Sri Chaitanya

JEE MAIN 2021

PHASE - IV

Key & Solutions

01-Sep-2021 | Shift - 2

Sri Chaitanya IIT Academy., India.

A.P, TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

A right Choice for the Real Aspirant

ICON Central Office – Madhapur – Hyderabad

Jee-Main_Final_1-September-2021_Shift-02

PHYSICS **Max Marks: 100**

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. Two resistors $R_1=(4\pm0.8)\Omega$ and $R_2=(4\pm0.4)\Omega$ are connected in parallel. The equivalent resistance of their parallel combination will be:

1) $(4\pm0.3)\Omega$ 2) $(4\pm0.4)\Omega$ 3) $(2\pm0.3)\Omega$ 4) $(2\pm0.4)\Omega$

Key: 3

Sol: $\frac{1}{R_1}+\frac{1}{R_2}=\frac{1}{R_q}$

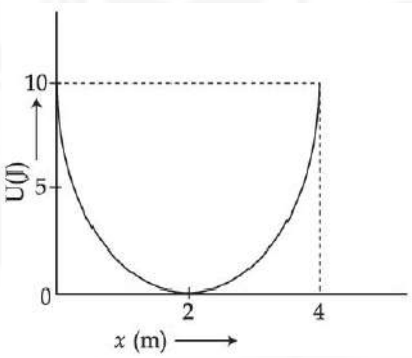
$$\frac{-dR_1}{R_1^2}-\frac{dR_2}{R_2^2}=\frac{-dR}{R_q^2}$$

$$\frac{0.8}{4^2}+\frac{0.4}{4^2}=\frac{dR}{2^2} \qquad R_q=2$$

$$dR=0.3$$

$$R_q=(2\pm0.3)$$

2. A mass of 5 kg is connected to a spring. The potential energy curve of the simple harmonic motion executed by the system is shown in the figure. A simple pendulum of length 4 m has the same period of oscillation as the spring system. What is the value of acceleration due to gravity on the planet where these experiments are performed?



1) $10m/s^2$ 2) $9.8m/s^2$ 3) $4m/s^2$ 4) $5m/s^2$

Key: 2

Sol: A = 2, m = 5 $PE_m=10$

$$10=\frac{1}{2}mw^2A^2 \quad w=1$$

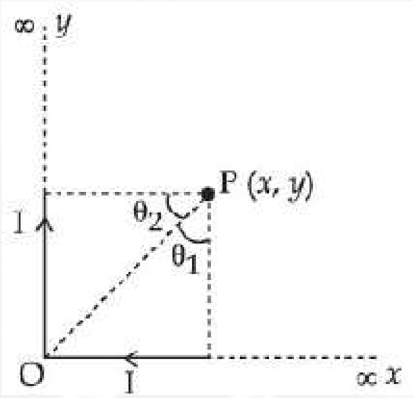
$T = \frac{2\pi}{w} = 2\pi$ ____(1)

$T_{spend} = 2\pi\sqrt{\frac{L}{g}}$ ___(2)

From (1) & (2)

$L = g = 9.8$

3. There are two infinitely long straight current carrying conductors and they are help at right angles to each other so that their common ends meet at the origin as shown in the figure given below. The ration of current in the both conductors is 1 : 1. The magnetic field at point P is _________

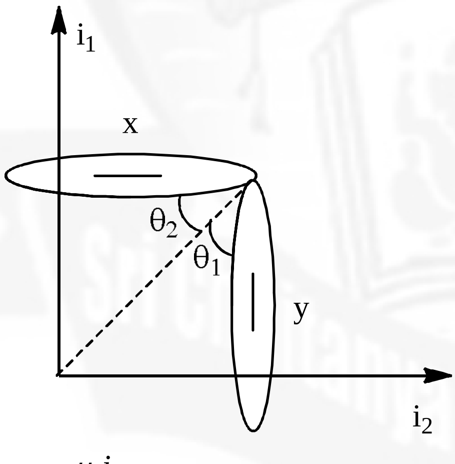


1) $\frac{\mu_0 I}{4\pi xy}\left[\sqrt{x^2+y^2}-(x+y)\right]$ 2) $\frac{\mu_0 Ixy}{4\pi}\left[\sqrt{x^2+y^2}+(x+y)\right]$

3) $\frac{\mu_0 Ixy}{4\pi}\left[\sqrt{x^2+y^2}-(x+y)\right]$ 4) $\frac{\mu_0 I}{4\pi xy}\left[\sqrt{x^2+y^2}+(x+y)\right]$

Key: 4

Sol:



$$B_1 = \frac{\mu_0 i}{4\pi x}(\sin\theta_2 + 1)$$

$$B_2 = \frac{\mu_0 i}{4\pi y}(\sin\theta_1 + 1)$$

$$B = B_1 + B_2 = \frac{\mu_0 i}{4\pi xy}\left(\sqrt{x^2+y^2} + x + y\right)$$

4. The temperature of an ideal gas in 3 – dimensions is 300 K. The corresponding de-Broglie wavelength of the electron approximately at 300 K, is

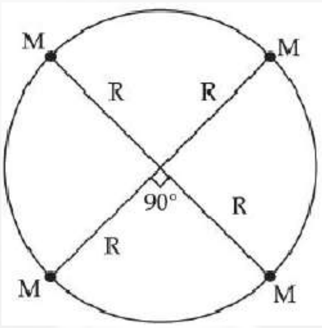
[m_e =mass of electron $=9\times10^{-31}kg$ h = Planck constant $=6.6\times10^{-34}\,J\ s$ $k_B=$ Boltzmann constant $=1.38\times10^{23}\,JK^{-1}$]

1) 6.26 nm 2) 2.26 nm 3) 3.25 nm 4) 8.46 nm

Key: 1

Sol: $\lambda=\dfrac{h}{\sqrt{3mkT}}=6.26nm$

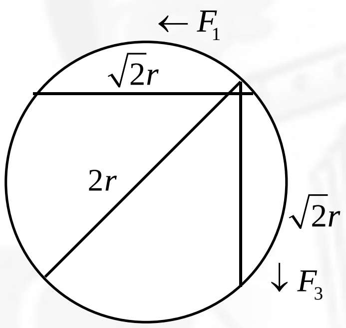
5. Four particles each of mass M, move along a circle of radius R under the action of their mutual gravitational attraction as shown in figure. The speed of each particle is:



1) $\dfrac{1}{2}\sqrt{\dfrac{GM}{R\left(2\sqrt{2}+1\right)}}$ 2) $\dfrac{1}{2}\sqrt{\dfrac{GM}{R}\left(2\sqrt{2}+1\right)}$ 3) $\sqrt{\dfrac{GM}{R}}$ 4) $\dfrac{1}{2}\sqrt{\dfrac{GM}{R}\left(2\sqrt{2}-1\right)}$

Key: 2

Sol:



$$F_3=F_1=\frac{Gm^2}{2r^2}$$

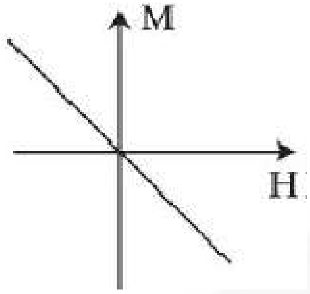
$$F_2=\frac{Gm^2}{(2r)^2}$$

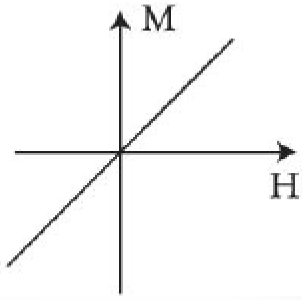
$$F=\bar{F}_3+\bar{F}_1+\bar{F}_2=\frac{\sqrt{2}Gm^2}{2r^2}+\frac{Gm^2}{(2r)^2}$$

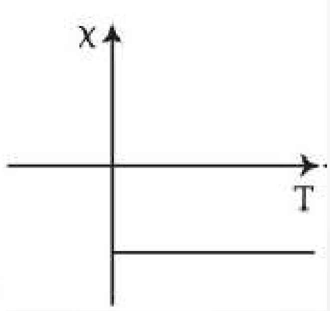
$$\frac{mv^2}{r}=\frac{Gm^2}{2r^2}\left(\sqrt{2}+\frac{1}{2}\right)$$

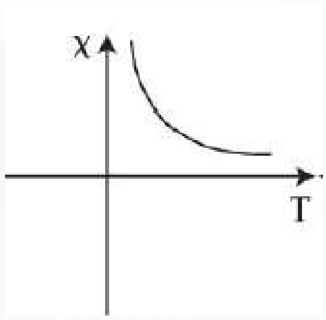
$$V=\sqrt{\frac{Gm}{2r}\left(\sqrt{2}+\frac{1}{2}\right)}=\frac{1}{2}\sqrt{\frac{Gm}{r}\left(2\sqrt{2}+1\right)}$$

6. Following plots show Magnetization (M) vs Magentising field (H) and Magnetic susceptibility (x) vs Temperature (T) graph:

a) 

b) 

c) 

d) 

Which of the following combination will be represented by a diamagnetic material?

1) b , c 2) a , d 3) a , c 4) b , d

Key: 4

Sol: $x = \frac{1}{T}; B = \mu_0 H$

7. A body of mass 'm' dropped from a height 'h' reaches the ground with a speed of 0.8 $\sqrt{gh}$.

1) 1.64 mgh 2) – 0.68 mgh 3) mgh 4) 0.64 mgh

Key: 2

Sol: $W_{fr} = E_1 - E_2 = mgh - \frac{1}{2}mv^2$

$= mgh - \frac{1}{2}m\left(0.8\sqrt{gh}\right)^2$

8. The half life period of a radioactive element x is same as the mean life time of another radioactive element y. Initially they have the same number of atoms. Then:

1) x and y decay at the same rate always. 2) y – will decay faster than x.

3) x – will decay faster than y

4) x and y same decay rate initially and later on different decay rate

Key: 2

Sol: $R_1 = -\lambda_x N_0$ _____(1)

$R_2 = -\lambda_y N_0$ ______(2)

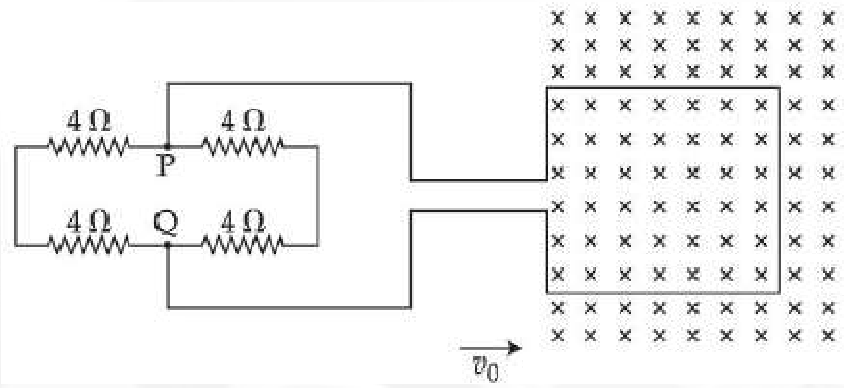
$T_{half} = T_{av}$

$\frac{\ln_2}{\lambda_x} = \frac{1}{\lambda_y}; \frac{\lambda_x}{\lambda_y} = \ln_2$____(3)

(1) & (2) in (3)

$\frac{R_1}{R_2} = 0.693 \qquad R_1 < R_2$

9. A square loop of side 20 cm and resistance 1Ω is moved towards right with a constant speed υ_0. The right arm of the loop is in a uniform magnetic field of 5 T. The field is perpendicular to the plane of the loop and is going into. The loop is connected to a network of resistors each of value 4Ω. What should be the value of υ_0 so that a steady current of 2 mA flows in the loop?



