Emits β^+ particles

3) Reacts just as hydrogen

4) Reacts slower than hydrogen

Key: 4

Sol: In case of isotopes chemical nature is same but reactivity is different

35. The major product of the following reaction is:

$$CH_3-\overset{CH_3}{\overset{|}{CH}}-CH_2-CH_2-\overset{O}{\overset{||}{C}}-Cl \xrightarrow[(iii)\,NaNO_2,HCl \quad (iv)\,H_2O]{(i)\,alcoholic\,NH_3 \quad (ii)\,NaOH,Br_2} Major\ Product$$

1) $CH_3-\underset{CH_3}{\underset{|}{CH}}-CH_2-CH_2OH$

2) $CH_3-\underset{CH_3}{\underset{|}{CH}}-CH_2-CH_2-Cl$

3) $CH_3-\underset{CH_3}{\underset{|}{CH}}-\overset{Br}{\overset{|}{CH}}-CH_2OH$

4) $CH_3-\underset{CH_3}{\underset{|}{CH}}-CH_2-CH_2-CH_2OH$

Key: 1

Sol:

$$CH_3-\underset{}{\overset{CH_3}{\overset{|}{C}}}H-CH_2-CH_2-\overset{O}{\overset{\|}{C}}-Cl \xrightarrow[NH_3]{Alcoholic} CH_3-\overset{CH_3}{\overset{|}{C}}H-CH_2-CH_2-\overset{O}{\overset{\|}{C}}-NH_2$$

$$\downarrow NaOH / Br_2$$

$$CH_3-\overset{CH_3}{\overset{|}{C}}H-CH_2-CH_2-NH_2$$

$$\downarrow NaNO_2 / HCl / H_2O$$

$$CH_3-\overset{CH_3}{\overset{|}{C}}H-CH_2-CH_2-OH$$

36. The gas ‘A’ is having very low reactivity reaches to stratosphere. It is non-toxic and non-flammable but dissociated by UV-radiations in stratosphere. The intermediates formed initially from the gas ‘A’ are:

1) $\dot{C}l+\dot{C}F_2Cl$ **2)** $Cl\dot{O}+\dot{C}H_3$ **3)** $Cl\dot{O}+\dot{C}F_2Cl$ **4)** $\dot{C}H_3+\dot{C}F_2Cl$

Key: 3

Sol:

$$CCl_3F+uvlight \rightarrow \dot{C}l+\dot{C}Cl_2F$$

$$CCl_2F_2+uvlight \rightarrow \dot{C}l+\dot{C}ClF_2$$

Cl atom produced destroying ozone layer.

$$\dot{C}l+O_3 \rightarrow \dot{C}lO+O_2$$

Hence intermediate are $\dot{C}lO$ & $\dot{C}F_2Cl$

37. Which of the following is not a correct statement for primary aliphatic amines?

1) Primary amines are less basic than the secondary amines

2) Primary amines can be prepared by the Gabriel phthalimide synthesis

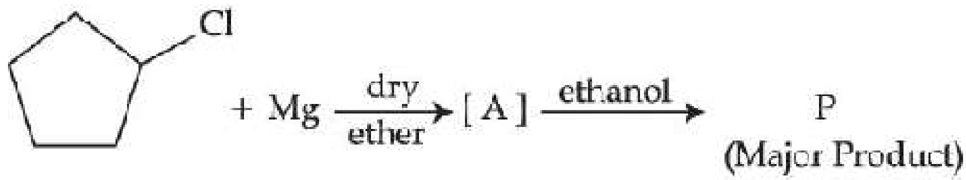
3) Primary amines on treating with nitrous acid solution form corresponding alcohols except methyl amine.

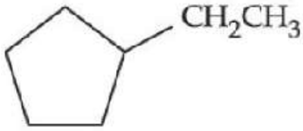
4) The intermolecular association in primary amines is less than the intermolecular association in secondary amines.

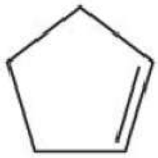
Key: 4

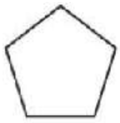
Sol: Primary amines inter molecular association is more than secondary amines because of more hydrogen bonds in primary amines

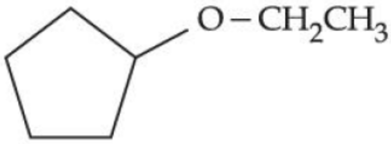
38. In the following sequence of reactions the P is:



1) 

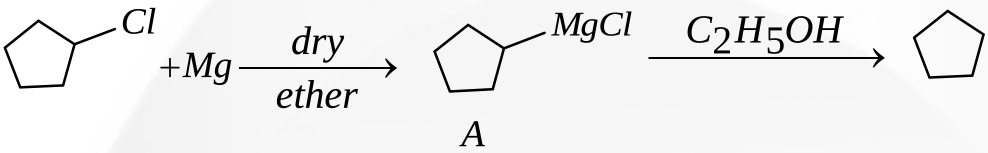
2) 

3) 

4) 

Key: 3

Sol:



39. The nature of oxides V_2O_3 and CrO is indexed as 'X' and 'Y' type respectively. The correct set of X and Y is:

1) X = basic Y = basic
2) X = basic Y=amphoteric
3) X=acidic Y=acidic
4) X=amphoteric Y=basic

Key: 1

Sol: V_2O_3 is Basic

CrO is Basic

Lower oxidation state metal oxides are basic nature.

40. Match List-I with List – II :

List-I (Species)	List-II (No.of lone pairs of electrons on the central atom)
(a) XeF_2	(i) 0
(b) XeO_2F_2	(ii) 1
(c) XeO_3F_2	(iii) 2
(d) XeF_4	(iv) 3

Choose the most appropriate answer from the options given below:

1) (a) – (iv), (b)-(i), (c)-(ii), (d)-(iii)
2) (a) – (iv), (b)-(ii), (c)-(i), (d)-(iii)
3) (a) – (iii), (b)-(ii), (c)-(iv), (d)-(i)
4) (a) – (iii), (b)-(iv), (c)-(ii), (d)-(i)

Key: 2

Sol: In XeF_2 No. of lone pairs on Xe atom = 3

In XeO_2F_2 No. of lone pairs on Xe atom =1

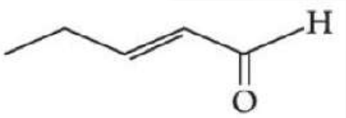
In XeO_3F_2 No. of lone pairs on Xe atom =0

In XeF_4 No. of lone pairs on Xe atom =2

41. The structure of the starting compound P used in the reaction given below is:

1) 

