Sri Chaitanya

IIT Academy., India

JEE Main 2020

07 Jan 2020, Slot - 1

(9.30 PM - 12.30 PM)

Question Paper

Solutions

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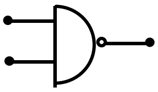
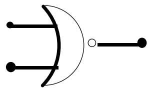
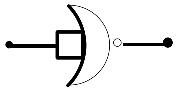
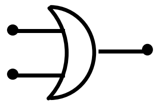
PHYSICS

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. Which of the following gives a reversible operation?

1)  2)  3)  4) 

Ans: 3

Sol: A logic gate is reversible if we can recover input data from the output

Eg. NOT gate

2. A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000N, the speed of the elevator at full load is close to: $(1HP = 746W, g = 10ms^{-2})$

1) 1.9m/s 2) 1.7m/s 3) 2.0m/s 4) 1.5m/s

Ans: 1

Sol: $4000 \times V + mg \times V = P$ $\quad \dfrac{60 \times 746}{4000 + 20000} V$ $\quad V = 1.86 m/s \approx 1.9 m/s$

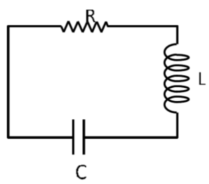
3. A LCR circuit behaves like a damped harmonic oscillator. Comparing it with a physical spring mass damped oscillator having damping constant 'b', the correct equivalence would be

1) $L \leftrightarrow m, C \leftrightarrow \dfrac{1}{K}, R \leftrightarrow b$ 2) $L \leftrightarrow \dfrac{1}{b}, C \leftrightarrow \dfrac{1}{m}, R \leftrightarrow \dfrac{1}{k}$

3) $L \leftrightarrow K, C \leftrightarrow b, R \leftrightarrow m$ 4) $L \leftrightarrow m, C \leftrightarrow K, R \leftrightarrow b$

Ans: 1

Sol: In damped oscillation $\quad ma + bv + kx = 0$



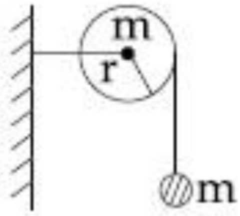
$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \_\_\_\_\_ (i)$$

In the circuit $\quad -iR - L\dfrac{di}{dt} - \dfrac{q}{c} = 0$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{c}.q = 0.....(ii)$$

Comparing equation $(i) \& (ii)$ $\quad m = L, b = R, k = \dfrac{1}{c}$

4. As shown in the figure, a bob of mass m is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius r and mass m. when released from rest the bob starts falling vertically. When it has covered a distance of h, the angular speed of the wheel will be



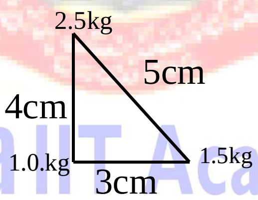
1) $\frac{1}{r}\sqrt{\frac{4gh}{3}}$ 2) $\frac{1}{r}\sqrt{\frac{2gh}{3}}$ 3) $r\sqrt{\frac{3}{2gh}}$ 4) $r\sqrt{\frac{3}{4gh}}$

Ans: 1

Sol: $mgh=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$ $v=\omega R$ (no slipping)

$mgh=\frac{1}{2}m\omega^2R^2+\frac{1}{2}\frac{mR^2}{2}\omega^2$ $mgh=\frac{3}{4}m\omega^2R^2$ $\omega=\sqrt{\frac{4gh}{3R^2}}=\frac{1}{R}\sqrt{\frac{4gh}{3}}$

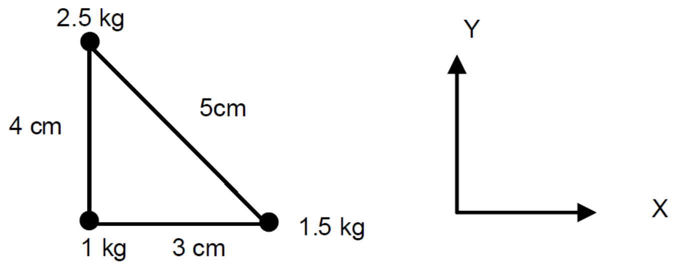
5. Three points particles of masses 1.0kg, 1.5kg and 2.5kg are placed at three corners of right angle triangle of sides 4.0cm, 3.0cm and 5.0cm as shown in the figure. The centre of mass of the system is at a point.



1) 0.6cm right and 2.0cm above 1kg mass
2) 0.9cm right and 2.0cm above 1kg mass
3) 2.0cm right and 0.9cm above 1kg mass
4) 1.5cm right and 1.2cm above 1kg mass

Ans: 2

Sol: Take 1kg mass at origin



$X_{cm}=\frac{1\times0+1.5\times3+2.5\times0}{5}=0.9cm$ $Y_{cm}=\frac{1\times0+1.5\times0+2.5\times4}{5}=2cm$

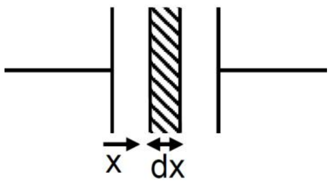
6. A parallel plate capacitor has plates of area A separated by distance 'd' between them. It is filled with a dielectric which has a dielectric constant that varies as $k(x) = K(1+\alpha x)$ where 'x' is the distance measured from one of the plates. If $(\alpha d) << 1$, the total capacitance of the system is best given by the expression

1) $\frac{A \in_0 K}{d}\left[1+\left(\frac{\alpha d}{2}\right)^2\right]$ 2) $\frac{A \in_0 K}{d}\left[1+\frac{\alpha^2 d^2}{2}\right]$

3) $\frac{AK \in_0}{d}(1+\alpha d)$ 4) $\frac{AK \in_0}{d}\left[1+\frac{\alpha d}{2}\right]$

Ans: 4

Sol: Capacitance of element $\frac{K\varepsilon_0 A}{dx}$



Capacitance of element $C' = \frac{K_0(1+\alpha x)\varepsilon_0 A}{dx}$

$\sum \frac{1}{C'} = \int_0^d \frac{dx}{K_0 \varepsilon_0 A(1+\alpha x)}$ $\frac{1}{C} = \frac{1}{K_0\varepsilon_0 A\alpha}\ln(1+\alpha d)$

Given $-\alpha d << 1$ $\frac{1}{C} = \frac{1}{K_0\varepsilon_0 A\alpha}\left(\alpha d - \frac{\alpha^2 d^2}{2}\right)$

$\frac{1}{C} = \frac{d}{K_0\varepsilon_0 A}\left(1-\frac{\alpha d}{2}\right)$ $C = \frac{K_0\varepsilon_0 A}{d}\left(1+\frac{\alpha d}{2}\right)$

7. The time period of revolution of electron in its ground state orbit in a hydrogen atom is 1.6×10^{-16} s. The frequency of revolution of the electron in its first excited state $(in\, s^{-1})$ is

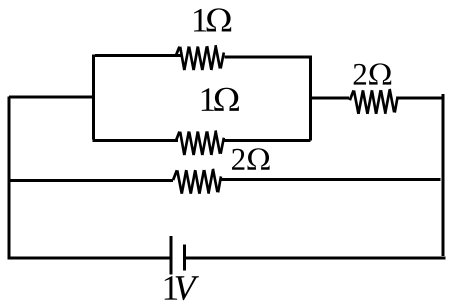
1) 7.8×10^{14} 2) 5.6×10^{12} 3) 6.2×10^{15} 4) 1.6×10^{14}

Ans: 1

Sol: $T \propto \frac{r}{v} \propto \frac{n^2}{z}\times\frac{n}{z} \propto \frac{n^3}{z^2}$ $\frac{T_1}{T_2} = \frac{n_1^3}{n_2^3} = \frac{1}{8}$ $T_2 = 8T_1$ $= 8\times1.6\times10^{-16} = 12.8\times10^{-16}$

$$f_2 = \frac{1}{12.8\times10^{-16}} \approx 7.8\times10^{14}$$

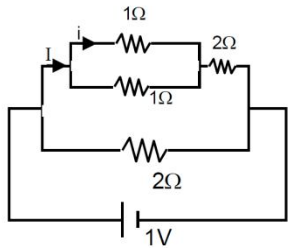
8. The current I_1 (in A) flowing through 1Ω resistor in the following circuit is