1) $1 m/s$ 2) $10^{-2}\ cm/s$ 3) $10^{2}\ m/s$ 4) $1 cm/s$

Key: 4

Sol: $V_0 = 10^{-2} m/s; V_0 = 1cm/s$

10. A capacitor is connected to a 20 V battery through a resistance of 10Ω. It is found that the potential difference across the capacitor rises to 2 V in $1\mu s$. The capacitance of the capacitor is ________ μF.

1) 1.85 2) 0.95 3) 9.52 3) 0.105

Key: 2

Sol: $V = V_0\left(1-e^{-t/RC}\right) \Rightarrow 2 = 20\left(1-e^{-t/RC}\right) = \frac{9}{10} = e^{-t/RC}; \ln\frac{9}{10} = \frac{-t}{RC} \Rightarrow \ln\frac{10}{9} = \frac{t}{RC} \Rightarrow C = 0.95\mu F$

11. A glass tumbler having inner depth of 17.5 cm is kept on a table. A student starts pouring water $(\mu = 4/3)$ into it while looking at the surface of water from the above. When he feels that the tumbler is half filled, he stops pouring water. Up to what height, the tumbler is actually filled?

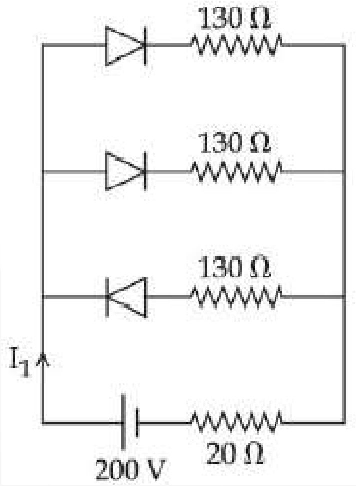
1) 11.7 cm 2) 8.75 cm 3) 10 cm 4) 7.5 cm

Key: 1

Sol: A.D = $\frac{t}{\mu}$

$\frac{h}{2} = \frac{t}{\mu}; t = \mu\frac{h}{2} = \frac{4}{3}\times\frac{17.5}{2} = 11.7m$

12. In the given figure, each diode has a forward bias resistance of $30\,\Omega$ and infinite resistance in reverse bias. The current I_1 will be:



1) 2.35 A 2) 2 A 3) 3.75 A 4) 2.73 A

Key: 2

Sol: $R_q = \frac{160}{2} = 80$

$R_T = 80 + 20 \quad i = \frac{V}{R} = \frac{200}{100} = 2$

13. A student determined Young's Modulus of elasticity using the formula $Y = \frac{MgL^3}{4bd^3\delta}$. The value of g is taken to be $9.8\,m/s^2$, without any significant error, his observation are as following

Physical Quantity	**Least count of the Equipment used for measurement**	**Observed value**
Mass (M)	1 g	2 kg
Length of bar (L)	1 mm	1 m
Breadth of bar (d)	0.1 mm	4 cm
Thickness of bar (d)	0.01 mm	0.4 cm
Depression (δ)	0.01 mm	5 mm

Then the fractional error in the measurement of Y is:

1) 0.083 2) 0.0083 3) 0.155 4) 0.0155

Key: 4

Sol: $\frac{\Delta y}{y} = \frac{\Delta M}{M} + \frac{3\Delta L}{L} + \frac{\Delta b}{b} + \frac{3\Delta d}{d} + \frac{\Delta 8}{8}$

$= 1.55 \times 10^{-2}$

14. Due to cold weather a 1 m water pipe of cross-sectional are 1 cm^2 is filled with ice at $-10^0 C$. Resistive heating is used to melt the ice. Current of 0.5 A is passed through $4\,k\Omega$ resistance. Assuming that all the heat produced is used for melting what is the minimum time required?

(Given latent heat of fusion for water /ice $=3.33\times10^5\ J\,kg^{-1}$, specific heat of ice $=2\times10^3\ J\,kg^{-1}$ and density of ice $=10^3\ kg/m$)

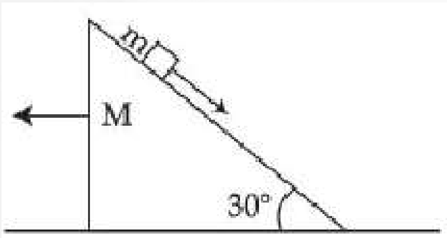
1) 70.6 s 2) 35.3 s 3) 3.53 s 4) 0.353 s

Key: 2

Sol: $m=(A)(l)(d)=10^{-1}kg$; $i^2RT = ms\Delta\theta + mL$

15. A block of mass m slides on the wooden wedge, which in turn slides backward on the horizontal surface. The acceleration of the block with respect to the wedge is : Given $m=8\,kg.\ M=16\,kg$

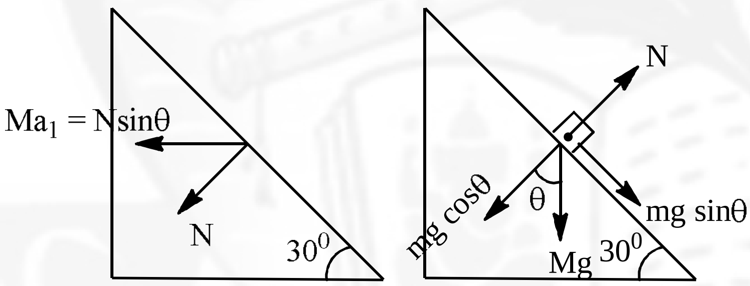
Assume all the surfaces shown in the figure to be frictionless.



1) $\frac{3}{5}g$ 2) $\frac{4}{3}g$ 3) $\frac{2}{3}g$ 4) $\frac{6}{5}g$

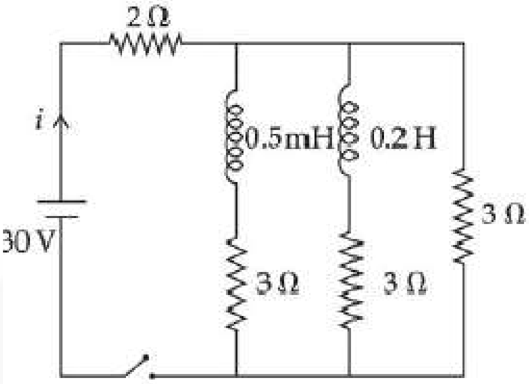
Key: 3

Sol:



$N\sin\theta = 16a_1 \Rightarrow a_1 = \frac{\sqrt{3}}{8}g; a_2 = \frac{g}{2}; a_{12} = \frac{2g}{3}$

16. For the given circuit the current I through the battery when the key in closed and the steady state has been reached is _______

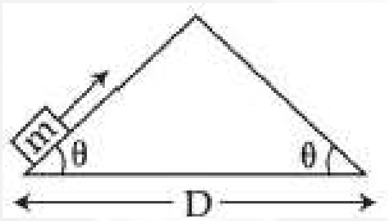


1) 0 A 2) 25 A 3) 10 A 4) 6 A

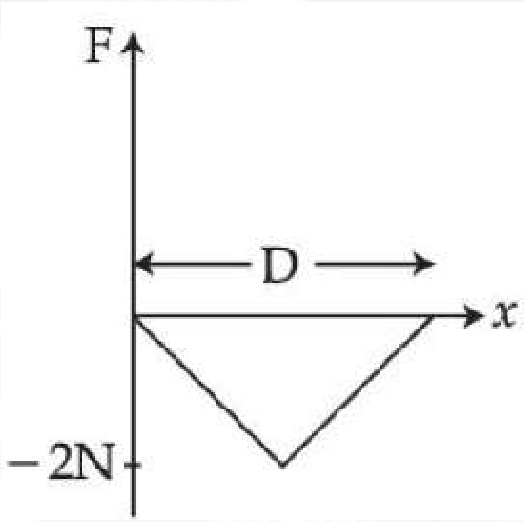
Key: 3

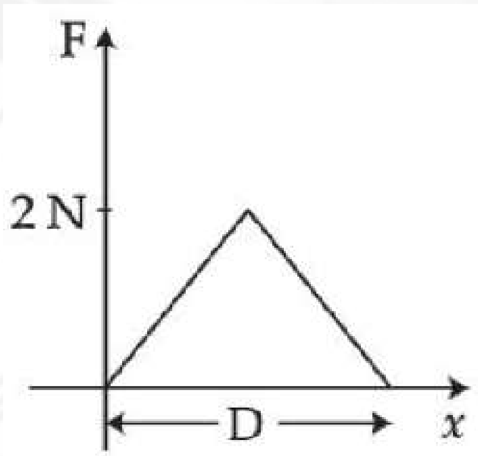
Sol: $R_q = 3 \quad V = 30 \quad i = 10$

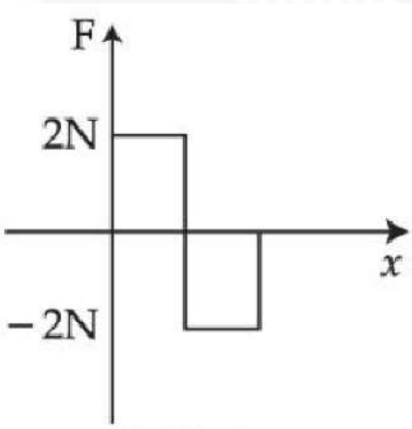
17. An object of mass 'm' is being moved with a constant velocity under the action of an applied force of 2 N along a frictionless surface with following surface profile.

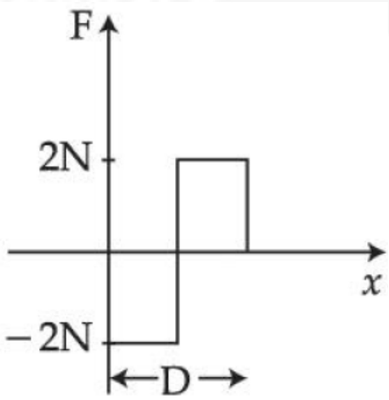


The correct applied force vs distance graph will be:

1) 

2) 

3) 

4) 

Key: 3

Sol: Since F = constant ; vol = constant.

