Sri Chaitanya

IIT Academy., India

JEE Main 2020

07 Jan 2020, Slot - 2

(2.30 PM - 5.30 PM)

Question Paper

Solutions

Corporate Office : Plot No-304, Kasetty Heights, Ayyappa Society Madhapur, Hyderabad-500081

www.srichaitanya.net

PHYSICS

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. If weighs 196N on a spring balance at the north pole. Its weight recorded on the same balance if it is shifted to the equator is close to(Take g = 10 ms^2 at the north pole and the radius of the earth = 64000km)

1) 194.32 N 2) 194.66 N 3) 195.32 N 4) 195.66 N

Key: 3

Sol: at pole, weight $= mg = 196$ $m = 19.6kg$

At equator, $= mg - m\omega^2 R = 196 - (19.6)\left[\frac{2\pi}{24\times 3600}\right]^2 \times 6400\times 10^3$ =195.32N

2. In a building there are 15 Bulbs of 45 W, 15 bulbs of 100 W, 15 small fans of 10W and 2 heater of 1 kW. The voltage of electric main is 220V. the minimum fuse capacity (rated value) of the building will be:

1) 10 A 2) 20 A 3) 25 A 4) 15 A

Ans: 2

Sol: Total power is $(15\times 45)+(15\times 100)+(15\times 10)+(2\times 1000) = 4325W$

So current is $= \frac{4325}{220} = 19.66A$

Is 20amp

3. Under an adiabatic process, the volume of an ideal gas gets doubled. Concequently the mean collision time between the gas molecule changes from τ_1 to τ_2. If $\frac{C_P}{C_V} = \gamma$ for this gas then a good estimate for $\frac{\tau_2}{\tau_1}$ is given by:

1) $\frac{1}{2}$ 2) 2 3) $\left(\frac{1}{2}\right)^{\gamma}$ 4) $\left(\frac{1}{2}\right)^{\frac{\gamma+1}{2}}$

Ans: 4

Sol: $t \propto \frac{V}{\sqrt{T}}$ .....(1)

$TV^{\gamma-1} = \text{constant}$ .....(2)

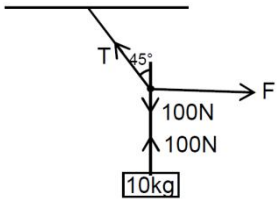
$\therefore t \propto V^{\frac{\gamma+1}{2}}$

4. A mass of 10kg is suspended by a rope of length 4m, from the ceiling. A force F is applied horizontally at the mid-point of the rope such that the top half of the rope makes an angle of 45° with the vertical. Then F equals: (Take g = 10 ms^{-2} and the rope to be massless)

1) 100N 2) 90N 3) 75N 4) 70N

Ans: 1

Sol: $\frac{T}{\sqrt{2}}=100$



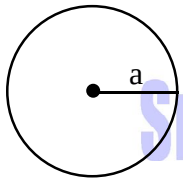
$\frac{T}{\sqrt{2}}=F$ $F=100N$

5. Mass per unit area of a circular disc of radius a depends on the distance r from its centre as $\sigma(r)=A+Br$. The moment of inertia of the disc about the axis, perpendicular to the plane and passing through its centre is:

1) $2\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$ 2) $2\pi a^4\left(\frac{aA}{4}+\frac{B}{5}\right)$ 3) $\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$ 4) $2\pi a^4\left(\frac{A}{4}+\frac{B}{5}\right)$

Ans: 1

Sol: $\alpha=A+Br$



$\int dm=\int(A+Br)2\pi r dr$ $I=\int dm\,r^2=\int_0^a(A+Br)2\pi r^3 dr=2\pi\left(A\frac{a^2}{4}+B\frac{a^5}{5}\right)=2\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$

6. Two ideal Carnot engines operate in cascade(all heat given up by one engine is used by the other engine to produce work) between temperatures, T_1 and T_2. The temperature of the hot reservoir of the first engine is T_1 and the temperature of the cold reservoir of the second engine is T_2. T is temperature of the sink of first engine which is also the source for the second engine. How is t related to T_1 and T_2, if both the engines perform equal amount of work?