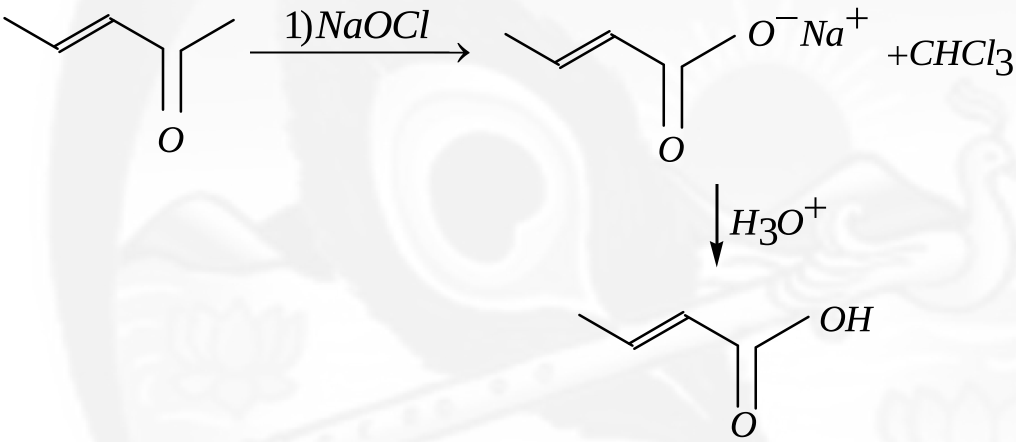
2) 

3) 

4) 

Key: 2

Sol:



42. Match items of List-I with those of List-II:

List-I (Property)	List-II (Example)
(a) Diamagnetism	(i) MnO
(b) Ferrimagnetism	(ii) O_2
(c) Paramagnetism	(iii) NaCl
(d) Antiferromagnetism	(iv) Fe_3O_4

Choose the most appropriate answer from the options given below:

1) $(a)-(ii),(b)-(i),(c)-(iii),(d)-(iv)$

2) $(a)-(i),(b)-(iii),(c)-(iv),(d)-(ii)$

3) $(a)-(iii),(b)-(iv),(c)-(ii),(d)-(i)$

4) $(a)-(iv),(b)-(ii),(c)-(i),(d)-(iii)$

Key: 3

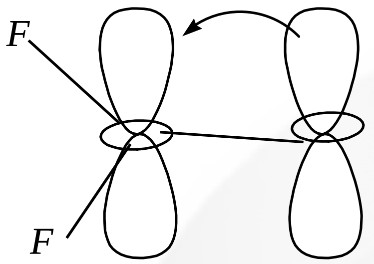
Sol: Examples from NCERT

43. In which one of the following molecules strongest back donation of an electron pair from halide to boron is expected?

1) BCl_3 **2)** BI_3 **3)** BF_3 **4)** BBr_3

Key: **3**

Sol: In BF_3



In BF_3 B atom is Sp^2 hybridized the un hybrid 2p orbital receives e^- pair from F atom forming strong $2P_\pi - 2P_\pi$ overlap.

44. Tyndall effect is more effectively shown by :

1) Suspension **2)** Lyophobic colloid

3) True solution **4)** Lyophilic colloid

Key: **2**

Sol: T for tyndall effect. There must be more difference in refractive indices of dispersion medium and dispersion phase

45. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as

Reason (R)

Assertion (A) : Synthesis of ethyl phenyl ether may be achieved by Williamson synthesis.

Reason (R) : Reaction of bromobenzene with sodium ethoxide yields ethyl phenyl ether.

In the light of the above statements, choose the most appropriate answer from the options given below:

1) (A) is correct but (R) is not correct

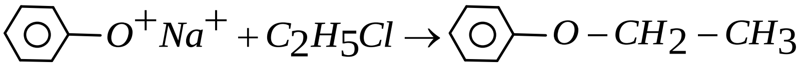
2) Both (A) and (R) are correct and (R) is the correct explanation of (A)

3) Both (A) and (R) are correct but (R) is NOT the correct explanation of (A)

4) (A) is not correct but (R) is correct

Key: **1**

Sol:



William son synthesis

Aryl halides are not involve in $ArSN^2$ reaction

46. Acidic ferric chloride solution on treatment with excess of potassium ferrocyanide gives a Prussian blue coloured colloidal species. It is:

1) $K_5Fe\left[Fe(CN)_6\right]_2$ **2)** $Fe_4\left[Fe(CN)_6\right]_3$

3) $HFe\left[Fe(CN)_6\right]$ **4)** $KFe\left[Fe(CN)_6\right]$

Key: 2

Sol: $FeCl_3 + K_4[Fe(CN)_6] \rightarrow Fe_4[Fe(CN)_6]_3$ Prussian blue

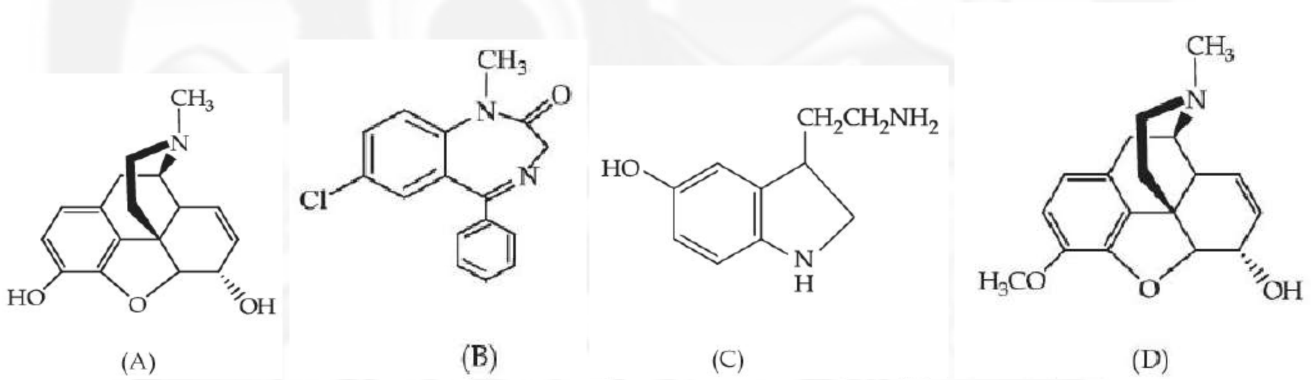
47. The unit of the van der Waals gas equation parameter 'a' in $\left(P+\frac{an^2}{V^2}\right)(V-nb)=nRT$ is:

1) $kg\ ms^{-2}$ **2)** $kg\ ms^{-1}$ **3)** $dm^3\ mol^{-1}$ **4)** $atm\ dm^6\ mol^{-2}$

Key: 4

Sol: $\frac{an^2}{v^2}$ and P must have same units as pressure so a has units at $Lt^2 / mole^2$

48.