18. The ranges and heights for two projectiles projected with the same initial velocity at angles 42^0 and 48^0 with the horizontal are R_1, R_2 and H_1, H_2 respectively, Choose the correct option

1) $R_1 < R_2$ and $H_1 < H_2$
2) $R_1 = R_2$ and $H_1 < H_2$
3) $R_1 = R_2$ and $H_1 = H_2$
4) $R_1 > R_2$ and $H_1 = H_2$

Key: 2

Sol: $\theta_1\,\theta_2$ are complimentary angles

$$\therefore R_1 = R_2 \qquad \frac{H_1}{H_2} = \frac{u^2 \sin^2\theta/2g}{u^2\cos^2\theta/2g} = \tan^2 42$$

$$\tan 42 < 1$$

$$H_1 < H_2$$

19. Electric field of a plane electromagnetic wave propagation through a non-magnetic medium is given by $E = 20\cos\left(2\times10^{10}t - 200x\right)V/m.$ The dielectric constant of the medium is equal to: (take $\mu_r = 1$)

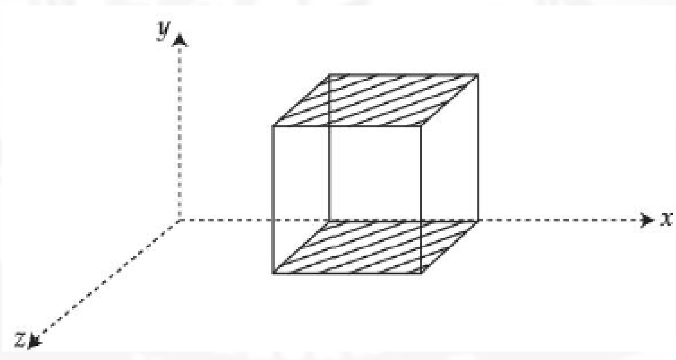
1) 2
2) 9
3) 3
4) $\frac{1}{3}$

Key: 2

Sol: $\frac{C}{V} = \frac{\sqrt{\mu\in}}{\mu_0\in_0} \Rightarrow V = \frac{10}{k} = 1\times10^8$

$$\frac{3\times10^8}{1\times10^8} = \sqrt{\in_r} \quad ; \in_r = 9$$

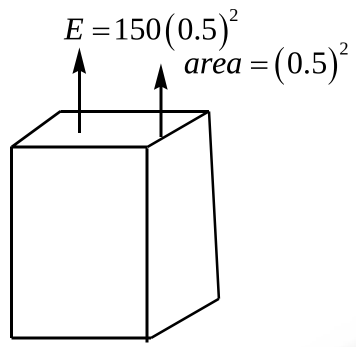
20. A cube is place inside an electric field, $\vec{E} = 150y^2\hat{j}.$ The side of the cube is 0.5 m and is placed in the field as shown in the given figure. The charge inside the cube is:



1) $8.3\times10^{-12}C$
2) $3.8\times10^{-12}C$
3) $3.8\times10^{-11}C$
4) $8.3\times10^{-11}C$

Key: 1

Sol:



$$\phi = E.A = \frac{q}{\epsilon_0}$$

$$150\times(0.5)^2 = \frac{q}{8.85\times10^{-12}}$$

$$q = 8.3\times10^{-12}$$

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

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21. The width of one of the two slits in Young's double slit experiment is three times the other slit. If the amplitude of the light coming from a slit is proportional to the silt-with, the ratio of minimum of maximum intensity in the interference pattern is $x:4$ where x is _______

Key: 1

Sol: $\frac{A_1}{A_2} = 3$ $\frac{I_{\min}}{I_{\max}} = \left(\frac{A_1 - A_2}{A_1 + A_2}\right)^2 = \left(\frac{2}{4}\right)^2 = \frac{1}{4}$

22. When a body slides down from rest along a smooth inclined plane making an angle of 30^0 with the horizontal, it takes time T. When the same body slides down from the rest along a rough inclined plane making the same angle and through the same distance, it takes time $\alpha T,$ where α is a constant greater than I. The co-efficient of friction between the body and the rough plane is $\frac{1}{\sqrt{x}}\left(\frac{\alpha^2 - 1}{\alpha^2}\right)$ where x = ______

Key: 3

Sol: $t_{rough} = \alpha t_{smooth}$

$$\sqrt{\frac{2L}{g(\sin\theta - \mu\cos\theta)}} = \alpha\sqrt{\frac{2L}{g(sin\theta)}}$$

$$\mu = \left(1 - \frac{1}{n^2}\right)\tan\theta$$

23. A 2 kg steel rod of length 0.6 m is clamped on a table vertically at its lower end and is free to rotate in vertical plane. The upper end is pushed so that the rod falls under gravity. Ignoring the friction due to clamping at is lower and, the speed of the free end of rod when it passes through is lowest position is ____ ms^{-1}, (Take $g = 10\,ms^{-1}$)

Key: 6

Sol: Apply conservation of energy $\Delta U = \Delta KE_{rot}$

$$mgL = \frac{1}{2}I\omega^2 \quad I = \frac{ml^2}{3}$$

$$V = L\omega = 6$$

24. The average translation kinetic energy of N_2 gas molecules at _______ ^0C becomes equal to the K.E. of an electron accelerated from rest through a potential difference of 0.1 volt, (Given $k_B = 1.38 \times 10^{-23} J/K$) (Fill the nearest integer)

Key: 500

Sol: $3\left(\frac{1}{2}kT\right) = eV$

$$T = 773K = 500^0C$$

25. The temperature of 3.00 mol of an ideal diatomic gas is increased by 40.0^0C without changing the pressure of the gas. The molecules in the gas rotate but do not oscillate. If the ratio of change in internal energy of the gas to the amount of working by the gas is $\frac{x}{10}$. Then the value of x (round off to the nearest integer) is _______ (Given $R = 8.31\ J\ mol^{-1}\ K^{-1}$)

Key: 25

Sol: $\gamma = \frac{7}{5}; \frac{du}{dw} = \frac{x}{10} \Rightarrow \frac{nc_v dT}{nRdT} = \frac{x}{10} \Rightarrow \frac{5}{2} = \frac{x}{10} \Rightarrow \therefore x = 25$

26. A uniform heating wire of resistance 30Ω is connected across a potential difference of 240 V. The wire is then cut into hall and a potential difference of 240 V is applied across each half separately. The ratio of power dissipation in first case to the total power dissipation in the second case would be 1 : x, where x is__

Key: 4

Sol: $\frac{36\Omega}{240V} \quad \frac{18\Omega}{240V}$

$$P = \frac{240^2}{36} \quad P_1 + P_2 = \frac{240^2}{18} + \frac{240^2}{18}$$

$$\frac{P_1}{P_2} = \frac{\frac{1}{36}}{1/9} = \frac{9}{36} = \frac{1}{4}$$

27. Two satellites revolve around a planet in coplanar circular orbits in anticlockwise direction. Their period revolutions are 1 hour and 8 hours respectively. The radius of the orbit of nearer satellite is $2 \times 10^3\ km.$ The angular speed of the father satellite as observed from the nearer satellite at the instant when both the satellites are closest is $\frac{\pi}{x}$ rad h^{-1} where x is

Key: 3

Sol: $w = \frac{v_1 - v_2}{r_1 - r_2} - - - - - (1)$.

Keplers 3rd law

$T^2 \propto r^3; r \propto T^{2/3} \Rightarrow \frac{r_1}{r_2} = \left(\frac{1}{8}\right)^{2/3} = \frac{1}{4}$

$r_1 = 2\times10^3; r_2 = 8\times10^3$; $V_1 = \sqrt{\frac{GM}{r_1}}; V_2 = \sqrt{\frac{GM}{r_2}}$

$\therefore w = \frac{V_1 - V_2}{r_1 - r_2} = \frac{\pi}{3}$

28. A steel rod with $y = 2.0\times10^{11}\ Nm^{-2}$ and $\alpha = 10^{-5}\ {}^0C^{-1}$ of length 4 m and area of cross-section $10\,cm^2$ is heated from $0\ {}^0C$ to 400^0C without being allowed to extend. The tension produced in the rod is $x\times10^5 N$ where the value of x is _____

Key: 8

Sol: $\frac{F}{A} = Y\alpha\Delta\theta; F = YA\alpha\Delta\theta$

$F = 8\times10^5$

29. An engine is attached to a wagon through a shock absorber of length 1.5 m. The system with a total mass of 40,000 kg is moving with a speed of 72 kmh^{-1} when the brakes are applied to bring it to rest. In the process of the system being brought to rest the spring of the shock absorber gets compressed by 1.0 m. If 90 % of energy of the wagon is lost due to friction, the spring constant is _________$\times10^5 N/m$

Key: 16

Sol: $\frac{10}{100}\left(\frac{1}{2}mv^2\right) = \frac{1}{2}kx^2$

$k = 16\times10^5$

Ans : 16

30. A carrier wave with amplitude of 250 V is amplitude modulated by a sinusoidal base band singal of amplitude 150 V. The ration of minimum amplitude to maxim amplitude for the amplitude modulated wave is 50 : x, the value of x is ________.

Key: 200

Sol: $\frac{A_c + A_m}{A_c - A_m} = \frac{A_{max}}{A_{min}} \Rightarrow \frac{250-150}{250+150} = \frac{100}{400} = \frac{1}{4} or \frac{50}{200}$

$\therefore x = 200$.

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

31. Hydrogen peroxide reacts with iodine in basic medium to give

1) IO^- 2) IO_3^- 3) IO_4^- 4) I^-

Key: 4

Sol: H_2O_2 reduces halogens to halideions in basic medium

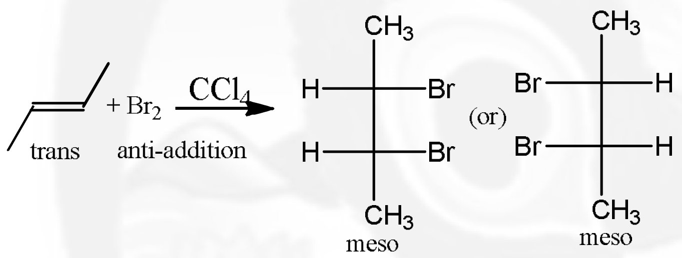
$I_2 + H_2O_2 + 2NaOH \rightarrow 2NaI + 2H_2O + O_2$

32. The stereoisomers that are formed by electrophilic addition of bromine to trans–but–2–ene is/are:

1) 1 racemic and 2 enantiomers 2) 2 enantiomers and 2 mesomers

3) 2 enantiomers 4) 2 identical mesomers.

Key: 4



Sol:

4 two identical mesomers.

