36 Years of Excellence 1986-2021

Sri Chaitanya

JEE MAIN 2021

PHASE - IV

Key & Solutions

27-Aug-2021 | Shift - 2

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A right Choice for the Real Aspirant

ICON Central Office – Madhapur – Hyderabad

Jee-Main_Final_27-Aug-2021_Shift-02

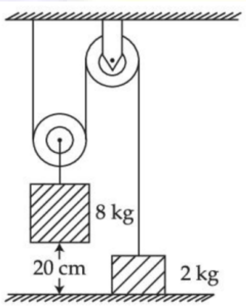
PHYSICS **Max Marks: 100**

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

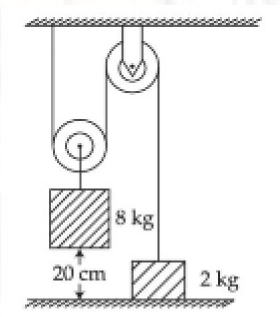
1. The boxes of masses 2 kg and 8 kg are connected by a massless string passing over smooth pulleys. Calculate the time taken by box of mass 8 kg to strike the ground starting from rest. (use $g = m/s^2$):



1) 0.34s 2) 0.2s 3) 0.25s 4) 0.4 s

Key: 4

Solution:



Force equations of each block are

8g – 2T = 8a

T – 2g = 2(2a)

$4g = 16a \Rightarrow a = \frac{5}{2}\,m/s^2$

Thus

$s = ut + \frac{1}{2}at^2$

$$20\times10^{-2} = 0 + \frac{1}{2}\times\frac{5}{2}\times t^2$$

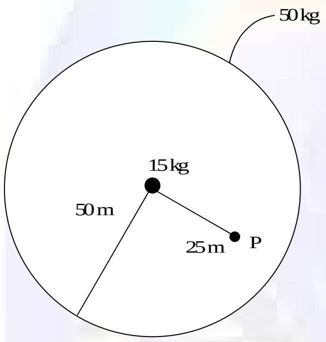
t = 0.4s.

2. A mass of 50 kg is placed at the centre of a uniform spherical shell of mass 100 kg and radius 50 m. If the gravitational potential at a point, 25 m from the centre is V kg/m. The value of V is:

1) +2 G 2) -20 G 3) -60 G 4) -4 G

Key: 4

Solution:



$$V_P = -\frac{G(50)}{25} - \frac{G(100)}{50}$$

$$= -2G - 2G = -4G$$

3. If force (F), length (L) and time (T) are taken as the fundamental quantities. The what will be the dimension of density:

1) $\left[FL^{-4}T^2\right]$ **2)** $\left[FL^{-3}T^2\right]$ **3)** $\left[FL^{-3}T^3\right]$ **4)** $\left[FL^{-5}T^2\right]$

Key: 1

Solution:

$$d \propto F^a L^b T^c$$

$$\left[M\ L^{-3}\right] = \left[M\ LT^{-2}\right]^a \left[L^b\right]\left[T^c\right]$$

$$= \left[M^a L^{a+b} T^{-2a+c}\right]$$

Comparing:

a = 1 …..(i)

a + b = –3 …..(ii)

–2a + c =0 …..(iii)

c = 2 ; b = –4 ; $[d] = [F\ L^{-4}T^2]$

4. An antenna is mounted on a 400 m tall building. What will be the wavelength of signal that can be radiated effectively by the transmission tower up to a range of 44 km?

1) 37.8m 2) 75.6 m 3) 605 m 4) 302 m

Key: 3

Solution:

$$h=\frac{d^2}{2R}$$

$$h=\frac{\lambda}{4}$$

h – height of antenna

λ – wave length of signal

r – radius of earth

d – transmission range

$$\lambda=\frac{d^2}{R}=\frac{2\times44\times10^3\times44\times10^3}{6400\times10^3}$$

$=605\,\text{m}$

5. Curved surfaces of a plano-convex lens of refractive index μ_1 and a plano-concave lens of refractive index μ_2 have equal radius of curvature as shown in figure. Find the ratio of radius of curvature to the focal length of the combined lenses.

1) $\mu_2-\mu_1$ **2)** $\mu_1-\mu_2$ **3)** $\frac{1}{\mu_2-\mu_1}$ **4)** $\frac{1}{\mu_1-\mu_2}$

Key: 2

Solution:

$$\frac{1}{f_1}=(\mu_1-1)\left[\frac{1}{R}\right]$$

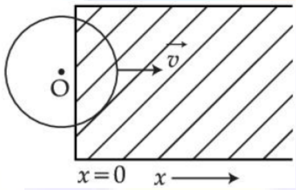
$$\frac{1}{f_2}=(\mu_2-1)\left[\frac{1}{-R}\right]$$

$$\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{(\mu_1-1)}{R}-\frac{(\mu_2-1)}{R}$$

$$\frac{1}{f_{eq}}=\frac{(\mu_1-\mu_2)}{R}$$

$$\frac{R}{f_{eq}}=\mu_1-\mu_2$$

6. A constant magnetic field of 1 T is applied in the $x>0$ region. A metallic circular ring of radius 1 m is moving with a constant velocity of 1m/s along the *x*-axis. At $t=0s$, the centre O of the ring is at $x=-1\text{m}$. What will be the value of the induced emf in the ring at $t=1s$? (Assume the velocity of the ring does not change.)



1) 2π V **2)** 1 V **3)** 2 V **4)** 0 V

Key: 3

Solution:

$\xi = \text{B l v}$

Here at t = 1s

l = 2R = 2m

B = 1T

V = 1 m/s

The induced emf = (2) (1) (1) = 2V

7. Two discs have moments of inertia I_1 and I_2 about their respective axes perpendicular to the plane and passing through the centre. They are rotating with angular speeds, ω_1 and ω_2 respectively and are brought into contact face to face with their axes of rotation coaxial. The loss in kinetic energy of the system in the process is given by:

1) $\frac{I_1I_2}{(I_1+I_2)}(\omega_1-\omega_2)^2$ **2)** $\frac{I_1I_2}{2(I_1+I_2)}(\omega_1-\omega_2)^2$

3) $\frac{(I_1-I_2)^2\omega_1\omega_2}{2(I_1+I_2)}$ **4)** $\frac{(\omega_1-\omega_2)^2}{2(I_1+I_2)}$

Key: 2

Solution:

Angular momentum conservation

$$I_{1\omega 1} - I_{2\omega 2} = (I_1 + I_2)\omega$$

$$\omega = \frac{I_{1\omega 1} + I_{2\omega 2}}{I_1 + I_2}$$

Loss = $\frac{1}{2}I_{1\omega 1^2} + \frac{1}{2}I_{2\omega 2^2} - \frac{1}{2}(I_1 + I_2)\omega^2$

$$\frac{1}{2}I_{1\omega 1^2} + \frac{1}{2}I_{2\omega 2^2} - \frac{1}{2}(I_1 + I_2)\left(\frac{I_{1\omega 1} - I_{2\omega 2}}{I_1 + I_2}\right)^2$$

$$= \frac{1}{2}\frac{I_1 I_2}{(I_1 + I_2)}(\omega_1 - \omega_2)^2$$

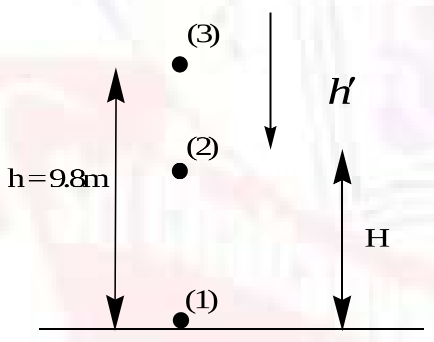
$$E_i - E_f = \frac{I_1 I_2 (\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$$

8. Water drops are falling from a nozzle of a shower onto the floor, from a height of 9.8 m. The drops fall at a regular interval of time. When the first drop strikes the floor, at that instant, the third drop begins to fall. Locate the position of second drop from the floor when the first drop strikes the floor.

1) 4.18 m **2)** 2.45 m **3)** 7.35 m **4)** 2.94 m

Key: 3

Solution:



For first drop.

$$h = \frac{1}{2}g(2n)^2$$

For 2nd drop

$$h' = \frac{1}{2}g(n)^2$$

$$\frac{h}{h'} = \frac{4}{1}$$

$$h' = \frac{h}{4} = \frac{9.8}{4}$$

So height of 2nd drop

$$H = h - h' = 9.8 - \frac{9.8}{4} = \frac{3}{4} \times 9.8 = 7.35\,\text{m}.$$

9. If the rms speed of oxygen molecules at 0^{o}C is $160\ \text{m/s}$, find the rms speed of hydrogen molecule at 0^{o}C.

1) $80\,\text{m/s}$ **2)** $332\,\text{m/s}$ **3)** $40\,\text{m/s}$ **4)** $640\,\text{m/s}$

Key: 4

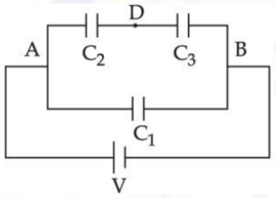
Solution:

$$V_{rms} \propto \frac{1}{\sqrt{M}}$$

$$\frac{(V_{rms})O_2}{(V_{rms})H_2} = \sqrt{\frac{m_{H_2}}{m_{O_2}}} = \sqrt{\frac{2}{32}} = \frac{1}{4}$$

$$V_2 = 4V_1 = 4(160) = 640\,\text{m/s}$$

10. Three capacitors $C_1 = 2\mu F, C_2 = 6\mu F$ and $C_3 = 12\mu F$ are connected as shown in figure. Find the ratio of the charges on capacitors C_1, C_2 and C_3 respectively:



1) $1:2:2$ **2)** $3:4:4$ **3)** $2:3:3$ **4)** $2:1:1$

Key: 1

Solution:

$$Ceq = C_1 + \frac{C_2 C_3}{C_2 C_3}$$

$$Ceq = 2 + \frac{6 \times 12}{6 + 12} = 6\text{F}$$

then

$$\frac{Q_{C_1}}{Q_{C_2}} = \frac{C_1 V}{\frac{C_2 C_3}{C_2 + C_3} V} = \frac{2 \times (6+12)}{6 \times 12} = \frac{1}{2}$$

And charge on C_2 = charge on C_3

$$\therefore Q_{C_1} : Q_{C_2} : Q_{C_3} = 1:2:2$$

11. A monochromatic neon lamp with wavelength of 670.5 nm illuminates a photo-senstitive material which has a stopping voltage of 0.48 V. What will be the stopping voltage if the source light is changed with another source of wavelength of 474.6 nm?