The correct statement about (A),(B),(C) and (D) is :

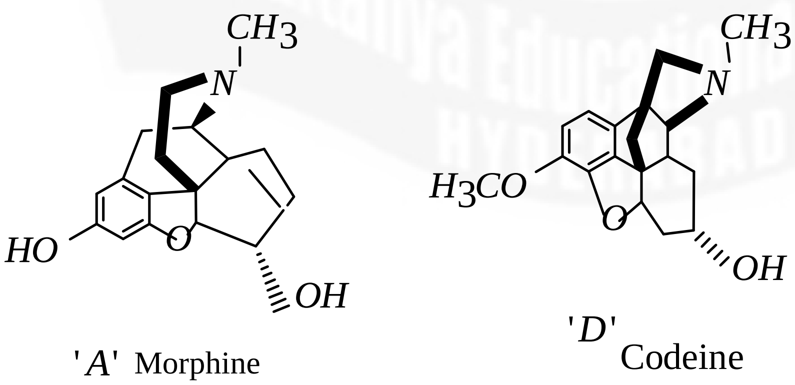
1) (B) and (C) are tranquilizers 2) (B),(C) and (D) are tranquillizers

3) (A),(B) and (C) are narcotic analgesics

4) (A) and (D) are tranquillizers

Key: 1

Sol:



- Narcotic drugs

B and C are tranquilisers NCERT

49. Which refining process is generally used in the purification of low melting metals?

1) Electrolysis **2)** Liquation

3) Chromatographic method **4)** Zone refining

Key: 2

Sol: metals which have low melting point can be purified by liquation technique

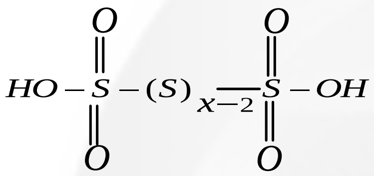
50. In polythionic acid, $H_2S_xO_6\left(x=3to5\right)$ the oxidation state(s) of sulphur is/are:

1) 0 and +5 only **2)** 0 and +5 only

3) +5 only **4)** +3 and +5 only

Key: 2

Sol:



Central sulphur atom has zero oxidation state, terminal sulphur has +5 oxidation state

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

---

51. The kinetic energy of an electron in the second Bohr orbit of a hydrogen atom is equal to $\frac{h^2}{xma_0^2}$. The value of 10 x is ________ (a_0 is radius of Bohr's orbit)

(Nearest integer) [Given : $\pi=3.14$]

Key: 3155

Sol: $KE=\frac{1}{2}mv^2$ But $mvr=\frac{nh}{2\pi}$

$$v=\frac{nh}{2\pi mr}$$

$$KE=\frac{1}{2}m.\frac{n^2h^2}{4\pi^2m^2r^2}=\frac{h^2}{8\pi^2mr_0^2.n^2}$$

So $x=8\pi^2n^2=8\times(2)^2\times(3.14)^2=315.50$

As we have to give $10x$

Ans 3155

52. The number of f electron in the ground state electronic configuration of Np (Z=93) is _________. (Integer answer)

Key: 4

Sol: $Np(93)=\{Rn\}7S^2 6d^1 5f^4$

No of e^- present in f orbital is 4

53. The number of moles of NH_3, that must be added to 2 L of 0.80 M $AgNO_3$ in order to reduce the concentration of Ag^+ ions to $5.0\times10^{-8}M$ ($K_{formation}$ for $\left[Ag(NH_3)_2\right]^+ = 1.0\times10^8$ is ________. (Nearest integer)

[Assume no volume change on adding NH_3]

Key: 4

Sol:

$Ag^+ + 2NH_3 \rightleftharpoons [Ag(NH_3)^2)^+$

0.8 a 0

5×10^{-8} a–1–6 0.8

So $1\times10^8 = \dfrac{0.8}{5\times10^{-8}(a-1.6)^2}$

$(a-1.6)^2 = \dfrac{0.8}{5} = 0.16$

$a-1.6 = 0.4 \Rightarrow a = 2$

But for 2lts, no of mole required = 4

54. The reaction that occurs in a breath analyser, a device used to determine the alcohol level in a person’s blood stream is,

$2K_2Cr_2O_7 + 8H_2SO_4 + 3C_2H_6O \to 2Cr_2(SO_4)_3 + 3C_2H_4O_2 + 2K_2SO_4 + 11H_2O$

If the rate of appearance of $Cr_2(SO_4)_3$ is _________ mol $\min^{-1}$. (Nearest integer)

Key: 4

Sol: Here the reaction

$\dfrac{-1}{3}\dfrac{d}{dt}(C_2H_6O) = \dfrac{1}{2}\dfrac{d}{dt}[Cr(SO_4)3]$

So $\dfrac{d}{dt}(C_2H_6O) = \dfrac{3}{2}\times2.67 = 4.005 = 4$

55. When 10 mL of an aqueous solution of $KMnO_4$ was titrated in acidic medium, equal volume of 0.1 M of an aqueous solution of ferrous sulphate was required for complete discharge of colour. The strength of $KMnO_4$ in grams per litre is _________ $\times10^{-2}$. (Nearest integer) [Atomic mass of $K = 39$, Mn=55, O=16]

Key: 316

Sol: No of equivalent of $KMnO_4 =$ no.of equivalent of $FeSO_4$

So $10\times10^{-3}\times M\times5 = 10\times10^{-3}\times0.1\times1 \Rightarrow m = 0.02$

So $0.02 = \frac{wt}{158} \times \frac{1000}{1000}$

$wt = 3.16 gr = 316 \times 10^{-2} gr$

56. 200mL of 0.2M HCl is mixed with 300 mL of 0.1M NaOH. The molar heat of neutralization of this reaction is -57.1 kJ. The increase in temperature in °C of the system on mixing is $x \times 10^{-2}$. The value of x is _______. (Nearest integer)

[Given : Specific heat of water = 4.18 J $g^{-1}K^{-1}$ Density of water = 1.00 $g\,cm^{-3}$]

(Assume no volume change on mixing)

Key: 82

Sol: Here NaOH is limiting reagent and 0.03 mole of NaOH was naturalized So heat released = $0.03 \times 57.1 kJ$ = 1.713

$2\theta = ms\Delta T$

$1.713 = 500 \times 4.18 \times \Delta T$

$\Delta T = 0.8196 \cong 82 \times 10^{-2}$

57. 1 mol of an octahedral metal complex with formula $MCl_3.2L$ on reaction with excess of $AgNO_3$ gives 1 mol of AgCl. The dentisity of Ligand L is ______. (Integer answer)

Key: 2

Sol: Octahedral complex has 6 lone pair donax

$\because$ One mole complex has 2 mole of Cl^- ions inside the co-ordination sphere and one Cl^- ion is present out side the complex

$ML_2Cl_3 \Rightarrow [ML_2Cl_2]Cl$

Hence density is 2

58. In Carius method for estimation of halogens, 0.2g of an organic compound gave 0.188 g of AgBr. The percentage of bromine in the compound is ______. (Nearest integer)

[Atomic mass: Ag=108, Br=80]

Key: 40

Sol: $\%Br = \frac{80}{188} \times \frac{wt.of\ AgBr}{wt\,of\ O.C} \times 100$

$\frac{80}{188} \times \frac{0.188}{0.2} \times 100 = 40\%$

59. The number of moles of CuO, that will be utilized in Dumas method for estimating nitrogen in a sample of 57.5 g of N,N-dimethylaminopentane is _________ $\times 10^{-2}$. (Nearest integer)

Key:

Sol: $C_xH_yN_z + (2x+\frac{y}{2})CuO \rightarrow XCO_2 + \frac{y}{2}H_2O + \frac{z}{2}N_2 + (2x+\frac{y}{2})Cu$

$C_7H_{17}N \rightarrow CuO$

1 mole 22.5 mole

115 gm 22.5 mole

57.5 gm ?

11.25 mole CuO

$1125\times10^{-2} mole$

60. 1 kg of 0.75 molal aqueous solution of sucrose can be cooled up to -4°C before freezing. The amount of ice (in g) that will be separated out is ________. (Nearest integer) [Given: $K_f(H_2O) = 1.86\,K\,kg\,mol^{-1}$]

Key: 518

Sol: $(Tf^0 - T) = \frac{wt}{mwt} \times \frac{1000}{(w - w_{ice})} \times kf$

$4 = \frac{204.14}{342} \times \frac{1000}{795.86 - w_{ice}} \times 1.86$

$w_{ice} = 518.301 \cong 518$

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases. 1.