1) $T=\frac{2T_1T_2}{T_1+T_2}$ 2) $T=\frac{T_1+T_2}{2}$ 3)T= 0 4) $T=\sqrt{T_1T_2}$

Ans: 2

Sol: Q_H : Heat input to Ist engine

Q_L :: Heat rejected from Ist engine

Q_L : Heat rejected from II[nd] engine

Work done by I[st] engine = work done by II[nd] engine

$$Q_H - Q_L = Q_L - Q_L \qquad 2Q_L = Q_H + Q_L \qquad 2 = \frac{T_1}{T} + \frac{T_2}{T}$$

$$T = \frac{T_1 + T_2}{2} \qquad Q_H - Q_L = Q_L - Q_L \quad 2Q_L = Q_H + Q_L \qquad 2 = \frac{T_1}{T} + \frac{T_2}{T} \quad T = \frac{T_1 + T_2}{2}$$

7. The activity of a radioactive sample falls from $700s^{-1}$ to $500s^{-1}$ in 30 minute. Its half-life is close to:

1) 66 2) 62 3) 52 4) 72

Ans: 2

$$\ln\left[\frac{A_0}{A_1}\right] = \lambda t \qquad \Rightarrow \ln 2 = \lambda t_{1/2} \_\_\_\_\_ (i)$$

$$\Rightarrow \ln\left[\frac{700}{500}\right] \times \lambda(30\,\text{min}) \_\_\_\_\_ (ii)$$

$$(i)/(ii) \Rightarrow \frac{\ln 2}{\ln(7/5)} = \frac{t_{1/2}}{(30\,\text{min})} \qquad \Rightarrow (2.06004)30 = t_{1/2} = 61.8\,\text{min}\,.$$

8. In a Young's double slit experiment, the separation between the slits is 0.15 mm. In the experiment, a source of light of wavelength 589 nm is used and the interference pattern is observed on a screen kept 1.5 m away. The separation between the successive bright fringes on the screen is:

1) 5.9 mm 2) 3.9 mm 3) 4.9 mm 4) 6.9 mm

Ans: 1

Sol: $\beta = \frac{\lambda D}{d} = \frac{589 \times 10^{-9} \times 1.5}{0.15 \times 10^{-3}} = 5.9mm$

9. An ideal fluid flows(laminar flow) through a pipe of non-uniform diameter. The maximum and minimum diameters of the pipes are 6.4 cm and 4.8 cm, respectively. The ratio of the minimum and the maximum velocities of fluid in this pipe is:

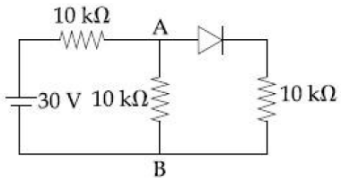
1) $\frac{81}{256}$ 2) $\frac{9}{16}$ 3) $\frac{3}{4}$ 4) $\frac{\sqrt{3}}{2}$

Ans: 2

Sol: Using equation of continuity

$$A_1V_1 = A_2V_2 \qquad \frac{V_1}{V_2} = \frac{A_2}{A_1}\left(\frac{4.8}{6.4}\right)^2 = \frac{9}{16}$$

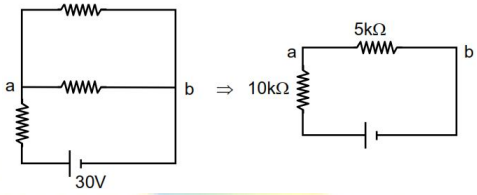
10. In the figure, potential difference between A and B is:



1) zero 2) 15V 3) 10V 4) 5V

Ans: 3

Sol: Diode is in forward bias, so it will behave as simple wire so



$$V_{ab} = \frac{30}{5+10} \times 5 = 10V$$

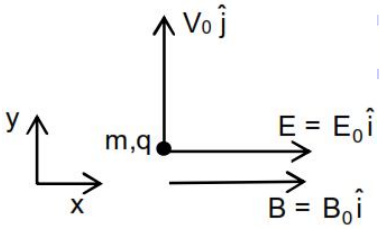
11. A particle of mass m and q has an initial velocity $\vec{v} = v_0 \hat{j}$. If an electric field $\vec{E} = E_0 \hat{i}$ and magnetic field $\vec{B} = B_0 \hat{i}$ act on the particle, its speed will double after a time:

1) $\frac{\sqrt{3}mv_0}{qE_0}$ 2) $\frac{2mv_0}{qE_0}$ 3) $\frac{3mv_0}{qE_0}$ 4) $\frac{\sqrt{2}mv_0}{qE_0}$

Ans: 1

Sol:

As $\vec{V} = v_0 \hat{j}$ (magnitude of velocity does not change in y-x plane)



$$(2v_0)^2 = v_0^2 + v_x^2; \qquad v_x = \sqrt{3}v_0$$

$$\therefore \qquad \sqrt{3}v_0 = 0 + \frac{qE}{m}t; \qquad t = \frac{\sqrt{3}mv_0}{qE_0}$$

12. A stationary observer receives sound from two identical tuning forks, one of which approaches and the other one recedes with the same speed (much less than the speed of sound). The observer hears 2 beats/sec. The oscillation frequency of each tuning fork is V_0 = 1400Hz and the velocity of sound in air is 350 m/s. The speed of each tuning fork is close to:

1) $\frac{1}{4}m/s$ 2) 1m/s 3) $\frac{1}{8}m/s$ 4) $\frac{1}{2}m/s$

Ans: 1

Sol: 

$$f_0\left(\frac{C}{C-V}\right)-f_0\left(\frac{c}{C+V}\right)=2 \qquad V=\frac{1}{4}m/s$$

13. An electron(of mass m) and a photon have same energy E in the range of a few eV. The ratio of the de-Broglie wavelength associated with the electron and the wavelength of the photon is (c=speed of light in vacuum)

1) $\frac{1}{C}\left(\frac{E}{2m}\right)^{\frac{1}{2}}$ 2) $\frac{1}{c}\left(\frac{2E}{m}\right)^{\frac{1}{2}}$ 3) $c(2mE)^{1/2}$ 4) $\left(\frac{E}{2m}\right)^{\frac{1}{2}}$

Ans: 1

Sol: λ_d for electron $=\frac{h}{\sqrt{2mE}}$ λ for photon $\frac{hC}{E}$

Ratio $=\frac{h}{\sqrt{2mE}}\frac{E}{hC}=\frac{1}{C}\sqrt{\frac{E}{2m}}$

14. A planar loop of wire rotates in a uniform magnetic field. Initially, at t=0, the plane of the loop is perpendicular to the magnetic field. If it rotates with a period of 10s about an axis in its plane then the magnitude of induced emf will be maximum and minimum, respectively at:

1) 2.5 s, 5.0 s 2) 5.0 s, 7.5 s 3) 2.5 s, 7.5 s 4) 5.0 s, 10.0 s

Ans: 1

Sol: $\therefore \quad \omega=\frac{2\pi}{T}=\frac{\pi}{5}$

When $\omega t=\frac{\pi}{2}$ $\because$ ϕ will have minimum

$\therefore e$ will be maximum $\qquad t=\frac{\frac{\pi}{2}}{\frac{\pi}{5}}=2.5\text{sec}$

When $\omega t=\pi$ $\because$ ϕ will have maximum

$\therefore e$ will be minimum $\qquad t=\frac{\pi}{\pi/5}=\text{sec}$

15. The Electric field of a plane electromagnetic wave is given by $\vec{E}=E_0\frac{\hat{i}+\hat{j}}{\sqrt{2}}\cos(Kz+\omega t)$. At t=0, a positively charged particle is at the point $(x,y,z)=\left(0,0,\frac{\pi}{k}\right)$ .If itsinstantaneous velocity at (t=0) is $v_0\hat{k}$, the force acting on it due to the wave is:

1) zero 2) Antiparallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

3) Parallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ 4) Parallel to $\hat{k}$

Ans: 2

Sol: Force due to electric field is in direction $-\frac{\left(\hat{i}+\hat{j}\right)}{\sqrt{2}}$

Because at $t=0, E=-\frac{\left(\hat{i}+\hat{j}\right)}{\sqrt{2}}E_0$

Force due to magnetic field is in direction $q\left(\vec{v}\times\vec{B}\right)$ and $\vec{v}\parallel\hat{k}$

$\therefore$ it is parallel to $\vec{E}$

$\therefore$ net force is antiparallel to $\frac{\left(\hat{i}+\hat{j}\right)}{\sqrt{2}}$