1) 1.5 V **2)** 1.25 V **3)** 0.24 V **4)** 0.96 V

Key: 2

Solution:

Equation of photo electric effect

$E = W_o + eV_o$

$E_1 = W_o + eV_{o1}$ … (1)

$E_2 = W_o + eV_{o2}$ ….(2)

$(2) - (1) \Rightarrow E_2 - E_1 = e\,(V_{o2} - V_{o1})$

$V_{o2} = \dfrac{E2 - E1}{e} + V_{o1}$ ….(3)

$E_1 = \dfrac{hc}{\lambda} = \dfrac{12400}{4746} eV$

$V_{o1} = 0.48V$

Substituting these values in (3) we get $V_{o2} = 1.25$ V

12. A player kicks a football with an initial speed of $25ms^{-1}$ at an angle of 45^o from the ground. What are the maximum height and the time taken by the football to reach at the highest point during motion? (Take $g = 10ms^{-2}$)

1) $h_{max} = 10$m T=2.5 s **2)** $h_{max} = 3.54$m T=0.125 s

3) $h_{max} = 15.625$m T=3.54 s **4)** $h_{max} = 15.625$m T=1.77 s

Key: 4

Solution:

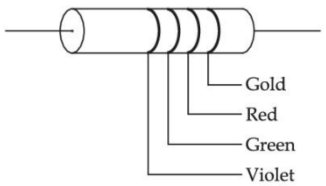
$\theta = 45^\circ$

$H = \dfrac{u^2 \sin^2\theta}{2g} = \dfrac{(25)^2 \times \left(\dfrac{1}{2}\right)}{2\times 10} = \dfrac{125}{8} m$

and time $t = \dfrac{T}{2}$

$= \dfrac{u\sin\theta}{g} = \dfrac{25\left(\dfrac{1}{\sqrt{2}}\right)}{10} = \dfrac{25}{10\sqrt{2}} = \dfrac{5}{2\sqrt{2}}$ s

13. The colour coding on a carbon resistor is shown in the given figure. The resistance value of the given resistor is:



1) $(5700 \pm 375)\Omega$ **2)** $(5700 \pm 285)\Omega$

3) $(7500 \pm 750)\Omega$ **4)** $(7500 \pm 375)\Omega$

Key: 4

Solution:

A color code is used to indicate the resistance value of a carbon and its percentage accuracy

Colour	Letter as an ald to memory	Number	Multiplier	Colour	Tolerance
Black Brown Red Orange Yellow Green Blue Violet Grey White	B B R O Y G B V G W	0 1 2 3 4 5 6 7 8 9	10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8 10^9	Gold Silver No fourth band	5% 10% 20%

A set of coloured co-axial rings or bands is printed on the resistor which reveals the following facts:

1. The first band indicates the first significant figure.
2. The second band indicates the second significant figure.
3. The third band indicates the power of ten with which the above two significant figures must be multiplied to get the resistance value in ohms.
4. The fourth band indicates the tolerance or possible variation in percent of the indicated value. If the fourth band is absent, it implies a tolerance of ± 20%

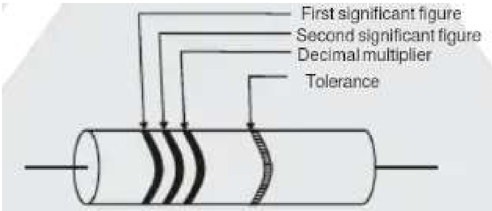
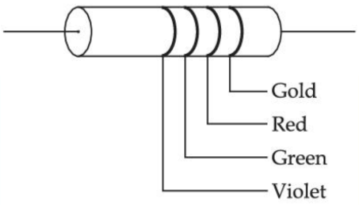


Figure. Meanings of four bands.



R = $(7500 \pm 750)\Omega$

14. For full scale deflection of total 50 divisions, 50 mV voltage is required in galvanometer. The resistance of galvanometer if its current sensitivity is 2 div/mA will be:

1) 2Ω **2)** 5Ω **3)** 4Ω **4)** 1Ω

Key: 1

Solution:

C.S. $= \frac{\theta}{I_g}$

$$I_g = \frac{\theta}{C.S.} = \frac{50div}{2div/mA} = 25mA$$

Potential difference = 50 mV

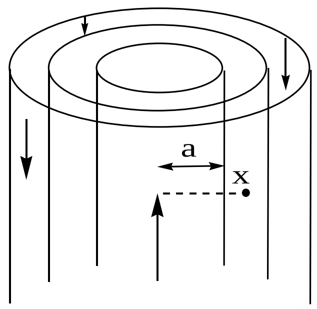
Resistance = $\frac{V}{I} = \frac{50mV}{25mA} = 2\Omega$

15. A coaxial cable consists of an inner wire of radius ‘a’ surrounded by an outer shell of inner and outer radii ‘b’ and ‘c’ respectively. The inner wire carries an electric current i_0, which is distributed uniformly across cross-sectional area. The outer shell carries an equal current in opposite direction and distributed uniformly. What will be the ratio of the magnetic field at a distance x from the axis when (i) $x < a$ and (ii) $a < x < b$?

1) $\frac{x^2}{b^2 - a^2}$ **2)** $\frac{b^2 - a^2}{x^2}$ **3)** $\frac{a^2}{x^2}$ **4)** $\frac{x^2}{a^2}$

Key: 4

Solution:



$B_{(i)} = B_{inner} + B_{outer}$

$= \dfrac{\mu i x}{2\pi a^2} + 0$

$B_{(ii)} = B_{inner} + B_{outer}$

$= \dfrac{\mu i}{2\pi x}$

Now $\dfrac{B_i}{B_{ii}} = \dfrac{x^2}{a^2}$

16. For a transistor α and β are given as $\alpha = \dfrac{I_C}{I_E}$ and $\beta = \dfrac{I_C}{I_B}$. Then the correct relation between α and β will be:

1) $\alpha = \dfrac{\beta}{1-\beta}$ **2)** $\alpha\beta = 1$ **3)** $\beta = \dfrac{\alpha}{1-\alpha}$ **4)** $\alpha = \dfrac{1-\beta}{\beta}$

Key: 3

Solution:

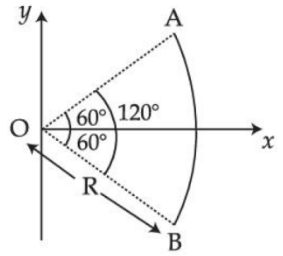
$i_e = i_b + i_c$

$\dfrac{i_e}{i_c} = \dfrac{i_b}{i_c} + 1$

$\dfrac{1}{\alpha} = \dfrac{1}{\beta} + 1$

$\Rightarrow \beta = \dfrac{\alpha}{1-\alpha}$

17. Figure shows a rod AB, which is ben in a 120^o circular arc of radius R. A charge $(-Q)$ is uniformly distributed over rod AB. What is the electric field $\vec{E}$ at the centre of curvature O?



1) $\frac{3\sqrt{3}Q}{16\pi^2\varepsilon_0 R^2}(\hat{i})$ **2)** $\frac{3\sqrt{3}Q}{8\pi^2\varepsilon_0 R^2}(-\hat{i})$

3) $\frac{3\sqrt{3}Q}{8\pi^2\varepsilon_0 R^2}(\hat{i})$ **4)** $\frac{3\sqrt{3}Q}{8\pi\varepsilon_0 R^2}(\hat{i})$

Key: 3

Solution:

$$\delta = \frac{2k\lambda}{R}\sin\frac{\theta}{2}\hat{i}$$

$$= \frac{2k}{R}\left(\frac{Q}{2R\frac{\pi}{3}}\right)\sin 60°\hat{i}$$

$$= \frac{3kQ}{\pi R^2}\frac{\sqrt{3}}{2}\hat{i}$$

$$= \frac{3\sqrt{3}}{2\pi}\frac{kQ}{R^2}\hat{i}$$

$$= \frac{3\sqrt{3}Q}{8\pi^2 \in_0 R^2}$$

18. The light waves from two coherent sources have same intensity $I_1 = I_2 = I_0$. In interference pattern the intensity of light at minima is zero. What will be the intensity of light at maxima?

1) I_0 **2)** $5I_0$ **3)** $2I_0$ **4)** $4I_0$

Key: 4

Solution:

The resultant intensity due to two waves of intensity I_0 each

$$= 4I_0\cos^2\left(\frac{\phi}{2}\right)$$

Where ϕ = phase difference between waves. At constructive interference $\phi = 0$

$\therefore I_{max} = 4I_0$

19. The height of victoria falls is 63 m. What is the difference in temperature of water at the top and at the bottom of fall?

[Given 1 cal=4.2 J and specific heat of water =1 cal $g^{-1o}C^{-1}$]

1) 0.147^{o}C **2)** 0.014^{o}C **3)** 14.76^{o}C **4)** 1.476^{o}C

Key: 1

Solution:

mgh = ms $\Delta\theta$

g x 63 = $\Delta\theta \times 4.2 \times 10^{3}$

$\Delta\theta = 0.147°$ C

20. Match List-I with List-II.

List-I	**List-II**
a) R_H (Rydberg constant)	i) kg $m^{-1}s^{-1}$
b) h(Planck's constant)	ii) kg $m^{2}s^{-1}$
c) μ_B (Magnetic field energy density)	iii) m^{-1}
d) η (coefficient of viscocity)	iv) kg $m^{-1}s^{-2}$

Choose the most appropriate answer from the options given below:

1) a-iii, b-ii, c-iv, d-i **2)** a-ii, b-iii, c-iv, d-i

3) a-iii, b-ii, c-i, d-iv **4)** a-iv, b-ii, c-i, d-iii

Key: 1

Solution:

Units of R_H (Rydberg constant) = m^{-1}

Units of h(Planck's constant) = kg $m^{2}s^{-1}$

Units of μ_B (Magnetic field energy density) = kg $m^{-1}s^{-2}$

Units of η (coefficient of viscocity) = kg $m^{-1}s^{-1}$

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. An ac circuit has an inductor and a resistor of resistance R in series, such that $X_L = 3R$. Now, a capacitor is added in series such that $X_C = 2R$. The ratio of new power factor with the old power factor of the circuit is $\sqrt{5} : x$. The value of x is ______.

Key: 1

Solution:

$$\text{Power Factor} = \frac{R}{\sqrt{X^2 + R^2}}$$

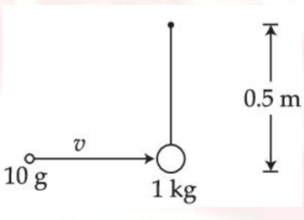
$$\text{Power Factor}_{\text{initial}} = \frac{R}{\sqrt{9R^2 + R^2}} = \frac{1}{\sqrt{10}}$$

$$\text{Power Factor}_{\text{final}} = \frac{R}{\sqrt{(3-2)^2 R^2 + R^2}} = \frac{1}{\sqrt{2}}$$

$$\frac{PF_{final}}{PF_{initial}} = \sqrt{5}$$

22. A bullet of 10 g, moving with velocity v, collides head-on with the stationary bob of a pendulum and recoils with velocity 100 m/s. The length of the pendulum is 0.5 m and mass of the bob is 1 kg. The minimum value of $v =$_______m/s so that the pendulum describe a circle.

(Assume the string to be inextensible and $g = 10\,m/s^2$)



Key: 400

Solution:

To complete the vertical circle final velocity of 1kg ball V_1 should be $> \sqrt{5gR}$

Velocity of 1 Kg ball after collision V_1 according to low of conservation of momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$10 \times 10^{-3} \times V + 1(0) = 10 \times 10^{-3}(-100) + 1(V_1)$$

$$V_1 = 10^{-2}(V) + 10^{-2}(100)$$

$$V_1 = \frac{V}{100} + 1$$

This V_1 should be greater than $\sqrt{5gR}$

$$\frac{V}{100} + 1 > \sqrt{5 \times 10 \times \frac{1}{2}}$$

$$\frac{V}{100} + 1 > 5$$

$$\frac{V}{100} + 14 \Rightarrow V > 400\,\text{m/s}$$

23. A plane electromagnetic wave with frequency of 30 MHz travels in free space. At particular point in space and time, electric field is 6 V/m. The magnetic field at this point will be $x \times 10^{-8}\text{T}$. The value of x is_____.

Key: 2

Solution:

$$B = \frac{E}{C} = \frac{6}{3 \times 10^8} = 2 \times 10^{-8} T$$

24. Two simple harmonic motion, are represented by the questions

$$y_1 = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$$

$$y_2 = 5\left(\sin 3\pi t + \sqrt{3}\cos 3\pi t\right)$$

Ratio of amplitude of y_1 to $y_2 = x:1$. The value of x is _______.

Key: 1

Solution:

$$y_1 = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$$

$$y_2 = 5\left|\sin 3\pi t + \sqrt{3}\cos \pi t\right|$$

$$y_2 = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$$

$$A_1 = 10, A_2 = 10$$

$$\frac{A_2}{A_1} = \frac{10}{10} = 1$$

25. A heat engine operates between a cold reservoir at temperature $T_2 = 400K$ and a hot reservoir at temperature T_1. It takes 300 J of heat from the hot reservoir and delivers 240 J of heat to the cold reservoir in a cycle. The minimum temperature of the hot reservoir has to be _______K.

Key: 500

Solution:

Efficiency of heat engine = $\eta = 1 - \frac{T_{cold}}{T_{Hot}} = \frac{W}{Q}$

$= 1 - \frac{400}{T_{Hot}} = \frac{300-240}{300} = \frac{60}{300}$

$= 1 - \frac{400}{T_{Hot}} = \frac{1}{5}$

$\frac{400}{T_{Hot}} = 1 - \frac{1}{5} = \frac{4}{5}$

$\therefore T_{Hot} = 500k$

26. A tuning fork is vibrating at 250 Hz. The length of the shortest closed organ pipe that will resonate with the tuning fork will be _______cm.