33. Experimentally reducing a functional group cannot be done by which one of the following reagents?

1) Zn / H_2O 2) Na / H_2 3) $Pd - C / H_2$ 4) $Pt - C / H_2$

Key: 1

Sol: $2Na + H_2 \xrightarrow{673K} \underset{\text{strong reducinga agent}}{2NaH}$

$NaH + R - OH \rightarrow RONa + H_2$

$Pd - C + H_2$ reduces double bond

$Pd - C + H_2$ reduces double bond $No_2 + ONH_2$; $C = N$ to $CH_2 - NH_2$

$Pd - C + H_2$ reduces double bond

Metal + acid $\Rightarrow$ reduced $-\overset{\overset{\displaystyle O}{\|}}{C}-$ to $-\overset{\overset{\displaystyle OH}{\|}}{CH}-$

$\left(Zn + HCl_1 Zn + H_2SO_4\ etc\right)$

$Zn + H_2O \Rightarrow$ does not reduces functional group Zn does not react with water because if forms a protective layer of insoluble $Zn(OH)_2$

34. Number of paramagnetic oxides among the following given oxides is ____

$Li_2O, CaO, Na_2O_2, KO_2, MgO$ and K_2O

1) 1 2) 3 3) 0 4) 2

Key: 1

Sol: $Li_2O \Rightarrow$ Diamagnetic

$CaO \Rightarrow$ Diamagnetic

$NaNO_2 \Rightarrow$ Diamagnetic

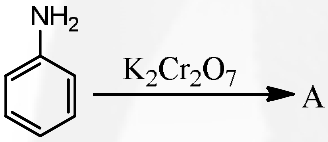
$KO_2 \Rightarrow$ Paramagnetic

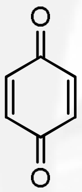
$MgO \Rightarrow$ Diamagnetic

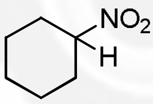
$K_2O \Rightarrow$ Diamagnetic

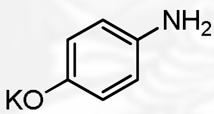
And oxides, peroxides are diamagnetic & suproxides are para magnetic due to odd no of $e^{-1}s$

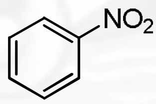
35. Identify A in the following reaction



1) 

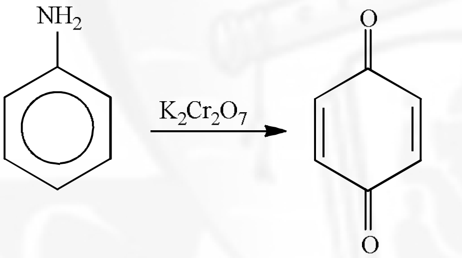
2) 

3) 

4) 

Key: 1

Sol:

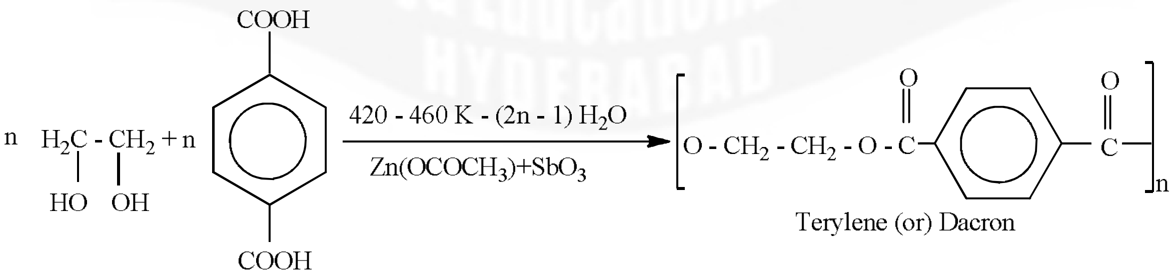


p - benzoguinone

36. Monomer units of Dacron polymer are:

1) glycerol and phthalic acid
2) glycerol and terephthalic acid
3) ethylene glycol and phthalic acid
4) ethylene glycol and terephthalic acid.

Key: 4



Sol:

Monomers of Dacron polymers: ethylene glycol + Terephthalic acid

37. Identify the element for which electronic configuration in +3oxidation state is $[Ar]3d^5$:

1) Ru 2) Fe 3) Co 4) Mn

Key: 2

Sol: $Fe:\underbrace{1s^2 2s^2 2p^6 3s^2 3p^6}_{Ar} 4s^2 3d^6$

$Fe^{+3}:[Ar]3d^5$

38. Water sample is called cleanest on the basis of which one of the BOD values given below.

1) 21 ppm 2) 3 ppm 3) 15 ppm 4) 11 ppm.

Key: 2

Sol: Pure water has $BOD < 5PPm$ polluted water had $BOD < 17PPm$

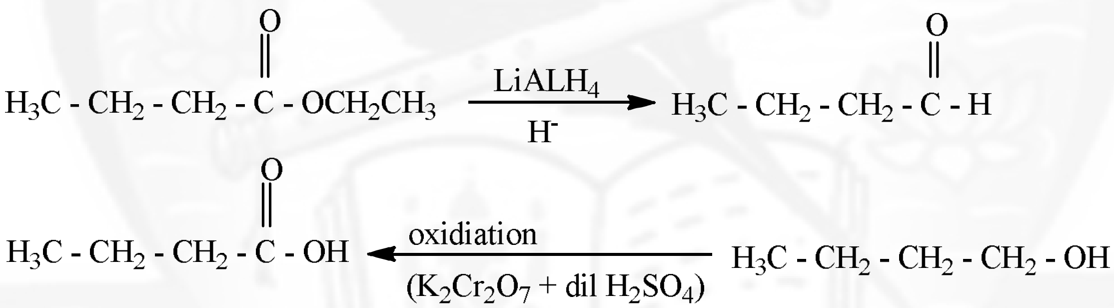
39. In the following sequence of reaction a compound A, (molecular formula $C_6H_{12}O_2$) with a straight chain structure gives a C_4 carboxylic acid. A is:

1) $CH_3-CH_2-COO-CH_2-CH_2-CH_3$ 2) $CH_3-CH_2-\underset{}{\overset{OH}{\overset{|}{C}}}H-CH_2-O-CH=CH_2$

3) $CH_3-CH_2-CH_2-O-CH=CH-CH_2-OH$ 4) $CH_3-CH_2-CH_2-COO-CH_2-CH_3$

Key: 4

Sol: $\underset{C_6H_{12}O_2}{A} \xrightarrow[H_3O^+]{LiAdH_4} B \xrightarrow{\text{oxidation}} C_4-\text{carboxylic acid}$

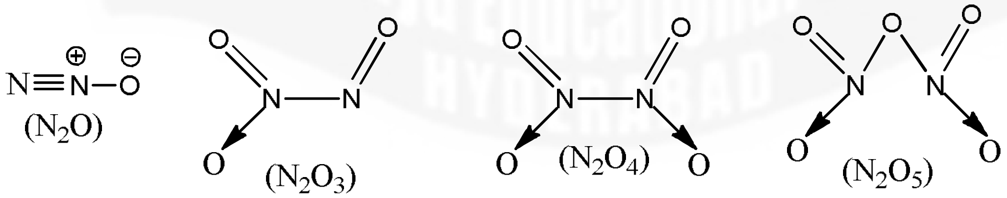


40. The oxide without nitrogen – nitrogen bond is:

1) N_2O_4 2) N_2O_5 3) N_2O 4) N_2O_3

Key: 2

Sol:



41. The potassium ferrocyanide solution gives a Prussian blue colour, when added to:

1) $FeCl_2$ 2) $FeCl_3$ 3) $CoCl_3$ 4) $CoCl_2$

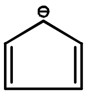
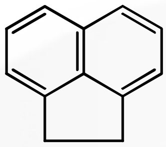
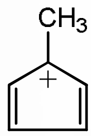
Key: 2

Sol: $4FeCl_3 + 3\underset{\text{Pot ferrocyanide}}{K_4\left[Fe^{2+}(CN)_6\right]} \rightarrow \underset{\text{ferric ferrocyanide}}{Fe_4^{+3}\left[Fe^{+2}(CN)_6\right]_3} + 12KCl$

(Or)

$FeCl_3 + K_4\left[Fe(CN)_6\right] \rightarrow \underset{\substack{\text{Pot ferric ferrocyanide}\\ \text{(Prussian blue)}}}{KFe^{3+}\left[Fe^{2+}(CN)_6\right]} + 3KCl$

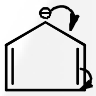
42. Which of the following compounds is aromatic in nature?

1)  2)  3) 

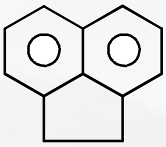
4) 

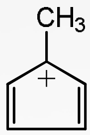
Key: 1

Sol:

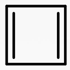


$6\pi e^-$ and Aromatic





Anti Aromatic



Anti Aromatic

43. Match List–I with List–II.

	List–I (Colloid Preparation Method)		List–II (Chemical reaction)
(a)	Hydrolysis	(i)	$2AuCl_3 + 3HCHO + 3H_2O \rightarrow 2Au(sol) + 3HCOOH + 6HCl$
(b)	Reduction	(ii)	$As_2O_3 + 3H_2S \rightarrow As_2S_3(sol) + 3H_2O$
(c)	Oxidation	(iii)	$SO_2 + 2H_2S \rightarrow 3S(sol) + 2H_2O$
(d)	Double Decomposition	(iv)	$FeCl_3 + 3H_2O \rightarrow Fe(OH)_3(sol) + 3HCl$

Choose the most appropriate answer from the options given below:

1) $(a)-(iv),(b)-(i),(c)-(iii),(d)-(ii)$ 2) $(a)-(i),(b)-(iii),(c)-(ii),(d)-(iv)$

3) $(a)-(i),(b)-(ii),(c)-(iv),(d)-(iii)$ 4) $(a)-(iv),(b)-(ii),(c)-(iii),(d)-(i)$

Key: 1

Sol: Double decomposition: $As_2O_3 + 3H_2S \rightarrow \underset{\substack{\text{(Colliodial}\\ \text{Solution)}}}{As_2S_3} + 3H_2O$

Reduction: Au solution can obtained by reduction $AuCl_3$ solution with H–CHO.

$2AuCl_3 + 3H-CHO + 3H_2O \rightarrow 2Au + 3H-COOH + 6HCl$

Hydrolysis: $FeCl_3 + 3H_2O \rightarrow \underset{\substack{\text{(Colliodial}\\ \text{Solution)}}}{Fe(OH)_3} + 3HCl$

Oxidation : $2H_2S^{-2} + SO_2 \rightarrow 2H_2O + 3S^0$.

44. Calamine and Malachite, respectively, are the ores of :

1) Nickel and Aluminium 2) Aluminium and Zinc

3) Zinc and Copper 4) Copper and Iron.

Key: 3

Sol: Malachite : $CuCO_3.Cu(OH)_2$

Calamine : $ZnCO_3$

45. The Crystal field stabilization Energy (CFSE) and magnetic moment (spin–only) of an octahedral aqua complex of a metal ion (M^{Z+}) are $-0.8\Delta_0$ and 3.87 BM, respectively.

Identify (M^{Z+}):

1) Co^{2+} 2) V^{3+} 3) Mn^{4+} 4) Cr^{3+}

Key: 1

Sol: d^7 in weak field ligand d^7. $\begin{matrix} \uparrow\uparrow & e_g \\ \uparrow\downarrow,\uparrow\downarrow,\uparrow & t_{2g} \end{matrix}$; $t_{2g}^5 e_g^2$

Number of unpaired electrons = 3, $\mu = 3.87$

CFSE = $\left[-0.4\times t_{2g}e^{-}s + 0.6eg\, e^{-}s\right]\Delta_0 = -0.4\times5+0.6\times2 = -2+1.2 = -0.8\Delta_0$

$Co: 3d^7 4s^2;\ Co^{+2}: 3d^7, \left[Co(H_2O)_6\right]^{+2}$

46. In the following sequence of reactions, $C_3H_6 \xrightarrow{H^+/H_2O} A \xrightarrow[dil\,KOH]{KIO} B + C$.

The compounds B and C respectively, are:

1) $CH_3I, HCOOK$ 2) CHI_3, CH_3COOK 3) $CI_3COOK, HCOOH$ 4) CI_3COOK, CH_3I

Key: 2

Sol: $C_3H_6 \xrightarrow{H^+/H_2O} A \xrightarrow[dil\,KOH]{KIO} B + C$

$H_3C-CH=CH_2 \xrightarrow{H^+/H_2O} H_3C-\overset{OH}{\overset{|}{C}}H-CH_3$

$2KOH + I_2 \rightarrow \underset{Pot\ hypoiodite}{KOI} + KI + H_2O$

$H_3C-\overset{OH}{\overset{|}{C}}H-CH_3 \xrightarrow[\text{by halogens}]{\text{oxidation}} H_3C-\overset{O}{\overset{||}{C}}-CH_3$

$H_3C-\overset{O}{\overset{||}{C}}H-CH_3 \xrightarrow[\text{by halogens}]{\text{oxidation}} CH_3-\overset{O}{\overset{||}{C}}-\overset{\ominus}{O}\overset{\oplus}{K} + CHI_3 + 2NaOH$

47. Given below are two statements

Statement-I: The nucleophlic addition of sodium hydrogen sulphite to an aldehyde or a ketone involves proton transfer to form a stable ion

Statement-II: The nucleophilic addition of hydrogen cyanide to an aldehyde or a ketone yields amine as final product.