1) 0.2 2) 04 3) 0.5 4) 0.25

Ans: 1

Sol: $I=\frac{1}{2.5}=0.4A$ $\quad i=\frac{1}{2}=0.2A$



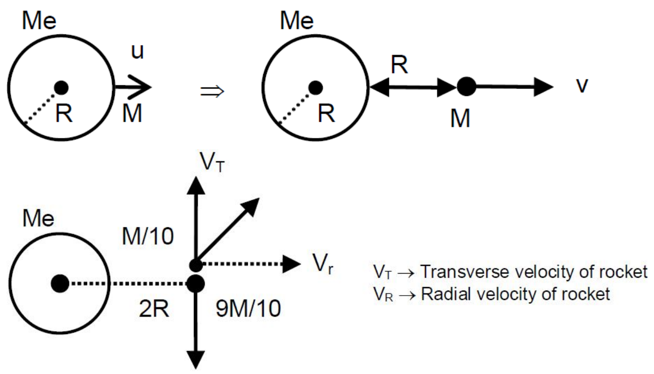
9. A satellite of mass m is launched vertically upwards with as initial speed u from the surface of the earth. After if reaches height $R\left(R=\text{radius of the earth}\right)$, it ejects a rocket of mass $\frac{m}{10}$ so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G is the gravitational constant; M is the mass of the earth):

1) $\frac{3m}{8}\left[u+\sqrt{\frac{5GM}{6R}}\right]^2$ 2) $\frac{m}{20}\left[u-\sqrt{\frac{2GM}{3R}}\right]^2$

3) $5m\left(u^2-\frac{119}{200}\frac{GM}{R}\right)$ 4) $\frac{m}{20}\left(u^2+\frac{113}{200}\frac{GM}{R}\right)$

Ans: 3

Sol: $\frac{-GM_eM}{R}+\frac{1}{2}Mu^2=\frac{f-GM_eM}{2R}+\frac{1}{2}Mv^2$



$v=\sqrt{u^2-\frac{GM_e}{R}}$ $\qquad \frac{M}{10}V_T=\frac{9M}{10}\sqrt{\frac{GM_e}{2R}}$ $\qquad \frac{M}{10}V_r=M\left(u^2-\frac{GM_e}{R}\right)$

Kinetic energy $=\frac{1}{2}\frac{M}{10}\left(V_T^2+V_r^2\right)=\frac{M}{20}\left(81\frac{GM_e}{2R}+100u^2-100\frac{GM_e}{R}\right)$

$=\frac{M}{20}\left(100u^2-\frac{119GM_e}{2R}\right)$ $\qquad =5M\left(u^2-\frac{119GM_e}{200R}\right)$

10. A long solenoid of radius R carries a time (t) - dependent current $I(t)=I_0t(1-t)$. A ring of radius 2R is placed coaxially near its middle. During the time interval $0\le t\le 1$, the inducted current (I_R) and the induced EMF (V_R) in the ring changes as:

1) Direction of I_R remains unchanged and V_R is maximum at t=0.5

2) At t=0.5 direction of I_R reverses and V_R is zero

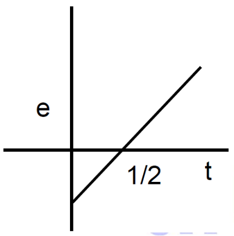
3) Direction of I_R remains unchanged and V_R is zero at t=0.25

4) At t=0.25 direction of I_R reverses and V_R is maximum

Ans: 2

Sol: $I=I_0t-I_0t^2$ $\qquad \phi=BA$ $\qquad \phi=\mu_0 nIA$

$V_R=-\frac{d\phi}{dt}=-\mu_0 nAI_0(1-2t)$ $\qquad V_R=0$ at $t=\frac{1}{2}$ $\qquad$ And $I_R=\frac{V_R}{\text{Resistance of loop}}$

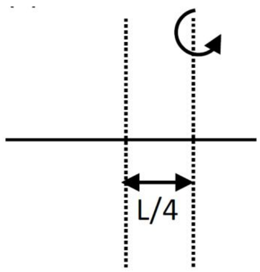


11. The radius of gyration of a uniform rod of length l, about an axis passing through a point $\frac{l}{4}$ away from the centre of the rod, and perpendicular to it is

1) $\sqrt{\frac{7}{48}}l$ $\qquad$ 2) $\sqrt{\frac{3}{8}}l$ $\qquad$ 3) $\frac{1}{8}l$ $\qquad$ 4) $\frac{1}{4}l$

Ans: 1

Sol: $\frac{ML^2}{12}+M\left(\frac{L}{4}\right)^2=MK^2$



$\frac{L^2}{12}+\frac{L^2}{16}=K^2$ $\qquad K=\sqrt{\frac{7}{48}}L$

12. Two moles of an ideal gas with $\frac{C_P}{C_V}=\frac{5}{3}$ are mixed with 3 moles of another ideal gas with $\frac{C_P}{C_V}=\frac{4}{3}$. The values of $\frac{C_P}{C_V}$ for the mixture is

1) 1.50 2) 1.42 3) 1.47 4) 1.45

Ans: 2

sol: $\gamma_{mixture}=\frac{n_1C_{P_1}+n_2C_{P_2}}{n_1C_{V_1}+n_2C_{V_2}}=\frac{n_1\frac{\gamma_1 R}{\gamma_1-1}+n_2\frac{\gamma_2 R}{\gamma_2-1}}{\frac{n_1R}{\gamma_1-1}+\frac{n_2R}{\gamma_2-1}}$

no rearranging we get

$\frac{n_1+n_2}{\gamma_{mix}-1}=\frac{n_1}{\gamma_1-1}+\frac{n_2}{\gamma_2-1}$; $\quad \frac{5}{\gamma_{mix}-1}=\frac{3}{1/3}+\frac{2}{2/3}$

$\frac{5}{\gamma_{mix}-1}=9+3=12 \Rightarrow \gamma_{mixure}=\frac{17}{12}=1+\frac{5}{12}$; $\quad \gamma_{mix}=1.42$

13. A litre of dry air at STP expands adiabatically to a volume of 3litres. If γ=1.40, the work done by air is: ($3^{1.4}$=4.6555) [Take air to be an ideal gas)

1) 100.8J 2) 90.5J 3) 60.7J 4) 48J

Ans: 2

Sol: $P_1=1atm, T_1=273K \quad P_1V_1^{\gamma}=P_2V_2^{\gamma} \quad P_2=P_1\left[\frac{V_1}{V_2}\right]^{\gamma} \quad =1atm\left(\frac{1}{3}\right)^{1.4}$

Now work done $=\frac{P_1V_1-P_2V_2}{\gamma-1}=88.7J$

Closest ans is 90.5J

14. If the magnetic field in a plane electromagnetic wave is given by $\vec{B}=3\times10^{-8}\sin\left(1.6\times10^{3}x+48\times10^{10}t\right)\hat{j}T$; then what will be the expression for the electric field?

1) $\vec{E}=\left(9\sin\left(1.6\times10^{3}x+48\times10^{10}t\right)\hat{k}V/m\right)$

2) $\vec{E}=\left(3\times10^{-8}\sin\left(1.6\times10^{3}x+48\times10^{10}t\right)\hat{j}V/m\right)$

3) $\vec{E}=\left(3\times10^{-8}\sin\left(1.6\times10^{3}x+48\times10^{10}t\right)\hat{i}V/m\right)$

4) $\vec{E}=\left(60\sin\left(1.6\times10^{3}x+48\times10^{10}t\right)\hat{k}V/m\right)$

Ans:1

Sol: $\frac{E_0}{B_0} = C$ (speed of light in vacuum)

$E_0 = B_0 C = 3\times10^{-8}\times3\times10^{8}$ $= 9N/C$

15. A polarizer-analyser set is adjusted such that the intensity of light coming out of the analyser is just 10% of the original intensity. Assuming that the polarizer -analyser set does not absorb any light, the angle by which of the analyser need to be rotated further to reduce the output intensity to be zero is

1) 45º 2) 71.6º 3) 90º 4) 18.4º

Ans: 4

Sol: $I = I_0\cos^2\theta$ $\frac{I_0}{10} = I_0\cos^2\theta$

$\cos\theta = \frac{1}{\sqrt{10}} = 0.31 < \frac{1}{\sqrt{2}}$ which is 0.707

So $\theta > 45^\circ$ and $90-\theta < 45^\circ$ so only one option is correct

i.e 18.4º

angle rotated should be =90º-71.6º=18.4º

16. Speed of transverse wave of a straight wire (mass 6.0g, length 60cm and area of cross section 1.0 mm^2) is 90 ms^{-1}. If the young's modulus of wire in $16\times10^{11}\,\mathrm{Nm}^{-2}$ the extension of wire over its natural length is.