(Take speed of sound in air as $340\,\text{ms}^{-1}$)

Key: 34

Solution:

For closed organ pipe

$F_{-0} = (2n + 1)\ \frac{V}{4\ell}$

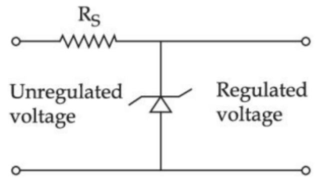
For minimum length, n = 0

$F_0 = \frac{V}{4\ell}$

$\Rightarrow \quad \ell \frac{V}{4 f_0}$

$= \frac{340}{4 \times 250} = 34\,\text{cm}$

27. A zener diode of power rating 2 W is to be used as a voltage regulator. If the zener diode has a breakdown of 10 V and it has to regulate voltage fluctuated between 6 V and 14 V, the value of R_S for safe operation should be _______Ω.



Key: 20

Solution:

For diode

$P = Vi$

$i = \frac{P}{V} = \frac{2}{10} = 0.2mA$

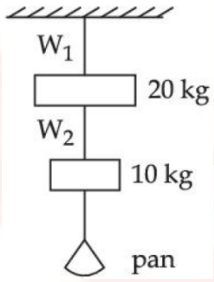
Now for $V_S = 14V$

Voltage drop across $R_L = 14 - 10 = 4V$

Thus, $V = iR_S$

$R_L = \frac{4}{0.2 \times 10^{-3}} = 20m\Omega$

28. Wires W_1 and W_2 are made of same material having the breaking stress of 1.25×10^9 N/m^2. W_1 and W_2 have cross-sectional area of 8×10^{-7}m^2 and 4×10^{-7}m^2, respectively. Masses of 20 kg and 10 kg hang from them as shown in the figure. The maximum mass that can be placed in the pan without breaking the wires is ______kg. (Use $g = 10$ m/s^2)



Key: 40

Solution:

Breaking force of a wire = (Breaking stress) Area

$\therefore$ Breaking force of W_1 = 1.25 x 10^9 x 8 x 10^{-7}

= 1.25 x 8 x 10^2

= 1000N

Breaking force of W_2 = 1.25 x 10^9 x 4 x 10^2 = 500N

Tension in W_1 = 20g + 10g + mg [where m is load we applied]

$T_1 = (30 + m)g$

Tension in $W_2 = T_2 = 10g + mg = (10 + m)g$

Now not to break $T_1 < W_1$, $T_2 < W_2$

$(30 + m)g < 1000$

$30 + m < 100$

$M < 70$ kg

And $T_2 < 500$

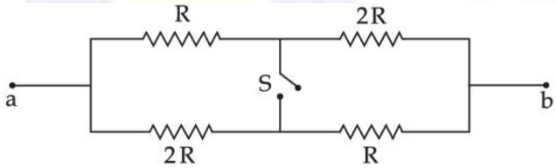
$(10 + m)g < 500$

$10 + m < 50$

$M < 40$ kg

$\therefore$ m should be less than 40kg.

29. The ratio of the equivalent resistance of the network (shown in figure) between the points a and b when switch is open and switch is closed is $x:8$. The value of x is _____.



Key: 9

Solution:

When switch is closed

$R_1 = \frac{2R}{3} + \frac{2R}{3} = \frac{4R}{3}$

When open $R_2 = \frac{3R\,.\,3R}{3R + 3R} = \frac{3R}{2}$

$\frac{R_c}{R_o} = \frac{4R}{3} \times \frac{2}{3R} = \frac{8}{9}$

30. X different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are excited to states with principal quantum number n=6? The value of X is____.

Key: 15

Solution:

The No. of spectral lines emitted when electron moves to n to 1 = $\frac{n(n-1)}{2}$

Given n = 6

$\frac{6(6-1)}{2} = 15$

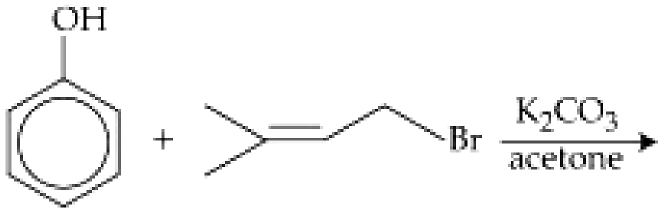
CHEMISTRY **Max Marks: 100**

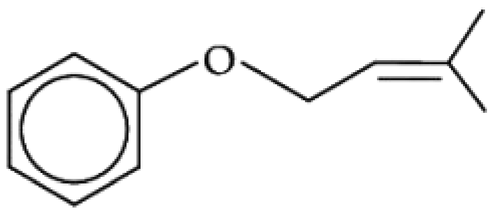
(SINGLE CORRECT ANSWER TYPE)

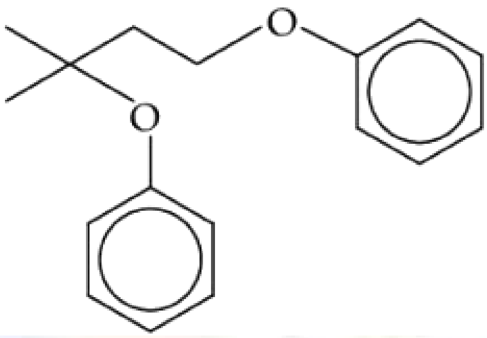
This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

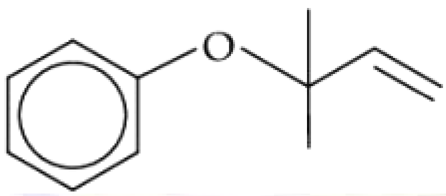
Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

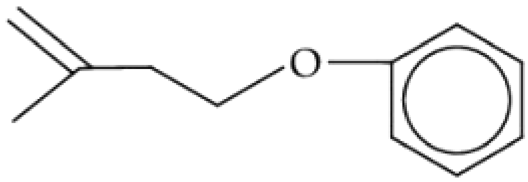
31. The major product of the following reaction, if it occurs by S_N2 mechanism us:



1) 

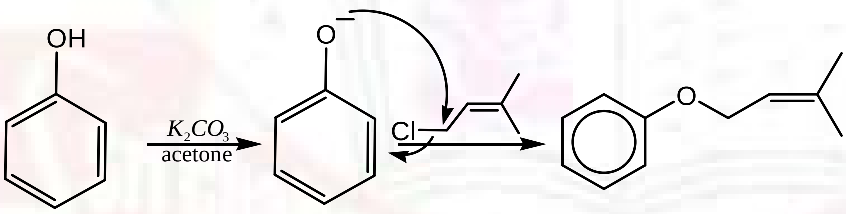
2) 

3) 

4) 

Key: 1

Solution:



32. Which one of the following (mainly) when red phosphorus is heated in a sealed tube at 803K?

1) $\alpha -$ Black phosphorus **2)** Yellow phosphorus

3) $\beta -$ Black phosphorus **4)** White phosphorus

Key: 1

Solution:

$$\text{Red - P} \xrightarrow[\text{803k}]{\text{In a sealed tube}} \alpha - Black$$

33. Lyophilic sols are more stable then lyophobic sols because.

1) The colloidal particles have no charge.

2) There is a strong electrostatic repulsion between the negatively charged colloidal particles.

3) The colloidal particles are solvated.

4) The colloidal particles have positive charge.

Key: 3

Solution:

Due to higher extent of solvation lyophilic sols are more stable

34. Which one of the following reactions will not yield propionic acid?

1) $CH_3CH_2CH_3 + KMno_4(Heat), OH^- / H_3O^+$

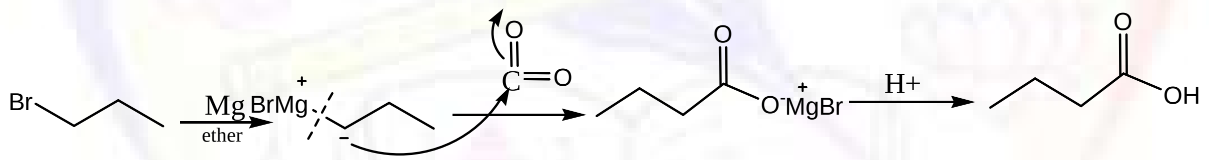
2) $CH_3CH_2CH_2Br + Mg, CO_2\ dry\ ether / H_3O^+$

3) $CH_3CH_2COCH_3 + OI^- / H_3O^+$

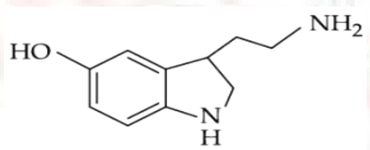
4) $CH_3CH_2CCl_3 + OH^- / H_3O^+$

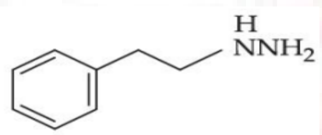
Key: 2

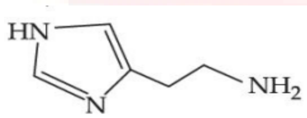
Solution:

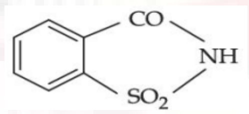


35. Which one of the following chemicals is responsible for the production of HCl in stomach leading to irritation and pain?

1) 

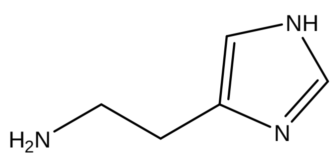
2) 

3) 

4) 

Key: 3

Solution:



histamine

36. Hydrolysis of sucrose gives:

1) $\alpha-D-(-)$- Glucose and $\alpha-D-(+)$- Fructose

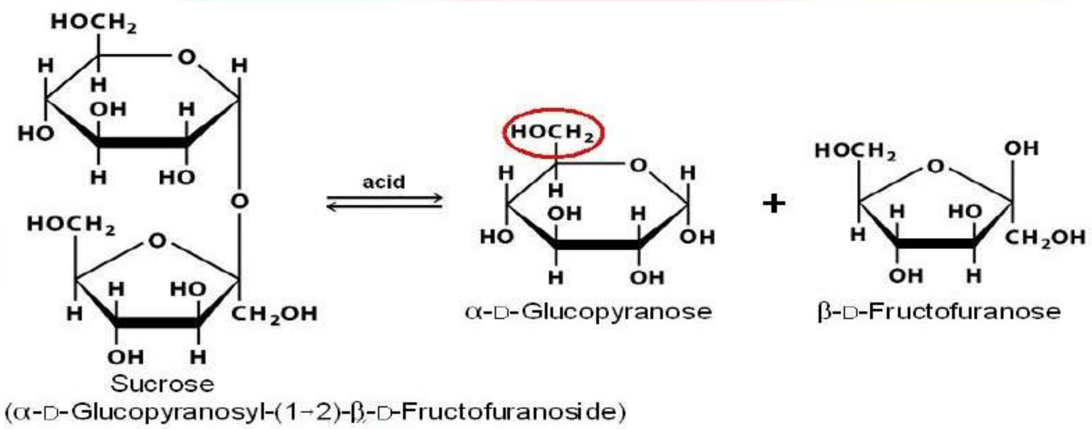
2) $\alpha-D-(-)$- Glucose and $\beta-D-(-)$- Fructose

3) $\alpha-D-(+)$- Glucose and $\beta-D-(-)$- Fructose

4) $\alpha-D-(+)$- Glucose and $\alpha-D-(-)$- Fructose

Key: 3

Solution:



37. Choose the correct statement from the following:

1) The standard enthalpy of formation for alkali metal bromides becomes less negative on descending the group.

2) LiF has least negative standard enthalpy of formation among alkali metal fluorides.

3) The low solubility of CsI in water is due to its high lattice enthalpy.

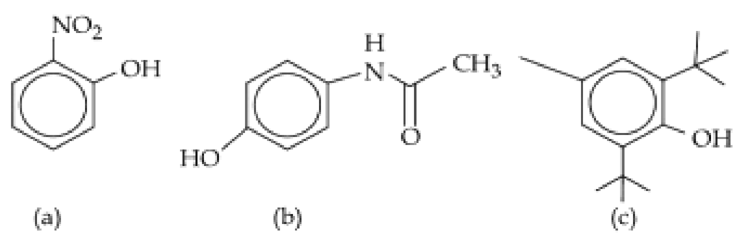
4) Among the alkali metal halides, LiF is least soluble water.