61. Let us consider a curve, $y = f(x)$ passing through the point $(-2,2)$ and the slope of the tangent to the curve at any point $(x, f(x))$ is given by $f(x) + xf'(x) = x^2$. Then :

1) $x^3 + x\,f(x) + 12 = 0$ **2)** $x^2 + 2x\,f(x) + 4 = 0$

3) $x^3 - 3x\,f(x) - 4 = 0$ **4)** $x^2 + 2x\,f(x) - 12 = 0$

Key: 3

Sol: $y + x\dfrac{dy}{dx} = x^2$

$\Rightarrow ydx + xdy = x^2 dx$

$\Rightarrow d(xy) = x^2 dx \Rightarrow xy = \dfrac{x^3}{3} + C$

The curve ranges through (-2, 2)

$\Rightarrow -4 = \dfrac{-8}{3} + C$

$\Rightarrow C = \dfrac{-4}{3}$

$\therefore$ The curve $xy = \dfrac{x^3}{3} - \dfrac{4}{3} = x^3 - 3xy - 4 = 0$

62. Equation of a plane at a distance $\sqrt{\dfrac{2}{21}}$ from the origin, which contains the line of intersection of the planes $x - y - z - 1 = 0$ and $2x + y - 3z + 4 = 0$, is

1) $3x - 4z + 3 = 0$ **2)** $4x - y - 5z + 2 = 0$

3) $-x + 2y + 2z - 3 = 0$ **4)** $3x - y - 5z + 2 = 0$

Key: 2

Sol: The required plane is $(x - y - z - 1) + \lambda(2x + y - 3z + 4) = 0$

$$\frac{|-1+4\lambda|}{\sqrt{(1+2\lambda^2)+(\lambda-1)^2+(-1-3\lambda)^2}} = \sqrt{\frac{2}{21}}$$

$21(4\lambda-1)^2 = 2[(2\lambda+1)^2 + (\lambda-1)^2 + (3\lambda+1)^2]$

$= 21[16\lambda^2 - 8\lambda + 1] = 2[4\lambda^2 + 4\lambda + 1 + \lambda^2 - 2\lambda + 1 + 9\lambda^2 + 6\lambda + 1]$

$= 21(16\lambda^2 - 8\lambda + 1) = 2[14\lambda^2 + 18\lambda + 3]$

$= 336\lambda^2 - 168\lambda + 21 = 28\lambda^2 + 16\lambda + 6$

$$= 308\lambda^2 - 184\lambda + 15 = 0$$
$$\lambda = 1/2$$
$$\therefore (x - y - z - 1) + \frac{1}{2}(2x + y - 3z + 4) = 0$$

Hence the equation of plane is $4x - y - 5z + 2 = 0$

63. $\sum_{k=0}^{20} \left({}^{20}C_k \right)^2$ is equal to :

1) ${}^{40}C_{20}$ **2)** ${}^{40}C_{21}$ **3)** ${}^{40}C_{19}$ **4)** ${}^{41}C_{20}$

Key: 1

Sol: We know that $\left({}^{n}C_0\right)^2 + \left({}^{n}C_1\right)^2 + ... + \left({}^{n}C_n\right)^2 = \left({}^{2n}C_n\right)$

$$\sum_{k=0}^{20} \left({}^{20}C_k\right)^2 = \left({}^{20}C_0\right)^2 + \left({}^{20}C_1\right)^2 + ... + \left({}^{20}C_{20}\right)^2 = {}^{2n}C_n = {}^{40}C_{20}$$

64. If α, β are the distinct roots of $x^2 + bx + c = 0$, then

$$\lim_{x \to \beta} \frac{e^{2(x^2+bx+c)} - 1 - 2(x^2 + bx + c)}{(x-\beta)^2}$$ is equal to

1) $b^2 - 4c$ **2)** $2\left(b^2 + 4c\right)$ **3)** $2\left(b^2 - 4c\right)$ **4)** $b^2 + 4c$

Key: 3

Sol: $$\lim_{x \to \beta} \frac{e^{2(x^2+bx+c)} - 1 - 2(x^2 + bx + c)}{(x-\beta)^2}$$

$$= \lim_{x \to \beta} \frac{e^{2(x^2+bx+c)} - 1 - 2(x^2 + \beta x + c)}{[(x-\alpha)(x-\beta)]^2}(x-\alpha)^2$$

$$= \lim_{t \to 0} \frac{e^{2t} - 1 - 2t}{t^2} \times \lim_{x \to \beta} (x-\alpha)^2$$

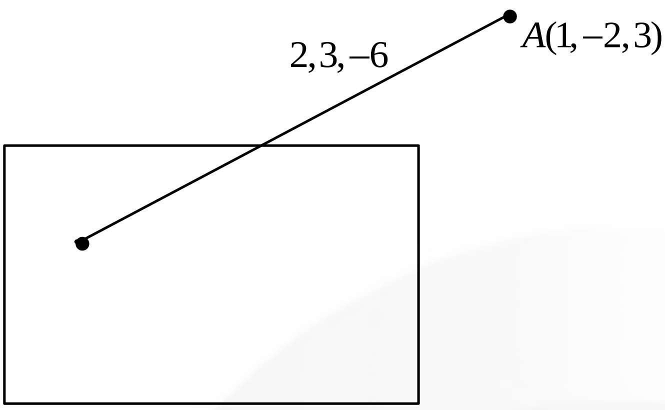
$$= 2(\beta - \alpha)^2 = 2[(\alpha+\beta)^2 - 4\alpha\beta] = 2(b^2 - 4c)$$

65. The distance of the point (1, -2, 3) from the plane x – y + z = 5 measured parallel to a line, whose direction rations are 2, 3, □6 is:

1) 3 **2)** 2 **3)** 5 **4)** 1

Key: 4

Sol:



$$x-y+z=5$$

$$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6}=t$$

$$<2t+1,3t-2,-6t+3>$$

$$2t+1-3t+2-6t+3-5=0$$

$$=-7t+1=0\rightarrow t=1/7$$

$$\left\langle\frac{9}{7},\frac{-11}{7},\frac{15}{7}\right\rangle(1,-2,3)$$

$$\left(\frac{2}{7}\right)^2+\left(\frac{3}{7}\right)^2+\left(\frac{6}{7}\right)^2=\frac{49}{49}=1$$

66. Let A be a fixed point (0, 6) and B be a moving point (2t, 0). Let M be the mid-point of AB and the perpendicular bisector of AB meets the y – axis at C. The locus of the mid-point P of MC is

1) $3x^2+2y-6=0$ **2)** $3x^2-2y-6=0$

3) $2x^2+3y-9=0$ **4)** $2x^2-3y+9=0$

Key: 3

Sol: $A(0,6)\ B(2t,0)\ M(t,3)$

Slope of $AB=\frac{6}{-2t}=\frac{-3}{t}\Rightarrow$ slope perpendicular bisector = $t/3$

Mod point of $MC=P\left(\frac{t}{2},\frac{6-t^2}{2}\right)=\left(\frac{t}{2},3-\frac{t^2}{6}\right)$