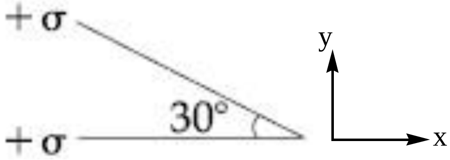
1) 0.03mm 2) 0.02mm 3) 0.01mm 4) 0.04mm

Ans: 1

Sol: $V = \sqrt{\frac{T}{\mu}}$ $T = \mu v^2$ $\frac{\mu V^2}{A} = Y\frac{\Delta l}{l}$ $\Delta l = \frac{\mu V^2 l}{AY}$

After substituting value of $\mu, v, l,$ A and Y we get $\Delta l = 0.03mm$

17. Two infinite planes each with uniform surface charge density $+\sigma$ are kept in such a way that the angle but them is 30^0. The electric field in the region shown between them is given by

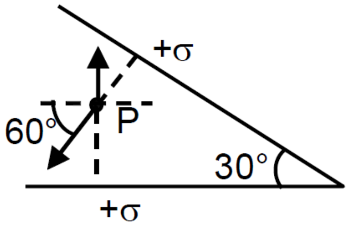


1) $\frac{\sigma}{2\varepsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{y}-\frac{\hat{x}}{2}\right]$ 2) $\frac{\sigma}{2\varepsilon_0}\left[\left(1+\sqrt{3}\right)\hat{y}+\frac{\hat{x}}{2}\right]$

3) $\frac{\sigma}{\varepsilon_0}\left[\left(1+\frac{\sqrt{3}}{2}\right)\hat{y}+\frac{\hat{x}}{2}\right]$ 4) $\frac{\sigma}{2\varepsilon_0}\left[\left(1+\sqrt{3}\right)\hat{y}-\frac{\hat{x}}{2}\right]$

Ans: 1

Sol: $\vec{E}=\frac{\sigma}{2\varepsilon_0}\cos 60^{\circ}\left(-\hat{x}\right)+\left[\frac{\sigma}{2\varepsilon_0}-\frac{\sigma}{2\varepsilon_0}\sin 60^{\circ}\right]\left(\hat{y}\right)$



$=\frac{\sigma}{2\varepsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{y}-\frac{\hat{x}}{2}\right]$

18. If we need a magnification of 375 from a compound microscope of tube length 150 mm and an objective of focal length 5mm, the focal length of the eye-piece, should be close to :

1) 12 mm 2) 33mm 3) 22m 4) 2 mm

Ans: 3

Sol: Case-I

If final image at least distance of clear vision

$M.P.=\frac{L}{f_0}\left(1+\frac{D}{f_e}\right);375=\frac{150}{5}\left[1+\frac{25}{f_e}\right]\quad \frac{375}{30}=1+\frac{25}{f_e}\quad \frac{345}{30}=\frac{25}{f_e}$

$f_e=\frac{750}{345}=2.17cm;\ f_e\approx 22mm$

Case-II

If final image is at infinity

$M.P=\frac{L}{f_0}\left(\frac{D}{f_e}\right)=375\qquad f_e=22mm$

19. Visible light of wavelength 6000×10^{-8} cm falls normally on a single slit and produces a diffraction pattern. It is found that the second diffraction minimum is at 60^{0} from the central maximum. If the first minimum is produced at θ_1, then θ_1, is close to,

1) 25^0 2) 45^0 3) 20^0 4) 30^0

Ans: 1

Sol: For 2nd minima $d\sin\theta=2\lambda$

$\sin\theta=\frac{\sqrt{3}}{2}(given)\quad \Rightarrow\frac{\lambda}{d}=\frac{\sqrt{3}}{4}\_\_\_\_\_(i)$

So for 1st minima is $d\sin\theta=\lambda$

$\sin\theta=\frac{\lambda}{d}=\frac{\sqrt{3}}{4}$ from equation (i)

$\theta=25.65^{\circ}$

$\theta\approx25^{\circ}$

20. Consider a circular coil of wire carrying constant current I, forming a magnetic dipole. The magnetic flux through an infinite plane that contains the circular coil and excluding the circular coil area is given by ϕ_i. The magnetic flux through the area of the circular coil is given by ϕ_0. Which of the following option is correct?

1) $\phi_i = -\phi_0$ 2) $\phi_i > \phi_0$ 3) $\phi_i < \phi_0$ 4) $\phi_i = \phi_0$

Ans: 1

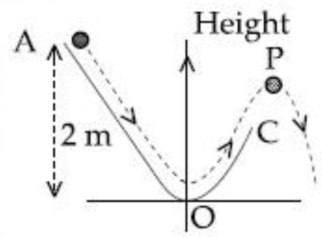
Sol: As magnetic field lines always form a closed loop, hence every magnetic field line creating magnetic flux in the inner region must be passing through the outer region. Since flux in two regions are in opposite direction $\therefore \phi_i = -\phi_0$

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value(in decimal notation, truncated/rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer , 0 if not attempted and 0 in all other cases.

21. A particle (m = 1kg) slides down a frictionless track (AOC) starting from rest at a point A (height 2 cm) After reaching C, the particle continues to move freely in air as a projectile. When it reaching its highest point P(height 1m), the kinetic energy of the particle (in J) is:(Figure drawn is schematic and not to scale; take g = 10 ms^{-2})



Ans: 10

Sol: $KE = PEl1 - PE_2 = mgh_1 - mgh_2$ $= 1\times10\times2 - 1\times10\times1 - 10J$

22. A Carnot engine operates between two reservoirs of temperature 900 K and 300 K. The engine performs 1200 J of work per cycle. The heat energy (in J) delivered by the engine to the low temperature reservoir, in a cycle is_____________.

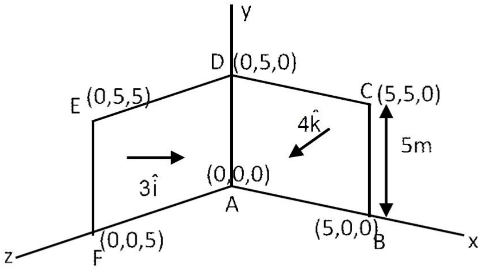
Ans: 600

Sol: $\eta = \frac{W}{Q_h} = 1 - \frac{300}{900} = \frac{2}{3}$ $Q_h = \frac{3}{2}W = 1800J$ $Q_L = Q_h - Q = 600J$

23. A loop ABCDEFA of straight edges has six corner points A(0,0,0), B(5,0,0), C(5,5,0), D(0,5,0), E(0,5,5) and F(0,0,5). The magnetic field in this region is $\vec{B} = (3\hat{i} + 4\hat{k})T$. The quantity of flux through the loop ABCDEFA (in Wb) is____.

Ans: 175

Sol: $\phi = \vec{B}.\vec{A} = \left(3\hat{i}+4\hat{k}\right).\left(25\hat{i}+25\hat{k}\right)$ $\phi = (3\times 25)+(4\times 25) = 175\,\text{weber}$



24. A beam of electromagnetic radiation of intensity 6.4×10^{-5} W/cm^2 is comprised of wavelength, $\lambda = 310\,\text{nm}$. It falls normally on a metal(work function $\varphi = 2\text{eV}$) of surface area 1cm^2. If one in 10^3photons ejects an electron, total number of electrons ejected in 1s is 10^x (hc = 1240 eVnm, 1eV = 1.6×10^{-19}J) , then x is ______

Ans: 11

Sol: Power incident $P = I\times A$

No=no.of photons incident / second

$$nE_{ph} = IA \qquad n = \frac{IA}{E_{ph}}$$

$$n = \frac{IA}{\left(\frac{hc}{\lambda}\right)} = \frac{6.4\times10^{-5}\times1}{\frac{1240}{310}\times1.6\times10^{-19}}$$

$n = 10^{+14}$ per second

Since efficiency $=10^{-3}$

No.of electrons emitted $=10^{+11}$ per second.