16. A thin lens made of glass (refractive index = 1.5) of focal length f=16 cm is immersed in a liquid of refractive index 1.42. If its focal length in liquid f_l then the ratio f_l/f is closest to the integer:

1) 9 2) 17 3) 1 4) 5

Ans: 1

Sol: $\frac{1}{f_a}=\left(\frac{\mu_0}{\mu_a}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$\frac{1}{f_m}=\left(\frac{\mu_0}{\mu_m}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$\Rightarrow \frac{f_a}{f_m}=\frac{\left(\frac{\mu_0}{\mu_m}-1\right)}{\left(\frac{\mu_0}{\mu_a}-1\right)}=\frac{\left(\frac{1.50}{1.42}-1\right)}{\left[\frac{1.50}{1}-1\right]}=\frac{0.08}{(1.92)(0.5)}$

$\frac{f_m}{f_a}=\frac{(1.42)(0.5)}{0.08}=8.875\approx 9$

17. An elevator in a building can carry a maximum of 10 persons, with the average mass of each person being 68 kg. The mass of the elevator itself is 920 kg and it moves with a constant speed of 3m/s. The frictional force opposing the motion is 6000N. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator (g = 10 m/s^2) must be at least ;

1) 66000 W 2) 62360 W 3) 48000 W 4) 56300 W

Ans: 1

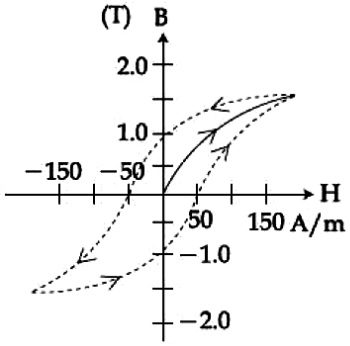
Sol: Net force on motor will be

$F_m\left[920+68(10)\right]g+6000 \quad = 22000N$

So, required power for motor

$P_m=\overrightarrow{F_m},\vec{v} \quad = 22000\times 3 \quad = 66000 watt$

18. The figure gives experimentally measured B vs. H variation in a ferromagnetic material. The retentivity, co-ercivity and saturation, respectively, of the material are.



1) 1.5T, 50 A/m and 1.0T
2) 1.5 T, 50 A/m and 1.0T
3) 1.0 T, 50 A/m and 1.5 T
4) 150 A/m, 1.0 T and 1.5 T

Ans: 3

Sol: x=retentivity

Y=coercivity

Z=saturation magnetization

19. An emf of 20V is applied at time t = 0 to a circuit containing in series 10 mH inductor and 5Ω resistor. The ratio of currents at time $t=\infty$ and t = 40 s is close to: (Take e^2 =7.389).

1) 1.06
2) 1.48
3) 1.15
4) 0.84

Ans: 1

Sol: $i=i_0\left(1-e^{-Rt/L}\right) \quad \frac{i_0}{i}=\frac{1}{1-e^{-2\times 10^4}} \quad \frac{i_0}{i}=1$

20. The dimension of $\frac{B^2}{2\mu_0}$, where B is magnetic field and μ_0 is the magnetic permeability of vacuum, is:

1) $ML^{-1}T^{-2}$
2) ML^2T^{-2}
3) ML^2T^{-1}
4) MLT^{-2}

Ans: 1

Sol: Energy density in magnetic field $=\frac{B^2}{2\mu_0}$

$=\frac{\text{Force}\times\text{displacement}}{(\text{displacement})^3}=\frac{MLT^{-2}L}{L^3}=ML^{-1}T^{-2}$

(NUMERICAL VALUE TYPE)

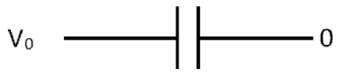
This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value(in decimal notation, truncated/rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

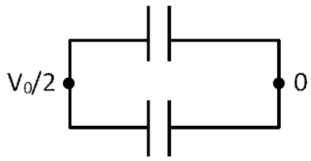
Marking scheme: +4 for correct answer , 0 if not attempted and 0 in all other cases.