In the light of the above statements, choose the most appropriate answer from the options given below:

1) Both statement I and statement II are true

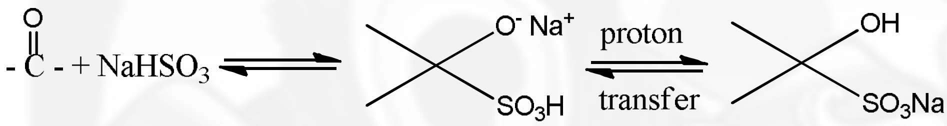
2) Statement I is true but statement II is false

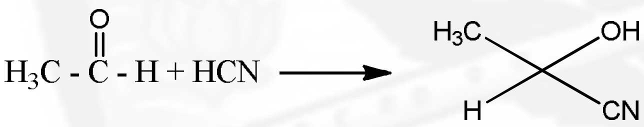
3) Statement I is false but statement II is true

4) Both statement I and statement II are false.

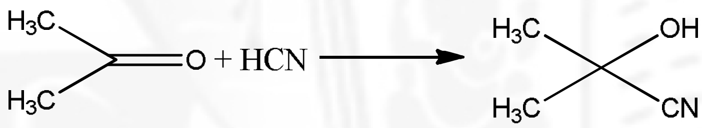
Key: 2

Sol:





acetaldehyde
cyanohydrine



Statement – I is correct. Statement – II wrong.

48. In the given chemical reaction, colors of the Fe^{2+} and Fe^{3+} ions, are respectively:

$$5Fe^{2+} + MnO_4^- + 8H^+ \rightarrow Mn^{2+} + 4H_2O + 5Fe^{3+}$$

1) Green, Yellow 2) Green, Orange 3) Yellow, Green 4) Yellow, Orange

Key: 1

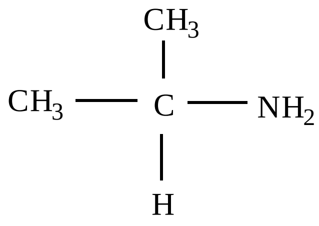
Sol: $5Fe^{+2} + MnO_4^- + 8H^+ \rightarrow Mn^{2+} + 5Fe^{3+} + 4H_2O$

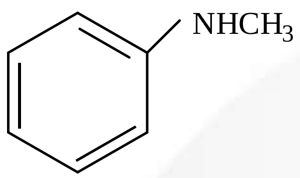
(Or)

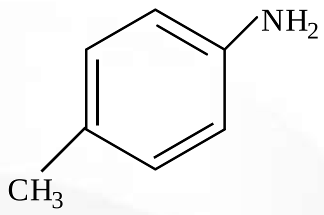
$$2KMnO_4 + 8H_2SO_4 + 10\underset{(Green)}{FeSO_4} \rightarrow K_2SO_4 + 2MnSO_4 + 5\underset{Yellow}{Fe_2\left(SO_4\right)_3} + 8H_2O$$

49. Which one of the following gives the most stable Diazonium salt?

1) $CH_3-CH_2-CH_2-NH_2$

2) 

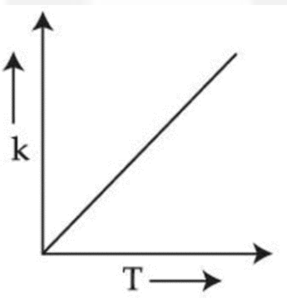
3) 

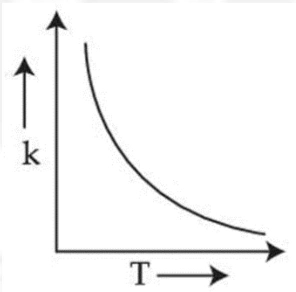
4) 

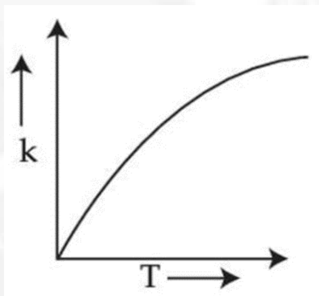
Key: 4

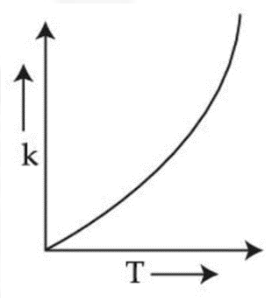
Sol: Primary aliphatic amines form highly unstable alkanediazonium salts.
Aromatic diazonium salts are much more stable than aliphatic diazonium salts.

50. Which one of the following given graphs represents the variation of rate constant (k) with temperature (T) for an endothermic reaction?

1) 

2) 

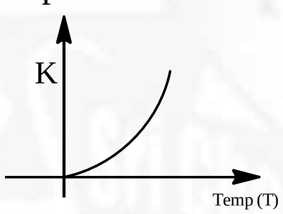
3) 

4) 

Key: 4

Sol: Arrhenius equation, $K = Ae^{-Ea/RT}$

Exponential increases of rate constant with temperature.



(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

51. The spin-only magnetic moment value of B_2^+ species is ______ $\times 10^{-2}$ BM. (Nearest integer) [Given: $\sqrt{3} = 1.73$]

Key: 173

Sol: B_2^+. Total no. of electrons = 9

M.O.C: $\sigma_{1s}^2 \overset{*}{\sigma}{}_{1s}^{2} \sigma_{2s}^2 \overset{*}{\sigma}{}_{2s}^{2} \pi_{2_{p_x}} \pi_{2_{p_y}} \sigma_{2_{p_z}}$

No. of unpaired electrons (n) = 1. $\mu = \sqrt{n(n+2)} = \sqrt{1+(1+2)} = \sqrt{3} = 1.73 = 173 \times 10^{-2}$

52. If 80 g of copper sulphate $CuSO_4.5H_2O$ is dissolved in deionised water to make 5 L of solution. The concentration of the copper sulphate solution is $x \times 10^{-3} mol\, L^{-1}$. The value of x is _____. [Atomic masses Cu: 6354 u , S: 32 u, O:16 u, H: 1u]

Key: 64.117

Sol: $(GMW)_{CuSO_4.5H_2O} = 249.54$; $M = \frac{W}{GMW} \times \frac{1}{V(lit)} = \frac{80}{249.54} \times \frac{1}{5} = 0.064117 = 64.117 \times 10^{-3}$

53. The number of atoms in 8 g f sodium is $x \times 10^{23}$. The value of x is _____. (Nearest integer) [Given: $N_A = 6.02 \times 10^{23} mol$ Atomic mass of $Na = 23.0u$]

Key: 2.0939

Sol: 23 g of $Na = 6.023 \times 10^{23}$ atoms of Na

$8g = x$

$x = \frac{8 \times 6.02 \times 10^{23}}{23}$

$1 = 2.0939 \times 10^{23}$

54. A peptide synthesized by the reactions of one molecule each of Glycine, Leucine, Aspartic acid and Histidine will have ____ peptide linkages.

Key: 3

Sol: No of peptide linkages = No of aminoacids – 1

Glycine, Leucine, aspartic acid & Histidine

= 4 – 1

= 3

55. The molar solubility of $Zn(OH)_2$ in 0.1 M $NaOH$ solution is $x \times 10^{-18} M$. The value of x is ____. (Nearest integer) (Given: the solubility product of $Zn(OH)_2$ is 2×10^{-20})

Key: 2

Sol: $Zn(OH)_2 \rightleftharpoons Zn^{+2} + 2OH^-$

$\underset{0.1}{NaOH} \rightarrow \underset{0.1}{Na^+} + \underset{0.1}{OH^-}$

$Zn(OH)_2 \rightleftharpoons Zn^{+2} + \underset{2s+0.1}{2OH^-}$

K_{sp} is very small, $2s <<<<< 0.1$

$2s + 01 \approx 0.1$

$(K_{sp})_{Zn(OH)_2} = [Zn^{+2}][OH^{-1}]^2$

$2 \times 10^{-20} = [Zn^{+2}](0.1)^2$

$[Zn^{+2}] = \frac{2 \times 10^{-20}}{10^{-2}} = 2 \times 10^{-18}$

$= x \times 10^{-18}$

56. An empty LPG cylinder weighs 14.8 kg. When full, it weighs 29.0 kg and shows a pressure of 3.47 atm. In the course of use at ambient temperature, the mass of the cylinder is reduced to 23.0 kg. The final pressure inside the cylinder is ______ atm. (nearest integer) (Assume LPG to be an ideal gas)

Key: 3.45

Sol: LPG cylinder weight $=14.8\,kg$

Full weight $=29\,kg$

Pressure $=3.47\,atom$

Mass of gas in cylinder $=29-14.8=14.2\,kg$

$=14.2\times10^{3}\,g$

P = 3.47

Decrease in the amount of LPG = 29-23

= 6 kg

LPG gas is n-butane = 58

$=6\times10^{3}\,g$

$=\dfrac{6\times10^{3}}{58}$

$=103.44\,moles$

Volume of $103.44\,moles$ at 1 atm

$PV=nRT$

$V=\dfrac{nRT}{P}=\dfrac{103.44\times0.0821\times298.15}{1}$

= 2532.20 L

= $2.532\times10^{-2}\,m^{3}$

$=2.532\times10^{-3}\,m^{3}$

= 2.532

As the cylinder contains liquefied petroleum gas in equilibrium with its vapours There fore so long as temperature remains constant some LPG is present, pressure will remains constant as the cylinder still contains LPG = 23-14.8 = 8.2 kg, pressure inside the cylinder will be same 3.45 atm

57. A 50 watt bulb emits monochromatic red light of wavelength of 795 nm. The number of photons emitted per second by the bulb is 2×10^{-20}. The value of x is _____ . (Nearest integer) [Given: $h=6.63\times10^{-34}\,Jc$ and $c=3.0\times10^{8}\,ms^{-1}$]