$x = 11$.

25. A non- isotropic solid metal cube has coefficients of linear expansion as: $5\times10^{-5}\,/\,^0\text{C}$ along the x-axis and $5\times10^{-6}\,/\,^0\text{C}$ along the y and the z-axis. If coefficient of volume expansion of the solid $\text{C}\times10^{-6}\,/\,^0\text{C}$ then the value of C is __________

Ans: 60

Sol: $V = 2\alpha_2 + \alpha_1 \quad = 10\times10^{-6} + 5\times10^{-5}$

$= 60\times10^{-6}\, I\,{}^{o}C$

CHEMISTRY

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

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26. The relative strength of interionic/intermolecular forces in decreasing order is:

1) ion-dipole > dipole –dipole > ion-ion

2) dipole-dipole>ion – dipole > ion – ion

3) ion-ion > ion –dipole > dipole – dipole

4) ion-dipole > ion-ion > dipole-dipole

Ans: 3

Sol: Ion – ion interactions are the strongest type

27. Oxidation number of potassium in K_2O, K_2O_2 and KO_2, respectively is:

1) +1,+2 and +4 2) +1,+4 and +2 3) +2, +1 and $+\frac{1}{2}$ 4) +1,+1 and +1

Ans: 4

Sol: Potassium, being an alkali metal has the same oxidation state of +1 in all the three compounds given.

28. At 35°C, the vapour pressure of CS_2, is 512 mm Hg and that of acetone is 344mm Hg. A solution of CS_2 in acetone has a total vapour pressure of 600mm Hg. The false statement amongst the following is:

1) CS_2 and acetone are less attracted to each other than to themselves

2) heat must be absorbed in order to produce the solution at 35°C

3) Raoult's law is not obeyed by this system

4) a mixture of 100 mL CS_2 and 100 mL acetone has a volume of <200 mL

Ans: 4

Sol: A mixture of acetone and CS_2 exhibits positive deviation from ideal behaviour. This should lead to $\Delta V_{mix} = +ve$. So option (d) is incorrect

29. The atomic radius of Ag is closest to:

1) Ni 2) Cu 3) Au 4) Hg

Ans: 3

Sol: 4d series elements and corresponding 5d series elements have similar atomic radii as a consequence of lanthanoid contraction. This results in Au having the closest atomic radius to Ag.

30. The dipole moments of CCl_4,$CHCl_3$ and CH_4 are in the orders:

1) $CH_4 < CCl_4 < CHCl_3$
2) $CHCl_3 < CH_4 = CCl_4$
3) $CH_4 = CCl_4 < CHCl_3$
4) $CCl_4 < CH_4 < CHCl_3$

Ans: 3

Sol: $CHCl_3$ is the only polar compound given among the two.

$CH_4 = CCl_4 < CHCl_3$

31. In comparison to the zeolite process for the removal of permanent hardness, the synthetic resin method is:

1) less efficient as the resins cannot be regenerated
2) more efficient as it can exchange only cations
3) less efficient as it exchanges only anions
4) more efficient as it can exchange both cations as well as anions

Ans: 4

Sol: Synthetic resins are advantageous for removal of permanent hardness as they exchange both cations and anions.

32. Among the following statements that which was not proposed by Dalton was

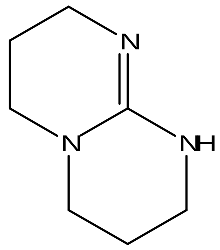
1) chemical reactions involve reorganization of atoms. These are neither created nor destroyed in a chemical reaction.
2) when gases combine or reproduced in a chemical reaction they do so in a simple ratio by volume provided all gases are at the same T & P.
3) matter consists of indivisible atoms.
4) all the atoms of a given element have identical properties including identical mass. Atoms of different elements differ in mass.

Ans: 2

Sol: (b) is Gay Lussac's law of combining gas volumes.

33. The increasing orders of pK_b for the following compounds will be:

$NH_2-CH=NH$ (A) ,



(B)

, CH_3NHCH_3 (C)

1) $(B)<(C)<(A)$ 2) $(B)<(A)<(C)$ 3) $(C)<(A)<(B)$ 4) $(A)<(B)<(C)$

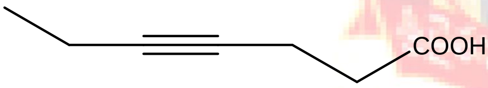
Ans: 2

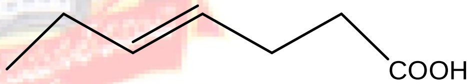
Sol: (B) is guanidine type and hence is the most basic followed by (A) which is an amidine which must be more basic than the simple secondary amine (C)

Therefore the order of pKa is B<A<C

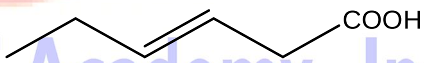
34. What is the product of following reaction?

$$HeX-3-ynal \xrightarrow[\substack{ii)PBr_3\\ iii)Mg/ether\\ iv)CO2/H_3O^+}]{i)NaBH_{44}}$$

1) 

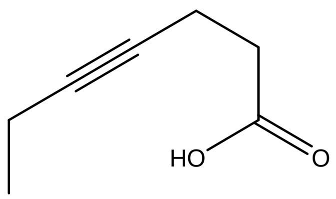
2) 

3) 

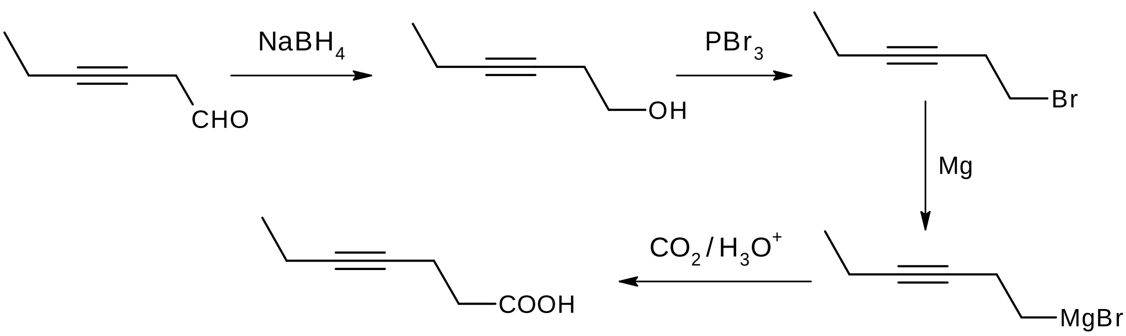
4) 

Ans:1

Sol:



hept-4-yn oic acid



35. The number of orbitals associated with quantum numbers $n=5, m_5=+\frac{1}{2}$ is:

1) 11 2) 50 3) 25 4) 15

Ans: 3

Sol: $n=5 \Rightarrow$ possible orbitals $= n^2 = 25 \Rightarrow$ there can be 25 electrons with n=5, $m_s = +\frac{1}{2}$

36. The purest form of commercial iron is:

1) wrought iron 2) pig iron 3) cast iron 4) scrap iron and pig iron

Ans: 1

Sol: Wrought iron is the purest form of commercial iron.

37. The theory that can completely/properly explain the nature of bonding in $\left[Ni(Co)_4\right]$ is:

1) Werner's theory 2) Crystal field theory
3) Molecular orbital theory 4) Valence bond theory

Ans: 3

Sol: Bonding in metal carbonyls is completely explained by Molecular orbital theory.