$t=2x,$ $6(y-3)=-t^2\Rightarrow 6(y-3)=-4x^2$

$$\Rightarrow 3(y-3)=-2x^2$$

$$\Rightarrow 2x^2+3y-9=0$$

67. When a certain biased die is rolled, a particular face occurs with probability $\frac{1}{6}-x$ and is opposite face occurs with probability $\frac{1}{6}+x$. All other faces occur with probability $\frac{1}{6}$. Note that opposite faces sum to 7 in any die. If $0<x<\frac{1}{6}$, and the probability of obtaining total sum = 7, when such a die is rolled twice, is $\frac{13}{96}$, then the value of x is :

1) $\frac{1}{8}$ **2)** $\frac{1}{12}$ **3)** $\frac{1}{9}$ **4)** $\frac{1}{16}$

Key: 1

Sol: The required probability is $2\left(\frac{1}{6}+x\right)\left(\frac{1}{6}-x\right)+\frac{4}{36}=\frac{13}{96}$

$$2\left(\frac{1}{36}-x^2\right)=\frac{13}{96}-\frac{4}{36}=\frac{1}{6}\left[\frac{13}{16}-\frac{4}{6}\right]=\frac{1}{12}\left[\frac{13}{8}-\frac{4}{3}\right]$$

$$2\left(\frac{1}{36}-x^2\right)=\frac{1}{12}\left(\frac{39-32}{24}\right)=\frac{1}{12}.\frac{7}{24}$$

$$=\frac{1}{18}-\frac{7}{12.24}=2x^2=\frac{1}{6}\left[\frac{1}{3}-\frac{7}{48}\right]=2x^2$$

$$=\frac{1}{18}\left[1-\frac{7}{16}\right]=2x^2$$

$$=\frac{1}{18}\left(\frac{9}{16}\right)=2x^2\Rightarrow\frac{1}{32}=2x^2\Rightarrow x^2=\frac{1}{64}$$

$$\Rightarrow x=\frac{1}{8}$$

68. **If** $x^2+9y^2-4x+3=0,\ x,y\in\mathbb{R}$, then x any y respectively lie in the intervals :

1) $[1,3]$ and $[1,3]$ **2)** $\left[-\frac{1}{3},\frac{1}{3}\right]$ and $\left[-\frac{1}{3},\frac{1}{3}\right]$

3) $\left[-\frac{1}{3},\frac{1}{3}\right]$ and $[1,3]$ **4)** $[1,3]$ and $\left[-\frac{1}{3},\frac{1}{3}\right]$

Key: 4

Sol: Treating quadratic in x

$$x^2-4x+3(3y^2+1)=0$$

$$\Delta\geq 0\ 16-4.3(3y^2+1)\geq 0$$

$$4-9y^2-3\geq 0$$

$$1-9y^2\geq 0\Rightarrow 9y^2-1\leq 0\Rightarrow y\in\left[\frac{-1}{3},\frac{1}{3}\right]$$

Treating quadratic in Y, $9y^2 = -(x^2 - 4x + 3) \geq 0$

$\Rightarrow (x^2 - 4x + 3) \leq 0 \Rightarrow x \in [1,3]$

69. If for $x, y \in R,\ x > 0.\ y = \log_{10} x + \log_{10} x^{1/3} + \log_{10x} 1/9 + ...\text{upto } \infty$ terms and $\frac{2+4+6+....+2y}{3+6+9+....+3y} = \frac{4}{\log_{10} x}$, then the ordered pair (x, y) is equal to :

1) $\left(10^6, 6\right)$ **2)** $\left(10^4, 6\right)$ **3)** $\left(10^6, 9\right)$ **4)** $\left(10^3, 3\right)$

Key: 3

Sol: $\left[1 + \frac{1}{3} + \frac{1}{9} + ... + \infty\right] \log_{10} x$

$$\log_{10} x \left[\frac{1}{1 - \frac{1}{3}}\right] = \frac{3}{2} \log_{10} x = y$$

$$\frac{2[1+2+...+y]}{3[1+2+...+y]} \Rightarrow \frac{2}{3} = \frac{4}{\log_{10} x} = \log_{10} x = 6$$

$x = 10^6$
$y = 9$

70. $\int_6^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e \left(x^2 - 44x + 484\right)} dx$ is equal to:

1) 6 **2)** 10 **3)** 5 **4)** 8

Key: 3

Sol: $\int_6^{16} \frac{\log x^2}{\log x^2 + \log_e (x-22)^2} dx$

$$2I = \int_6^{16} 1 dx \Rightarrow I = \frac{10}{2} = 5$$

71. If the matrix $A = \begin{pmatrix} 0 & 2 \\ K & -1 \end{pmatrix}$ satisfies $A\left(A^3 + 3I\right) = 2I$, then the value of K is :

1) 1 **2)** $\frac{1}{2}$ **3)** -1 **4)** $-\frac{1}{2}$

Key: 2

Sol: $A = \begin{pmatrix} 0 & 2 \\ k & -1 \end{pmatrix} A^2 = \begin{pmatrix} 2k & -2 \\ -k & 2k+1 \end{pmatrix}$

$$A^4 = \begin{pmatrix} 4k^2+2k & -8k-2 \\ -4k^2-k & 4k^2+6x+1 \end{pmatrix}$$

$$A^4+3A = \begin{pmatrix} 4k^2+2k & -8k+4 \\ -4k^2-2k & 4k^2+6x-2 \end{pmatrix} = 2I \Rightarrow k = \frac{1}{2}$$

72. If $U_n = \left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)^2 \ldots\left(1+\frac{n^2}{n^2}\right)^n$, then $\lim_{n\to\infty}(U_n)^{\frac{-4}{n^2}}$ is equal to :

1) $\frac{16}{e^2}$ **2)** $\frac{e^2}{16}$ **3)** $\frac{4}{e^2}$ **4)** $\frac{4}{e}$

Key: 2

Sol: $-\frac{4}{n^2}\log U_n = \frac{-4}{n^2}\left[\ln\left(1+\frac{1}{n^2}\right)+2\ln\left(1+\frac{2^2}{n^2}\right)+\ldots+n\ln\left(1+\frac{n}{n^2}\right)\right]$

$= -\frac{4}{n}\left[\sum\frac{r}{n}\ln\left(1+\frac{r^2}{n^2}\right)\right]$

$= -\frac{4}{n}\left[\int_1^1 x\ln(1+x^2)dx\right]$

$= -4\{\frac{1}{2}.\int_1^1 \ln t\,dt - 2\left[t\ln t - t\right]_1^2$

$= -2\left[(2\ln 2-2)-(1\ln 1-1\right]$

$= -2\left[\ln 4-2+1\right]$

$= -2\left[\ln 4-1\right]$

$= 2-\ln 16 = \ln_e x^2 - \ln_e 16 = \ln\left(\frac{e^2}{16}\right)$

$\therefore L = \frac{e^2}{16}$

73. Let $\frac{\sin A}{\sin B} = \frac{\sin(A-C)}{\sin(C-B)}$, where, A, B, C are angles of a triangle ABC. If the lengths of the sides opposite these angles are a, b, c respectively, then :

1) a^2, b^2, c^2 are in A.P. **2)** b^2, c^2, a^2 are in A.P.