Key: 2

Sol: Power of the bulb = 50 watt

$=50\,Js^{-1}$

Energy one photon (E) $=\dfrac{hc}{\lambda}$

$$E=\frac{6.626\times10^{-34}J.S\times3\times10^{8}ms^{-1}}{795\times10^{-9}m}$$

$0.025003 \times 10^{-17} = 25.00 \times 10^{-20}$

No of photons emitted $= \dfrac{50 J.S^{-1}}{25.00 \times 10^{-20} J}$

$= 1.999 \times 10^{20}$

$\approx 2 \times 10^{20}$

58. For the reaction $2NO_2(g) \rightleftharpoons N_2O_4(g)$, when $\Delta S = -176.0\, J\, K^{-1}$ and $\Delta H = --57.8\, K\, J\, mol^{-1}$ the magnitude of ΔG at 298 K for the reaction is _____ $K\, J\, mol^{-1}$ (Nearest integer)

Key: 52.4

Sol: $\Delta S = -176\, JK^{-1} = -176 \times 10^{-3}\, KJK^{-1}$

$\Delta H = -57.8\, KJ\, mol^{-1}$

$\Delta G = \Delta H - T\Delta S$

$= -57.8 - \left(298 \times -176 \times 10^{-3}\right)$

$= -57.8 + 52,448 \times 10^{-3}$

$= 52,390.2 \times 10^{-3}$

$= 52.2902$

$= 52.4 = 5.$

59. The sum of oxidation states of two silver ions in $\left[Ag(NH_3)_2\right]\left[Ag(CN)_2\right]$ complex is______ .

Key: 2

Sol: $\left[Ag^{+1}(NH_3)_2\right]^{+}\left[Ag^{+1}(CN)_2\right]^{-}$

$1 + 1 = 2$

60. If the conductivity of mercury at $0^0 C$ is $1.07 \times 10^6\, Sm^{-1}$ and the resistance of a cell containing mercury is 0.243 Ω, then the cell constant of the cell is $x \times 10^4\, m^{-1}$. The value of x is ____ . (Nearest Integer)

Key: 26.001

Sol: $K = 1.07 \times 10^6\, s\, m^{-1}$

$R = 0.243\Omega$

$\dfrac{l}{a} = x \times 10^4$

$K = \dfrac{1}{R} \times \dfrac{l}{a}$

$1.07 \times 10^6 \times 0.243 = \dfrac{l}{a}$

$\dfrac{l}{a} = 0.26001 \times 10^6$

$= 26.001 \times 10^4$

MATHEMATICS **Max Marks: 100**

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

61. Consider the system of linear equations

$-x+y+2z=0$

$3x-ay+5z=1$

$2x-2y-az=7$

Let S_1 be the set of all $a \in R$ for which the system is inconstant and S_2 be the set of all $a \in R$ for which the system has infinitely many solutions. If $n(S_1)$ and $n(S_2)$ denote the number of elements in S_1 and S_2 respectively, then

1) $n(S_1)=1, n(S_2)=0$ 2) $n(S_1)=2, n(S_2)=0$

3) $n(S_1)=0, n(S_2)=2$ 4) $n(S_1)=2, n(S_2)=2$

Key: 2

Sol: $\begin{vmatrix} -1 & 1 & 2 & 0 \\ 3 & -a & 5 & 1 \\ 2 & -2 & -a & 7 \end{vmatrix} = \begin{vmatrix} -1 & 1 & 2 & 0 \\ 0 & 3-a & 11 & 1 \\ 0 & 0 & 4-a & 7 \end{vmatrix} \begin{matrix} R_2+3R_1 \\ R_3+2R_1 \end{matrix}$

Clearly when a = 4, the system is inconsistent and when a = 3, $2=\frac{1}{11}$ & $3=7$ which is contradiction and no solution.

$\therefore$ The system is inconsistent for both a = 3, a = 4.

62. If n is the number of solutions of the equation

$2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right)=1, x\in[0,\pi]$ and S is the sum of all these solutions, then the ordered pair (n,S) is:

1) $\left(2,\frac{8\pi}{9}\right)$ 2) $\left(2,\frac{2\pi}{3}\right)$ 3) $\left(3,\frac{5\pi}{3}\right)$ 4) $\left(3,\frac{13\pi}{9}\right)$

Key: 4

Sol: $2\cos x\left(4\left(\sin^2\frac{\pi}{4}-\sin^2 x\right)-1\right)=1 \Rightarrow 2\cos x\left(2-4\sin^2 x-1\right)=1 \Rightarrow 2\cos x\left(1-4\sin^2 x\right)=1 \Rightarrow 2\cos x\left(4\cos^2 x-3\right)=1$

$\cos 3x=\frac{1}{2}$; $3x=\frac{\pi}{3},\frac{5\pi}{3},\frac{7\pi}{3}$, $x=\frac{\pi}{9},\frac{5\pi}{9},\frac{7\pi}{9}$

$\therefore n=3, S=\frac{13\pi}{9}$

63. Which of the following is equivalent to the Boolean expression $p \wedge \sim q$?

1) $\sim(p \rightarrow \sim q)$ 2) $\sim(p \rightarrow q)$ 3) $\sim(q \rightarrow p)$ 4) $\sim p \rightarrow \sim q$

Key: 2

Sol:

P	q	$\sim q$	$(p \to q)$	$\sim(p \to q)$	$(p \wedge \sim q)$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

$\sim(p \Rightarrow q) \equiv p \wedge (\sim q)$

64. The function $f(x) = x^3 - 6x^2 + ax + b$ is such that $f(2) = f(4) = 0$. Consider two statements.

$(S1)$ there exists $x_1, x_2 \in (2,4), x_1 < x_2$, such that $f'(x_1) = -1$ and $f'(x_2) = 0$

$(S2)$ there exists $x_3, x_4 \in (2,4), x_3 < x_4$, such that f is decreasing in $(2, x_4)$, increasing in $(x_4, 4)$ and $2f'(x_3) = \sqrt{3} f(x_4)$. Then

1) $(S1)$ is false and $(S2)$ is true
2) $(S1)$ is true and $(S2)$ is false
3) both (S1) and (S2) are false
4) both (S1) and (S2) are true.

Key: 4

Sol: Let α, β, γ be the roots of $f(x)$. $\alpha + \beta + \gamma = 6 \Rightarrow 2 + 4 + \gamma = 6 \Rightarrow \gamma = 0$

$\therefore b = 0, a = \Sigma\alpha\beta = 8 \qquad \therefore f(x) = x^3 - 6x^2 + 8x = x(x-2)(x-4)$

$f'(x) = 3x^2 - 12x + 8 = 0$; $x = \dfrac{12 \pm 4\sqrt{3}}{6} \Rightarrow x = 2 + \dfrac{2}{\sqrt{3}} \in (2,4)$

$(S1)\ f'(x_1) = -1 \Rightarrow 3x^2 - 12x + 9 = 0 \Rightarrow x^2 - 4x + 3 = 0$; $\therefore x_1 = 3 \in (2,4)$ $f'(x_2) = 0$ *for* $x_2 = 2 + \dfrac{2}{\sqrt{3}} > 3$

$(S2)\ x_4 = 2 + \dfrac{2}{\sqrt{3}} \Rightarrow \sqrt{3} f(x_4) = \dfrac{-16}{3} \Rightarrow 2f'(x_3) = \dfrac{-16}{3} \Rightarrow f'(x_3) = \dfrac{-8}{3} \Rightarrow x_3 = \dfrac{8}{3} < x_4$

65. Let $P_1, P_2, ..., P_{15}$ be 15 points on a circle. The number of distinct triangles formed by points P_i, P_j, P_k such that $i + j + k \neq 15$, is:

1) 455
2) 443
3) 419
4) 12

Key: 2

Sol: $i + j + k = 15$. When $i = 1, j + k = 14 \Rightarrow (2,12)(3,11)(4,10)(5,9)(6,8) = 5$

$i = 2, j + k = 13 \Rightarrow (3,10).......(6,7) = 4$

$i = 3, j + k = 12 \Rightarrow (4,8)(5,7) = 2$

$i = 4, j + k = 11 \Rightarrow (5,6) = 1 \Rightarrow 12$ *ways*

$\therefore$ The number of possible triangles using the vertices P_i, P_j, P_k such that $i + j + k \neq 15$ is equal to $^{15}C_3 - 12 = 455 - 12 = 443$.

66. Let $f:R\to R$ be a continuous function. Then $\lim_{x\to\frac{\pi}{4}}\frac{\frac{\pi}{4}\int_2^{\sec^2 x} f(x)\,dx}{x^2-\frac{\pi^2}{16}}$ is equal to

1) $4f(2)$ 2) $2f(\sqrt{2})$ 3) $2f(2)$ 4) $f(2)$

Key: 3

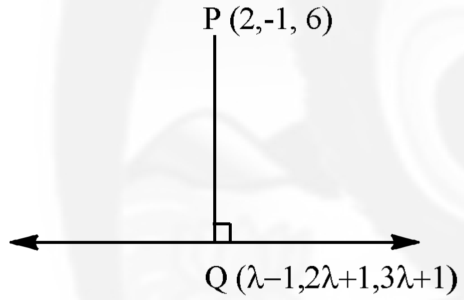
Sol: $\lim_{x\to\frac{\pi}{4}}\frac{\frac{\pi}{4}f(\sec^2 x)2\sec^2 x\tan x}{2x}=\frac{\frac{\pi}{4}f(2).2(2)(1)}{2\times\frac{\pi}{4}}=2f(2)$.

67. The distance of line $3y-2z-1=0=3x-z+4$ from the point $(2,-1,6)$ is:

1) $4\sqrt{2}$ 2) $2\sqrt{6}$ 3) $\sqrt{26}$ 4) $2\sqrt{5}$

Key: 2

Sol: Put $z=1\Rightarrow y=1, x=-1$ and dr's of line are (1,2,3). The given line is $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-1}{3}=\lambda$



PQ is perpendicular to given line $1(\lambda-3)+2(2\lambda+2)+3(3\lambda-5)=0\Rightarrow 14\lambda=14\Rightarrow\lambda=1$

$Q=(0,3,4)$; $\therefore PQ=\sqrt{4+16+4}=2\sqrt{6}$.

