



JEE MAIN 2021 PHASE - IV



Key & Solutions 31-Aug-2021 | Shift - 1

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A right Choice for the Real Aspirant

ICON Central Office – Madhapur – Hyderabad

Jee-Main_Final_31-August-2021_Shift-01

PHYSICS

Max Marks: 100

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 in all other cases.

1. In an ac circuit, an inductor, a capacitor and a resistor are connected in series with $X_L = R = X_C$. Impedance of this circuit is

1) zero	2) $2R^2$	3) R	4) $R\sqrt{2}$
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Key: 3

Sol:

- $Z = \sqrt{R^2 + (x_L x_c)^2}$ Given $X_L = R = X_c$ $Z = \sqrt{R^2}$ Z = R
- 2. A moving proton and electron has the same de-Broglie wavelength. If K and P denote the K.E. and momentum respectively. Then choose the correct option:

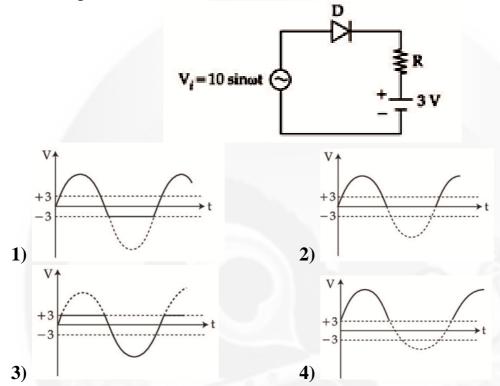
1) $K_p < K_e$ and $P_p = P_e$	2) $K_p = K_e$ and $P_p = P_e$
3) $K_p > K_e$ and $P_p = P_e$	4) $K_p < K_e$ and $P_p < P_e$

Sol:
$$K = \frac{P^2}{2m}$$

 $P^2 = 2mK$
 $\frac{h^2}{\lambda^2} = 2mK$
 $K = \frac{h^2}{2m\lambda^2}$
 $K\alpha \frac{1}{m} \Longrightarrow K_p < K_e$
 $P = \frac{h}{\lambda}$

 $P_p = P_e$ $h \rightarrow \text{constant}$ $\lambda \rightarrow \text{constant}$ $P \rightarrow \text{constant}$

3. Choose the correct waveform that can represent the voltage across R of the following circuit.



- Sol: Half wave rectifier (up to 3 volts D.C Domenats Ac so upto 3 volts diode is Reverse bias, from 3 volts diode is forwards bias
- 4. An object is placed at the focus of concave lens having focal length f. What is the magnification and distance of the image from the optical centre of the lens?

1)
$$1,\infty$$

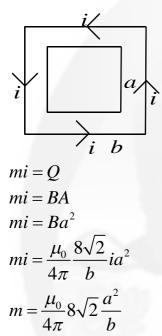
2) very high, ∞
3) $\frac{1}{4}, \frac{f}{4}$
4) $\frac{1}{2}, \frac{f}{2}$
Key: 4
Sol: $m = \frac{v}{u} = \frac{-f/2}{-f}$
 $= 1/2$
 $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$
 $-\frac{1}{f} = \frac{1}{v} + \frac{1}{f} \Rightarrow \frac{1}{v} = -\frac{1}{f} - \frac{1}{f}$
 $\frac{1}{v} = \frac{-2}{f}$

v = -f / 2

5. A small square loop of side 'a' and one turn is placed inside a larger square loop of side b and one turn (b>>a). The two loops are coplanar with their centres coninciding. If a current I is passed in the square loop of the side 'b', then the coefficient of mutual inductance between the two loops is

1)
$$\frac{\mu_0}{4\pi} 8\sqrt{2} \frac{a^2}{b}$$
 2) $\frac{\mu_0}{4\pi} 8\sqrt{2} \frac{b^2}{a}$ **3**) $\frac{\mu_0}{4\pi} \frac{8\sqrt{2}}{a}$ **4**) $\frac{\mu_0}{4\pi} \frac{8\sqrt{2}}{b}$

Key: 1 Sol:



6. For an ideal gas the instantaneous change in pressure 'p' with volume 'v' is given by the equation $\frac{dp}{dv} = -ap$. If $p = p_0$ at v = 0 is the given boundary condition, then the maximum temperature one mole of gas can attain is : (Here R is the gas constant) 1) $\frac{ap_0}{eR}$ 2) infinity 3) $0^{\circ}C$ 4) $\frac{p_0}{aeR}$

Sol:
$$\frac{ap}{dv} = -ap$$

 $\frac{dp}{p} = -adv$
 $ln\left(\frac{p}{p_o}\right) = -av$
 $\frac{p}{p_o} = e^{-av}$
 $p = p_o e^{-av}$
 $pv = nRT$

$$P_{o}ve^{-av} = nRT$$

$$P_{o}[ve^{-av}(-a) + e^{-av}(1)] = nR\frac{dT}{dv}$$

$$\frac{P_{o}e^{-av}(1-av)}{nR} = \frac{dT}{dv}$$

$$\frac{P_{o}e^{-av}(1-av)}{nR} = 0 \quad \left(\frac{dT}{dv} = 0\right)$$
Finally $T = \frac{P_{o}}{enaR}$

$$n = 1, T = \frac{P_{o}}{aeR}$$

- 7. Two particles A and B having charges $20 \ \mu C \ and -5\mu C$ respectively are held fixed with a separation of 5cm. At what position a third charged particle should be placed so that it does not experience a net electric force?
 - 1) At midpoint between two charges
 - **2)** At 5cm from 20 μ *C* on the left side of system
 - **3**) At 5 cm from $-5\mu C$ on the right side
 - **4**) At 1.25 cm from a $-5\mu C$ between two charges

Key: 3

20µc -5µc Q

Sol:

$$\begin{array}{l}
\overleftarrow{5cm} \rightarrow x \rightarrow \\
K \frac{20\mu c \times Q}{\left(\frac{5}{100} + x\right)^2} = K \frac{5\mu c \times Q}{x^2} \\
\frac{4}{\left(\frac{5}{100} + x\right)^2} = \frac{1}{x^2} \\
4x^2 = \left(\frac{5}{100} + x\right)^2 \\
2x = \frac{5}{100} + x \\
x = 5cm \\
At 5cm \text{ from - } 5\mu c \text{ on right side}
\end{array}$$

8. Angular momentum of a single particle moving with constant speed along circular parth

1) changes in magnitude but remains same in the direction

- 2) is zero
- 3) remains same in magnitude but changes in the direction
- 4) remains same in magnitude and direction

Key: 4

Sol: Angular momentum = $m(\vec{r} \times \vec{v})$ Here direction of angular momentum perpendicular

to the plane of \vec{r} and \vec{v} so it will remains constant magnitude of velocity is not changing so magnitude angular momentum will also remains constant

9. A uniform heavy rod of weight 10 kg ms^{-2} , cross-sectional area 100 cm^2 and length 20cm is hanging from a fixed support. Young modulus of the material of the rod is $2 \times 10^{11} Nm^{-2}$. Neglecting the lateral contraction, find the elongation of rod due to its own weight

List – II

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1) 5 \ge 10^{-8} m 2) 5 \ge 10^{-10} m 3) 4 \ge 10^{-8} m 4) 2 \ge 10^{-9} m
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Key: 2

Sol: $y = \frac{Mgl}{2Ae}$ $e = \frac{Mgl}{2Ay}$ $e = \frac{10 \times 0.2}{2 \times 10^{-2} \times 2 \times 10^{11}} = 5 \times 10^{-10} m$

10. Match List – I with List – II. List – I

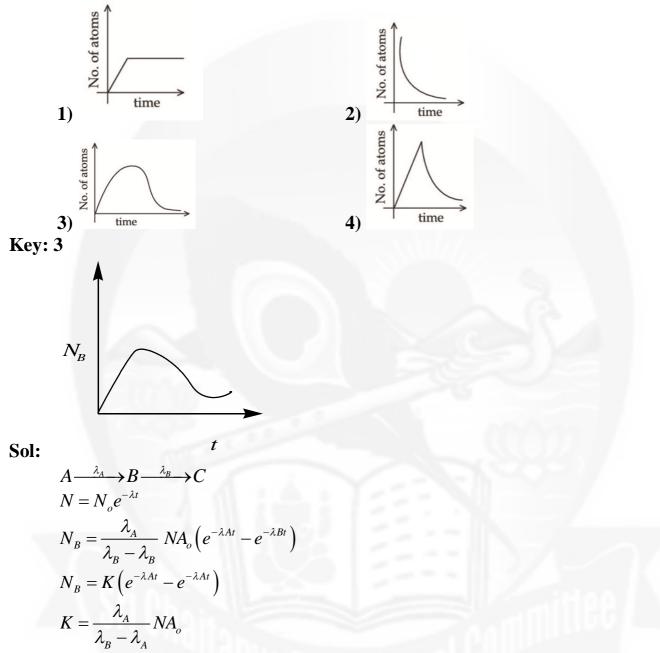
(a) Torque	$(\mathbf{i}) MLT^{-1}$
(b) Impulse	(ii) MT^{-2}
(c) Tension	(iii) ML^2T^{-2}
(d) Surface Tension	$(iv) MLT^2$

Choose the most appropriate answer from the option given below:

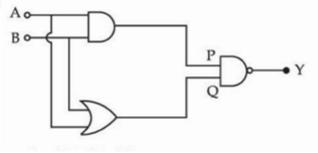
1) (a)–(iii), (b)-(iv), (c)-(i), (d)-(ii)	2) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)
3) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)	4) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

Sol:	a) Torque	$\vec{F \times r}$ [ML^2T^{-2}]
	b) Impulse	$J = Ft \left[MLT^{-1} \right]$
	c) Tension	$T = F = \left[MLT^{-2} \right]$
	d) Surface Tension $T = \frac{F}{L} = \left[MT^{-2}\right]$	