3) $b^2-a^2 = a^2+c^2$ **4)** c^2, a^2, b^2 are in A.P

Key: 2

Sol: $\sin(B+C)\sin(B-C)=\sin(C+A)\sin(C-A)$

$\sin^2 B-\sin^2 C=\sin^2 C-\sin^2 A$

$b^2-c^2=c^2-a^2$

$b^2+a^2=2c^2$

$\Rightarrow a^2, c^2, b^2$ AP

74. Let $y=y(x)$ be the solution of the differential equation $\frac{dy}{dx}=2\left(y+2\sin x-5\right)x-2\cos x$ such that $y\left(0\right)=7$. Then $y\left(\pi\right)$ is equal to :

1) $e^{\pi^2}+5$ **2)** $3e^{\pi^2}+5$ **3)** $2e^{\pi^2}+5$ **4)** $7e^{\pi^2}+5$

Key: 3

Sol: $\frac{dy}{dx}-2\cos x=2(y+2\sin x-5)x$

$(dy+2\cos x dx)=(2xdx)(y+2\sin x-5)$

$\int\frac{dy+2\cos x dx}{(y+2\sin x-5)}=2xdx$

$=\ln(y+2\sin x-5)=x^2+C$

$y(0)=7\Rightarrow\ln(7+0-5)=0+C\Rightarrow C=\ln 2$

$\ln(y+2\sin x-5)=x^2+\ln 2$

75. If $\left(\sin^{-1}x\right)^2-\left(\cos^{-1}x\right)^2=a;\ 0<x<1,\ a\neq 0$, then the value of $2x^2-1$ is :

1) $\cos\left(\frac{2a}{\pi}\right)$ **2)** $\cos\left(\frac{4a}{\pi}\right)$ **3)** $\sin\left(\frac{2a}{\pi}\right)$ **4)** $\sin\left(\frac{4a}{\pi}\right)$

Key: 3

Sol: $(Sin^{-1}x+Cos^{-1}x)(Sin^{-1}x-Cos^{-1}x)=a$

$Sin^{-1}x-Cos^{-1}x=\frac{2a}{\pi}$

$\frac{\pi}{2}-2\cos^{-1}x=\frac{2a}{\pi}$

$\left(\frac{\pi}{2}-\frac{2a}{\pi}\right)=2Cos^{-1}x$

$\left(\frac{\pi}{4}-\frac{a}{\pi}\right)=Cos^{-1}x$

$x=\cos\left(\frac{\pi}{4}-\frac{a}{\pi}\right)=\cos\theta$

$2x^2 - 1 = 2\cos^2\theta - 1$

$= \cos 2\theta$

$= \cos\left(\frac{\pi}{2} - \frac{2a}{\pi}\right)$

$= \sin\left(\frac{2a}{\pi}\right)$

76. If $S = \left\{ z \in C : \frac{z-i}{z+2i} \in \mathbb{R} \right\}$, then :

1) S contains only one element **2)** S contains exactly two elements

3) S is circle in the complex plane **4)** S is a straight line in the complex plane

Key: 4

Sol: $S = \{Z \in Z; \frac{Z-i}{Z+2i} \in R]$

$\frac{x+iy-i}{x+iy+2i} = \frac{x+i(y-1)}{x+i(y+2)} \times \frac{x-i(y+2)}{x-i(y+2)}$ is real

$x(y-1) - x(y+2) = 0$

$xy - x - xy - 2x = 0$

$x = 0 \Rightarrow$ S is a straight

77. A wire of length 20 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a regular hexagon. Then the length of the side (in meters) of the hexagon, so that the combined area of the square and the hexagon is minimum, is :

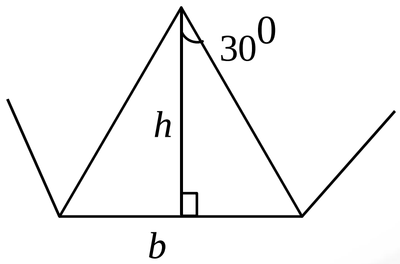
1) $\frac{10}{2+3\sqrt{3}}$ **2)** $\frac{10}{3+2\sqrt{3}}$ **3)** $\frac{5}{3+\sqrt{3}}$ **4)** $\frac{5}{2+\sqrt{3}}$

Key: 2

Sol: $4a + 6b = 20$

$s = a^2 + \frac{3\sqrt{3}}{2}b^2$

$= \left(\frac{20-64}{4}\right)^2 + \frac{3\sqrt{3}}{2}b^2$



$$\frac{2\pi}{6}=\frac{\pi}{3}$$

$$\tan 30=\frac{b/2}{4}$$

$$\frac{1}{\sqrt{3}}=\frac{b}{2h}$$

$$\sqrt{3}b=2h$$

$$=\left(5-\frac{3b}{2}\right)^2+\frac{3\sqrt{3}}{2}b^2 \qquad A=6\times\frac{1}{2}bh=3.b.\frac{\sqrt{3}}{2}b=\frac{3\sqrt{3}}{2}b^2$$

$$\frac{ds}{db}=0$$

$$2\left(5-\frac{3}{2}b\right)\left(\frac{-3}{2}\right)+\frac{3\sqrt{3}}{2}.2b=0$$

$$-15+\frac{9}{2}b+3\sqrt{3}b=0$$

$$15=\left(\frac{9}{2}+3\sqrt{3}\right)b=3\left(\frac{3}{2}+\sqrt{3}\right)b$$

$$\Rightarrow 5=\left(\frac{3+2\sqrt{3}}{2}\right)b\Rightarrow b=\frac{10}{3+2\sqrt{3}}$$

78. The statement $(p\wedge(p\to q)\wedge(q\to r))\to r$ is :

1) equivalent to $q\to\sim r$ **2)** equivalent to $p\to\sim r$

3) a fallacy **4)** a tautology

Key: 4

Sol:

P	**q**	**r**	$p\wedge(p\to q)\wedge(q\to r)\to r$
T	**T**	**T**	**T**
T	**F**	**F**	**T**
F	**T**	**F**	**T**
F	**F**	**T**	**T**
F	**F**	**T**	**T**
F	**T**	**F**	**T**
F	**F**	**T**	**T**
F	**F**	**F**	**T**

79. If $0<x<1$, then $\frac{3}{2}x^2\frac{5}{3}x^3+\frac{7}{4}x^4+...,$ is equal to :

1) $x\left(\frac{1+x}{1-x}\right)+\log_e(1-x)$ **2)** $\frac{1+x}{1-x}+\log_e(1-x)$

3) $x\left(\frac{1-x}{1+x}\right)+\log_e(1-x)$ **4)** $\frac{1-x}{1+x}+\log_e(1-x)$

Key: 1

Sol: $\frac{3}{2}x^2+\frac{5}{3}x^3+\frac{7}{4}x^4+....$

$$\log(1-x)=-(x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+.....)$$

$$=-x-\frac{x^2}{7}-\frac{x^3}{3}-\frac{x^4}{4}.....$$

$$\frac{(1+x)}{1-x}=(1+x)(1+x+x^2+x^3+x^4+....)$$

$$=1+2x+2x^2+2x^3+2x^4+.....$$

$$\frac{x(1+x)}{1-x}=x+2x^2+2x^3+2x^4+....$$

adding $\frac{x(1+x)}{1-x}+\log(-x)=\frac{3}{2}x^2+\frac{5}{3}x^3+...$

80. A tangent and a normal are drawn at the point $P(2,-4)$ on the parabola $y^2=8x$, which meet the directrix of the parabola at the points A and B respectively. If $Q(a,b)$ is a point such that AQBP is a square, then $2a+b$ is equal to