68. The range of the function

$f(x)=\log_{\sqrt{5}}\left(3+\cos\left(\frac{3\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right)-\cos\left(\frac{3\pi}{4}-x\right)\right)$ is

1) [0,2] 2) [–2,2] 3) $(0,\sqrt{5})$ 4) $\left[\frac{1}{\sqrt{5}},\sqrt{5}\right]$

Key: 1

Sol: $f(x)=\log_{\sqrt{5}}\left(3+2\cos\frac{\pi}{4}\cos x-2\sin\frac{3\pi}{4}\sin x\right)=\log_{\sqrt{5}}\left(3+\sqrt{2}(\cos x-\sin x)\right)$

As $\cos x-\sin x\in\left[-\sqrt{2},\sqrt{2}\right]$, $f(x)\in[0,2]$

69. If $y=y(x)$ is the solution curve of the differential equation $x^2dy+\left(y-\frac{1}{x}\right)dx=0; x>0,$ and $y(1)=1,$ then $y\left(\frac{1}{2}\right)$ is equal to:

1) $\frac{3}{2}-\frac{1}{\sqrt{e}}$ 2) $3+e$ 3) $3-e$ 4) $3+\frac{1}{\sqrt{e}}$

Key: 3

Sol: $x^2dy+ydx=\frac{dx}{x}\Rightarrow\frac{dy}{dx}+\frac{y}{x^2}=\frac{1}{x^3}$

$I.F=e^{\int\frac{1}{x^2}dx}=e^{\frac{1}{x}}\Rightarrow y.e^{\frac{1}{x}}=\int e^{\frac{1}{x}}.\frac{1}{x^3}dx+C$

$\frac{-1}{x}=t\Rightarrow\frac{1}{x^2}dx=dt\Rightarrow y.e^{\frac{1}{x}}=\int -te^t.dt+C=-\left[te^t-e^t\right]+C\Rightarrow y.e^{\frac{1}{x}}=\frac{1}{x}e^{\frac{1}{x}}+e^{\frac{1}{x}}+C$

Put x = 1 $\Rightarrow(1).e^{-1}=\frac{e^{-1}}{1}+e^{-1}+C\Rightarrow C=-e^{-1}$

Equation = $y.e^{\frac{1}{x}}=\frac{1}{x}e^{\frac{1}{x}}+e^{\frac{1}{x}}-e^{-1}\Rightarrow y=\frac{1}{x}+1-\frac{e^{\frac{1}{x}}}{e}\,;x=\frac{1}{2}\Rightarrow y\left(\frac{1}{2}\right)=2+1-\frac{e^2}{e}\Rightarrow y=3-e$

70. Let the acute angle bisector of the two planes $x-2y-2z+1=0\,and\,2x-3y-6z+1=0$ be the plane P. Then which of the following points lies on P?

1) $\left(-2,0,-\frac{1}{2}\right)$ 2) $(4,0,-2)$ 3) $(0,2,-4)$ 4) $\left(3,1,-\frac{1}{2}\right)$

Key: 1

Sol: $a_1a_2+b_1b_2+c_1c_2=2+6+12>0$

$\therefore$ plane of acute angle bisector is $\frac{x-2y-2z+1}{3}=+\left(\frac{2x-3y-6z+1}{7}\right)\Rightarrow x-5y+4z+4=0$

By inspection, $\left(-2,0,-\frac{1}{2}\right)$ lies on it.

71. $\cos^{-1}(\cos(-5))+\sin^{-1}(\sin(6))-\tan^{-1}(\tan(12))$ is equal to:

(The inverse trigonometric functions take the principal values)

1) $4\pi-11$ 2) $3\pi+1$ 3) $4\pi-9$ 4) $3\pi-11$

Key: 1

Sol: $\cos^{-1}(\cos(-5))=\cos^{-1}(\cos 5)=2\pi-5$; $\sin^{-1}(\sin 6)=6-2\pi$; $\tan^{-1}(\tan 12)=12-4\pi$

Given exp. = $(2\pi-5)+(6-2\pi)-(12-4\pi)=4\pi-11$.

72. Let $a_1,a_2,....,a_{21}$ be an AP such that $\sum_{n=1}^{20}\frac{1}{a_na_{n+1}}=\frac{4}{9}$. If the sum of this AP is 189, then a_6a_{16} is equal to:

1) 57 2) 72 3) 36 4) 48

Key: 2

Sol: $\sum_{n=1}^{20}\frac{1}{a_na_{n+1}}=\sum\frac{1}{a_n(a_n+d)}=\frac{4}{9}=\frac{1}{d}\sum_{n=1}^{20}\left(\frac{1}{a_n}-\frac{1}{a_n+d}\right)\Rightarrow\frac{1}{d}\left[\left(\frac{1}{a_1}-\frac{1}{a_2}\right)+\left(\frac{1}{a_2}-\frac{1}{a_3}\right)+....+\left(\frac{1}{a_{20}}-\frac{1}{a_{21}}\right)\right]=\frac{4}{9}$

$\Rightarrow \frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_{21}}\right)=\frac{4}{9} \Rightarrow \frac{1}{d}=\left(\frac{a_{21}-a_1}{a_1.a_{21}}\right)=\frac{4}{9} \Rightarrow a_1a_{21}=45$; $a_1(a_1+20d)=45$ ....(1)

Now sum of first 21 terms = $\frac{21}{2}(2a_1+20d)=189 \Rightarrow a_1+10d=9$ ....(2)

By using equation (1) and (2) we get $a_1=3, d=\frac{3}{5}$ otherwise $a_1=15$, $d=-\frac{3}{5}$.

So, $a_6.a_{16}=(a_1+5d)(a_1+15d)=72$.

73. Consider the parabola with vertex $\left(\frac{1}{2},\frac{3}{4}\right)$ and the directrix $y=\frac{1}{2}$. Let P be the point where the parabola meets the line $x=-\frac{1}{2}$. If the normal to the parabola at P intersects the parabola again at the point Q, then $(PQ)^2$ is equal to.

1) 75/8 2) 25/2 3) 15/2 4) 125/16

Key: 4

Sol: Vertex $\left(\frac{1}{2},\frac{3}{4}\right)$

Directrix is $y=\frac{1}{2}$

$\therefore$ Equation of parabola is $\left(y-\frac{3}{4}\right)=\left(x-\frac{1}{2}\right)^2$

Sift the (0,0) to $\left(\frac{1}{2},\frac{3}{4}\right)$

They it becomes $x^2=y$ and given lines $x=\frac{1}{2}$ become $x=-1$

$\therefore p=(-1,1)$

Length of the focal chord $PQ=\frac{(1+4)^{3/4}}{4}$

$\therefore PQ^2=\frac{125}{16}$

74. Let $S_n=1.(n-1)+2.(n-2)+3.(n-3)+....+(n-1).1, n\geq 4$. The sum $\sum_{n=4}^{\infty}\left(\frac{2S_n}{n!}-\frac{1}{(n-2)!}\right)$ is equal to

1) $\frac{e}{3}$ 2) $\frac{e-1}{3}$ 3) $\frac{e-2}{6}$ 4) $\frac{e}{6}$

Key: 2

Sol: $t_r=r(n-1)=nr-r^2$

$$f_n=\sum_{r-1}^{n}t_r=\frac{n^2(n+1)}{2}=\frac{n(n+1)(2n+1)}{6}=\frac{n(n^2-1)}{6}$$

$$\sum_{n=4}^{\infty}\left(\frac{2f_n}{n!}-\frac{1}{(n-2)!}\right)=\sum_{n=4}^{\infty}\frac{1}{3(n-3)!}$$

$$=\frac{1}{3}\left(4+\frac{1}{2!}+\frac{1}{3!}+.....\right)$$

$$=\frac{e-1}{3}$$

75. Let θ be the acute angle between the tangents to the ellipse $\frac{x^2}{9}+\frac{y^2}{1}=1$ and the circle $x^2+y^2=3$ at their point of intersection in the first quadrant. Then $\tan\theta$ is equal to:

1) $\frac{5}{2\sqrt{3}}$ 2) $\frac{2}{\sqrt{3}}$ 3) 2 4) $\frac{4}{\sqrt{3}}$

Key: 2

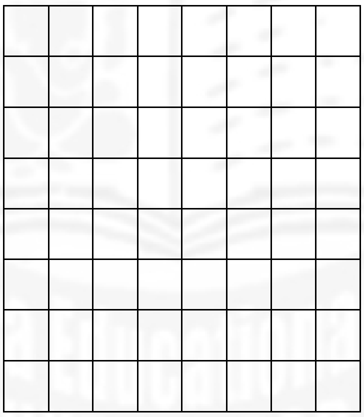
Sol: The point of intersection of the curves $\frac{x^2}{9}+\frac{y^2}{1}=1$ and $x^2+y^2=3$ in the first quadrant is $\left(\frac{3}{2},\frac{\sqrt{3}}{2}\right)$

Now slope of tangent to the ellipse $\frac{x^2}{9}+\frac{y^2}{1}=1$ at $\left(\frac{3}{2},\frac{\sqrt{3}}{2}\right)=$ $m_1=-\frac{1}{3\sqrt{3}}$

And slope of tangent to the circle $x^2+y^2=3$ at $\left(\frac{3}{2},\frac{\sqrt{3}}{2}\right)=$ $m_2=-\sqrt{3}$

So. If angle between both curves is θ then $\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=\left|\frac{-\frac{1}{3\sqrt{3}}+\sqrt{3}}{1+\left(\frac{1}{3\sqrt{3}}\right)\left(-\sqrt{3}\right)}\right|=\left(\frac{2}{\sqrt{3}}\right)$

76. Two squares are chosen at random on a chessboard. The probability that they have a side in common is:



1) $\frac{1}{7}$ 2) $\frac{1}{18}$ 3) $\frac{1}{9}$ 4) $\frac{2}{7}$

Key: 2

Sol: In total, there are 64 squares on a chess board.

If we differentiate them with respect to the number of squares they are adjacent to, there are 3 types of squares:

Squares adjacent to 4 other squares

Out of the 64 squares, all the squares expect for the corner ones and edge ones are adjacent to 4 other squares

There are a total of 36 of these squares, P(selecting such a square) = 36/64

Then, we need the probability of selecting a square that is adjacent to this square.

Let's call this event A

P(A) = 4/63

Squares adjacent to 3 other squares

The squares that lie on the edges of the chess board (neglecting the corner ones) are adjacent to 3 other squares

There are 24 such squares,

P(selecting such a square) = 24/64

Similarly, we need the probability of selecting a square that is adjacent to this square.

Let this be event B

P(B) = 3/63

Squares adjacent to 2 other squares

The corner squares are the ones,

There are 4 such squares,

P(selecting such a square) = 4/64

Here also, let the event of selecting an adjacent square be C,

P(C) = 2/63

Now, solving all the three cases and adding them will give us the answer.

Answer, P(selecting a square adjacent to 4 squares)*P(A) + P(selecting a square adjacent to 3 squares)*P(B) + P(selecting a square adjacent to 2 squares)*P(C)

(34/64)*(4/63) + 24/64)*(3/63) + (4/64)*(2/63)

77. The number of pairs (a,b) of real numbers, such that whenever α is a root of the equation $x^2+ax+b=0, \alpha^2-2$ is also a root of this equation, is:

1) 8 2) 2 3) 4 4) 6

Key: 4

Sol: Case 1:

Suppose $\alpha=\beta$, so that α is a double root,

Since α^2-2 is also, a root, the only possibility is $\alpha=\alpha^2-2$

This reduces to $(\alpha+1)(\alpha-2)=0$.

Hence $\alpha=-1, or\ \alpha=2$

Observe that $\alpha=-2\alpha$ and $\beta=\alpha^2$

Thus (a, b) = (2, 1) or (-4, 4)

Case 2:

Suppose $\alpha\neq\beta$. These are four possibilities

i) $\alpha=a^2-2\,and\,\beta^2-2;$

ii) $\alpha=\beta^2-2\,and\,\beta=\alpha^2-2;$

iii) $\alpha = \alpha^2 - 2\, and\, \beta = \beta^2 - 2;\, and\, \alpha \neq \beta;$ or

iv) $\beta = \beta^2 - 2 = \alpha^2 - 2;\, and\, \alpha \neq \beta;$

$\alpha \neq -\beta$ is identical to (iii), so that we get exactly same pairs (a, b).

Thus we get 6 pairs; (a, b)

= (-4, 4), (2, 1), (-1, -2), (1, -1) , (0, -4), (0, -1).