11. A sample of radioactive nucleus A disintegrates to another radioactive nucleus. B, which in turn disintegrates to some other stable nucleus C. Plot of a graph showing the variation of number of atoms of nucleus B versus time is : (Assume that at t = 0, there are no B atoms in the sample)



12. In the following logic circuit the sequence of the inputs A, B are (0, 0), (0, 1), (1, 0) and (1, 1). The output Y for this sequence will be:



Key: 4

- **Sol:** Boolean expression for given circuit $Y = \overline{(A.B).(A+B)}$ Check given values
- 13.Consider a galvanometer shunted with 5Ω resistance and 2% of current passes
through it. What is the resistance of the given galvanometer?
1) 245Ω 2) 300Ω 3) 344Ω 4) 226Ω

Key: 1

Sol:

$$S = \frac{G}{\frac{1}{i_g} - 1}$$

$$S = \frac{G}{\frac{100}{2} - 1} = \frac{G}{\frac{98}{2}} = \frac{G}{49}$$

$$G = 5 \times 49 = 245$$

14. Which of the following equations is dimensionally incorrect? Where t = time, h = height, s = surface tension, θ = angle, ρ = density, r = radius, acceleration due to gravity, v = volume, p = pressure, W = work done, Γ = torque, ϵ = permittivity, E = electric field, J = current density, L = length

1)
$$h = \frac{2s\cos\theta}{prg}$$
 2) $v = \frac{\pi pa^4}{8\eta L}$ **3**) $J = \in \frac{\partial E}{\partial t}$ **4**) $W = \Gamma \theta$

Key: 2

Sol: Poiseulilles equation

Rate of liquid flow $\left(\frac{V}{t}\right) = \frac{\pi \operatorname{Pr}^4}{8\eta l}$

But in the given equation $v = \frac{\pi \operatorname{Pr}^4}{8\eta l}$ There no time so it's dimensionally wrong

15. A coil having N turns is wound tightly in the form of a spiral with inner and outer radii 'a' and 'b' respectively. Find the magnetic field at centre, when a current I passes through coil

1)
$$\frac{\mu_0 I}{8} \left(\frac{a-b}{a+b} \right)$$
 2) $\frac{\mu_0 IN}{2(b-a)} \log_e \left(\frac{b}{a} \right)$

3)
$$\frac{\mu_0 I}{8} \left(\frac{a+b}{a-b} \right)$$
 4) $\frac{\mu_0 I}{4(a-b)} \left[\frac{1}{a} - \frac{1}{b} \right]$

Key: 2

Sol: $(b-a) \to N$

$$dx \to n = ?$$
$$n = \frac{N}{(b-a)}dx$$

Magnetic field by small element of radius x is

$$dB = \frac{\mu_o ni}{2x}$$
$$dB = \frac{\mu_o i}{2} \frac{N}{(b-a)} \frac{dx}{x}$$
$$Total field \qquad B = \frac{\mu_o N}{2}$$

Total field $B = \frac{\mu_o Ni}{2(b-a)} \int_a^b \frac{dx}{x}$

$$B = \frac{\mu_o Ni}{2(b-a)} \log(x)_a^b$$
$$B = \frac{\mu_o Ni}{2(b-a)} \log\left(\frac{b}{a}\right)$$

16. A helicopter is flying horizontally with a speed 'v' at an altitude 'h' has to drop a food packet for a man on the ground. What is the distance of helicopter from the man when the food packet is dropped?

1)
$$\sqrt{\frac{2v^2h}{g} + h^2}$$
 2) $\sqrt{\frac{2gh}{v^2}} + h^2$ **3**) $\sqrt{\frac{2ghv^2 + 1}{h^2}}$ **4**) $\sqrt{2ghv^2 + h^2}$

Sol:

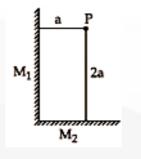
$$\frac{l_1}{d} = \sqrt{l_1^2 + \upsilon^2 t^2}$$

$$= \sqrt{l_1^2 + \frac{\upsilon^2 2h}{g}}$$

$$l_1 = \frac{1}{2}gt^2$$

$$t^2 = \frac{2h}{g}$$

17. Two plane mirrors M_1 and M_2 are at right angle to each other shown. A point source 'P' is placed at 'a' and '2a' meter away from M_1 and M_2 respectively. The shortest distance between the images thus formed is : $(Take \sqrt{5} = 2.3)$



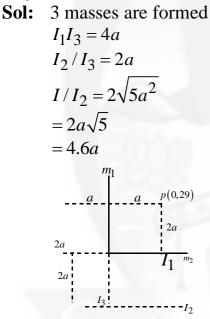
3) $2\sqrt{10}$ a

1) 4.6a

2) 2.3a

4) 3a

Key: 1



18. The masses and radii of the earth and moon are (M_1, R_1) and (M_2, R_2) respectively. Their centres are at a distance 'r' apart. Find the minimum escape velocity for a particle of mass 'm' to the projected from the middle of these two masses:

1)
$$V = \frac{1}{2}\sqrt{\frac{2G(M_1 + M_2)}{r}}$$

2) $V = \frac{\sqrt{2G(M_1 + M_2)}}{r}$
3) $V = \frac{1}{2}\sqrt{\frac{4G(M_1 + M_2)}{r}}$
4) $V = \sqrt{\frac{4G(M_1 + M_2)}{r}}$

Key: 4
Sol:
$$\frac{1}{2}m\upsilon^2 = \frac{GM_1m}{rh} + \frac{GM_2m}{rh}$$

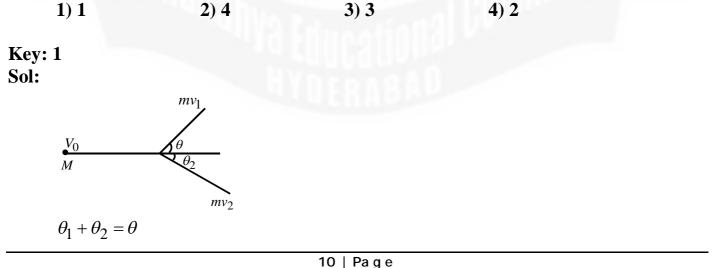
$$\upsilon = \sqrt{\frac{4G}{r} (M_1 + M_2)}$$

19. A reversible engine has an efficiency of ¹/₄. If the temperature of the sink is reduced by 58°C, its efficiency becomes double. Calculate the temperature of the sink:

1) $280^{\circ}C$ **2)** $174^{\circ}C$ **3)** $180.4^{\circ}C$ **4)** $382^{\circ}C$

Key: 2 Sol: $\eta_1 = \frac{T_1 - T_2}{T_1} = \frac{1}{4}$ (1) $\eta_2 = 2\eta_1 = \frac{1}{2} = \frac{T_1 - (T_2 - 58)}{T_1}$ $\frac{1}{2} = \frac{T_1 - T_2}{T_1} + \frac{58}{T_1}$ $\frac{1}{2} = \frac{1}{4} + \frac{58}{T_1} \Rightarrow \frac{58}{T_1} = \frac{1}{4}$ From (1) $T_1 = 232$ $\frac{1}{4} = \frac{232 - T_2}{232} \Rightarrow 232 - T_2 = \frac{232}{4}$ $T_2 = 232.58$ = 174

20. A body of mass M moving at speed V_0 collides elastically with a mass 'm' at rest. After the collision, the two masses move at angles θ_1 and θ_2 with respect to the initial direction of motion of the body of mass M. The largest possible value of the ratio M/m, for which the angles θ_1 and θ_2 will be equal, is:



Solving,
$$\sin \theta = \frac{M}{m}$$

For max $\theta = 90^{0}$
Value of $\sin \theta$
 $mv_{1} \sin \theta_{1} + mv_{2} \sin \theta_{2}$
 $= mv_{0}$ (1)
 $mv_{1} \sin \theta_{1} = mv_{2} \sin \theta_{2}$ (2)
 $\frac{1}{2}nv_{0}^{2} = \frac{1}{2}mv_{1}^{2} + \frac{1}{2}mv_{2}^{2}$
 $\left(\frac{M}{m}\right)_{max} = 1$

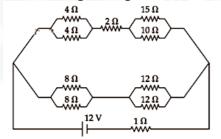
(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.

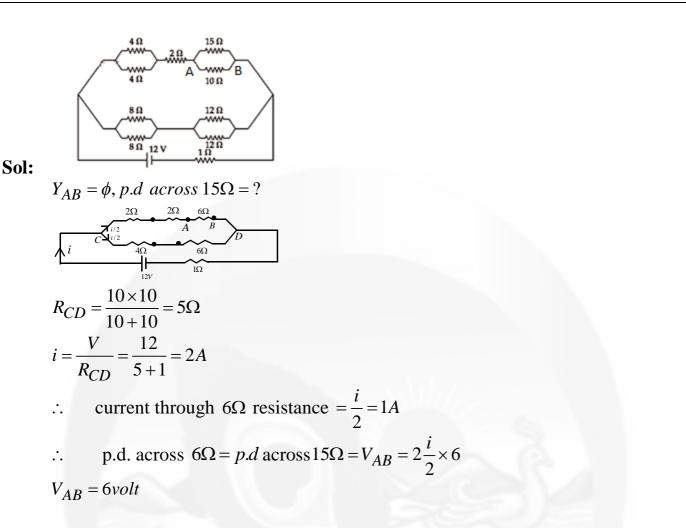
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. When a rubber ball is taken to a depth of ______ m in deep sea, its volume decreases by 0.5%. (The bulk modulus of rubber = 9.8 x 10⁸ NM⁻²) Density of sea water = 10³kgm⁻³ g = 9.8 m/s²) Key: 0500.00 Sol: $K = \frac{\Delta P}{-\Delta V} = \frac{h\rho g}{-\Delta V} \Rightarrow 9.8 \times 10^8 = \frac{h \times 10^3 \times 9.8}{-\left(\frac{-0.5}{100}\right)}$ h = 500m