1) **–12** **2) –16** **3) – 18** **4) - 20**

Key: 2

Sol: $y^2=8x$ $4a=8$

$2y\frac{dy}{dx}=8$ $a=2$

$$\frac{dy}{dx}=\frac{4}{y}$$

$$m=\frac{4}{-4}=-1$$

1st : $y+4=-1(x-2)$

$=-x+2\Rightarrow x+y+2=0$

Equation of dir : $x=-2$

$y=0$

Normal

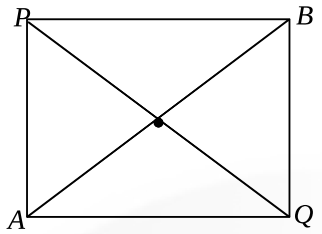
$y+4=1(x-2)=x-y-6=0$

$-2-y-6=0\Rightarrow -y=8$

$y=-8$

$A(-2,0)$ $\qquad\qquad$ $B=(-2,-8)$

$Q(a,b)$



$$\left(\frac{a+2}{2},\frac{b-4}{2}\right)=(-2,-4)$$

$$a+2=4 \quad b-4=-8$$

$$a=-6 \qquad b=-4$$

$$2a+b=-12-4=-16$$

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

81. If $\int \frac{dx}{\left(x^2+x+1\right)^2}=a\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+b\left(\frac{2x+1}{x^2+x+1}\right)+c,\ x>0$ where C is the constant of integration, then the value of $9\left(\sqrt{3}a+b\right)$ is equal to ______

Key: 15

Sol: $\int \frac{1}{(x^2+x+1)^2}$

$$\int \frac{1}{\left(\left(x^2+\frac{1}{2}\right)+\frac{3}{4}\right)^2} \qquad x+\frac{1}{2}=\frac{\sqrt{3}}{2}\tan\theta \qquad fn=\frac{\sqrt{3}}{2}\sec^2\theta\tan\theta$$

$$=\int \frac{1}{\frac{9}{16}\sec^4\theta}.\frac{\sqrt{3}}{2}\sec^2\theta d\theta \qquad \tan\theta=\frac{2x+1}{\sqrt{3}}$$

$$=\frac{8}{3\sqrt{3}}\int\cos^2\theta d\theta$$

$$=\frac{8}{3\sqrt{3}}\int\left(\frac{1+\cos 2\theta}{2}\right)d\theta$$

$$=\frac{4}{3\sqrt{3}}\left(\frac{\theta+\sin 2\theta}{2}\right)+C$$

$$=\frac{4}{3\sqrt{3}}Tan^{-1}(\frac{2x+1}{\sqrt{3}})+\frac{4}{3\sqrt{3}}\sin\theta\cos\theta+C$$

$$=\frac{4}{3\sqrt{3}}Tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{4}{3\sqrt{3}}\left(\frac{\sqrt{3}(2x+1)}{(2x+1)^2+3}\right)$$

$$=\frac{4}{3\sqrt{3}}Tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{4}{3}\frac{2x+1}{x^2+x+1}+C$$

$$a=\frac{4}{3\sqrt{3}},b=1/3$$

$$9\left(\sqrt{3}.\frac{4}{3\sqrt{3}}+\frac{1}{3}\right)=15$$

82. Let $\vec{a}=\hat{i}+5\hat{j}+a\hat{k}, \vec{b}=\hat{i}+3\hat{j}+\beta\hat{k}$ and $\vec{c}=-\hat{i}+2\hat{j}-3\hat{k}$ be three vectors such that, $\left|\vec{b}\times\vec{c}\right|=5\sqrt{3}$ and $\vec{a}$ is perpendicular to $\vec{b}$. Then the greatest amongst the values of $|\vec{a}|^2$ is ____

Key: 90

Sol: $\bar{a}-\bar{b}=0$

$1+15+\alpha\beta=0\Rightarrow\alpha\beta=-16$

$$\bar{b}\times\bar{c}=\begin{vmatrix} i & j & k \\ 1 & 3 & \beta \\ -1 & 2 & -3 \end{vmatrix}$$

$i(-9-2\beta)-j(-3+\beta)+5k$

$(9+2\beta)^2+(\beta-3)^2+25=75$

$(2\beta+9)^2+(\beta-3)^2=50$

$5\beta^2+30\beta+40=0$

$\beta^2+6\beta+8=0$

$(\beta+2)(\beta+4)=0$

$\beta=-2$ or -4

$\alpha=8$ or 4

$|a|=\sqrt{1+25+\alpha^2}=\sqrt{26+64}=\sqrt{90}$

$|\bar{a}|^2=90$

83. A number is called a palindrome if it reads the same backward as well as forward. For example 285582 is a six digit palindrome. The number of six digit palindromes, which are divisible by 55, is _______

Key: 100

Sol: $5\ a\ b\ b\ a\ 5$

$(5+b+a)-(a+b+5)=11k$

$a,b=10\times 10=100$

84. Let the equation $x^2+y^2+px+(1-p)y+5=0$ represent circles of varying radius $r\in(0,5]$. Then the number of elements in the set $S=\{q:q=p^2 \text{ and } q \text{ is an integer}\}$ is _______

Key: 7

Sol:

$x^2+y^2+Px+(1-P)y+5=0$

$\left(-\frac{P}{2},\frac{P-1}{2}\right)$

$r^2-\frac{P^2}{4}+\frac{(P-1)^2}{4}-5>0$

$P^2+P^2-2P+1>20$

$2P^2-2P-19>0$

$P=\frac{2\pm\sqrt{4+8.19}}{4}=\frac{1\pm\sqrt{39}}{2}$

$P<\frac{1-\sqrt{39}}{2},P>\frac{1+\sqrt{39}}{2}$

$P<-2.6$ or $P>3.6$

$\frac{P^2}{4}+\frac{(P-1)^2}{4}-5\le 25$

$\Rightarrow\frac{P^2+P^2-2P}{4}\le 30$

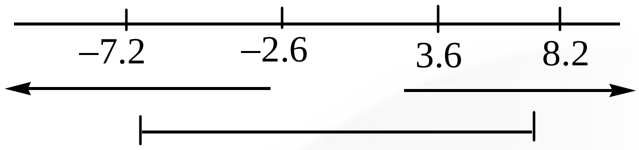
$2P^2-2P+1\le 120$

$2P^2-2P-119\le 0$

$P=\frac{2\pm\sqrt{4+4.2.119}}{4}=\frac{1\pm\sqrt{1+238}}{2}=\frac{1\pm\sqrt{1+239}}{2}$