Key: 4

Solution:

Least solubility of LiF is due to higher lattice energy which is attributed to smaller cation and anion

38. The compound/s which show significant intermolecular H-bonding is/are:



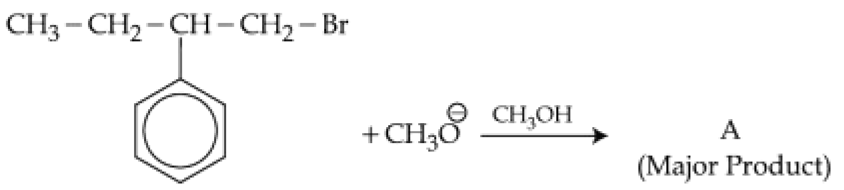
1) (a) and (b) only **2)** (b) only **3)** (a),(b) and (c) **4)** (c) only

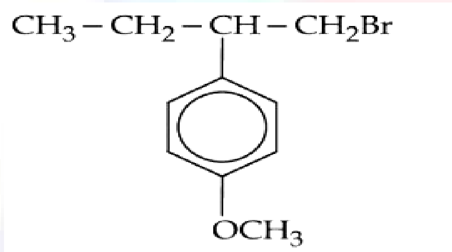
Key: 2

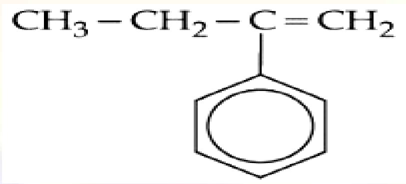
Solution:

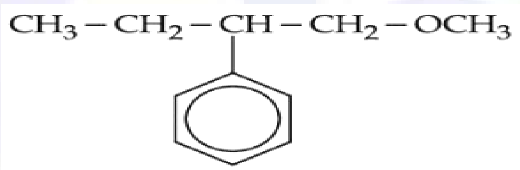
Ortho nitro phenol forms intramolecular hydrogen bond where as compound (C) do not form hydrogen bond due to steric hindrance. Option (b) will form intermolecular hydrogen bond.

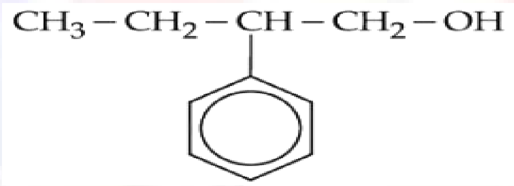
39. The major product (A) formed in the reaction given below is:



1) 

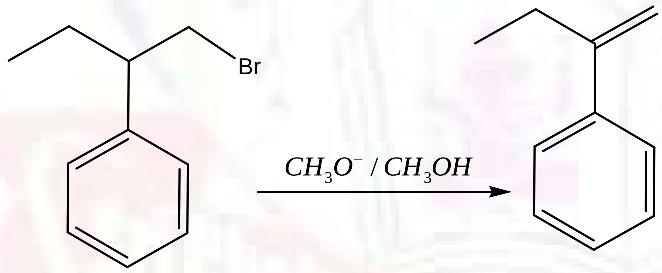
2) 

3) 

4) 

Key: 2

Solution:



40. The oxide that gives H_2O_2 most readily on treatment with H_2O is:

1) $BaO_2 8H_2O$ **2)** SnO_2 **3)** PbO_2 **4)** Na_2O_2

Key: 4

Solution:

$Na_2O_2 + 2H_2O \rightarrow 2NaOH + H_2O_2$

41. Potassium permanganate on heating at 513K gives a product which is:

1) Paramagnetic and colourless **2)** Diamagnetic and colourless

3) Paramagnetic and green **4)** Diamagnetic and green

Key: **1 &** 3

Solution:

$$2KMnO_4 \xrightarrow{\Delta} \underset{\substack{\text{(Green)}\\\text{(Para)}}}{K_2MnO_4} + \underset{\substack{\text{(Colourless)}\\\text{(Para)}}}{MnO_2} + O_2$$

42. The correct order of ionic radii for the ions $P^{3-}, S^{2+}, Ca^{2+}, K^{+}, Cl^{-}$ is:

1) $P^{3-} > S^{2-} > Ca^{2+} > K^{+}$

2) $Cl^{-} > S^{2-} > P^{3-} > Ca^{2+} > K^{+}$

3) $P^{3-} > S^{2-} > Cl^{-} > K^{+} > Ca^{2+}$

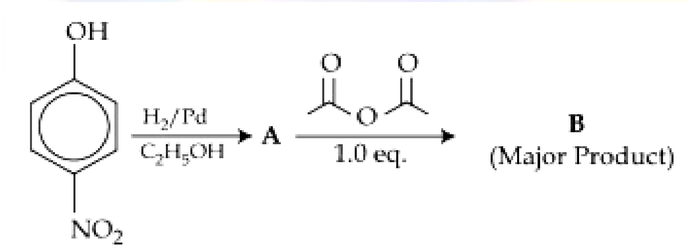
4) $K^{+} > Ca^{2+} > P^{3-} > S^{2-} > Cl^{-}$

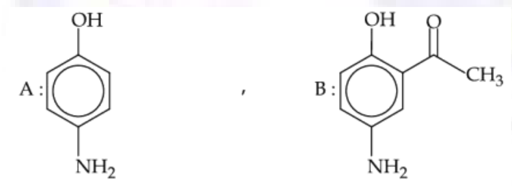
Key: 3

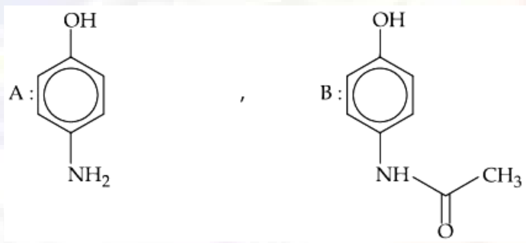
Solution:

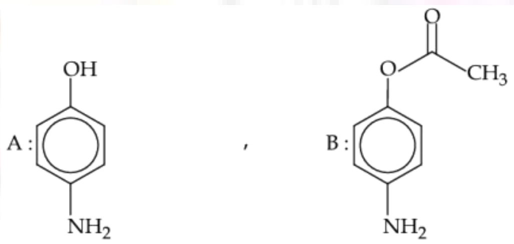
As charge increases the effective nuclear charge also increase thereby size decrease in the isoelectronic series

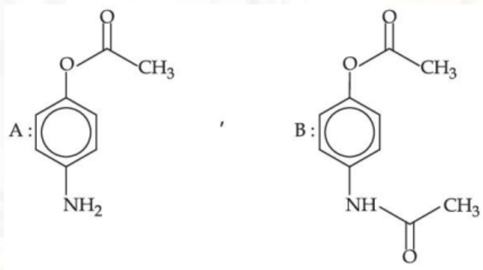
43. The correct structures of A and B formed in the following reactions are:



1) 

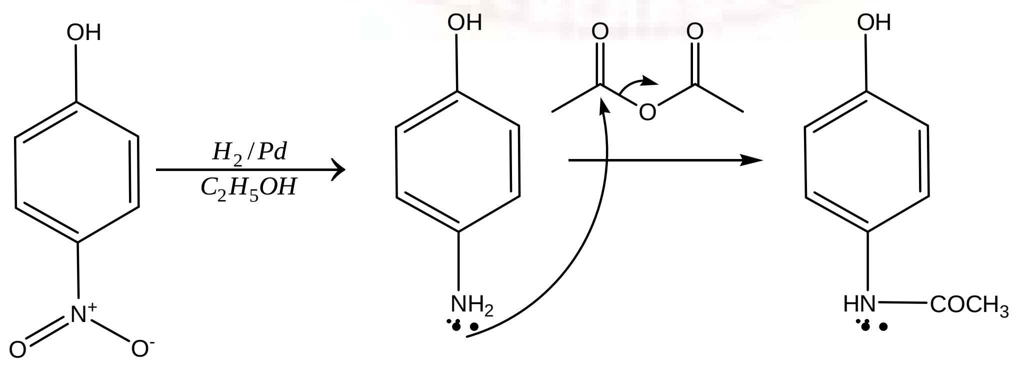
2) 

3) 

4) 

Key: 2

Solution:



44. Given below are two statements

Statement I: Ethyl pent-4-yn-oate on reaction with CH_3MgBr gives a 3^0- alcohol

Statement II: In this reaction one mole ethyl pent-4-yn-oate utilizes two moles of CH_3MgBr.

In the light of the above statements, choose the most appropriate answer from the options given below:

1) Both **Statement I** and **Statement II** are true

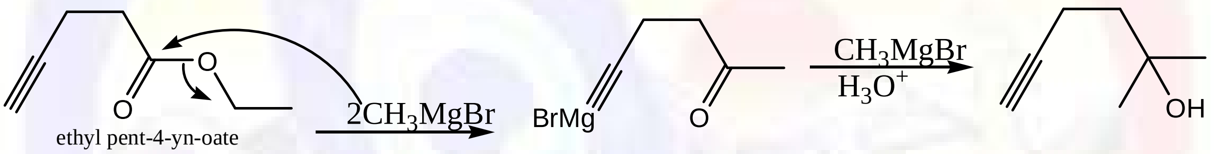
2) **Statement I** is false but **Statement II** is true

3) Statement I is true but **Statement II** is false

4) Both **Statement I** and **Statement II** are false

Key: 3

Solution:



45. Match List –I with List-II

List-I (Name of ore/mineral)	List-II (Chemical formula)
a) Calamine	i) ZnS
b) Malachite	ii) $FeCO_3$
c) Siderite	iii) $ZnCO_3$
d) Sphalerite	iv) $CuCO_3.Cu(OH)_2$

Choose the most appropriate answer from the options given below:

1) a)-(iii), b)-(iv), c)-(ii), d)-(i)   2) a)-(iii), b)-(ii), c)-(iv), d)-(i)

3) a)-(iv), b)-(iii), c)-(i), d)-(ii)   4) a)-(iii), b)-(iv), c)-(i), d)-(ii)

Key: 1

Solution:

Calamine - $ZnCO_3$

Malachite - $CuCO_3Cu(OH)_2$

Siderite - $FeCO_3$

Sphelerite - ZnS

46. In stratosphere most of the ozone formation is assisted by:

1) Visible radiations **2)** Cosmic rays.

3) ultraviolet radiation **4)** $\gamma -$rays.

Key: 3

Solution:

Ozone absorbs dangerous U.V rays there by protects the mankind

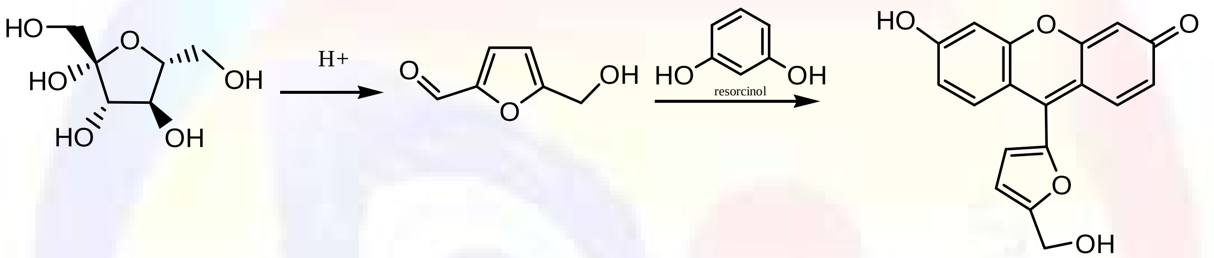
47. Which one of the following tests used for the identification of functional groups in organic compounds does not use copper reagent?