22. The voltage drop across 15Ω resistance in the given figure will be _____ V.



Key: 0006.00



23. If the sum of the heights of transmitting and receiving antennas in the line of sight of communication is fixed at 160 m, then the maximum range of LOS communication is _____ km. (Take radius of Earth = 6400 km)

Key: 0064.00

Sol:
$$h_r + h_t = 160m, R = \sqrt{2Rh_t} + \sqrt{2Rh_t}$$

 $R = \sqrt{2Rh_t} + \sqrt{2R(160 - ht)}$
 $R = \sqrt{2R} \left(\sqrt{h_t} + \sqrt{160 - h_t}\right)$
 $R_{max}, \frac{dl^2}{dh_t} = 0 \Rightarrow \sqrt{2R} \left(\frac{1}{2\sqrt{ht}} + \frac{1}{2\sqrt{160 - ht}}x - 1\right) = 0$
 $\frac{1}{\sqrt{ht}} = \frac{1}{\sqrt{160 - ht}} \Rightarrow \sqrt{h_t} = \sqrt{160 - ht}$
 $h_t = 80, h_r = 80 \because h_t + h_r = 160m$
 $R_{max} = R = \sqrt{2R} \left(\sqrt{h_t} + \sqrt{h_r}\right) = \sqrt{2 \times 6400 \times 10^3} \left(\sqrt{80} + \sqrt{80}\right)$
 $R_{max} = 64000m = 64 \ km$

24. The electric field in an electromagnetic wave is given by $E = (50) \sin \omega (t - x/c) NC^{-1}$ The energy contained in a cylinder of volume V is 5.5 x $10^{-12} J$. The value of V is _____ cm^3 . (given $\epsilon_0 = 8.8 \times 10^{-12} C^2 N^{-1} m^{-2}$)

Key: 00500.00

Sol: $E = 5DS_n \omega (t - x/c)$ $= 5.5 \times 10^{-12} J =$ $5.5 \times 10^{-12} = \frac{1}{2} E_0 E_0^2 \times V$ $5.5 \times 15^{12} \frac{1}{2} \times 8.8 \times 10^{-12} \times (50 \times 50) \times V$ $V = 500 cm^3$

25. A particle of mass 1 kg is hanging a spring of force constant 100 Nm^{-1} . The mass is pulled slightly downward and released so that it executes free simple harmonic motion with time period T. the time when the kinetic energy and potential energy

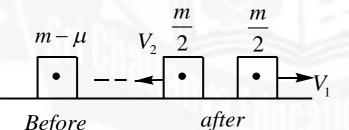
of the system will become equal, is $\frac{T}{x}$. The value of x is _____

Key: 0008.00

Sol: m=1kg, $m = 1kg, k = 100Nm^{-1}$, $E_T = U + K = 2K \therefore U = K$ $E_T = 2 \times \frac{1}{2} M V^2$ $Y = A \cos \omega t$ $V = \omega A \sin \omega t$ $\frac{1}{2}m\omega A^2 = m.\omega A^2 S n^2 \omega t$ $S_n wt = \frac{1}{\sqrt{2}} \Longrightarrow \omega t = \frac{\pi}{4} \Longrightarrow \frac{2\lambda}{7}, t = \frac{\lambda}{4}$ $t = \frac{T}{8} = \frac{T}{r} \Longrightarrow x = 8$

A block moving horizontally on a smooth surface with a speed of 40 ms^{-1} splits into 26. two equal parts. If one of the parts moves at 60 ms^{-1} in the same direction, then the fractional change in the kinetic energy will be x:4 where x is

Key: 0001.00



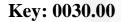
Sol:

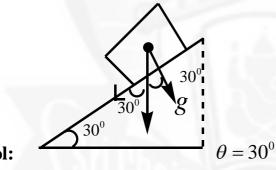
$$mu = m_1 v_1 + m_2 v_2$$
$$m(40) = \frac{m}{2}(60) - \frac{m}{2} V_2$$
$$V_2 = -20m/s$$

Before

$$\frac{\Delta K.B}{K} = \frac{R_f - K_i}{K_i} = \frac{\left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right) - \frac{1}{2}m\mu^2}{\frac{1}{2}m\mu^2}$$
$$= m_2 \left(\frac{V_1^2 + V_2^2 - 4^2}{\frac{m}{2}4^2}\right)$$
$$= \frac{\left(\frac{(60)^2 + (20)^2}{2}\right) - (40)^2}{(40)^2}$$
$$= \frac{\frac{3600 + 400}{2} - 1600}{\frac{1}{1600}}$$
$$= \frac{2000 - 1600}{1600} = \frac{400}{1600} = \frac{1}{4}$$
$$= \frac{x}{4} = \frac{1}{4}$$

27. A car is moving on a plane inclined at 30° to the horizontal with an acceleration of $10 \text{ } ms^{-2}$ parallel to the plane upward. A bob is suspended by a string from the roof of the car. The anlge in degrees which the string makes with the vertical is _____(Take $g = 10 \text{ } ms^{-2}$)





- Sol:
- 28. A wire having a linear mass density $9.0 \ge 10^{-1} \text{ kg/m}$ is stretched between two rigid supports with a tension of 900 N. The wire resonates at a frequency of 500 Hz. The next higher frequency at which the same wire resonates is 550 Hz. The length of the wire is _____ m

Key: 0010.00

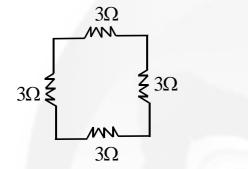
$$500 = \frac{P+1}{2l} \sqrt{\frac{T}{\mu}} \qquad (2)$$

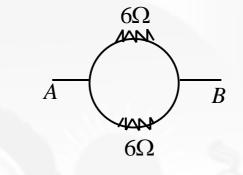
$$\frac{500}{550} = \frac{P-1}{P+1} \rightarrow 10P + 10 = 11P$$

$$P = 10$$

29. A square shaped wire with resistance of each side 3Ω is bent to form a complete circle. The resistance between two diametrically opposite points of the circle in unit of Ω will be _____







Sol:

$$R_{AB} = \frac{R_1 R_2}{R_1 + R_2} = \frac{6 \times 6}{6 + 6} = \frac{36}{12} = 3\Omega$$
$$R_{AB} = 3\Omega$$

30. A capacitor of 50 μF is connected in a circuit as shown in figure. The charge on the upper plate of the capacitor is _____ μC .

Key: 0100.00

Sol: potental diffrence acrm cpacity

$$V_{AB} = 2V$$

$$Q = C_V = 50 \times 10^{-6} \times 2 = 100 \mu C$$

$$i = \frac{V}{R} = \frac{6}{6 \times 10^3} = 10^{-3} A.$$

$$V_{AB} = 2V$$

$$Q = C_V = 50 \times 10^{-6} \times 2 = 100 \mu C$$

$$V_{AB} = i \times r = 10^{-2} \times 2 \times 10^{3} = 2V$$

$$Q = CV_{AB} = 50 \times 10^{-6} \times 2 = 100 \times 10^{-6} C$$

$$Q = 100 \mu C$$

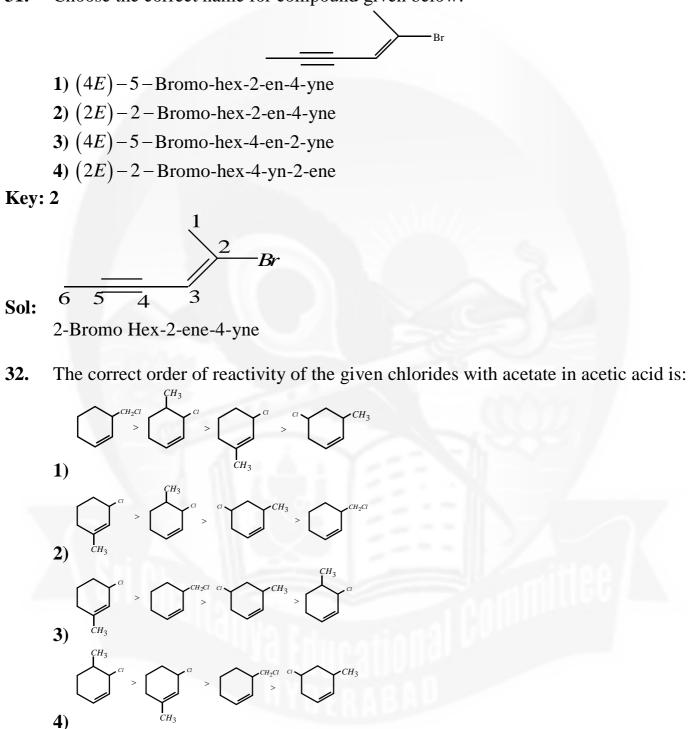
CHEMISTRY

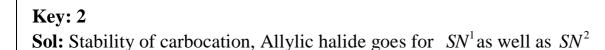
Max Marks: 100

(SINGLE CORRECT ANSWER TYPE)

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31. Choose the correct name for compound given below:





33. Given below are two statements: one is labeled as **Assertion** (**A**) and the other is labeled as **Reason** (**R**).

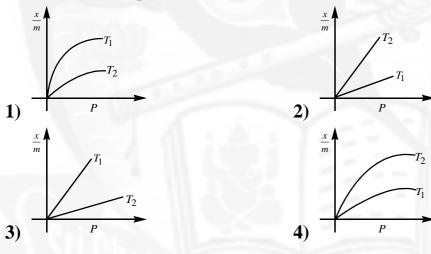
Assertion (A) : A simple distillation can be used to separate a mixture of propanol and propanone.

Reaction (**R**): Two liquids with a difference of more than $20^{\circ}C$ in their boiling points can be separated by simple distillations. In the light of the above statements, choose the most appropriate answer from the options given below:

- 1) (A) is false but (R) is true
- 2) (A) is true but (R) is false
- 3) Both (A) and (R) are correct but (R) is not the correct explanation of (A)
- 4) Both (A) (R) are correct and (R) is the correct explanation of (A).