38. The IUPAC name of the complex $\left[pt(NH_3)_2 Cl(NH_2CH_3)\right]Cl$ is:

1) Diammine(methanamine)chlorido platinum(II) chloride

2) Bisammine(methanamine)chlorido platinum(II) chloride

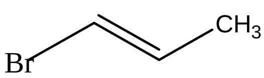
3) Diamminechlorido(aminomethane) platinum(II) chloride

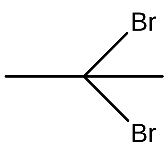
4) Diamminechlorido(methanamine) platinum(II) chloride

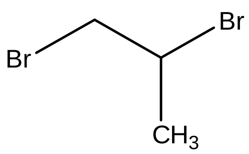
Ans: 4

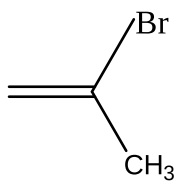
Sol: Diamminechlorido(methylamine)platinum(II) chloride

39. 1-methyl ethylene oxide when treated with an excess of HBr produces:

1) 

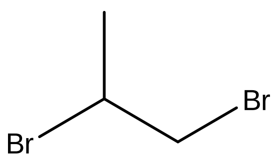
2) 

3) 

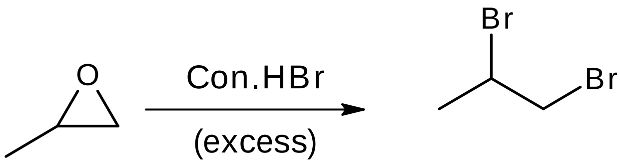
4) 

Ans: 3

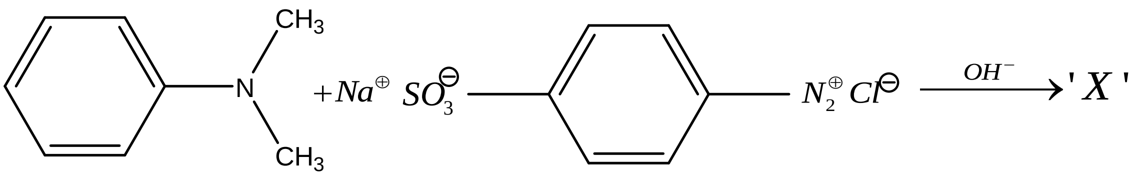
Sol:



1,2-dibromo propane



40. Consider the following reaction



The product 'X' is used

1) in proteins estimation as an alternative to ninhydrin

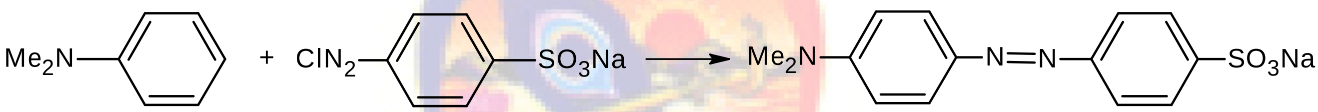
2) as food grade colourant

3) in laboratory test for phenols

4) in acid basetitration as an indicator

Ans: 4

Sol: The compound formed is methyl orange and is used as an acid-base indicator.



41. Math the following

i) Riboflavin a) Beriberi

ii) Thiamine b) Scurvy

iii) Pyridoxine c) Cheilosis

iv) Ascorbic acid d) Convulsions

1) (i) – (a) ; (ii) – (d); (iii) – (c) ; (iv) – (b)
2) (i) – (d) ; (ii) – (b); (iii) – (a) ; (iv) – (c)
3) (i) – (c) ; (ii) – (d); (iii) – (a) ; (iv) – (b)
4) (i) – (c) ; (ii) – (a); (iii) – (d) ; (iv) – (b)

Ans: 4

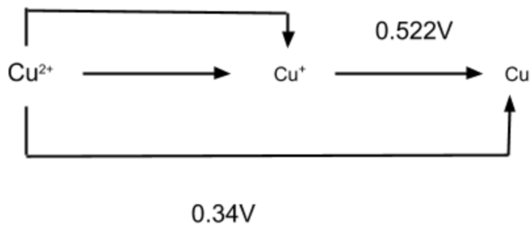
Sol: Riboflavin - cheilosis
Thiamine - beriberi
Pyridoxine - convulsions
Ascorbic acid – Scurvy

42. Given that the standard potential (E°) of Cu^{2+}/Cu and Cu^{+}/Cu are 0.34 V and 0.522 V respectively, the E° of Cu^{2+}/Cu^{+} is:

1) 0.182V 2) -0.158V 3) -0.182 V 4) +0.158 V

Ans: 4

Sol:



$$1\times E^{o}_{Cu^{2+}|Cu^{+}} + 1\times E^{o}_{Cu^{+}|Cu} = 2\times E^{o}_{Cu^{2+}|Cu} \Rightarrow E^{o}_{Cu^{2+}|Cu^{+}} = \frac{2\times 0.34 - 0.522}{1} = 0.158\text{V}$$

43. A solution of m-chloroaniline, m-chlorophenol and m-chlorobenzoic acid in ethyl acetate was extracted initially with a saturated solution of $NaHCO_3$ to give fraction A. The left over organic phase was extracted with dilute NaOH solution to give fraction B. The final organic layer was labelled as fraction C. Fractions A, B and C contain respectively:

1) m-chlorophenol, m-chlorobenoic acid and m-chloroaniline

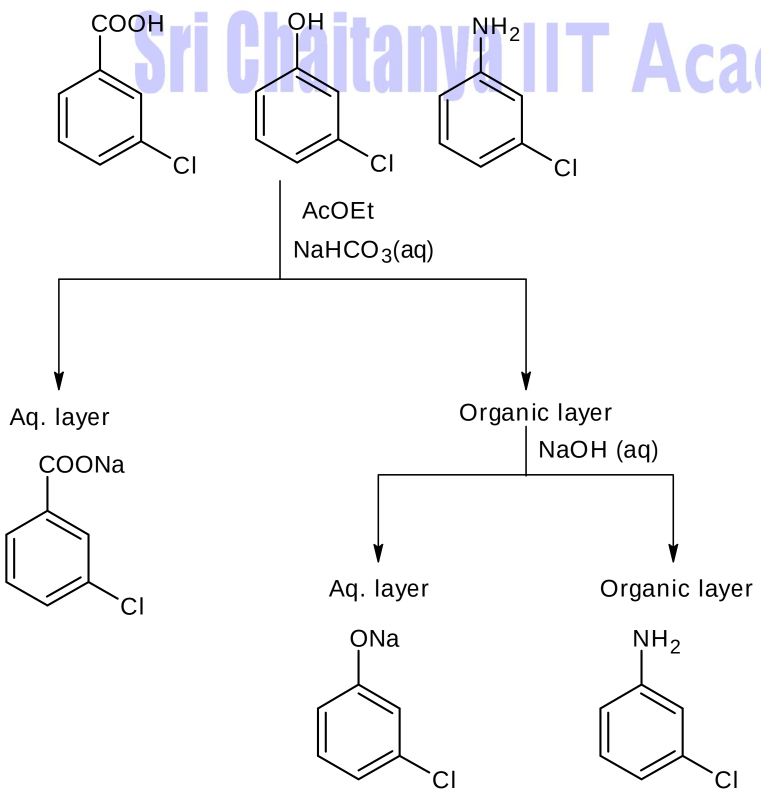
2) m-chlorobenzoic acid, m-chloroaniline and m-chlorophenol

3) m-chloroaniline, m-chlorobenzoic acid and m-chlorophenol

4) m-chlorobenzoic acid, m-chlorophenol and m-chloroaniline

Ans: 4

Sol:



44. The electron gain enthalpy (in kJ/mol) of fluorine, chlorine, bromine and iodine, respectively are:

1) -333, -325, -349 and -296 2) -333, -349, -325 and -296

3) -349, -333, -325 and -296 4) -296, -325, -333 and -349

Ans: 2

Sol: Electron gain enthalpy order: Cl>F>Br>I

45. Consider the following reaction:

a) $(CH_3)_3(CH(OH)CH_3) \xrightarrow{conc.H_2SO_4}$ b) $(CH_3)_2CHCH(Br)CH_3 \xrightarrow{alc.KOH}$

c) $(CH_3)_2CHCH(Br)CH_3 \xrightarrow{(CH_3)_3O^-K^+}$ d) $(CH_3)_2\underset{\underset{OH}{|}}{C}-CH_2-CHO \xrightarrow{\Delta}$

Which of these reaction(s) will not produce Saytzeff product?

1) a) ,c) and d) 2) d) only 3) b) and d) 4) c) only

Ans: 4

Sol: Reaction C) gives Hoffman product as the major product.