$P \in \left[\frac{1-\sqrt{239}}{2}, \frac{1+\sqrt{239}}{2}\right]$

$P \in [-7.2, 8.2]$



$P \in [-7.2, -2.6] \cup [3.6, 8.2]$

$P \in [-7, 6, -5, -4, -3, -2, 4, 5, 6, 7, 8]$

$q = P^2 \in \{4, 9, 16, 25, 36, 49, 64\}$

85. The number of distinct real roots of the equation $3x^4 + 4x^3 - 12x^2 + 4 = 0$ is _____

Key: 4

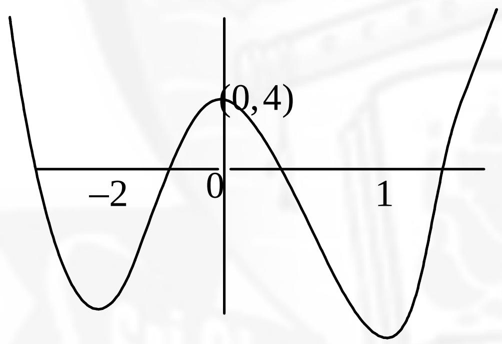
Sol: $3x^4 + 4x^3 - 12x^2 + 4 = 0$

$f'(x) = 12x^3 + 12x^2 - 24x$

$= 12x(x^2 + x - 2)$

$= 12x(x+2)(x-1)$

$x = 0, -2, 1$



$f(-2) = 48 - 32 - 48 + 4$

$f(1) = 3 + 4 - 12 + 4 = -1$

No of real roots = 4

86. If $A = \{x \in R : |x-2| > 1\}$, $B = \left\{x \in R : \sqrt{x^2 - 3} > 1\right\}$, $C = \{x \in R : |x-4| \ge 2\}$ and Z is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^e \cap Z$ is ____

Key: 2^8

Sol: $A=\{x\in R:|x-2|>1\}, B=\{x\in R:\sqrt{x^2-3}>1\}$

$|x-2|>1$

$x-2<-1 or\, x-2>1$

$x<1 of\; x>3$

$x^2-3>1$

$\Rightarrow x^2>4$

$\Rightarrow (x+2)(x-2)>0$

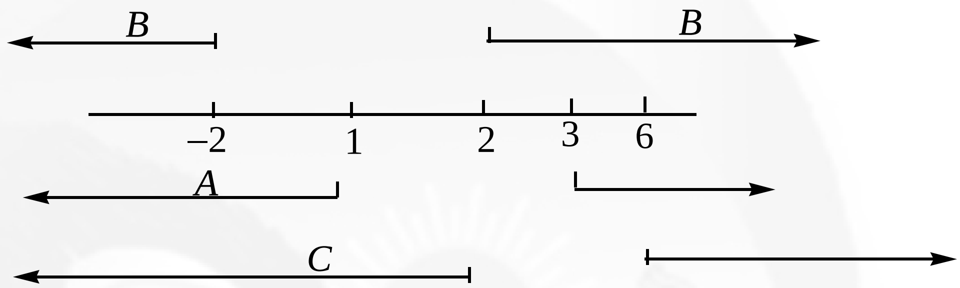
$x<-7\, or\, x>2$

$|x-4|\geq 2$

$x-4\leq -2$ or $x-4\geq 2$

$x\leq 2$ or $x\geq 6$

$A\cap B\cap C$



$=(-\infty,-2)\cup[6,\infty)$

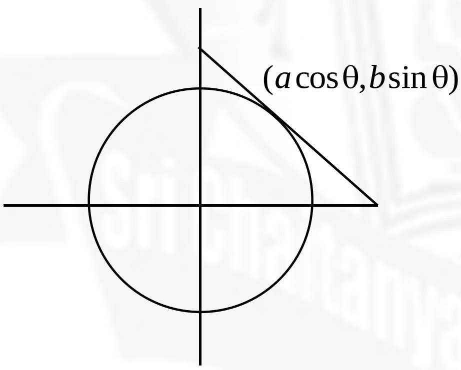
$(A\cap B\cap C)^c=[-2,6)$

$(A\cap B\cap C)^c nZ=\{-2,-1,0,1,2,3,4,5\}$

87. If the minimum area of the triangle formed by a tangent to the ellipse $\frac{x^2}{b^2}+\frac{y^2}{4a^2}=1$ and the co-ordinate axis is kab, then k is equal to

Key: 1

Sol:



$$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$$

$$\frac{x}{a/\cos\theta}+\frac{y}{b/\sin\theta}=1$$

$$A=\frac{1}{2}\frac{ab}{\sin\theta\cos\theta}=\frac{ab}{\sin 2\theta}$$

$A_{\min} = ab$

$k = 1$

88. If the system of linear equations

$2x + y - z = 3$

$x - y - z = \alpha$

$3x + 3y + \beta z = 3$

Has infinitely many solution, then $\alpha + \beta - \alpha\beta$ is equal to ______

Key: 5

Sol:

$$\begin{bmatrix} 2 & 1 & -1 & 3 \\ 1 & -1 & -1 & \alpha \\ 3 & 3 & \beta & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & -1 & \alpha \\ 2 & 1 & -1 & 3 \\ 3 & 3 & \beta & 3 \end{bmatrix} \quad R_2 \to R_2 - 2R_1,\ R_3 \to R - 3R_1$$

$$\sim \begin{bmatrix} 1 & -1 & -1 & \alpha \\ 0 & 3 & 1 & 3-2\alpha \\ 0 & 6 & \beta+3 & 3-3\alpha \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\sim \begin{bmatrix} 1 & -1 & -1 & \alpha \\ 0 & 3 & 1 & 3-2\alpha \\ 0 & 0 & \beta+1 & \alpha-3 \end{bmatrix}$$

$\beta = -1, \alpha = 3$

$\alpha + \beta - \alpha\beta = 3 - 1 + 3 = 6 - 1 = 5$

89. Let n be an odd natural number such that the variance of 1, 2, 3, 4, …, n is 14. Then n is equal to

Key: 13

Sol: $\sigma^2 = \dfrac{\sum n_i^2}{n} - (\bar{n})^2$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4} = 14$$

$n^2 - 1 = 14 \times 12$

$n^2 = 169$

$n = 13$

90. **If** $y^{1/4}+y^{-1/4}=2x$ and $\left(x^2-1\right)\frac{d^2y}{dx^2}+\beta y=0$, then $|\alpha-\beta|$ is equal to ___

Key: 17

Sol: $y^{1/4}+y^{-1/4}=2x$

$$\left(y^{1/4}-y^{-1/4}\right)^2=4x^2-4$$

$$y^{1/4}-y^{-1/4}=2\sqrt{x^2-1}$$

$$2y^{1/4}=2x+2\sqrt{x^2-1}$$

$$y^{1/4}=x+\sqrt{x^2-1}$$

$$y=(x+\sqrt{x^2-1})^4$$

$$y_1=4(x+\sqrt{x^2-1})^3(1+\frac{x}{\sqrt{x^2-1}})$$

$$=4(x+\sqrt{x^2-1})^4$$

$$y_1\sqrt{x^2-1}=4y$$

$$y_1^2(x^2-1)=16y^2$$

$$y_1^2(2x)=(x^2-1)2y_1.y_2=32y.y_1$$

$$xy_1+(x^2-1)y_2=16y$$

$$(x^2-1)y_2+xy_1-16y=0$$

$$\alpha=1,\beta=-16$$

$$|\alpha-\beta|=17$$

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