Key: 4

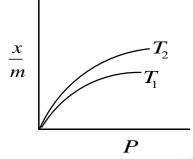
- **Sol:** Boiling point of propanol is 97[°]C Boiling point of propanone is 56[°]C The B.P difference is more, they can be separated by simple distillation.
- 34. Select the graph that correctly describes the adsorption isotherms at two temperatures T_1 and $T_2(T_1 > T_2)$ for a gas: (x mass of the gas adsorbed m mass of adsorbent P pressure)



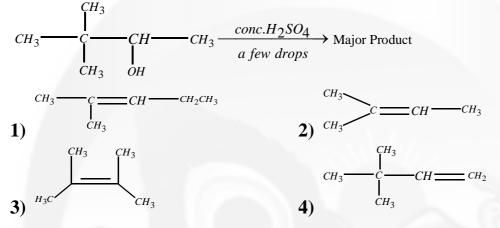
Key: 4

Sol: As Temperature increases $\frac{x}{m}$ value decreases

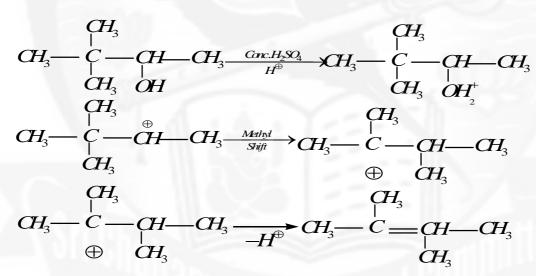
Since
$$T_1 > T_2$$
: $\frac{x_1}{m_1} < \frac{x_2}{m_2}$



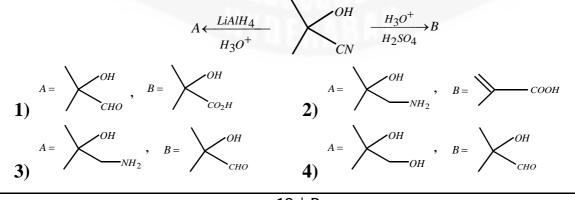
35. The major product formed in the following reaction is:



Key: 3 Sol:



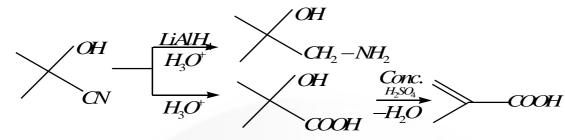
36. The major products A and Bin the following set of reactions are:



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Key: 2

Sol:



37. Given below are two statements: one is labeled as Assertion (A) and the other is labeled as Reason (R).

Assertion (A) : Metallic character decreases and non-metallic character increases on moving from left to right in a period.

Reason (\mathbf{R}) : It is due to increase in ionization enthalpy and decrease in electron gain enthalpy, when one moves from left to right in a period.

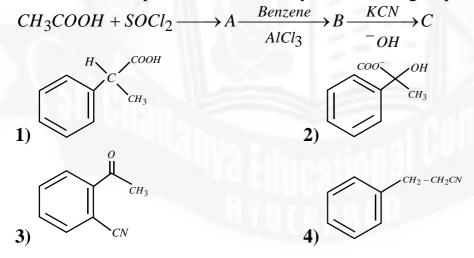
In the light of the above statements, choose the most appropriate answer from the options given below:

1) (A) is false but (R) is true

- 2) Both (A) and (R) are correct but (R) is not the correct explanation of (A)
- **3**) (A) is true but (R) is false
- 4) Both (A) and (R) are correct and (R) is the correct explanation of (A).

Key: 3

- **Sol:** On moving from left to right $I.E \uparrow$ and $E.A \uparrow$
- **38.** The structure of product C, formed by the following sequence of reactions is: Benzene = KCN



$$C_{H_{3}}-COOH+SOO_{2}\xrightarrow{P_{y}}{-SO_{2}}CH_{3}-COOI$$

$$OH$$

$$AIO_{3}|Benzene$$

$$C_{6}H_{5}-C-CH_{3}$$

$$C_{6}H_{5}-CO-CH_{3}+HO$$

$$OH$$

$$AIO_{3}|Benzene$$

$$C_{6}H_{5}-CO-CH_{3}+HO$$

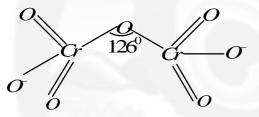
Sol:

39. In the structure of the dichromate ion, there is a:

- 1) non-linear unsymmetrical Cr O Cr bond
- 2) non-linear symmetrical Cr O Cr bond
- **3**) linear symmetrical Cr O Cr bond
- 4) linear unsymmetrical Cr O Cr bond

Key: 2

Sol: In $Cr_2O_7^{2-}$ ion two tetrahedral units are shared by on bridge oxygen atom



It is non linear and symmetrical

40. Which one of the following lanthanides exhibits +2 oxidation state with diamagnetic nature? (Given Z for Nd = 60, Yb = 70, La = 57, Ce = 58)

Key: 4

Sol: $yb^{+2} - [xe]4f^{14}$ - Dia magnetic $La^{+2} - [xe]5d^{1}$ Pare magnetic $Nd^{+2} - [xe]4f^{4}$ Pare magnetic $Ce^{+2} - [xe]4f^{2}$ Pare magnetic

41. Which one of the following 0.10*M* aqueous solutions will exhibit the largest freezing point depression?
1) glycine 2) glucose 3) hydrazine 4) *KHSO*₄

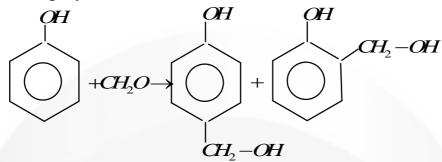
- **Sol:** $\Delta T_f = i \times k_f \times m$ $\Delta T_f \alpha i$ For *KHSO*₄; i = 2 For remaining Compounds i=1
- 42. Monomer of Novolac is:
 1) phenol and melamine
 2) 1,3 –Butadiene and styrene
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3) *o* – Hydroxymethylphenol

Key: 3

Sol: O – hydroxyl metyl phenol

Novolac is a co-polymer of Phenol + HCHO resin



43. Which one of the following compounds contains $\beta - C_1 - C_4$ glycosidic linkage?1) Maltose2) Lactose3) Amylose4) Sucrose

Key: 2

Sol: Lactose contains $C_1\beta - C_4\beta$ Glyco sidic linkage.

44. BOD values (in ppm) for clean water (A) and polluted water (B) are expected respectively as:

1) A < 5, B > 17 **2**) A > 25, B < 17 **3**) A > 15, B < 47 **4**) A > 50, B < 27

Key: 1

- Sol:For Clean waterBOD < 5PPM</th>Polluted waterBOD > 17PPM
- **45.** Given below are two statements: one is labeled as Assertion (A) and the other is labeled as Reaction (R).

Assertion (A): Aluminium is extracted from bauxite by the electrolysis of molten mixture of Al_2O_3 with cryolite

Reason (**R**): The oxidation state of Al in cryolite is +3.

In the light of the above statements, choose the most appropriate answer from the options given below:

1) (A) is true but (R) is false

- 2) (A) is false but (R) is true
- **3**) Both (A) and (R) are correct and (R) is the correct explanation of (A)
- 4) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

Key: 4

Sol: In the extraction of aluminium Electrolyte is Al_2O_3 + cryolite + Fluorospar Cryolite is added to Al_2O_3 Decrease M.P & Increase Electrical Conductivity Cryolite - Na_3AlF_6 [O.S of Al = +3]

 46. The denticity of an organic ligand, biuret is:
 1) 3
 2) 6
 3) 4
 4) 2

Key: 4

Sol:

$$\begin{array}{c} O & O \\ \parallel & \parallel \\ NH_2 - C - NH - C - NH_2 \text{ (Biuret)} \end{array}$$

It's denticity is two.

47. Given below are two statements:

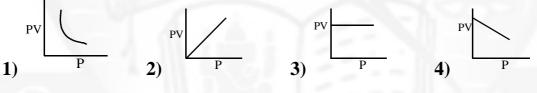
Statement I: The process of producing syn-gas is called gasification of coal **Statement II:** The composition of syn-gas is $CO + CO_2 + H_2(1:1:1)$

In the light of the above statements, choose the most appropriate answer from the options given below:

- 1) Both Statement I and Statement II are false
- 2) Statement I is false but Statement II is true
- 3) Statement I is true but Statement II is false
- 4) Both Statement I and Statement II is true

Key: 3

- Sol: Sun gas is mainly a mixture of CO (40-50%) and Hydrogen (45 50%)
- **48.** Which one of the following is the correct PV vs P plot at constant temperature for an ideal gas? (P and V stand for pressure and volume of the gas respectively)



Key: 3

- Sol: According to boylels law pv = constant at any pressure pv is constant. So pv versus p we get a straight line parallel to x axis.
- 49. The major component/ingredient of Portland Cement is:
 1) tricalcium aluminate
 3) dicalcium aluminate
 4) dicalcium silicate

Key: 2

- Sol: Major ingredient of Portland cement is tricalcium silicate (51%).
- **50.** Given below are two statements: one is labeled as Assertion (A) and the other is labeled as Reason (R).

Assertion (A): Treatment of bromine water with propene yields 1-bromopropan-2-ol.

Reason (R): Attack of water on bromonium ion follows Markovnikov rule and results in 1-bromopropan-2-ol.

In the light of the above statements, choose the most appropriate answer from the options given below:

- 1) (A) is true but (R) is false
- 2) Both (A) and (R) are true and (R) is the correct explanation of (A)
- 3) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)
- 4) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

Key: 2 $CH_3 - CH = CH_2 - \frac{Br - Br}{Br - Br}$ $H_2^{\gamma}O$ $CH_3 - CH - CH_2$ Sol: 1-bromo propan 2 - ol

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10. Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

 A_3B_2 is a sparingly soluble salt of molar mass $M(gmol^{-1})$ and solubility $x g L^{-1}$. 51.