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer , 0 if not attempted and 0 in all other cases.

46. Two solutions, A and B, each of 100 L was made by dissolving 4g of NaOH and 9.8 g of H_2SO_4 in water, respectively. The pH of the resultant solution obtained from mixing 40 L of solution A and 10 L of solution B is _____.

Ans: 10.6

Sol: $pH = 14 - \log\left[\frac{\frac{4}{40}}{100}\times 40 - \frac{\frac{9.8}{98}}{100}\times 10\times 2\right]\frac{1}{50} = 10.6$

47. During the nuclear explosion, one of the products is ^{90}Sr with half life of 6.93 years , if $1\mu g$ of ^{90}Sr was absorbed in the bones of a newly born baby in place of Ca, how much time, in years, is required to reduce it by 90% if it is not lost metabolically ____.

Ans: 23.00 to 23.03

Sol: $t = \dfrac{2.303}{\left(\dfrac{0.693}{6.93}\right)} \log \dfrac{100}{100-90} = 2.303 \times 10 = 23.03y$

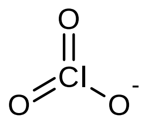
48. Chlorine reacts with hot and concentrated NaOH and produces compounds(X) and (Y). compound (X) gives white precipitate with silver nitrate solution. The average bond order between Cl and O atoms in (Y) is _____.

Ans: 1.66 to 1.67

Sol:

$$3Cl_2(g) + 6NaOH \xrightarrow{\Delta} \underset{(X)}{5NaCl} + \underset{(Y)}{NaClO_3} + 3H_2O$$

Structure of anion of Y is



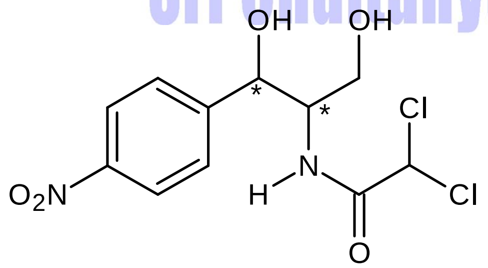
$$\text{Bond order of Cl-O bond} = 1 + \frac{2}{3} = 1.666... = 1.67$$

So 1.66 to 1.67

49. The number of chiral carbons in chloramphenicol is____

Ans: 2

Sol: Chloramphenicol has two chiral centres.



50. For the reaction $A(l) \rightarrow 2B(g)$

$\Delta U = 2.1Kcal, \Delta S = 20calK^{-1}$ at $300K$. Hence ΔG is Kcal is _____.

Ans: -2.7

Sol:

$$A(l) \longrightarrow 2B(g)$$
$$\Delta H = \Delta U + 2RT = 2100 + 2 \times 2 \times 300 = 3300cal$$
$$\Delta G = \Delta H - T\Delta S = 3300 - 300 \times 20 = -2700cal = -2.7kcal$$

MATHEMATICS

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

51. If $f(a+b+1-x)=f(x)$, for all x, where a and b are fixed positive real numbers, then $\frac{1}{a+b}\int_a^b x(f(x))+f(x+1)dx$ is equal to

1) $\int_{a+1}^{b+1} f(x)dx$ 2) $\int_{a+1}^{b+1} f(x+1)dx$ 3) $\int_{a-1}^{b-1} f(x+1)dx$ 4) $\int_{a-1}^{b-1} f(x)dx$

Ans: 1

Sol: $f(x+1)=f(a+b-x)$

$$I=\frac{1}{(a+b)}\int_a^b x\left(f(x)+f(x+1)\right)dx........(1)$$

$$I=\frac{1}{(a+b)}\int_a^b (a+b-x)\left(f(x+1)+f(x)\right)dx......(2)$$

From 1 and 2

$$2I=\int_a^b \left(f(x)+f(x+1)\right)dx$$

$$2I=\int_a^b f(a+b-x)dx+\int_a^b f(x+1)dx$$

$$2I=2\int_a^b f(a+1)dx \Rightarrow I=\int_a^b f(x+1)dx$$

$$=\int_{a+1}^{b+1} f(x)dx$$

52. let $\alpha \,\&\, \beta$ be two real roots of the equation $(k+1)\tan^2 x-\sqrt{2}.\lambda \tan x=(1-k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha+\beta)=50$ then a value of λ is:

1) 10 2) 5 3) $5\sqrt{2}$ 4) $10\sqrt{2}$

Ans: 1

Sol: $(k+1)\tan^2 x-\sqrt{2}\lambda \tan x+(k-1)=0$

$$\tan\alpha+\tan\beta=\frac{\sqrt{2}\lambda}{k+1}$$

$$\tan\alpha\tan\beta=\frac{k-1}{k+1}$$

$$\tan(\alpha+\beta)=(k-1)=0\frac{\frac{\sqrt{2}\lambda}{k+1}}{1-\frac{k-1}{k+1}}=\frac{\sqrt{2}\lambda}{2}=\frac{\lambda}{\sqrt{2}}$$

$$\tan^2(\alpha+\beta)=\frac{\lambda^2}{2}=50$$

$$\lambda=10$$

53. Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 appear is:

1) $6!$ 2) $\frac{5}{2}(6!)$ 3) $\frac{1}{2}(6!)$ 4) 5^6

Ans: 2

Sol: 1,3,5,7,9

For digit to repeat we have ${}^5C_1$ choice

And six digits can be arrange in $\frac{\lfloor 6}{\lfloor 2}$ ways

Hence total such number = $\frac{5\lfloor 6}{\lfloor 2}$

54. If y = mx + 4 is tangent to both the parabolas, $y^2 = 4x$ and $x^2 = 2by$, then b is equal to

1) -64 2) -32 3) -128 4) 128

Ans: 3

Sol: y = mx+4 ….(i)

$y^2 = 4x$ tangent $y = mx + \frac{a}{m} \Rightarrow y = mx + \frac{1}{m}$ ….(ii)

from (i) and (ii)

$4 = \frac{1}{m} \Rightarrow m = \frac{1}{4}$

So line $y = \frac{1}{4}x + 4$ is also tangent to parabola $x^2 = 2by$, so solve

$x^2 = 2b\left(\frac{x+16}{4}\right)$ $\Rightarrow 2x^2 - bx - 16b = 0 \Rightarrow D = 0 \Rightarrow b^2 - 4\times 2\times(-16b) = 0$

$\Rightarrow b^2 + 32\times 4b = 0$ $b = -128, b = 0$ (not possible)

55. Let α be a root of the equation $x^2 + x + 1 = 0$ and the matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}$, then the matrix A^{31} is equal to

1) A 2) A^2 3) A^3 4) I_3

Ans: 3

Sol: $A^2 = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$\Rightarrow A^4 = I \qquad \Rightarrow A^{30} = A^{28} \times A^3 = A^3$

56. If y = y(x) is the solution of the differential equation, $e^y\left(\frac{dy}{dx} - 1\right) = e^x$, such that y(0) = 0, then y(1) is equal to:

1) 2e 2) $2 + \log_e 2$ 3) $1 + \log_e 2$ 4) $\log_e 2$

Ans: 3

Sol: $e^y = t \qquad e^y \frac{dy}{dx} = \frac{dt}{dx}$

$IF = e^{\int -1.dx} = e^{-x} \qquad e^{y-x} = x + c$

Put $x = 0, y = 0$ then c = 1 $\quad e^{y-x} = x + 1$

$y = x \times \ell n(x+1)$ at $x = 1, y = 1 + \ell n(2)$

57. If $y(\alpha) = \sqrt{2\left[\frac{\tan\alpha + \cot\alpha}{1 + \tan^2\alpha}\right] + \frac{1}{\sin^2\alpha}}, \alpha \in \left[\frac{3\pi}{4}, \pi\right]$, then $\frac{dy}{d\alpha}$ at $\alpha = \frac{5\pi}{6}$ is:

1) 4 2) $-\frac{1}{4}$ 3) $\frac{4}{3}$ 4) -4

Ans: 1

Sol: $y = \sqrt{\frac{2\cos^2\alpha}{\sin\alpha\cos\alpha} + \frac{1}{\sin^2\alpha}} = \sqrt{2\cot\alpha + \operatorname{cosec}^2\alpha} = |1 + \cot\alpha| = -1 - \cot\alpha$