1) Biuret test for peptide bond **2)** Benedict's test

3) Barfoed's test **4)** Seliwanoff's test

Key: 4

Solution:



Cherry red

48. The addition of dilute $NaOH$ to Cr^{3+} salt solution will give:

1) Precipitate of $Cr_2O_3\left(H_2O\right)_n$ **2)** Precipitate of $Cr\left(OH\right)_3$

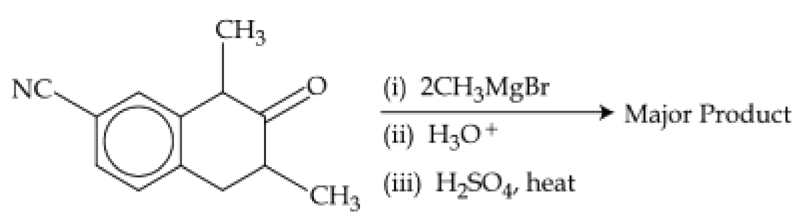
3) Precipitate of $\left[Cr\left(OH\right)_6\right]^{3-}$ **4)** a Solution of $\left[Cr\left(OH\right)_4\right]^{-}$

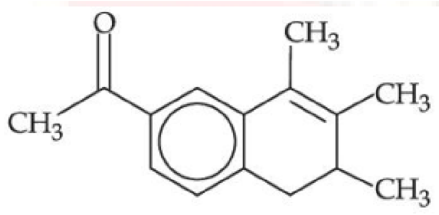
Key: 1 & 2

Solution:

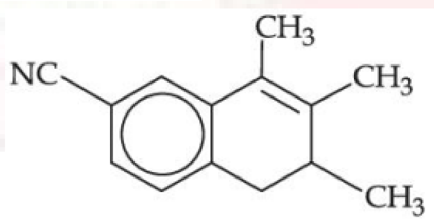
Cr^{3+} with dilute NaOH gives $Cr\left(OH\right)_3$ or Hydrated Cr_2O_3

49. Which one of the following is the major product of the given tion?





1)



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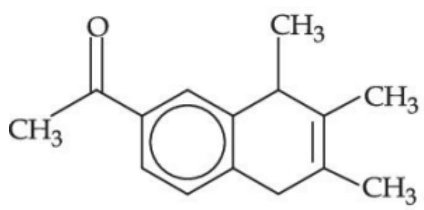
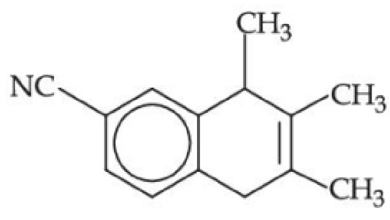


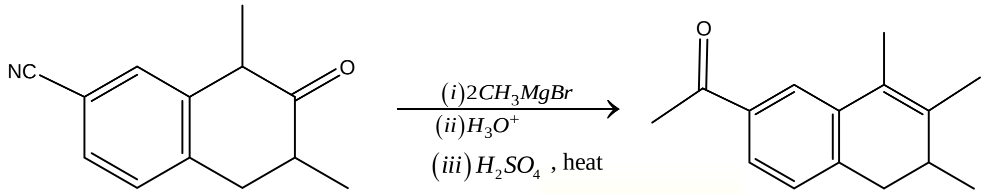
3)



4)

Key: 1

Solution:



50. Which one the following is used to remove most of plutonium from spent nuclear fuel?

1) ClF_3 **2)** BrO_3 **3)** O_2F_2 **4)** I_2O_5

Key: 3

Solution:

$$2O_2F_2 + Pu \rightarrow PuF_4 + 2O_2$$

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place.
(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

51. 100g of propane is completely reacted with 1000g of oxygen. The mole fraction of carbon dioxide in the resulting mixture is $x \times 10^{-2}$. The value of x is______________

Key: 19

Solution:

	C_3H_8	+	$5O_2$	$\rightarrow$	$3CO_2$	+	$4H_2O$
Moles	$\frac{100}{44}$		$\frac{1000}{32}$		O		O
Moles reacted & Produced	$\frac{-100}{44}$		$\frac{-5\times100}{44}$		$\frac{+3\times100}{44}$		$\frac{+4\times100}{44}$
Moles Left	0		$\frac{1000}{32} - \frac{5\times100}{44}$		$\frac{300}{44}$		$\frac{400}{44}$

Mole fraction of CO_2

$$= \frac{Moles\ of\ CO_2}{Total\ moles}$$

$$= \frac{\left(300/44\right)}{\left(\frac{1000}{32} - \frac{5\times100}{44} + \frac{300}{44} + \frac{400}{44}\right)}$$

$$= 0.1903 \quad \simeq 0.19 \quad = 19\times10^{-2}$$

52. When 5.1g of solid NH_4HS is introduced a two litre evacuated flask at $27^0C, 20\%$ of the solid decomposes into gaseous ammonia and hydrogen sulphide. The K_p for the reaction at 27^0C is $x \times 10^{-2}$, The value of x is________________.(Integer answer)

[Given $R = 0.082 L\ atm\ K^{-1} mol^{-1}$]

Key: 6

Solution:

$$NH_4HS_{(s)} \rightleftharpoons NH_{3(g)} + H_2S_{(g)}$$

	$NH_4HS_{(s)}$	$NH_{3(g)}$	$H_2S_{(g)}$
Initial	0.1mol	0	0
At equilibrium Concentration	Solid	$\frac{2\times10^{-2}}{2}$	$\frac{2\times10^{-2}}{2}$

$k_C = [NH_3][H_2S] = 10^{-4} mol^2 lit^{-2}$

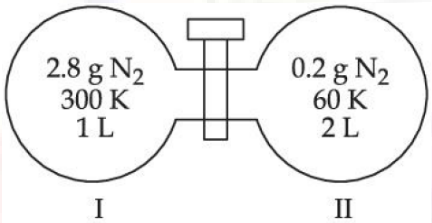
$k_P = k_C (RT)^{\Delta n}$

$= 10^{-4}(0.082 \times 300)^2$

$= 605.16 \times 10^{-4}$

$= 6.0516 \times 10^{-2}$

53. Tow flasks I and II shown below are connected by a vale of negligible volume.



When the valve is opened, the final pressure of the system in bar is $x \times 10^{-2}$. The value of x is ________________.(Integer answer)

[Assume-Ideal gas; 1bar$=10^5$ Pa; Molar mass of $N_2 = 28.0 g\ mol^{-1}; R = 8.31 J\ mol^{-1} K^{-1}$]

Key: 84

Solution:

Flask – I

$n_1 = \frac{2.8}{28} = 0.1$

Flask – II

$n_2 = \frac{0.2}{28}$

$= 7.143 \times 10^{-3}$

$P_1 = \frac{0.1\times 8.31\times 300}{10^{-3}} pa \quad P_2 = \frac{7.143\times 10^{-3}\times 8.31\times 60}{2\times 10^{-3}}$

$= 249300\,pa \qquad = 1780.75\,pa$

After opening the valve the gas from flask – I moves to flask – II until their pressures become equal i.e P_f

$P_{f(flask-I)} = P_{f(flask-II)}$

$$\frac{(0.1-x)\times 8.31\times 300}{10^{-3}} = \frac{(7.143\times 10^{-3}+x)\times 8.31\times 60}{2\times 10^{-3}}$$

$2(0.1-x)300 = (7.143\times 10^{-3}+x)60$

$1-10x = 7.143\times 10^{-3}+x$

$11x = 0.992857$

$x = 0.0902$

$$\therefore P_f = \frac{(0.1-x)\times 8.31\times 300}{10^{-3}} N/m^2$$

$$= \frac{(0.1-0.0902)\times 8.31\times 300}{10^{-3}}$$

$= 24431.4\,pa$

$= \frac{24431.4}{10^5} bar$

$= 24.431\times 10^{-2} bar$

$= 24$

54. Data given for following reaction is follows

$FeO_{(s)} + C_{(graphite)} \longrightarrow Fe_{(s)} + CO_{(g)}$

Substance	$\Delta_f H^\circ$ (kJ mol^{-1})	ΔS° (J mol^{-1} K^{-1})
$FeO_{(s)}$	– 266.3	57.49
$C_{(graphite)}$	0	5.74
$Fe_{(s)}$	0	27.28
$CO_{(g)}$	– 110.5	197.6

The minimum temperature in K at which the reaction becomes spontaneous is________ (Integer answer)

Key: 964

Solution:

$FeO_{(S)} + C_{(graphite)} \rightarrow Fe_{(S)} + CO_{(g)}$

$\Delta H^{0}_{f(reaction)} = \Delta H^{0}_{f(products)} - \Delta H^{0}_{+(reatant)}$

$= (0 - 110.5) - (-266.3 + 0)$

$= 155.8 kJ\ mol^{-1}$

$\Delta s^{0}(\text{Re}action) = \Delta S^{0}(\text{Pr}oduct) - \Delta S^{0}(\text{Re}actant)$

$= (27.28 + 197.6) - (57.49 + 5.74)$

$= 161.65 Jmol\, k^{-1}$

Minimum temperature required for

Spontaneity (T) $= \dfrac{\Delta H^{0}_{f}(\text{Re}action)}{\Delta S^{0}(\text{Re}action)}$

$= \dfrac{155.3 \times 10^{3}}{161.65}$

$= 963.8$

$\simeq 964\, \text{k}$

55. The resistance of a conductivity cell constant $1.14 cm^{-1}$, containing $0.001M$ KCl at $298K$ is 1500Ω. The molar conductivity of $0.001M$ KCl solution at $298K$ in $S\ cm^{-2} mol^{-1}$ is ___________(Integer answer)

Key: 760

Solution:

$R = 1500\Omega$

$G = 1.14 cm^{-1}$

$k = \dfrac{G}{R} = \dfrac{1.14}{1500}$

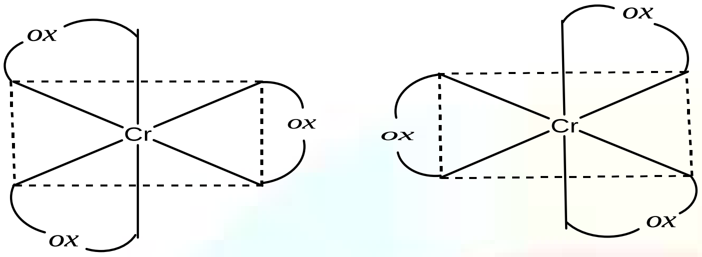
$\wedge_m = \dfrac{k \times 1000}{m} \qquad = \dfrac{\frac{1.14}{1500} \times 1000}{0.001}$

$= 760 Scm^{2} mol^{-1}$

56. The number of optical isomers possible for $\left[Cr\left(C_2O_4\right)_3\right]^{3-}$ is ___________

Key: 2

Solution:



Two optical isomers

57. The first order rate constant for the decomposition of $CaCO_3$ at $700K$ is $6.36\times10^{-3}s^{-1}$ and activation energy is $209kJ\,mol^{-1}$ Its rate constant (in s^{-1}) at $x\times10^{-6}$.The value of x is________.(Nearest integer)

[Given $R=8.31J\ K^{-1}mol^{-1}$;$\log 6.36\times10^{-3}=-2.19,10^{-479}=1.62\times10^{-5}$]

Key: 16

Solution:

$$\log\frac{k_2}{k_1}=\frac{E_0}{2.303R}\left[\frac{1}{T_1}-\frac{1}{T_2}\right]$$

$$\log\left(\frac{6.36\times10^{-3}}{x\times10^{-6}}\right)=\frac{209\times10^3}{2.303\times8.31}\left[\frac{1}{600}-\frac{1}{700}\right]$$

$$x=15.969\simeq16$$

58. 40g of glucose (Molar mass=180) is mixed with 200mL of water. The freezing point of solution is ______K.(Nearest integer)

[Given; $K=1.86K\,kg\,mol^{-1}$; Density of water $=1.00g\,cm^{-3}$; Freezing point of water $=273.15K$]

Key: 271

Solution:

$$\Delta T_f=k_f m$$

$$273.75-T=\left[1.86\times\frac{40}{180}\times\frac{1000}{200}\right]$$

$$T=271.08k$$

$$=271k$$

59. The number of photons emitted by a monochromatic (single frequency) infrared range finder of power 1mW and wavelength of 1000nm, in 0.1 second is $x \times 10^{13}$. The value of x is__________.(Nearest integer) $\left(h = 6.63 \times 10^{-34} Js, c = 3.00 \times 10^{8} ms^{-1}\right)$

Key: 50

Solution:

Energy = Power x time

$= 10^{-3} \times 0.1$

$= 10^{-4} J$

$$E = nhv = n\frac{hc}{\lambda}$$

$$n = \frac{E\lambda}{hc} = \frac{10^{-4} \times 1000 \times 10^{-9}}{\left(6063 \times 10^{-34} \times 3 \times 10^{8}\right)}$$

$= 5.0276 \times 10^{-14}$

$= 50.276 \times 10^{-13}$

$\simeq 50 \times 10^{13}$

60. The number of species having non-pyramidal shape among the following is

A) SO_3 B) NO_3^- C) PCl_3 D) CO_3^{2-}

Key: 3

Solution:

$SO_3 \rightarrow$ planar triangle

$NO_3^- \rightarrow$ Planar triangle

$PCl_3 \rightarrow$ pyramidal

$CO_3^{2-} \rightarrow$planar triangle

$\therefore$ Na – pyramidal

MATHEMATICS **MaxMarks: 100**

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

61. The set of all values of $k > -1$, for which the equation

$\left(3x^2+4x+3\right)^2-(k+1)\left(3x^2+4x+3\right)\left(3x^2+4x+2\right)^2+k\left(3x^2+4x+2\right)=0$ has real roots, is:

1) $(2,3]$ 2) $\left(\frac{1}{2},\frac{3}{2}\right]-\{1\}$ 3) $\left[-\frac{1}{2},1\right)$ 4) $\left(1,\frac{5}{2}\right]$

Key: 4

Solution:

Let $y=3x^2+4x+2$ has no real roots

$\therefore (y+1)^2-(k+1)^2-(k+1)(y+1)y+k.y^2=0$

$\Rightarrow \left(\frac{y+1}{y}\right)^2-(k+1)\left(\frac{y+1}{y}\right)+k=0$

$\Rightarrow \left(\frac{y+1}{y}-k\right)\left(\frac{y+1}{y}-1\right)=0$

$\Rightarrow \left(1+\frac{1}{y}-k\right)\left(\frac{1}{y}\right)=0$

$\Rightarrow 1+\frac{1}{y}-k=0 \qquad ----(1)$

$y=3\left[\left(x+\frac{2}{3}\right)^2+\frac{2}{9}\right]\geq\frac{2}{3}$

$\Rightarrow \frac{1}{y}\in\left(0,\frac{3}{2}\right]$

$\Rightarrow 1+\frac{1}{y}\in\left(1,\frac{5}{2}\right)$

$\Rightarrow k\in\left(1,\frac{5}{2}\right)$

62. Each of the persons A and B independently tosses three fair coins. The probability that both of them get the same number of heads is:

1) $\frac{5}{16}$ 2) $\frac{5}{8}$ 3) $\frac{1}{8}$ 4) 1

Key: 1

Solution:

$n=3 \qquad p=\frac{1}{2}$

Probability of A and B obtain the same number of heads is

$=P(x=0)P(y=0)+P(x=1)P(y=1)+P(x=2)P(y=2)+P(x=3)P(y=3)$

$=\left[3_{C_0}\left(\frac{1}{2}\right)^3\right]^2+\left[3_{C_1}\left(\frac{1}{2}\right)^3\right]^2+\left[3_{C_2}\left(\frac{1}{2}\right)^3\right]^2+\left[3_{C_3}\left(\frac{1}{2}\right)^3\right]^2$

$=\frac{5}{16}$

63. If two tangents drawn from a point P to the parabola $y^2=16(x-3)$ are at right angles, then the locus of point P is:

1) $x+1=0$ 2) $x+4=0$ 3) $x+3=0$ 4) $x+2=0$

Key: 1

Solution:

Two tangents drawn from point ‘P’ to the parabola

$y^2=4a(x-b)$ cuts at right angles.

The locus of ‘P’ is directrix of the parabola

$\Rightarrow x=b-a$

$\Rightarrow x=3-4$

$\Rightarrow x+1=0$

64. Let M and m respectively be the maximum and minimum values of the function

$f(x)=\tan^{-1}(\sin x+\cos x)$ in $\left[0,\frac{\pi}{2}\right]$. Then the value of $\tan(M-m)$ is equal to:

1) $3-2\sqrt{2}$ 2) $3+2\sqrt{2}$ 3) $2-\sqrt{3}$ 4) $2+\sqrt{3}$

Key: 1

Solution:

$m=\left(Tan^{-1}(1)\right)\Rightarrow\ Tan\,m=1$

$\Rightarrow=Tan^{-1}\left(\sqrt{2}\right)\Rightarrow M\,Tan\ M\ =\sqrt{2}$

$$Tan\left(M-m\right)\ =\frac{Tan\ M\ -Tan\ m}{1+Tan\ M.Tan\ m}$$

$$=\frac{\sqrt{2}-1}{1+\sqrt{2}}=3-2\sqrt{2}$$

65. Let $\left[\lambda\right]$ be the greatest integer less than or equal to λ. The set of all values of λ for which the system of linear equations $x+y+z=4$, $3x+2y+5z=3$, $9x+4y+\left(28+\left[\lambda\right]\right)z=\left[\lambda\right]$ has a solution is:

1) $\left(-\infty,-9\right)\cup\left[-8,\infty\right)$ 2) $\left[-9,-8\right)$

3) $\left(-\infty,-9\right)\cup\left(-9,\infty\right)$ 4) R

Key: 4

Solution:

$$\Delta=\begin{vmatrix}1 & 1 & 1\\ 3 & 2 & 5\\ 9 & 4 & \left(28+\left[\lambda\right]\right)\end{vmatrix}=1\left(2\left(28\right)+\left[\lambda\right]-20\right)+1\left(12-18\right)-1\left(84+3\left[\lambda\right]-45\right)$$

For no solv $\Delta=0$ and Δ_1,Δ_2 OR $\Delta_3\neq 0$

$$\left[\lambda\right]=-9\left(\because\Delta=0\right)$$

If we check with $\left[\lambda\right]=-9$ for Δ_1,Δ_2 and Δ_3

We get Δ_1,Δ_2 and $\Delta_3=0$

66. Let Z be the set of all integers,

$A=\left\{\left(x,y\right)\in\mathbb{Z}\times\mathbb{Z}:\left(x-2\right)^2+y^2\leq 4\right\}$,

$B=\left\{\left(x,y\right)\in\mathbb{Z}\times\mathbb{Z}:x^2+y^2\leq 4\right\}$ and

$C=\left\{\left(x,y\right)\in\mathbb{Z}\times\mathbb{Z}:\left(x-2\right)^2+\left(y-2\right)^2\leq 4\right\}$

If the total number of relations from $A\cap B$ to $A\cap C$ is 2^{p}, then the value of p is:

1) 16 2) 25 3) 9 4) 49

Key: 2

Solution:

$A = \{(0,0)(1,0)(1,1)(1,-1)(2,0)(2,2)(3,0)(4,0)(3,1)(3,-1)\}$

$B = \{(0,0)\ (\pm1,0)\ (\pm2,0)\ (0,\pm2)\}$

$A \cap B = \{(0,0)(1,0)(1,1)(1,-1)(2,0)\}$

$n(A \cap B) = 5$

$A \cap C = \{(1,1)(2,0)(2,1)(2,2)(3,1)\}$

$n(A \cap C) = 5$

Total number of relations

$\therefore p = 25$

67. The angle between the straight lines, whose direction cosines are given by the equations $2l + 2m - n = 0$ and $mn + nl + lm = 0$, is:

1) $\cos^{-1}\left(\frac{8}{9}\right)$ **2)** $\frac{\pi}{3}$ **3)** $\frac{\pi}{2}$ **4)** $\pi - \cos^{-1}\left(\frac{4}{9}\right)$

Key: 3

Solution:

$2l + 2m - n = 0$

$n = 2(l + m)$

$mn + nl + lm = 0$

$lm + n(l + m) = 0$

$\ln + 2(l + m)^2 = 0$

$2\left(\frac{l}{m}\right)^2 + 2 + 5\left(\frac{l}{m}\right) = 0$

$2t^2 + 5t + 2 = 0$

$t = -2, \frac{-1}{2}$

1. $\frac{l}{m} = -2$

$\frac{n}{m} = -2$

$(-2,1-2)$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$

2. $\frac{l}{m} = -\frac{1}{2}$

$N = -2l$

$(1,-2,-2)$

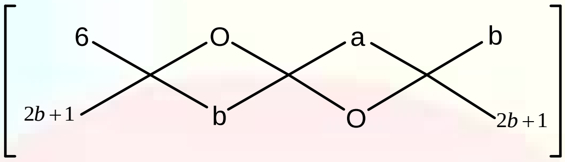
68. Let $A(a,0)$, $B(b,2b+1)$ and $C(0,b)$, $b \neq 0, |b| \neq 1$, be points such that the area of triangle ABC is 1 sq. unit, then the sum of all possible values of a is:

1) $\frac{-2b^2}{b+1}$ **2)** $\frac{-2b}{b+1}$ **3)** $\frac{2b}{b+1}$ **4)** $\frac{2b^2}{b+1}$

Key: 1

Solution:

Area of $\Delta^{1e} ABC = \frac{1}{2}$



$$1 = \frac{1}{2}\left|\left(b^2 + 2ab + a - ab\right)\right|$$

$$2 = \left|b^2 + ab + a\right|$$

Case – 1

$$b^2 + ab + a = 2$$

$$a = \frac{2-b^2}{6+1}$$

Case – 2

$$b^2 + ab + a = -2$$

$$a = \frac{-2-b^2}{6+1}$$

Sum of values of $a = \frac{-2b^2}{6+1}$

69. If the solution curve of the differential equation $\left(2x-10y^3\right)dy + ydx = 0$, passes through the points $(0,1)$ and $(2,\beta)$, then β is a root of the equation:

1) $2y^5 - y^2 - 2 = 0$ **2)** $2y^5 - 2y - 1 = 0$

3) $y^5 - 2y - 2 = 0$ **4)** $y^5 - y^2 - 1 = 0$

Key: 4

Solution:

$$\left(2x-10y^3\right)dy + ydx = 0$$

$$-y\frac{dx}{dy} = 2x - 10y^3$$

$$IF = e^{\int \frac{2}{y}dy} = e^{2dy} = y^2$$

$$x\left(y^2\right) = \int 10y^2\left(y^2\right)dy$$

$$xy^2 = \frac{10}{5}y^5 + c$$

$$= 2y^5 + c$$

Passes through $(0,1)$

$0 = 2 + c \Rightarrow c = -2$

$\therefore xy^2 = 2y^5 - 2$

Passes through $(2,\beta)$

$2\beta^2 = 2\beta^5 - 2$

$\beta^5 - \beta^2 - 1 = 0$

$\therefore y^5 - y^2 - 1 = 0$.

70. The area of the region bounded by the parabola $(y-2)^2 = (x-1)$, the tangent to it at the point whose ordinate is 3 and the *x*-axis is:

1) 9 **2)** 6 **3)** 10 **4)** 4

Key: 1

Solution:

$(y-2)^2 = (x-1)$

$2(y-2)\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2(y-2)}$

$m = \frac{1}{2(y-2)} = \frac{1}{2}$

$(y-3) = \frac{1}{2}(x-2)$

$\Rightarrow 2y - x - 4 = 0$

$$\int_0^3 \left[1+(y-2)^2\right]dy - \int_0^3 (2y-4)\,dy$$

$$\int_0^3 \left(y^2 + 4 - 4y + 1 - 2y + 4\right)dy$$

$$\int_0^3 \left(y^2 - 6y + a\right)dy$$

$$= \int_0^3 (y-3)^2\,dy = \left.\frac{(y-3)^3}{3}\right|_0^3 = |10-9| = 9 \text{ sq. units.}$$

71. Two poles, AB of length a metres and CD of length $a+b(b\neq a)$ metres are erected at the same horizontal level with bases at B and D. If $BD=x$ and $\tan\underline{|ACB}=\frac{1}{2}$, then:

1) $x^2+2(a+2b)x+a(a+b)=0$ **2)** $x^2+2(a+2b)x-b(a+b)=0$

3) $x^2-2ax+b(a+b)=0$ **4)** $x^2-2ax+a(a+b)=0$

Key: 3

Solution:

$$Tan\theta=\frac{1}{2}$$

$$Tan\alpha=\frac{x}{a+b} \qquad Tan(\alpha+\in)=\frac{x}{b}$$

$$\frac{Tan\alpha+Tan\theta}{1+Tan\alpha\ Tan\theta}=\frac{x}{b}$$

$$\frac{x}{a+b}+\frac{1}{2}=\frac{x}{b}\left(1-\frac{x}{2(a+b)}\right)$$

$$\Rightarrow x^2-2ax+b(a+b)=0$$

72. If $y(x)=\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right), x\in\left(\frac{\pi}{2},\pi\right)$, then $\left(\frac{dy}{dx},\pi\right)$, then $\frac{dy}{dx}$ at $x=\frac{5\pi}{6}$ is:

1) -1 **2)** $\frac{1}{2}$ **3)** $-\frac{1}{2}$ **4)** 0

Key: 3

Solution:

$$y=\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$$

$$\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}=\frac{1+\sin x+1-\sin x+2\sqrt{1-\sin^2 x}}{(1+\sin x)-(1-\sin x)}$$

$$=\frac{2+2|\cos x|}{2\sin x}=\frac{1-\cos x}{\sin x}=\tan\frac{x}{2}$$

$$y=\cot^{-1}\left(\tan\frac{x}{2}\right)$$

$$=\frac{\pi}{2}-\cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$y=\frac{\pi}{2}-\frac{x}{2}$$

$$\Rightarrow\frac{dy}{dx}=\frac{-1}{2}$$

73. If $0<x<1$ and $y=\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{3}{4}x^4+....,$ then the value of e^{1+y} at $x=\frac{1}{2}$ is:

1) $\frac{1}{2}\sqrt{e}$ **2)** $\frac{1}{2}e^2$ **3)** $2e$ **4)** $2e^2$

Key: 2

Solution:

$$y=\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{3}{4}x^4+.....$$

$$y=\left(x^2-\frac{x^2}{2}\right)+\left(x^3-\frac{x^3}{3}\right)+......$$

$$=x^2+x^3+x^4+.....-\left(\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}......\right)$$

$$=\frac{x^2}{1-x}+x+\log(1-x) \qquad \left[\because \log(1-x)=-x-\frac{x^2}{2}-\frac{x^3}{3}.....\right]$$

$$y=\frac{x}{1-x}+\log(1-x)$$

$$y\left(\frac{1}{2}\right)=1+\log\left(\frac{1}{2}\right)$$

$$e^{1+y}=e^{1+1+\log\left(\frac{1}{2}\right)}$$

$$=e^2\times e^{\log\frac{1}{2}}$$

$$y=\frac{e^2}{2}$$

74. Let $A=\begin{pmatrix}[x+1] & [x+2] & [x+3]\\ [x] & [x+3] & [x+3]\\ [x] & [x+2] & [x+4]\end{pmatrix}$, where $[t]$ denotes the greatest integer less than or equal to t. If $\det(A)=192$, then the set of values of x is the interval:

1) $[68,69)$ **2)** $[60,61)$ **3)** $[62,63)$ **4)** $[65,66)$

Key: 3

Solution:

$$\det A = \begin{vmatrix} [x]+1 & [x]+2 & [x]+3 \\ [x] & [x]+3 & [x]+3 \\ [x] & [x]+2 & [x]+4 \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\det A = \begin{vmatrix} [x]+1 & [x]+2 & [x]+3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$\det A = 3[x] + 6 = 192$

$[x] = 62$

$x \in [62, 63)$

75. If $\lim_{x\to\infty}\left(\sqrt{x^2 - x + 1} - ax\right) = b$, then the ordered pair (a,b) is:

1) $\left(1, -\frac{1}{2}\right)$ **2)** $\left(1, \frac{1}{2}\right)$ **3)** $\left(-1, \frac{1}{2}\right)$ **4)** $\left(-1, -\frac{1}{2}\right)$

Key: 1

Solution:

$\lim_{x\to\infty}\left(\sqrt{x^2 - x + 1} - ax\right)$

$\lim_{x\to\infty}\left(x\sqrt{1 - \frac{1}{x} + \frac{1}{x^2}} - ax\right)$

$\lim_{x\to\infty} x\left(1 - \frac{1}{2x} + \frac{1}{2x^2}\right) - ax$

$b = \lim_{x\to\infty}\left(x(1-a) - \frac{1}{2} + \frac{1}{2x}\right)$

$= x(1-a) - \frac{1}{2}$

$a = 1, b = \frac{-1}{2}$

$(a,b) = \left(1, \frac{1}{2}\right)$

76. The Boolean expression $(p \wedge q) \Rightarrow ((r \wedge q) \wedge p)$ is equivalent to:

1) $(p \wedge q) \Rightarrow (r \wedge q)$ **2)** $(q \wedge r) \Rightarrow (p \wedge q)$

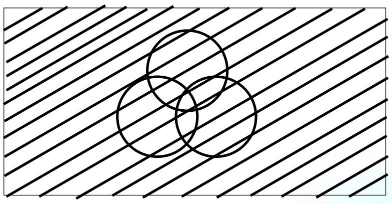
3) $(p \wedge r) \Rightarrow (p \wedge q)$ **4)** $(p \wedge q) \Rightarrow (r \vee q)$

Key: 1

Solution:

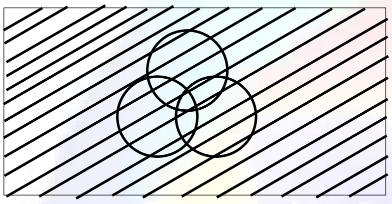
$(p \wedge q) \Rightarrow ((r \wedge q) \wedge p)$

$= \sim (p \wedge q) \vee ((r \wedge q) \wedge p)$



A) $(p \wedge q) \Rightarrow (r \wedge q)$

$= \sim (p \wedge q) \vee (r \wedge q)$



$\therefore$ is correct answer

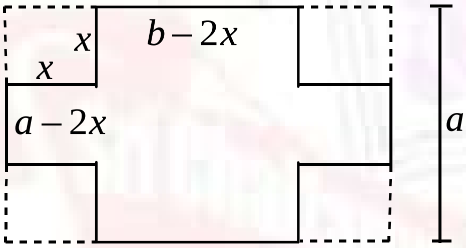
77. A box open from top is made from a rectangular sheet of dimension $a \times b$ by cutting square each of side x from each of the four corners and folding up the flaps. If the volume of the box is maximum, then x is equal to:

1) $\dfrac{a+b-\sqrt{a^2+b^2+ab}}{6}$ **2)** $\dfrac{a+b-\sqrt{a^2+b^2-ab}}{6}$

3) $\dfrac{a+b-\sqrt{a^2+b^2-ab}}{12}$ **4)** $\dfrac{a+b+\sqrt{a^2+b^2-ab}}{6}$

Key: 2

Solution:



Volume of cuboid $= (b-2x)(a-2x)x$

$\dfrac{dv}{dx} = \dfrac{d}{dx}\left(4x^3 - 2x^2(a+b) + abx\right)$

$= 12x^2 - 4x(a+b) + ab = 0$

$x = \dfrac{4(a+b) \pm \sqrt{[4(a+b)]^2 - 4(12)ab}}{24}$

$= \dfrac{(a+b) \pm \sqrt{a^2+b^2-ab}}{6}$

$$\frac{d^2v}{dx^2} = 24x - 4(a+b)$$

For $V_{\max} : \frac{d^2v}{dx^2} < 0$

$$\therefore x = \frac{a+b-\sqrt{a^2+b^2-ab}}{6}$$

$$A \cap B = \{(1,0)(1,1)(1,-1)\}$$

$$A \cap B = \{(1,1)\}$$

78. A different equation representing the family of parabolas with axis parallel to *y*-axis and whose length of latus rectum is the distance of the point $(2,-3)$ form the line $3x+4y=5$, is given by:

1) $11\frac{d^2y}{dx^2} = 10$ **2)** $10\frac{d^2y}{dx^2} = 11$ **3)** $10\frac{d^2x}{dy^2} = 11$ **4)** $11\frac{d^2x}{dy^2} = 10$

Key: 1

Solution:

Let $l = LLR = \frac{|2(3)+4(-3)-5|}{\sqrt{3^2+4^2}}$

$= \frac{11}{5}$

Let the vertex is (h,k)

Equation of parabola is

$$(x-h)^2 = \frac{11}{5}(y-k)$$

$$(y-k) = \frac{5}{11}(x-h)^2$$

$$\frac{dy}{dx} = \frac{5}{11}.2(x-h) = \frac{10x}{11} - \frac{5h}{11}$$

$$\frac{d^2y}{dx^2} = \frac{10}{11} \Rightarrow 11d^2y = 10dx^2$$

79. The equation of the plane passing through the line of intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k}) = 1$ and $\vec{r}.(2\hat{i}+3\hat{j}-\hat{k})+4=0$ and parallel to the *x*-axis is:

1) $\vec{r}.(\hat{i}+3\hat{k})+6=0$ **2)** $\vec{r}.(\hat{i}-3\hat{k})+6=0$

3) $\vec{r}.(\hat{j}-3\hat{k})-6=0$ **4)** $\vec{r}.(\hat{j}-3\hat{k})+6=0$

Key: 4

Solution:

$$x+y+z=1$$

$$2x+3y-z=-4$$

$$3x+4y=-3$$

Let $x=\lambda$

$$\therefore 3\lambda=-3-4y$$

$$\lambda=-\frac{(3+4y)}{3}$$

$$\Rightarrow y=\frac{-3}{4}(1+\lambda)$$

$$z=1-x-y=1-z+\frac{3}{4}(1+\lambda)$$

$$=\frac{7}{4}-\frac{\lambda}{4}$$

$$\lambda=7-4z$$

$$\frac{x-0}{1}=\frac{3+4y}{-3}=\frac{7-4z}{1}=\lambda$$

$$\frac{x-0}{1}=\frac{y+\frac{3}{4}}{\frac{-3}{4}}=\frac{z-\frac{7}{4}}{\frac{-1}{4}}$$

Equation of plane is given by

$$\begin{vmatrix} x & y+\frac{3}{4} & z-\frac{7}{4} \\ 1 & -\frac{3}{4} & -\frac{1}{4} \\ 1 & 0 & 0 \end{vmatrix}=0$$

$$-y+3z-6=0$$

$$y-3z+6=0$$

$$\therefore \vec{r}\left(\hat{j}-3\hat{k}\right)+6=0$$

80. The value of the integral $\int_0^1 \frac{\sqrt{x}dx}{(1+x)(1+3x)(3+x)}$ is:

1) $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{6}\right)$ **2)** $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{2}\right)$ **3)** $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{6}\right)$ **4)** $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$

Key: 4

Solution:

$$\int_0^1 \frac{\sqrt{x}\,dx}{(1+x)(1+3x)(3+x)}$$

$$px+\sqrt{x}=t$$

$$x=t^2$$

$$dx=2td\ t$$

$$\int_0^1 \frac{2t^2dt}{\left(1+t^2\right)\left(1+3t^2\right)\left(3+t^2\right)}$$

$$2\int_0^1 \frac{\left(1+t^2-1\right)dt}{\left(1+t^2\right)\left(1+3t^2\right)\left(3+t^2\right)}$$

$$=\int_0^1 \frac{2\,dt}{\left(1+3t^2\right)\left(3+t^2\right)}-\int_0^1 \frac{2dt}{\left(1+t^2\right)\left(1+3t^2\right)\left(3+t^2\right)}$$

$$=\int_0^1 \frac{2\,dt}{\left(1+3t^2\right)\left(3+t^2\right)}-\int_0^1 \frac{dt}{\left(1+3t^2\right)\left(1+t^2\right)}+\int_0^1 \frac{dt}{\left(1+3t^2\right)\left(3+t^2\right)}$$

$$=\int_0^1 \frac{3\,dt}{\left(1+3t^2\right)\left(3+t^2\right)}-\int_0^1 \frac{dt}{\left(1+3t^2\right)\left(1+t^2\right)}$$

$$=\frac{9}{8}\int_0^1 \frac{dt}{1+3t^2}-\frac{3}{8}\int_0^1 \frac{dt}{3+t^2}-\frac{3}{2}\int_0^1 \frac{1}{1+3t^2}+\frac{1}{2}\int_0^1 \frac{dt}{1+t^2}$$

$$=\frac{-3}{8}\int_0^1 \frac{dt}{1+3t^2}+\frac{1}{2}\int_0^1 \frac{dt}{1+t^2}-\frac{3}{8}\int_0^1 \frac{dt}{3+t^2}$$

$$=-\frac{\sqrt{3}}{8}\tan^{-1}\left(\sqrt{3}t\right)\Big|_0^1+\frac{1}{2}\tan^{-1}(t)\Big|_0^1-\frac{3}{8}\frac{1}{\sqrt{3}}Tan^{-1}\left(\frac{t}{\sqrt{3}}\right)$$

$$=\frac{\pi}{8}-\frac{\sqrt{3}}{8}\left(\frac{\pi}{2}\right)=\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$$

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place.
(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

81. The probability distribution of random variable X is given by:

X	1	2	3	4	5
P(X)	K	2K	2K	3K	K

Let $P = P\left(1 < X4 / X < 3\right)$. If $5P = \lambda K,$ then λ is equal to___________.