The solubility product satisfies $K_{sp} = a \left(\frac{x}{M}\right)^5$. The value of *a* is_____. (Integer

answer)

Key: 0108.00

Sol:
$$A_{3}B_{2(3)} \longrightarrow 3A_{(aq)}^{+2} + 2B_{(aq)}^{-3}$$

 $3S \quad 2S$
 $K_{sp} = \left[A^{+2}\right]^{3} \left[B^{-3}\right]^{2}$
 $= (3S)^{3}(2S)^{2} = 108S^{5}$
Solubility(s) $= \frac{x}{m} \times \frac{1}{1(L)} = \frac{x}{M} \text{ mole / } L$

$$K_{sp} = 108 \left(\frac{x}{m}\right)^5 \Longrightarrow K_{sp} = a \left(\frac{x}{m}\right)^5$$
$$a = 108$$

52. The total number of reagents from those given below, that can convert nitrobenzene into aniline is_____. (Integer answer)

I. Sn - HClII. $Sn - NH_4OH$ III. Fe - HClIV. Zn - HClV. $H_2 - Pd$ VI. H_2 – Raney Nickel

- Key: 0004.00
- Sol: Total no of Reagents from

$$\xrightarrow{\text{reduced}} \xrightarrow{\text{reduced}} \overbrace{\bigcirc}^{\text{reduced}}$$

 $Sn/BCl \rightarrow$ completely reduced $Fe/HCl \rightarrow$ completely reduced $H_2 - Pd \rightarrow$ completely reduced $H_2 - Raney - Nickel - completely reduced$

53. The number of halogen/(s) forming halic (V) acid is_____

Key: 0003.00

Sol: no of halogens forming halic (V) acids

$$XO_{3}^{-} = x + 3(-2) = -1$$

$$x = 5 \qquad 0.5$$

$$ClO_{3}^{-}, BrO_{3}^{-}, IO_{3}^{-}$$

$$Cd_{(s)} + HgSO_{4(s)} + \frac{9}{5}H_{2}O_{(\ell)} \rightleftharpoons CdSO_{4}\frac{9}{5}H_{2}O_{(s)} + 2$$

54. Consider the following cell reaction

$$Cd_{(s)} + Hg_2SO_{4(s)} + \frac{9}{5}H_2O_{(l)} \rightleftharpoons CdSO_4 \cdot \frac{9}{5}H_2O_{(s)} + 2Hg_{(l)}.$$

The value of E_{cell}^0 is 4.315 V at $25^0 C$. If $\Delta H^0 = -825.2 \text{ kJ mol}^{-1}$, the standard entropy change ΔS^0 in $J K^{-1}$ is _____. (Nearest integer)

[Given : Faraday constant = 96487 $C mol^{-1}$]

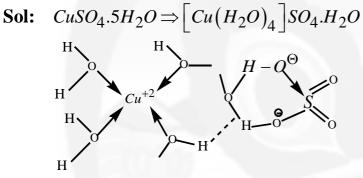
Key: 0024.83

Sol:
$$\Delta G^0 = nFE_{cell}^0$$

 $\Delta G^0 = -2 \times 96487 \times 4.314$
 $\Delta G^0 = -832,487.836 J = -8.32 \times 10^5 J$
 $= -8.32 \times 10^2 KJ$
 $\Delta G^0 = OH^0 - T\Delta S^0$
 $-8.32 \times 10^2 = -825 KJ - 298 \times \Delta S^0$
 $-8.32 \times 10^2 + 8.25 \times 10^2 - 298S^0$
 $\Delta S^0 = 24.83J$

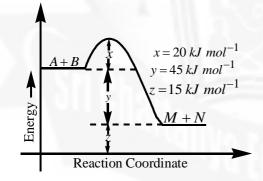
55. The number of hydrogen bonded water molecule(s) associated with stoichiometry $CuSO_4.5H_2O$ is_____.



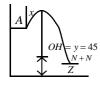


One H_2O form H – bond

56. According to the following figure, the magnitude of the enthalpy change of the reaction $A + B \rightarrow M + N$ in $kJ \mod^{-1}$ is equal to_____. (Integer answer)



Key: 0045.00 Sol: Magnitude of Enthalpy $OH_{R_{\chi n}} = E_P - E_R$



- 57. The molarity of the solution prepared by dissolving 6.3 g of oxalic acid $(H_2C_2O_4.2H_2O)$ in 250 mL of water in $mol L^{-1}$ is $x \times 10^{-2}$. The value of x is______. (Nearest integer) [Atomic mass: H:1.0, C:120, O:16.0] Key: 0020.00
- Sol: $M = \frac{wt}{mwt} \times \frac{1000}{V(ml)} = \frac{6.3}{126} \times \frac{1000}{250}$ = 0.2 $x \times 10^{-2} = 20 \times 10^{-2}$ x = 20
- **58.** For a first order reaction, the ratio of the time for 75% completion of a reaction to the time for 50% completion is_____. (Integer answer)

Sol:
$$T = \frac{2.303}{K} \log \frac{a}{a-x} =$$

$$T_{75\%} = \frac{2.303}{K} \log \frac{100}{100-50} = \frac{2.303}{K} \log \frac{100}{25}$$

$$= \frac{2.303}{K} \log 4$$

$$T_{50\%} = \frac{2.303}{K} \log \frac{100}{100-50} = \frac{2.303}{K} \log \frac{100}{50}$$

$$= \frac{2.303}{K} \log 2$$

$$\frac{T_{75\%}}{R_{5\%}} = \frac{\frac{2.303}{K} \log \log 4}{\frac{2.303}{K} \log 2} = 2$$

59. Ge(Z=32) in its ground state electronic configuration has x completely filled orbitals with $m_l = 0$. The value of x is_____.

Key: 0007.00

Sol: Ge(Z=32)=?

Completely filled orbitals with $m\ell = 0$

60. Consider the sulphides HgS, PbS, CuS, Sb_2S_3 , As_2S_3 and CdS. Number of these sulphides soluble in 50% HNO_3 is _____.

Key: 0005.00

Sol: 1) CdS, PbS, AS₂S₃, Bi₂S₃ & CuS are soluble in 50% HOO₃
2) HgS is insoluble in 50% HOO₃ but soluble in aqua rogia

MATHEMATICS

Max Marks: 100

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 in all other cases.

If the following system of linear equations **61**. 2x + y + z = 5x - y + z = 3x + y + az = bhas no solution, then: **1**) $a \neq \frac{1}{3}, b = \frac{7}{3}$ **2**) $a \neq -\frac{1}{3}, b = \frac{7}{3}$ **3**) $a = \frac{1}{3}, b \neq \frac{7}{3}$ **4**) $a = -\frac{1}{3}, b \neq \frac{7}{3}$ Key: 3 **Sol:** $AD \sim \begin{bmatrix} 2 & 1 & 1 & 5 \\ 1 & -1 & 1 & 3 \\ 1 & 1 & a & b \end{bmatrix}$ $R_2 \rightarrow 2R_2 - R_1, R_3 \rightarrow 2R_3 - R_1$ 0 1 2a - 1 2b - 5 $R_3 \rightarrow 3R_3 + R_2$ $0 \quad 0 \quad 6a - 2 \quad 6b - 14$ Given system of equations & has no solution $R(A) \neq R(AD)$ $6a - 2 = 0 \& 6b - 14 \neq 0$ $\therefore a = \frac{1}{3}, b \neq \frac{7}{3}$

62. Which of the following is not correct for relation R on the set of real numbers? 1) $(x, y) \in R \Leftrightarrow 0 < |x - y| \le 1$ is symmetric and transitive.

2) $(x, y) \in R \Leftrightarrow |x - y| \le 1$ is reflexive and symmetric.

3) $(x, y) \in R \Leftrightarrow 0 < |x| - |y| \le 1$ is neither transitive nor symmetric.

4) $(x, y) \in R \Leftrightarrow |x| - |y| \le 1$ is reflexive but not symmetric.

Key: 2

Sol: Reflexive $Let(x,x) \in R \Rightarrow |x-x| = 0 < 1$ is satisfied $\therefore R \text{ is reflexive}$ $Cet(x,y) \in R \Leftrightarrow |x-y| \le 1$ $Let \Leftrightarrow |y-x| \le 1$ $\Leftrightarrow (y-x) \in R$ <u>Transitive</u> :- Let $(x, y), (y, z) \in R$ $\Leftrightarrow |x - y| \le 1, |y - z| \le 1$ $\Rightarrow |x - z| = |x - y + y - z| \le |x - y| + |y - z| \le 2$ x = 0.1: y = 0.9: z = 1.6 $\Rightarrow |x - y| = 0.8 < 1$ $\Rightarrow |y - z| = 0.7 < 1$ $\Rightarrow |y - z| = 1.5 > 1$ $\Rightarrow (x, z) \notin R$ ∴ Ris not transitive

63. $\operatorname{cosec} 18^{0}$ is a root of the equation: 1) $x^{2} + 2x - 4 = 0$ 3) $x^{2} - 2x + 4 = 0$ (2) $4x^{2} + 2x - 1 = 0$ (3) $x^{2} - 2x + 4 = 0$ (4) $x^{2} - 2x - 4 = 0$

Key: 4

- Sol: One root $\cos ec18^{0} = \sqrt{5} + 1 = \alpha$ Other root $-\sqrt{5} + 1 = \beta$ Required quadratic eqn is $x^{2} - (\alpha + \beta)x + \alpha\beta = 0$ $x^{2} - 2x - 4 = 0$
- 64. The length of the latus rectum of a parabola, whose vertex and focus are on the positive x axis at a distance R and S (>R) respectively from the origin, is: 1) 4 (S-R) 2) 4(S+R) 3) 2(S-R) 4) 2(S+R)