$\frac{dy}{d\alpha} = \operatorname{cosec}^2\alpha \Rightarrow \left(\frac{dy}{d\alpha}\right) at\ \alpha = \frac{5\pi}{6}$ will be = 4

58. Let the function $f : [-7, 0] \to R$ be continuous on $[-7, 0]$ and differentiable on $(-7, 0)$. If $f(-7) = -3$ and $f'(x) \le 2$ for all $x \in (-7, 0)$, then for all such function f, $f(-1) + f(0)$ is in the interval:

1) $[-3, 11]$ 2) $(-\infty, 20]$ 3) $(-6, 20)$ 4) $(-\infty, 11]$

Ans: 2

Sol: Lets use LMVT for $x \in [-7,-1]$

$$\frac{f(-1)-f(-7)}{(-1+7)} \le 2 \qquad \frac{f(-1)+3}{6} \le 2 \Rightarrow f(-1) \le 9$$

Also use LMVT for $x \in [-7,0]$

$$\frac{f(0)-f(-7)}{(0+7)} \le 1$$

$$\frac{f(0)+3}{7} \le 2 \Rightarrow f(0) \le 11 \qquad \therefore f(0)+f(-1) \le 20$$

59. A vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}, (\alpha, \beta \in R)$ lies in the plane of the vectors $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$. If $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$. Then

1) $\vec{a}.\hat{i} + 1 = 0$ 2) $\vec{a}.\hat{i} + 3 = 0$ 3) $\vec{a}.\hat{k} + 4 = 0$ 4) $\vec{a}.\hat{k} + 2 = 0$

Ans: 4

Sol: angle bisector can be $\vec{a} = \vec{a} = \lambda(\hat{b} + \hat{c}) \, or \, \vec{a} = \mu(\hat{b} - \hat{c})$

$$\vec{a} = \lambda\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}} + \frac{\hat{i}+\hat{j}+4\hat{k}}{3\sqrt{2}}\right) = \frac{\lambda}{3\sqrt{2}}\left[3\hat{i}+3\hat{j}+\hat{i}-\hat{j}+4\hat{k}\right] = \frac{\lambda}{3\sqrt{2}}\left[4\hat{i}+2\hat{j}+4\hat{k}\right]$$

Compare with $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$

$$\frac{2\lambda}{3\sqrt{2}} = 2 \Rightarrow \lambda = 3\sqrt{2}$$

$$\vec{a} = 4\hat{i} + 2\hat{j} + 4\hat{k}$$

Not in option so now consider $\vec{a} = \mu\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}} - \frac{\hat{i}-\hat{j}+4\hat{k}}{3\sqrt{2}}\right)$

$$\vec{a} = \frac{\mu}{3\sqrt{2}}\left(3\hat{i}+3\hat{j}-\hat{i}+\hat{j}-4\hat{k}\right) \qquad = \frac{\mu}{3\sqrt{2}}\left(2\hat{i}+4\hat{j}-4\hat{k}\right)$$

Compare with $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$

$$\frac{4\mu}{3\sqrt{2}} = 2 \Rightarrow \mu = \frac{3\sqrt{2}}{2} \qquad \vec{a} = \hat{i} + 2\hat{j} - 2\hat{k}$$

$$\vec{a}.\hat{k} + 2 = 0 \qquad -2 + 2 = 0$$

60. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is:

1) $\sqrt{3}$ 2) $3\sqrt{2}$ 3) $\frac{3}{\sqrt{2}}$ 4) $2\sqrt{3}$

Ans: 2

Sol: $2ae=6$ and $\frac{2a}{e}=12$ $\Rightarrow ae=3$ and $\frac{a}{e}=6$ $\Rightarrow a^2=18$

$\Rightarrow b^2=a^2-a^2e^2=18-9=9 \Rightarrow L.R=\frac{2b^2}{a}=\frac{2\times 9}{3\sqrt{2}}=3\sqrt{2}$

61. The greatest positive integer k, for which 49^k+1 is a factor of the sum $49^{125}+49^{124}+......+49^2+49+1,$ is

1) 65 2) 32 3) 60 4) 63

Ans: 4

Sol: $\frac{(49)^{126}-1}{48}=\frac{\left((49)^{63}+1\right)\left(49^{63}-1\right)}{48}$

62. Let P be a plane passing through the points (2, 1, 0), (4, 1, 1) and (5, 0,1) and R be any point (2, 1,6). Then the image of R in the plane P is:

1) (6,5,-2) 2) (6,5,2) 3) (4,3,2) 4) (3,4,-2)

Ans: 1

Sol: Plane is $x+y-2z=3 \Rightarrow \frac{x-2}{1}=\frac{y-1}{1}=\frac{z-6}{-2}=\frac{-2(2+1-12-3)}{6} \Rightarrow (x,y,z)=(6,5,-2)$

63. If the system of linear equations

$2x+2ay+az=0$

$2x+3by+bz=0$

$2x+4cy+cz=0$

Where $a,b,c\in R$ are non-zero and distinct; has non-zero solution, then,

1) $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in A.P. 2) a,b,c are in A.P

3) a, b, c are in G.P 4) a + b + c = 0

Ans: 1

Sol: For non-zero solution

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0 \qquad \begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

$$(3bc-4bc)-(2ac-4ac)+(2ab-3ab)=0$$

$$-bc+2ac-ab=0$$

$$ab+bc=2ac$$

a,b,c in H.P

$\Rightarrow \frac{1}{a},\frac{1}{b},\frac{1}{c}$ in A.P.

64. The area of the region, enclosed by the circle $x^2+y^2=2$ which is not common to the region bounded by the parabola $y^2=x$ and the straight line y = x, is:

1) $\frac{1}{6}(24\pi-1)$ 2) $\frac{1}{6}(12\pi-1)$ 3) $\frac{1}{3}(6\pi-1)$ 4) $\frac{1}{3}(12\pi-1)$

Ans: 2

Sol: Total area – enclosed area $2\pi-\int_0^1 \sqrt{x}-x\,dx$

$$2\pi-\left(\frac{2x^{3/2}}{3}-\frac{x^2}{2}\right)_0^1 \; 2\pi-\left(\frac{2}{3}-\frac{1}{2}\right)\Rightarrow 2\pi-\left(\frac{1}{6}\right)\Rightarrow\frac{12\pi-1}{6}$$

65. The logical statement $(p\Rightarrow q)\wedge(q\Rightarrow\sim p)$ is equivalent to:

1) $\sim q$ 2) $\sim p$ 3) p 4) q

Ans: 2

Sol:

p	q	$p\to q$	$\sim p$	$q\to\sim p$	$(p\to q)\wedge(p\to\sim q)$
T	T	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	T	T

Cleary $(p\to q)\wedge(q\to\sim p)$ is equivalent to $\sim p$

66. An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value k when k consecutive heads are obtained for k = 3, 4, 5, other wise X takes the value -1.The expected value of X, is:

1) $\frac{1}{8}$ 2) $-\frac{1}{8}$ 3) $\frac{3}{16}$ 4) $-\frac{3}{16}$

Ans: 1

Sol:

k	0	1	2	3	4	5
P(k)	$\frac{1}{32}$	$\frac{12}{32}$	$\frac{11}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{1}{32}$

k = no. of times head occur consecutively

Now expectation

$$=\sum xP(k)=(-1)\times\frac{1}{32}+(-1)\times\frac{12}{32}+(-1)\times\frac{11}{32}+3\times\frac{5}{32}+4\times\frac{2}{32}+5\times\frac{1}{32}=+\frac{1}{8}$$

67. Let $x^k+y^k=a^k,(a,k>0)$ and $\frac{dy}{dx}+\left(\frac{y}{x}\right)^{\frac{1}{3}}=0$, then K is

1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) $\frac{4}{3}$ 4) $\frac{3}{2}$

Ans: 2

Sol: $k.x^{k-1}+k.y^{k-1}\frac{dy}{dx}=0$ $\quad \frac{dy}{dx}=-\left(\frac{x}{y}\right)^{k-1}$ $\quad \frac{dy}{dx}+\left(\frac{x}{y}\right)^{k-1}=0$ $\quad k-1=\frac{1}{3}$ $\quad k=1-\frac{1}{3}=\frac{2}{3}$