78. Let $J_{n,m} = \int_0^{1/2} \frac{x^n}{x^m - 1} dx, \forall n > m\ and\ n, m \in N$. Consider a matrix $A = \left[a_{ij}\right]_{3\times3}$ where $a_{ij} = \begin{cases} J_{6+i} - J_{i+3,3}, & i \le j \\ 0, & i > j \end{cases}$. Then $\left|adjA^{-1}\right|$ is :

1) $(105)^2 \times 2^{36}$ 2) $(105)^2 \times 2^{38}$ 3) $(15)^2 \times 2^{34}$ 4) $(15)^2 \times 2^{42}$

Key: 2

Sol: $J_{6+i,3} - J_{i+3,3};\ i \le j$

$\Rightarrow \int_0^{\frac{1}{2}} \frac{x^{6+i}}{x^3 - 1} - \int_0^{\frac{1}{2}} \frac{x^{i+3}}{x^3 - 1}$

$\Rightarrow \int_0^{\frac{1}{2}} \frac{x^{i+3}\left(x^3 - 1\right)}{x^3 - 1}$

$\frac{x^{3+i+1}}{3+i+1} = \left(\frac{x^{4+i}}{4+i}\right)^{\frac{1}{2}}$

$a_{ij} = j_{6+i,3} - j_{i+3,3} = \frac{\left(\frac{1}{2}\right)^{4+i}}{4+i}$

$a_{11} = \frac{1}{5.2^5}$

$a_{22} = \frac{1}{6.2^6}$

$a_{33} = \frac{1}{7.2^7}$

$|A| = a_{11}a_{22}a_{23}$ ($\because$ Below diagonal elements are 0's)

$= \frac{1}{5.6.7.2^{18}}$

$\left|AdjA^{-1}\right| = \frac{1}{|A|^2}$

$= (105)^2 \times 2^{38}$

79. The area, enclosed by the curve $y = \sin x + \cos x\ and\ y = \left|\cos x - \sin x\right|$ and the lines x = 0, $x = \frac{\pi}{2}$, is:

1) $4\left(\sqrt{2} - 1\right)$ 2) $2\sqrt{2}\left(\sqrt{2} - 1\right)$ 3) $2\sqrt{2}\left(\sqrt{2} + 1\right)$ 4) $2\left(\sqrt{2} + 1\right)$

Key: 2

Sol: Required Area $= 2\int_0^{\frac{\pi}{4}} (\sin x + \cos x) - (\cos x - \sin x)\, dx$

$$= 2\int_0^{\frac{\pi}{4}} 2\sin x dx$$

$$= -4[\cos x]_0^{\frac{\pi}{4}}$$

$$= -4\left[\frac{1}{\sqrt{2}} - 1\right]$$

$$= \frac{4\left(\sqrt{2}-1\right)}{\sqrt{2}}$$

$$= 2\sqrt{2}\left(\sqrt{2}-1\right)$$

80. The function $f(x)$, that satisfies the condition $f(x) = x + \int_0^{\pi/2} \sin x.\cos y f(y)\, dy$, is:

1) $x + (\pi + 2)\sin x$ 2) $x + \frac{\pi}{2}\sin x$ 3) $x + (\pi - 2)\sin x$ 4) $x + \frac{2}{3}(\pi - 2)\sin x$

Key: 1

Sol: $f(x) = x + x\sin x$ where $K = \int_0^{\frac{\pi}{2}} \cos y . f(y)\, dy$

$$\therefore K = \int_0^{\frac{\pi}{2}} \cos y.(y + K\sin y)\, dy$$

$$= \int_0^{\frac{\pi}{2}} y\cos y dy + K\int_0^{\frac{\pi}{2}} \sin y.\cos y\, dy$$

$$K = \frac{\pi}{2} - 1 + \frac{K}{2} \Rightarrow \frac{K}{2} = \frac{\pi - 2}{2}$$

$$\Rightarrow K = \pi - 2$$

$$\therefore f(x) = n + (\pi - 1)\sin x$$

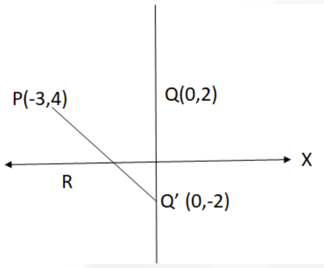
(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

81. A man starts walking from the point $P(-3,4)$, touches the x–axis at R, and then turns to reach at the point Q in the minimum time, then $50(PR)^2+(RQ)^2$ is equal to____.

Key: 1250



Sol:

Let $Q^1(0,-2)$ be the image of $Q(x,2)$ for minimum, P,R,R^1 are colixer equation of PQ^1 is $2x+y+2=0$

$\therefore R(-1,0)$

$50\left(PR^2+QR^2\right)=50(20+5)=1250$

82. Let the points of intersections of the lines $x-y+1=0$, $x-2y+3=0$ and $2x-5y+11=0$ are the mid points of the sides of a triangle ABC. Then the area of the triangle ABC is ____.

Key: 6

Sol: On solving, we get $D(1,2),E(7,5),F(2,3)$. Area of $\Delta DEF=\frac{3}{2}$.

Area of $\Delta ABC=4\times\frac{3}{2}=6$

83. Let $f(x)=x^6+2x^4+x^3+2x+3, x\in R$. Then the natural number n for which

$\lim\limits_{x\to1}\frac{x^n f(1)-f(x)}{x-1}=44$ is______.

Key: 7

Sol: $f(1)=9$

$$\lim_{x\to1}\frac{9x^n-f(x)}{x-1}=44\Rightarrow\lim_{x\to1}9nx^{n-1}-f'(x)=44\Rightarrow 9n-f'(1)=44\Rightarrow 9n=44+f'(1)=44+19=7$$

$\therefore n=7$.

84. Let $[t]$ denote the greatest integer $\le t$. The number of point where the function

$f(x)=[x]\left|x^2-1\right|+\sin\left(\frac{\pi}{[x]+3}\right)-[x+1], x\in(-2,2)$ I not continuous is ______

Key: 2

Sol: $f\left(1^+\right)=\frac{1}{\sqrt{2}}-2$ & $f\left(1^-\right)=\frac{\sqrt{3}}{2}-1$

$f\left(0^+\right)=\frac{\sqrt{3}}{2}-1$ & $f\left(0^-\right)=0$

$f\left(1^+\right)=f\left(-1^{-1}\right)=1$

$\therefore f$ is discontinuous of $n=0\&1$, i,e,. the number of point is 2

85. Let $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$. Let a vector $\vec{v}$ be in the plane containing $\vec{a}$ and $\vec{b}$. If $\vec{v}$ is perpendicular to the vector $3\hat{i}+2\hat{j}-\hat{k}$ and its projection on $\vec{a}$ is 19 units, then $|2\vec{v}|^2$ is equal to ____

Key: 1494

Sol: Let $\vec{a}=2\hat{i}-2\hat{j}+2\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$, $\vec{c}=3\hat{i}+2\hat{j}-\hat{k}$;

$\vec{v}=\lambda\vec{a}+\mu\vec{b}, \frac{\vec{v}.\vec{a}}{|\vec{a}|}=19 \Rightarrow \vec{v}.\vec{a}=57$

$\vec{v}.\vec{c}=\lambda(\vec{a}.\vec{c})+\mu\left(\vec{b}.\vec{c}\right)\Rightarrow\lambda+4\mu=0\to1$

$\vec{v}.\vec{a}=\lambda|\vec{a}|^2+\mu\left(\vec{a}.\vec{b}\right)\Rightarrow 9\lambda-2\mu=57\to2$

Solving 1 & 2 $\lambda=6,\mu=\frac{-3}{2}$

$\therefore\vec{v}=\left(\frac{21}{2}\right)\hat{i}-9\hat{j}+\left(\frac{27}{2}\right)\hat{k}$

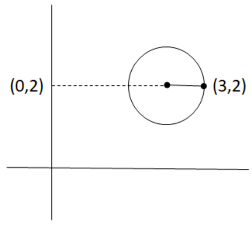
$\therefore|2\vec{v}|^2=441+324+729=1494$

86. If for the complex numbers z satisfying $|z-2-2i|\le1$, the maximum value of $|3iz-6|$ is attained at $a+ib$, then $a+b$ is equal to _____

Key: 5

Sol: $|Z-2-2i|\le1\Rightarrow$ z lies on or interior q circle whole centre is (2, 2) & radius 1

$|3iz+6|=3|z-2i|$



Clearly maximum distance from (0, 2) is at $3+2i$

$\therefore a+b=3+2=5$

87. All the arrangements, with or without meaning, of the word RARMER are written excluding any word that has two R appearing together. The arrangements are listed serially I the alphabetic order as in the English dictionary. Then the serial number of the word RARMER in this list is ______

Key: 77

Sol: The letters are $A.E.F.M.R.R$ no of words the start with 'A' & both R's are not together

$=\frac{5!}{2!}-4!=36$

Similarly start with 'E' & both R's are not gether = 36

No. of words start with 'AE' & R's are not together $=\frac{3!}{2!}-2!=1$

Start with 'FAM' = 1

Start with FARM $\rightarrow 2i=2$

Next word is FARMER

$\therefore$ Total $=36+36+1+1+2+1=77$

88. If the sum of the coefficients in the expansion of $(x+y)^n$ is 4096, then the greatest coefficient in the expansion is ____

Key: 926

Sol: Sum of coefficients $=2^n=4096$

$\Rightarrow n=12$

Greater binomial coefficient $={}^{12}C_6$

$=926$

89. Let $f(x)$ be a polynomial f degree 3 such that $f(k)=-\frac{2}{k}$ for $k=2,3,4,5$. Then the value of $52-10f(10)$ is equal to ___

Key: 26

Sol: $x\,f(x)+2=a(x-2)(x-3)(x-4)(x-5)......(i)$

Put $x=0$

$2=a(-2)(-3)(-4)(-5)$

$a=\dfrac{1}{60}$

Put $a=\dfrac{1}{60}$ in (i), we get

$x\,f(x)+2=\dfrac{1}{60}(x-2)(x-3)(x-4)(x-5)$

Now , Put $x=10$

$10\,f(10)+2=\dfrac{1}{60}8\times7\times6\times5$

$10\,f(10)=26$

$52-10\,f(10)=26$

90. Let X be a random variable with distribution.

X	-2	-1	3	4	6
P(X = x)	1/5	a	1/3	1/5	b

If the mean of X is 2, 3 and variance of X is σ^2, then $100\sigma^2$ is equal to:

Key: 781

Sol: $a+b+\dfrac{1}{5}+\dfrac{1}{5}+\dfrac{1}{3}=1\Rightarrow a+b=\dfrac{4}{15}\to1$

Can = 2.3 = $\sum xipi$

$-\dfrac{2}{5}-a+1+\dfrac{4}{5}6b=2.3\Rightarrow 6b-a=\dfrac{9}{10}\to(2)$

From 1 & 2 $a=\dfrac{1}{10},\ b=\dfrac{1}{6}$

$\sigma^2=\sum xi^2\rho i-\left(\sum xi^2\rho i\right)^2=\dfrac{131}{10}=\dfrac{529}{100}$

$\therefore 100\sigma^2=781$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