Key: 1

a = 0S - 0A

Sol:
$$a = S - R$$

L.L.R = 4*a* = 4(*S* - *R*)

65. If $a_r = \cos \frac{2r\pi}{9} + i \sin \frac{2r\pi}{9}$, $r = 1, 2, 3..., i = \sqrt{-1}$, Then the determinant $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is equal to: 1) a_5 2) a_5 3) $a_1a_9 - a_3a_7$ 4) $a_2a_6 - a_4a_8$

Key: 3

Sol: If
$$a_r = \cos \frac{2r\pi}{9} + i \sin \frac{2r\pi}{9}$$
, $r = 1, 2, 3, \dots$

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$$:: \begin{vmatrix} a_{1} & a_{2} & a_{3} \\ a_{4} & a_{5} & a_{6} \\ a_{7} & a_{8} & a_{9} \end{vmatrix} = \begin{vmatrix} e^{i\frac{2\pi}{9}} & e^{i\frac{4\pi}{9}} & e^{i\frac{6\pi}{9}} \\ e^{i\frac{8\pi}{9}} & e^{i\frac{10\pi}{9}} & e^{i\frac{12\pi}{9}} \\ e^{i\frac{4\pi}{9}} & e^{i\frac{16\pi}{9}} & e^{i\frac{18\pi}{9}} \\ e^{i\frac{4\pi}{9}} & e^{i\frac{6\pi}{9}} & e^{i\frac{8\pi}{9}} \end{vmatrix}$$
$$= e^{i\frac{2\pi}{9}} \left(e^{i\frac{20\pi}{9}} - e^{i\frac{20\pi}{9}} \right) + e^{i\frac{4\pi}{9}} \left(e^{i\frac{26\pi}{9}} - e^{i\frac{26\pi}{9}} \right)$$
$$+ e^{i\frac{6\pi}{9}} \left(e^{i\frac{24\pi}{9}} - e^{i\frac{24\pi}{9}} \right) = 0$$

now $a_1 a_9 - a_3 a_7$ = $e^{i\left(\frac{2\pi}{9}\right)} \cdot e^{i\left(\frac{18\pi}{9}\right)} - e^{i\left(\frac{6\pi}{9}\right)} \cdot e^{i\left(\frac{14\pi}{9}\right)}$ = $e^{i\left(\frac{20\pi}{9}\right)} - e^{i\left(\frac{20\pi}{9}\right)} = 0$

66. The function $f(x) = |x^2 - 2x - 3|$. $e^{|9x^2 - 12x + 4|}$ is not differentiable at exactly: 1) One Point 2) Four Points 3) Two Points 4) Three Points

Sol:
$$f(x) = |x^2 - 2x - 3| \cdot e^{|9x^2 - 12x + 4|}$$

 $f(x) = |x + 1| |x - 3| e^{(3x - 2)^2}$
 e^x is differentiable at every where
 $|x - a|$ is not differentiable at $x = a$
 f is not differentiable at $x = -1, 3$
 \therefore Number of non differentiable points is 2

67. If the function
$$f(x) = \begin{cases} \frac{1}{x} \log_e \left(\frac{1 + \frac{x}{a}}{1 - \frac{x}{b}} \right), & x < 0 \\ k, & x = 0 \\ \frac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2 + 1} - 1}, & x > 0 \end{cases}$$

is continuous at $x = 0$, then $\frac{1}{a} + \frac{1}{b} + \frac{4}{k}$ is equal to

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4) 5

Key: 1

Sol: left continuous
$$= \lim_{x \to 0^-} \frac{1}{x} \log \left(\frac{1 + \frac{x}{a}}{1 - \frac{x}{b}} \right)$$

 $= \lim_{x \to 0^-} \frac{\log \left(1 + \frac{x}{a} \right) - \log \left(1 - \frac{x}{b} \right)}{x}$
Using *L*, *H* rule $\lim_{x \to 0^+} \frac{\left(\frac{1}{1 + \frac{x}{a}} \right)^{\frac{1}{a}} + \frac{1}{\left(1 - \frac{x}{b} \right)} \left(\frac{1}{b} \right)}{1} = \frac{1}{a} + \frac{1}{b}$
Right continuous $\lim_{x \to 0^+} \frac{\cos^2 x - \sin^2 - 1}{\sqrt{x^2 + 1} - 1}$
 $\lim_{x \to 0^+} \frac{\cos 2x - 1}{\sqrt{x^2 + 1} - 1} \times \frac{\sqrt{x^2 + 1} + 1}{\sqrt{x^2 + 1} + 1}$
 $\lim_{x \to 0^+} \frac{-2\sin^2 x}{x^2 + 1 - 1} \cdot \lim_{x \to 0^+} \sqrt{x^2 + 1} + 1$
 $-2(2) = -4$
F is continuous at $x = 0 \Rightarrow LHL = RHL = f(0)$
 $k = f(0) = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = -4 \Rightarrow \frac{1}{a} + \frac{1}{b} = -4$
 $\frac{1}{a} + \frac{1}{b} + \frac{4}{k} = -4 - 1 = -5$

68. Let \vec{a} and \vec{b} be two vectors such that $|2\vec{a}+3\vec{b}| = |3\vec{a}+\vec{b}|$ and the angle between \vec{a} and \vec{b} is 60° . If $\frac{1}{8}\vec{a}$ is a unit vector, then $|\vec{b}|$ is equal to : 1) 6 2) 8 3) 4 4) 5

Key: 4
Sol:
$$|a| = 8$$

 $|2\overline{a} + 3\overline{b}| = |3\overline{a} + \overline{b}|$
S.o.B.S &
 $\Rightarrow 4|\overline{a}|^2 + 9|\overline{b}|^2 + 12\overline{a}.\overline{b} = 9|\overline{a}|^2 + |\overline{b}|^2 + 6\overline{a}.\overline{b}$
 $\Rightarrow 5|\overline{a}|^2 - 8|\overline{b}|^2 - 6.(\overline{a}.\overline{b}) = 0$

$$\Rightarrow 5|\overline{a}|^{2} - 8|\overline{b}|^{2} - 6|\overline{a}||5| \times \frac{1}{2} = 0$$

$$\Rightarrow 5(64) - 8|\overline{b}|^{2} - 24|\overline{b}| = 0$$

$$\Rightarrow 8|\overline{b}|^{2} + 24|\overline{b}| - 320 = 0$$

$$\Rightarrow |\overline{b}|^{2} + 3|\overline{b}| - 40 = 0$$

$$\Rightarrow |\overline{b}| = \frac{-3 \pm \sqrt{9 + 160}}{2} = \frac{-3 \pm 13}{2} = 5, -8 \therefore |\overline{b}| = 5$$

$$\therefore |b| = \frac{10}{2} = 5$$

69. Let f be a non-negative function in [0, 1] and twice differentiable in (0, 1). If $\int_{0}^{x} \sqrt{1 - (f^{1}(t))^{2}} dt = \int_{0}^{x} f(t) dt, 0 \le x \le 1 \text{ and } f(0) = 0, \text{ then } \lim_{x \to 0} \frac{1}{x^{2}} \int_{0}^{x} f(t) dt :$ 1) equals $\frac{1}{2}$ 2) equals 0 3) equals 1 4) does not exist

Key: 1
Sol:
$$:: \int_{0}^{x} \sqrt{1 - (F^{1}(t))^{2}} dt = \int_{0}^{x} F(t) dt$$

Diff w.r to x
 $\sqrt{1 - (F^{1}(x))^{2}} = F(x)$
 $\Rightarrow 1 - (F^{1}(x))^{2} = (F(x))^{2}$
 $\Rightarrow (F^{1}(x))^{2} = 1 - (F(x))^{2}$
 $\Rightarrow F^{1}(x) = \sqrt{1 - (F(x))^{2}}$
 $\frac{dy}{dx} = \sqrt{1 - y^{2}}$
By integration $\sin^{-1} y = x \Rightarrow y = \sin x$
Now $\lim_{x \to 0} \frac{1}{x^{2}} \int_{0}^{x} F(t) dt = \lim_{x \to 0} \frac{1}{x^{2}} \int_{0}^{x} \sin t dt$
 $\lim_{x \to 10} \frac{1 - \cos x}{x^{2}} = \frac{1}{2}$
70. The number of real roots of the equation $e^{4x} + 2e^{3x} - e^{x} - 6 = 0$ is:
 $1) 0$ 2) 4 3) 1 4) 2
Key: 3
Sol: Ut $e^{x} = t$

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 $t^{4} + 2t^{3} - t - 6 = 0, t > 0$ $t^{4} + 2t^{3} = t + 6$ Let g(t) = t + 6 g(t) = 1 > 0g is increasing $F(t) = t^{4} + 2t^{3}$ $F^{1}(t) = 4t^{3} + 6t^{2}$ = 2t(2t + 3) > 0F is increasing The number of roots is one

71. Three numbers are in an increasing geometric progression with common ration r. If the middle number is doubled, then the new numbers are in an arithmetic progression with common difference d. If the fourth term of GP is $3 r^2$, then r^2 -d is equal to :