Key: 30

Solution:

Given probability distance is:

X	1	2	3	4	5
P(X)	K	2K	2K	3K	K

$Wkt \sum P\left(X\right) = 1$

$\therefore k + 2k + 2k + 3k + k = 1$

$\Rightarrow k = \frac{1}{9}$

Given $P = P\left(1 < x < 4 / x < 3\right)$

$= \frac{P\left(\left(1 < x < 4\right) \cap \left(x < 3\right)\right)}{P\left(x < 3\right)}$

$= \frac{P\left(\{2,3\} \cap \{1,2\}\right)}{P\left(\{1,2\}\right)}$

$= \frac{P\left(\{2\}\right)}{P\left(\{1,2\}\right)} = \frac{2k}{k + 2k} = \frac{2}{3}$

$\Rightarrow P = \frac{2}{3}$

$5P = \lambda K$

$5\left(\frac{2}{3}\right) = \lambda\left(\frac{1}{9}\right)$

$\Rightarrow \lambda = 30$

82. Let S be the mirror image of the point $Q(1,3,4)$ with respect to the plane $2x-y+z+3=0$ and let $R(3,5,\gamma)$ be a point this plane. Then the square of the length of line segment SR is_______.

Key: 72

Solution:

Apply reflection formula

(image)

$$\frac{h-1}{2}=\frac{k-3}{-1}=\frac{l-4}{1}=\frac{-2(\cancel{6})}{\cancel{6}}=-2$$

$\Rightarrow h=-3$

$k=5$

$l=2$

$\therefore S(-3,5,2)$

Put $R(3,5,\gamma)$ in plane $2x-y+z+3=0$

$\Rightarrow \gamma=-4$

83. Let $S=\{1,2,3,4,5,6,9\}$. Then the number elements in the set $T=\{A\subseteq S: A\neq\phi$ and the sum of all the elements of A is not a multiple of 3}is_____________.

Key: 80

Solution:

Total number of subsets whose sum of

The elements is divisible by 3 is 48

$\therefore$ no. of subsets whose sum of

The elements not divisible by 3 is 128 – 48 = 80

84. If $\int\frac{2e^{x}+3e^{-x}}{4e^{x}+7e^{-x}}dx=\frac{1}{14}\left(ux+v\log\left(4e^{x}+7e^{-x}\right)\right)+C,$ where C is a constant of integration then $u+v$ is equal to____________.

Key: 7

Solution:

$2e^{x}+3e^{-x}=l\left(4e^{x}+7e^{-x}\right)+m\left(\frac{d}{dy}\left(4e^{x}+7e^{-x}\right)\right)$ Numerator=u(Denominator)+k($\frac{d}{dy}$ denominator)

$2e^{x}+3e^{-x}=l\left(4e^{x}+e^{-x}\right)+m\left(4e^{x}-7e^{-x}\right)-(1)$

From e^{x} coefficient $\rightarrow\ l+m=\frac{1}{2}$ - (2)

e^{-x} coefficient $\rightarrow l-m=\frac{3}{4}$ - (3)

On solving (2) and (3) $l=\frac{13}{28},\ \ m=\frac{1}{28}$

$$\rightarrow\int\frac{2e^{x}+3e^{-x}}{4e^{x}+e^{-x}}=\int\frac{\frac{13}{28}\left(4e^{x}+7e^{-x}\right)+\frac{1}{28}\left(4e^{x}-7e^{-x}\right)}{4e^{x}+7e^{-x}}dx$$

$$=\frac{13}{28}\int dx+\frac{1}{28}\int\frac{4e^{x}-7e^{-x}}{4e^{x}+7e^{-x}}dx$$

$$=\frac{13}{28}x+\frac{1}{28}\log\left(4e^{x}+7e^{-x}\right)+c$$

$$=\frac{1}{14}\left[\frac{13}{2}x+\frac{1}{2}\log\left(4e^{x}+7e^{-x}\right)\right]$$

$u=\frac{13}{2}\quad v=\frac{1}{2}$

$u+v=7$

85. Let Z_1 and Z_2 be two complex numbers such that arg $\left(Z_1-Z_2\right)=\frac{\pi}{4}$ *and* Z_1,Z_2 satisfy the equation $|z-3|=\operatorname{Re}(z)$. The imaginary part z_1+z_2 is equal to__________.

Key: 6

Solution:

$1z-31=\operatorname{Re}(z)$

$(x-3)^2+y^2=x^2$

$y^2-6x+9=0$

$y_1^2-y_2^2=6\left(x_1-x_2\right)$

$\Rightarrow y_1+y_2=6$

86. Let S be the sum of all solutions (in radians) of the equation $\sin^4\theta+\cos^4\theta=0$ in $[0,4\pi]$.

Then $\frac{8S}{\pi}$ is equal to __________.

Key: 56

Solution:

$$\sin^4\theta+\cos^4\theta-\sin\theta\cos\theta=0$$

$$\left(\sin^2\theta\right)^2+\left(\cos^2\theta\right)^2+2\left(\sin^2\theta\cos^2\theta\right)-\sin\theta\cos\theta=2\sin^2\theta\cos^2\theta$$

$$\left(\sin^2\theta+\cos^2\theta\right)^2=\sin\theta\cos\theta+2\left(\sin\theta\cos\theta\right)^2$$

$$\therefore \sin\theta\cos\theta=-\frac{1\pm\sqrt{(1)^2+4(2)(1)}}{2(2)}$$

$$=-1\ or\ +\frac{1}{2}$$

$$\sin^2\theta=2\sin\theta\cos\theta\quad\left[-1,3\ rejected\right]$$

$$=2\left(\frac{1}{2}\right)$$

$$2\theta=n\pi+(+1)^n\left[\frac{\pi}{2}\right]$$

$$\theta=\frac{n\pi}{2}+(-1)^n\left[\frac{\pi}{4}\right]$$

$$\theta\in[0,4\pi]$$

Put n values , θ can be $\frac{\pi}{4},\frac{5\pi}{4},\frac{9\pi}{4},\frac{13\pi}{4}$

$$S=sum=\frac{\pi+5\pi+9\pi+13\pi}{4}$$

$$=7\pi$$

$$\therefore \frac{\not{8}}{\not{\pi}}=56$$

$$\therefore \frac{8\times5}{\pi}=56$$

87. Let A $(\sec\theta,2\tan\theta)$ and B $(\sec\phi,2\tan\phi)$, where $\theta+\phi=\pi/2,$ be two points on the hyperbola $2x^2-y^2=2.$ If (α,β) is the point of the intersection of the normals to the hyperbola at A and B, Then $(2\beta)^2$ is equal to_____________.

Key: 36

Solution:

Given point not lie on the hyperbola.

Comment: no key

88. An online exam is attempted by 50 candidates out of which 20are boys. The average marks obtained by boys is 12 with a variance2. The variance of marks obtained by 30 girls is also 2. The average marks of all 50 candidates is 15, If μ is the average marks of girls and σ^2 is the variance of marks of 50 candidates, then $\mu+\sigma^2$ is equal to______.

Key: 25

Solution:

Given,

No.of boys $=n_1=20$

Average marks of boys $=\overrightarrow{x_1}=12$

$\Rightarrow$Total marks of boys $=\left[\sum xi\right]_1=\overline{x_1}n=12\times 20$

$=240$

Variance of boys score $=\sigma_1^2=2$

Let sum of squares of scores of boys $=\left[\sum xi^2\right]_1$

$$\Rightarrow \sigma_1^2=\frac{\left[\sum xi^2\right]_1}{n}-\left(\overline{x_1}\right)^2$$

$$2=\frac{\left[\sum xi^2\right]_1}{20}-12^2$$

$\Rightarrow \sum xi^2=146\times 20=2920$ - (1)

Given,

Total candidates $=50=N$

Mean of square $=\overline{x}=15$

$\Rightarrow$Total score put together $=N\overline{x}=50\times 15=750$

Let total sum of girls score be $=\left[\sum xi\right]_2$

$\Rightarrow \left[\sum xi\right]_1+\left[\sum xi\right]_2=750$

$\Rightarrow 240+\left[\sum xi\right]_2=750 \Rightarrow \left[\sum xi\right]_2=510$

$\therefore$ Mean of girls score $=\overline{x_2}=\frac{\left[\sum xi\right]_2}{n_2}=\frac{51\varnothing}{3\varnothing}=17=\mu$

Variance of girls score $=\sigma_2^2=2$ [Given]

$\Rightarrow \sigma_2^2=\frac{\left[\sum xi^2\right]_2}{30}-(17)^2$

$(291)30=\left[\sum xi^2\right]_2$

$\Rightarrow \left[\sum xi^2\right]_2=8730$

$\therefore$ Total variance $=\sigma^2=\frac{\left[\sum xi^2\right]_1+\left[\sum xi^2\right]_2}{n}-\left(\overline{x}\right)^2$

$\Rightarrow \sigma^2=\frac{8730+2920}{50}-225 \Rightarrow 8=\sigma^2$

89. $3\times 7^{22}+2\times 10^{22}-44$ when divided by 18 leaves the remainder______________.

Key: 15

Solution:

$3\times(7)^{22}=3\times(6+1)^{22}$

$=3\times\left({}^{22}C_0 6^{22}+{}^{22}C_1 6^{21}+....+{}^{22}C_{21}6+{}^{22}C_{22}\right)$

$=18\times 6^2\times\ {}^{22}C_0+18\times{}^{22}C_1\times 6^{26}+.......+18\times{}^{22}C_{21}+3$

$=18n+3,$ where n is some cohole number

$2\times(10)^{22}=2\times(9+1)^{22}$

$=2\times\left({}^{22}C_0 9^{22}+{}^{22}C_1 9^{21}+.....+{}^{22}C_{21}\times 9+{}^{22}C_{22}\right)$

$=18\times 9^{21}\times{}^{22}C_0+18\times{}^{22}C_1\times 9^{20}+......+18\times{}^{22}C_{21}+2$

$=18p+2,$ where P is some cohole n

Required value is $3\times 7^{22}+2\times 10^{22}-44$

$=18n+3+18p+2-44$

$=18k+5-44$ (k is a where no)

$=18k+5-18\times 310$

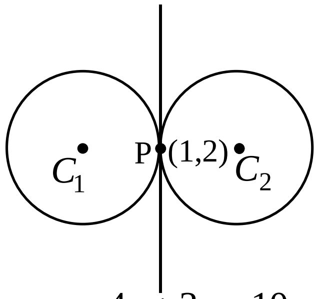
$=18(k-3)+15$

$\Rightarrow$ The required remainder is 15

90. Two circles each of radius 5 units touch each other at other at the point (1,2). If the equation of their common tangent is $4x+3y=10,$ and $C_1(\alpha,\beta)$ and $C_2(\gamma,\delta), C_1\neq C_2$ are their centres, then $\left|(\alpha+\beta)(\gamma+\delta)\right|$ is equal to______________.

Key: 40

Solution:



Given that,

$rC_1 = rC_2 = 5$ units

Equation of common tangent,

$l = 4x+3y=10$ at $P(1,2)$

$C_1(\alpha,\beta)$, $\qquad C_2 = (\gamma,\delta)$

Use know,

Foot of l^{lar} of C_2 on l is P

$$\Rightarrow \frac{1-8}{4} = \frac{2-\delta}{3} = \frac{-(48+3\delta-10)}{25} \quad - \ (1)$$

Also,

Distance $C_2 l = rC_2 = 5$

$$\Rightarrow \frac{|48+3\delta-10|}{\sqrt{25}} = 5$$

$$\Rightarrow 48+3\delta-10 = 5\times 5 = 25 \quad - \ (2)$$

Substituting (2) in (1), we get

$$\frac{1-8}{4} = \frac{2-\delta}{3} = \frac{-25}{25} = -1$$

$\Rightarrow \gamma = 5, \delta = 5 \qquad \therefore C_2 = (5,5)$

$\because C_1$ and C_2 meet at P, and $C_1P = C_2P = \gamma = 5$

P is mid point of C_1C_2

$$\Rightarrow (1,2) = \left[\frac{5+\alpha}{2}, \frac{5+\beta}{2}\right]$$

$\Rightarrow \alpha = -3, \beta = -1$

$\therefore |(\alpha+\beta)(\gamma+\delta)| = |(-4)(10)| = |-40| = 40$

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for securing a perfect score in **JEE Main 2021 (July)**, as per the NTA Results

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