68. If $\operatorname{Re}\left(\frac{z-1}{2z+i}\right)=1$, where z = x + iy, then the point (x, y) lies on a

1) straight line whose slope is $-\frac{2}{3}$ 2) circle whose diameter is $\frac{\sqrt{5}}{2}$

3) straight line whose slope is $\frac{3}{2}$ 4) circle whose centre is at $\left(-\frac{1}{2},-\frac{3}{4}\right)$

Ans: 2

Sol: $z=x+iy$

$$\left(\frac{z-1}{2z+i}\right)=\frac{(x-1)+iy}{2(x+iy)+i}=\frac{(x-1)+iy}{2x+(2y-1)i}\times\frac{2x-(2y+1)i}{2x-(2y+1)}$$

$$\operatorname{Re}\left(\frac{z+1}{2z+i}\right)=\frac{2x(x-1)+y(2y+1)}{(2x)^2+(2y+1)^2}=1$$

$$\Rightarrow 2x^2+2y^2-2x+y=4x^2+4y^2+4y+1 \qquad \Rightarrow 2x^2+2y^2+2x+3y+1=0$$

$$\Rightarrow x^2+y^2+x+\frac{3}{2}y+\frac{1}{2}=0 \quad \text{circle with centre } \left(-\frac{1}{2}-\frac{3}{4}\right)$$

$$r=\sqrt{\frac{1}{4}+\frac{9}{16}-\frac{1}{2}}=\sqrt{\frac{4+9-8}{16}}=\frac{\sqrt{5}}{4}$$

69. If $g(x)=x^2+x-1$ and $(g\,o\,f)(x)=4x^2-10x+5$, then $f\left(\frac{5}{4}\right)$ is equal to:

1) $\frac{1}{2}$ 2) $-\frac{1}{2}$ 3) $-\frac{3}{2}$ 4) $\frac{3}{2}$

Ans: 2

Sol: $g(f(x))=f^2(x)+f(x)-1$ $\qquad g\left(f\left(\frac{5}{4}\right)\right)=4\left(\frac{5}{4}\right)^2-10.\frac{5}{4}+5=-\frac{5}{4}$

$$g\left(f\left(\frac{5}{4}\right)\right)=f^2\left(\frac{5}{4}\right)+f\left(\frac{5}{4}\right)-1 \qquad -\frac{5}{4}=f^2\left(\frac{5}{4}\right)+f\left(\frac{5}{4}\right)-1$$

$$f^2\left(\frac{5}{4}\right)+f\left(\frac{5}{4}\right)+\frac{1}{4}=0 \qquad \left(f\left(\frac{5}{4}\right)+\frac{1}{2}\right)^2=0 \qquad f\left(\frac{5}{4}\right)=-\frac{1}{2}$$

70. Five numbers are in A.P. whose sum is 25 and product is 2520,if one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is:

1) $\frac{21}{2}$ 2) 16 3) 27 4) 7

Ans: 2

Sol: Let terms be $-2d, a-d, a, a+d, a+2d$ Sum = $25 \Rightarrow 5a=25 \Rightarrow a=5$

Product = 2520 $\quad (5-2d)(5-d)5(5+d)(5+d)=2520$

$$\Rightarrow (25-4d^2)(25-d^2)=504 \qquad \Rightarrow 625-100d^2-25d^2+4d^4=504$$

$$\Rightarrow 4d^4-125d^2+625-504=0$$

$\Rightarrow 4d^4 - 125d^2 + 121 = 0 \quad \Rightarrow (d^2-1)(4d^2-121) = 0 \quad \Rightarrow d = \pm 1, \quad d = \pm\frac{11}{2}$

$d = \pm 1$, does not give $\frac{-1}{2}$ as a term

$\therefore d = \frac{11}{2}$ $\qquad$ $\therefore$ Largent term = 5+ 2d = 5 + 11 = 16

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

71. Let A(1, 0), B(6, 2) and $C\left(\frac{3}{2}, 6\right)$ be the vertices of a triangle ABC. If P is a point inside the ΔABC such that $\Delta APC, \Delta APB \& \Delta BPC$ have equal areas, then the length of the line segment PQ, where Q is the point $\left(-\frac{7}{6}, -\frac{1}{3}\right)$ is______.

Ans: PQ = 5

Sol: P will be centroid of ΔABC

$P\left(\frac{17}{6}, \frac{8}{3}\right) \quad \Rightarrow PQ = \sqrt{\left(\frac{24}{6}\right)^2 + \left(\frac{9}{3}\right)^2} = 5$

72. If the variance of the first 'n' natural numbers is 10 and the variance of the first 'm' even natural numbers is 16, then m + n is equal to _____.

Ans: 18

Sol: $Var(1,2,........,n) = 10 \Rightarrow \frac{1^2+2^2+.......+n^2}{n} - \left(\frac{1+2+......+n}{n}\right)^2 = 10$

$\Rightarrow \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2 = 10$

$\Rightarrow n^2 - 1 = 120 \qquad \Rightarrow n = 11$

$Var(2,4,6,.......,2m) = 16 \Rightarrow \text{var}(1,2,.....,m) = 4$

$\Rightarrow m^2 - 1 = 48 \Rightarrow m = 7 \Rightarrow m + n = 18$

73. $\lim_{x\to 2} \frac{3^x + 3^{3-x} - 12}{3^{\frac{-x}{2}} - 3^{1-x}}$ is equal to ________.

Ans: 36

Sol: $\lim_{x\to 2} \frac{3^x + 3^{3-x} - 12}{3^{-x/2} - 3^{1-x}} \Rightarrow \lim_{x\to 2} \frac{3^{2x} - 12.3^x + 27}{3^{x/2} - 3}$

$= \lim_{x\to 2} \frac{(3^8-9)(3^8-3)}{(3^{8/2}-3)}$

$$= \lim_{x\to 2} \frac{\left(3^{x/2}+3\right)\left(3^{x/2}-3\right)\left(3^{x}-3\right)}{\left(3^{x/2}-3\right)}$$

=36

74. If the sum of the coefficients of all even powers of x in the product $\left(1+x+x^2+.....+x^{2n}\right)\left(1-x+x^2-x^3+.....+x^{2n}\right)$ is 61, then n is equal to ______.

Ans: 30

Sol: Let $\left(1-x+x^2.....\right)\left(1+x+x^2.....\right)=a_0+a_1x+a_2x^2+....$

Put x = 1

$1\left(2n+1\right)=a_0+a_1+a_2+....a_{2n}$ ....(i)

Put x = -1

$\left(2n+1\right)\times 1=a_0-a_1+a_2+...a_{2n}$ ..(ii)

From (i) + (ii)

$4n+2=2\left(a_0+a_2+...\right)$

$=2\times 61$

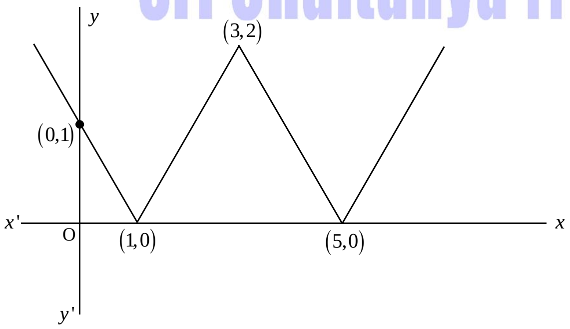
$\Rightarrow 2n+1=61 \Rightarrow n=30$

75. Let S be the set of points where the functions $f\left(x\right)=\left|2-\left|x-3\right|\right|$, $x\in R$, is not differentiable. Then $\sum_{x\in S} f\left(f\left(x\right)\right)$ is equal to _____.

Ans: 3

Sol: $f\left(x\right)=\left|2-\left|x-3\right|\right|$

F is not differentiable at



$x=1,3,5 \quad \Rightarrow \sum_{x\in s} f\left(f\left(x\right)\right)=f\left(f\left(1\right)\right)+f\left(f\left(3\right)\right)+f\left(f\left(5\right)\right)=f\left(0\right)+f\left(2\right)+f\left(10\right)$ =1+1+1=3

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