1) $7 + 3\sqrt{3}$ 2) $7 - 7\sqrt{3}$ 3) $7 - \sqrt{3}$ 4) $7 + \sqrt{3}$

Sol:
$$\frac{a}{r}, a, ar, 3r^2 \rightarrow \text{in G.P}$$

 $\frac{a}{r}, 2a, ar, ar^2 \rightarrow \text{in AP}$
 $\Rightarrow ar^2 = 3r^2$
 $\Rightarrow a = 3$
 $\Rightarrow 4a = ar + \frac{a}{r}$
 $\Rightarrow 4 = r + \frac{1}{r}$
 $\Rightarrow r^2 + 1 = 4r$
 $\Rightarrow r^2 - 4r + 1 = 0$
 $\Rightarrow r = \frac{4 \pm \sqrt{12}}{2}$
 $r = 2 + \sqrt{3}, 2 - \sqrt{3} \text{ (rejected)}$
 $\therefore d = ar - 2a$
 $= a(r - 2)$
 $= a(2 + \sqrt{3} - 2)$
 $= 3\sqrt{3}(a = 3)$
 $\therefore r^2 - d = (2 + \sqrt{3})^2 - 3\sqrt{3}$

$$= 4 + 3 + 4\sqrt{3} - 3\sqrt{3}$$
$$= 7 + \sqrt{3}$$

72.
$$\lim_{x \to 0} \frac{\sin^2(\pi \cos^4 x)}{x^4} \text{ is equal to :}$$

1) 4π 2) $4\pi^2$ 3) $2\pi^2$ 4) π^2

Key: 2

Sol:
$$= \lim_{x \to 0} \frac{\sin^{2} \left(\pi \left(1 - \sin^{2} x \right)^{2} \right)}{x^{4}}$$
$$= \lim_{x \to 0} \frac{\sin^{2} \left(\pi \left(1 + \sin^{4} x - 2\sin^{2} x \right) \right)}{x^{4}}$$
$$= \lim_{x \to 0} \frac{\sin^{2} \left[\pi - \pi \left(2\sin^{2} x - \sin^{4} x \right) \right]}{x^{4}}$$
$$= \lim_{x \to 0} \frac{\sin^{2} \left[\pi (2\sin^{2} x - \sin^{4} x) \right]}{x^{4}}$$
$$= \lim_{x \to 0} \left[\frac{\sin \left[\pi \left(2\sin^{2} x - \sin^{4} x \right) \right]}{\pi \left(2\sin^{2} x - \sin^{4} x \right)} \right]^{2} \cdot \frac{\pi^{2} \left(2\sin^{2} x - \sin^{4} x \right)^{2}}{x4}$$
$$= \pi^{2} \lim_{x \to 0} \left(\frac{\sin x}{x} \right)^{4} \left(2 - \sin^{2} x \right)^{2}$$
$$\therefore \pi^{2} \left(2 - 0^{2} \right) = 4\pi^{2}$$

73. If
$$\frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y}$$
, $y(0) = 1$, then $y(1)$ is equal to :
1) $\log_2(1+e^2)$ 2) $\log_2(2+e)$ 3) $\log_2(2e)$ 4) $\log_2(1+e)$

Sol:
$$\frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y}$$
$$= \frac{2^x (2^y - 1)}{2^y}$$
$$\int \frac{2^y dy}{2^y - 1} = \int 2^x dx$$
Let
$$2^y \log 2 \, dy = dt$$
$$2^y \log 2 \, dy = dt$$

$$\int \frac{dt}{t} \frac{1}{\log 2} = \int 2^x dx$$

$$\frac{\log t}{\log 2} = \frac{2^x}{\log 2} + \frac{c}{\log 2}$$

$$\log t = 2^x + c$$

When $x = 0, y = 1, c = -1$

$$\Rightarrow \log (2^y - 1) = 2^x - 1$$

$$2^y - 1 = e^{2^x - 1}$$

$$2^y = e^{2^x - 1} + 1$$

if $x = 1, 2^y = e + 1 \Rightarrow y = \log_2(e + 1)at x = 1$
i.e $f(1) = \log_2(e + 1)$

74. The sum of 10 terms of the series $\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \dots is$:1) 12) $\frac{143}{144}$ 3) $\frac{120}{121}$ 4) $\frac{99}{100}$

Key: 3

Sol:
$$\sum_{r=1}^{10} \frac{2r+1}{r^2 (r+1)^2} = \sum_{r=1}^{10} \frac{(r+1)^2 - r^2}{r^2 (r+1)^2}$$
$$\sum_{r=1}^{10} \frac{1}{r^2} - \frac{1}{(r+1)^2}$$
$$= \left(\frac{1}{1^2} - \frac{1}{2^2}\right) + \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \dots - \left(\frac{1}{10^2} - \frac{1}{11^2}\right)$$
$$= 1 - \frac{1}{121}$$
$$= \frac{121 - 1}{121}$$
$$= \frac{120}{121}$$

75. If p and q are the lengths of the perpendiculars from the origin on the lines, $x \operatorname{cosec} \alpha - y \sec \alpha = k \cot 2\alpha$ and $x \sin a + y \cos \alpha = k \sin 2\alpha$ respectively, then k^2 is equal to 1) $4p^2 + q^2$ 2) $p^2 + 2q^2$ 3) $p^2 + 4q^2$ 4) $2p^2 + q^2$

Key: 1

Sol: $x coace\alpha - y \sec \alpha = k \cot 2\alpha$

Perpendicular Distance from origin

$$P = \frac{1k \cot 2\alpha}{\sqrt{conec^2 \alpha} + \sec^2 \alpha}$$

= $\frac{k \cot 2\alpha}{\sqrt{\frac{1}{\sin^2 \alpha} + \frac{1}{\cos^2 \alpha}}}$
= $\frac{(k \cot 2\alpha) \sin \alpha \cos \alpha}{1}$
= $\frac{(k \cot 2\alpha) \sin \alpha \cos \alpha}{1}$
 $P = \frac{k}{2} \left(\frac{\cos 2\alpha}{\sin 2\alpha}\right) (\sin 2\alpha)$
 $2p = k \cos 2\alpha$
 $x \sin \alpha + y \cos \alpha = k \sin 2\alpha$
Perpendicular distance form origin
 $q = \frac{|k \sin 2\alpha|}{\sqrt{\sin^2 \alpha + \cos^2 \alpha}}$
 $q = k \sin 2\alpha$
 $1^2 + 2^2 \Rightarrow 4p^2 + q^2 = k^2 (\sin^2 2\alpha + \sin^2 2\alpha) = k^2 (1)$
 $\therefore 4p^2 + q^2 = k^2$

76. Let *, $\Box \in \{\land,\lor\}$ be such that the Boolean expression $(p * negation q) \Rightarrow (p \Box q)$ is a tautology 1) *= $\lor,\Box = \lor$ 2) *= $\land,\Box = \land$ 3) *= $\land,\Box = \lor$ 4) *= $\lor,\Box = \land$

Key: 3 Sol: Verification

77. A vertical pole fixed to the horizontal ground is divided in the ratio 3:7 by a mark on it with lower part shorter then the upper part. If the two parts subtend equal angles at a point on the ground 18m away from the base of the pole, then the height of the pole (in meters) is :

1)
$$12\sqrt{10}$$
 2) $6\sqrt{10}$ 3) $8\sqrt{10}$ 4) $12\sqrt{15}$
Key: 1
Sol:
 $7x$
 $3x$
 $7x$
 $3x$
 18
 $Tan\alpha = \frac{3x}{18}$
 $\tan 2\alpha = \frac{10x}{18}$

 $\frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{5x}{9}$ $=\frac{x}{6}$ $\frac{2\left(\frac{x}{6}\right)}{1-\frac{x^2}{36}} = \frac{5x}{9}$ $\Rightarrow x = \frac{6\sqrt{2}}{\sqrt{5}}$ Height = $10x = 12\sqrt{10}$ -----(1) Let the equation of the plane, that passes through the point (1,4,-3) and contains the **78**. line of intersection of the planes 3x - 2y + 4z - 7 = 0 and x + 5y - 2z + 9 = 0, be $\alpha x + \beta y + \gamma z + 3 = 0$ then $\alpha + \beta + \gamma$ is equal to : 1) 23 3) -23 4) - 152) 15 Key: 3 Sol: equation of required plane is $(3x-2y+4z-7) + \lambda(x+5y-2z+9) = 0) - \dots (1)$ It passing through (1,4,-3) $\Rightarrow (3 - 8 - 12 - 7) + \lambda (1 + 20 + 6 + 9) = 0$ \Rightarrow 36 $\lambda = 2y$ $\Rightarrow \lambda = 2/3$ Subsuiting λ value in 1 $\Rightarrow (3x-2y+4z-7) + \frac{2}{3}(x+5y-2z+9) = 0$ 11x + 4y + 8z - 3 = 0 $\Rightarrow -11x - 4y - 8z + 3 = 0$ $\alpha + \beta + \gamma = -11 - 4 - 8$ = -23The line $12x\cos\theta + 5y\sin\theta = 60$ is tangent to which of the following curves? **79**. 1) $25x^2 + 12y^2 = 3600$ 2) $x^2 + y^2 = 169$ 3) $x^2 + y^2 = 60$ 4) $144x^2 + 25y^2 = 3600$ Key: 4 Equation of tangent at $P(\theta)$ to ellipse is $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$ Sol: Given $\frac{x\cos\theta}{5} + \frac{\sin\theta}{12} = 1$ a = 5, b = 12Equation of ellipse is $\frac{x^2}{x^2} + \frac{y^2}{y^2} = 1$

$$25 \quad 144 \\ \implies 144x^2 + 25y^2 = 3600$$

80. The integral
$$\int \frac{1}{\sqrt{(x-1)^3(x+2)^5}} dx$$
 is equal to :
1) $\frac{4}{3} \left(\frac{x-1}{x+2} \right)^{\frac{1}{4}} + C$ 2) $\frac{4}{3} \left(\frac{x-1}{x+2} \right)^{\frac{5}{4}} + C$ 3) $\frac{3}{4} \left(\frac{x+2}{x-1} \right)^{\frac{1}{4}} + C$ 4) $\frac{3}{4} \left(\frac{x+2}{x-1} \right)^{\frac{5}{4}} + C$

Key: 1
Sol:
$$\int \frac{1}{\left(\frac{x+1}{x+2}\right)^{3/4} \cdot (x+2)^2} dx$$

Put $\frac{x-1}{x+2} = t$
 $\Rightarrow \frac{(x+2)-(x-1)}{(x+2)^2} dx = dt$
 $= \int \frac{1}{t^{3/4}} \cdot \frac{dt}{3}$
 $= \frac{1}{3} \cdot \frac{t^{1/4}}{\frac{1}{4}} + C$
 $= \frac{4}{3} \cdot \left(\frac{x-1}{x+2}\right)^{\frac{1}{4}} + C$

Varia 1

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

81. Let [t] denote the greatest integer $\leq t$. Then the value of 8. $\int ([2x]+[x]) dx$

is____

Key: 0005.00

Sol:
$$8 \int_{-1/2}^{1} ([2x] + 1x1) dx$$
$$= 8 \int_{-1/2}^{1} [2x] dx + 8 \int_{-1/2}^{1} |x| dx$$
Put $2x = t \Rightarrow dx = \frac{1}{2} dt = \lim ts : -1/2, 1$ New : -1,2
$$= 8 \int_{-1}^{2} [t] \cdot \frac{1}{2} dt + 8 \left[\int_{-1/2}^{0} (-x) dx + \int_{0}^{1} x dx \right]$$

$$=4\left(\int_{-1}^{0} (-1)dt + \int_{0}^{1} (0)dt + \int_{1}^{2} (1)dt\right)$$
$$+8\left(-\frac{x^{2}}{2}\right)_{-1/2}^{0} + \left(\frac{x^{2}}{2}\right)_{0}^{1}$$
$$=4\left(-x\right)_{-1}^{0} + 0 + \left(x\right)_{1}^{2} + 8\left(0 + \frac{1}{8} + \frac{1}{2}\right)$$
$$=4\left(0 - 1 + (2 - 1)\right) + (1 + 4) = 0 + 5 = 5$$

82. An electric instrument consists of two units. Each unit must function independently for the instrument to operate. The probability that the first unit functions is 0.9 and that of the second unit is0.8. The instrument is switched on and it fails to operate. If the probability that only the first unit failed and second unit is functioning is p, then 98 p is equal to _____

Key: 0028.00

Sol:
$$P(I) = 0.9$$
, $P(II) = 0.8$
 $P(\bar{I}) = 0.1$, $P(\bar{II}) = 0.2$
IF. II P IP.IIF IF II F
 $0.1 \ge 0.8$ $0.9 \ge 0.2$ $0.1 \ge 0.2$
 $=0.08$ 0.18 0.02
F: FAIR, P: PASS
 $P = \frac{0.08}{0.08 + 0.18 + 0.02} = \frac{0.08}{0.28} = \frac{8}{28} = \frac{2}{7}$
Now, $98P = 98 \times \frac{2}{7} = 28$

83. If
$$x\phi(x) = \int_{5}^{5} (3t^2 - 2\phi'(t)) dt$$
, $x > -2$, and $\phi(0) = 4$, then $\phi(2)$ is

Key: 0004.00

Sol:
$$x\phi(x) = \int_{5}^{x} (3t^2 - 2\phi^1(t))dt, x > -2 \text{ and } \theta(0) = 4$$

Diff w.r.t x
 $\Rightarrow x\phi^1(x) + \phi(x) = 3x^2 - 2\phi^1(x)$
 $\Rightarrow (x+2)\phi^1(x) + \phi(x) = 3x^2$
 $\Rightarrow (x+2)\frac{dy}{dx} + y = 3x^2$
 $\Rightarrow \frac{dy}{dx} + \frac{1}{x+2}y = \frac{3x^2}{x+2}$
I.F $e^{\int \frac{1}{x+2}dx} = e^{\log(x+2)} = x+2$
Solutions,

$$y(x+2) = \int (x+2) \cdot \frac{3x^2}{(x+2)} dx + c$$

$$\Rightarrow y(x+2) = 3 \cdot \frac{x^3}{3} + C$$

$$\phi(0) = 4 \Rightarrow 4(0+2) = O + C \Rightarrow C = 8$$

$$y(x+2) = x^3 + 8 \Rightarrow y = \frac{x^3 + 8}{x+2}$$

$$\phi(x) = \frac{x^3 + 8}{x+2}$$

$$\phi(2) = \frac{8+8}{2+2} = \frac{16}{4} = 4$$

84. The number of six letter words (with or without meaning),formed using all the letters of the word 'VOWELS', so that all the consonants never come together, is

Key: 0576.00

Sol: Vowels

V,O,E , LOUGONENTH : W, Z, S The no of arrangements that all the consonants never come together = Total – all the consonants come together = $6!-(4!)\times(3)!=576$

85. If the variable line $3x + 4y = \alpha$ lies between the two circles $(x-1)^2 + (y-1)^2 = 1$ and $(x-9)^2 + (y-1)^2 = 4$, without intercepting a chord on either circle, then the sum of all the integral values of α is ______

Key: 0165.00

Sol:
$$(x+1)^2 + (y-1)^2 = 12$$

 $3x+4y-\alpha=0$
 $r \le d$
 $\Rightarrow \alpha -7 \le -5(or)\alpha -7 \ge 5$
 $\alpha \le 2 \text{ or } \alpha \ge 12 \dots (1)$
 $(x-9)^2 + (y-1)^2 = 2^2$
 $3x+4y-\alpha=0$
 $r \le d$
 $|31-\alpha|\ge 10$
 $\alpha \le 21$ (or) $\alpha \ge 41$
 $\alpha \in [12,21]$
Sum of integral values $= 12+13+\dots+21$
 $= 165$

86. If 'R' is the least value of 'a' such that the function $f(x) = x^2 + ax + 1$ increasing on [1,2] and 'S' is the greatest value of 'a' such that the function $f(x) = x^2 + ax + 1$ is decreasing on [1,2], then the value of |R - S| is_____

Key: 0002.00

Sol: Given
$$f(x) = x^2 + ax + 1, x \in [1,2]$$

w.k.T f' $(x) = 2x + a$
given $1 \le x \le 2 \Rightarrow 2 \le 2x \le 4$
 $\Rightarrow -4 \le -2x \le -2$
 $f(x)$ is increasing on $[1,2]$
 $\Rightarrow f'(x) > 0$
 $\Rightarrow 2x + a > 0 \Rightarrow a > -2x$
 $\therefore R = \text{Least value of } a = -2$
 $f(x)$ is decreasing on $[1,2]$
 $\Rightarrow f'(x) < 0$
 $\Rightarrow 2x + a < 0$
 $\Rightarrow 2x + a < 0$
 $\Rightarrow a < -2x$
 $\therefore S = \text{greatest value of } a = -4$
Now $|R - S| = |-2 + 4| = 2$

87. A point z moves in the complex plane such that $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$, then the minimum

value of
$$\left|z - 9\sqrt{2} - 2i\right|^2$$
 is equal to

Key: 0098.00

Sol:
$$z = x + iy$$

Given $\operatorname{ang} \left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$
 $\Rightarrow \quad Tan^{-1} \left(\frac{y}{x-2}\right) - Tan^{-1} \left(\frac{y}{x+2}\right) = \frac{\pi}{4}$
 $\Rightarrow \quad Tan^{-1} \left(\frac{\frac{y}{x-2} - \frac{y}{x+2}}{1 + \frac{y}{x-2} \cdot \frac{y}{x+2}}\right) = \frac{\pi}{4}$
 $\Rightarrow \quad y(x+2) - y(x-2) = (x^2 - 4) + y^2$
 $\Rightarrow \quad 4y = x^2 + y^2 - 4$

 $x^{2} + y^{2} - 4y - 4 = 0$ $\Rightarrow x^2 + y^2 - 4y + 4 = 8$ $x^{2} + (y-2)^{2} = (2\sqrt{2})^{2}$ \Rightarrow circle with centre (0,2) \Rightarrow Radius $2\sqrt{2}$ Locus of z is $|z-2i|=2\sqrt{2}$ Consider $\left|z-9\sqrt{2}-2i\right|$ $\geq \left| z - 2i \right| - 9\sqrt{2} \right|$ $\geq \left| 2\sqrt{2} - 9\sqrt{2} \right|$ $\geq \left|-7\sqrt{2}\right|$ $\geq 7\sqrt{2}$ $\therefore \qquad \left|z - 9\sqrt{2} - 2i\right|^2 \ge \left(7\sqrt{2}\right)^2 \ge 49 \times 2$ ≥ 98

Minimum value of $\left|z - 9\sqrt{2} - 2i\right|^2 = 98$

88. The mean of 10 numbers $7 \times 8,10 \times 10,13 \times 12,16 \times 14,...$ is

Key: 0398.00

- Sol: The mean of 10 numbers $7 \times 8,10 \times 10,13 \times 12,16 \times 14,\dots, \text{is}$ $= \frac{7(8) + 10(10) + 13(12) + 16(14) + \dots}{10}$ $= \frac{1}{10} \sum_{r=1}^{10} (3r+4)(2r+6)$ $= \frac{1}{10} \sum_{r=1}^{10} (6r^2 + 26r + 24)$ $= \frac{1}{10} [2310 + 1430 + 240]$ $= \frac{3980}{10} = 398$
- 89. The square of the distance of the point of intersection of the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6}$ and the plane 2x - y + z = 6 from the point (-1, -1, 2) is _____

Key: 0061.00

Key: 0055.00

Sol: Given
$$\left(\frac{x}{4} - \frac{12}{x^2}\right)^{12}$$

Indebent terms $\rightarrow r = \frac{np}{p+q}$
 $r = \frac{(12)(1)}{1+2} = \frac{12}{3} = 4$
Also given $12_{C_4} \cdot \left(\frac{1}{4}\right)^8 (-12)^4 = \frac{3^6}{4^4} K$
 $\Rightarrow \frac{12 \times 11 \times 10 \times 9}{1 \times 2 \times 3 \times 4} \times \frac{3^4 \times 4^4}{4^8} = \frac{3^6}{4^4} \cdot K$
 $\Rightarrow 55 \times 3^2 \times 3^4 = 3^6 \times K$
 $\Rightarrow K = 55$

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