21. A 60 pF capacitor is fully charged by a 20 V supply. It is then disconnected from the supply and is connected to another uncharged 60 pF capacitor in parallel. The electrostatic energy that is lost in the process by the time the charge is redistributed between them is (in nJ) ____

Ans: 6

Sol: $V_0 = 20V$





Heat loss $U_1 - U_f = \frac{1}{2}CV_0^2 - 2\left[\frac{1}{2}C\left(\frac{V_0}{2}\right)^2\right] = \frac{CV_0^2}{4}$

$= \frac{\left(60\times10^{-12}\right)(20)^2}{4} J \qquad = 6\times10^{-9} J = 6nJ$

22. M gram of steam at 100°C is mixed with 200 g of ice at its melting point in a thermally insulated container. If it produces liquid water at 40ºC [heat of vaporization of water is 540 cal/g and heat and heat of fusion of ice is 80 cal/ g], the value of M is_____

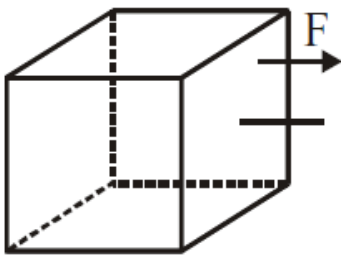
Ans: 40

Sol: $M_{ice}L_f + M_{ice}(40-0)C_w = m_{steam}L_V + m_{steam}(100-40)C_w$

$\Rightarrow 200\left[80 + 40(1)\right] = m\left[540 + 60(1)\right]$

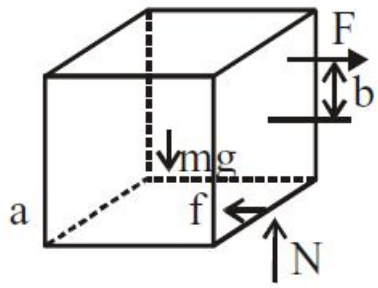
$\Rightarrow 200(120) = m(600) \qquad m = 40gm$

23. Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum possible value of $100\times\frac{b}{a}$ for box not to topple before moving is _____



Ans: 75

Sol:



$F = \mu mg$ …. (1)

$F\left(b+\frac{a}{2}\right) = mg\frac{a}{2}$ ….(2)

$\mu mg\left(b+\frac{a}{2}\right) = mg \times \frac{a}{2}$

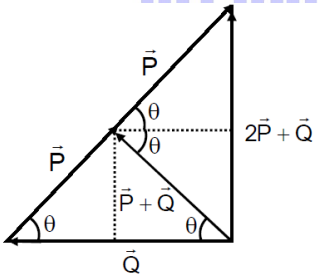
$\left(b+\frac{a}{2}\right)\mu = \frac{a}{2}$ $0.4 = \mu = \frac{a}{2b+a}$

$0.8b + 0.4a = a$ $0.8b = 0.6a$ $\frac{b}{a} = \frac{3}{4}$

24. The sum of two forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$ such that $|\vec{R}| = |\vec{P}|$. The angle θ (in degrees) that the resultants of $2\vec{P}$ and $\vec{Q}$ will make with $\vec{Q}$ is ____

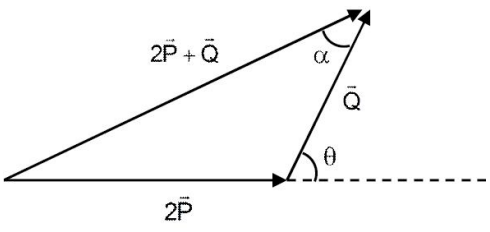
Ans: 90

Sol: So angle between $\left(2\vec{P}+\vec{Q}\right)$ and $\vec{Q}$ is 90º



Alternate solution $|\vec{P}+\vec{Q}| = |\vec{P}|$

$P^2 + Q^2 + 2PQ\cos\theta = p^2$ $\Rightarrow Q + 2P\cos\theta = 0$ $\Rightarrow \cos\theta = \frac{Q}{2P}$



$$\tan\alpha = \frac{2P\sin\theta}{2P\cos\theta + Q} = \infty \quad \because [2P\cos\theta + Q = 0]$$

$\alpha = 90^o$

So angle between $2\overline{P} + \overline{Q}$ and $\overline{Q}$ is 90^o

25. The balancing length for a cell is 560 cm in a potentiometer experiment. When an external resistance of 10Ω is connected in parallel to the cell, the balancing length changes by 60 cm. If the internal resistance of the cell is $\frac{N}{10}\Omega$, where N is an integer then value of N is _____

Ans: 12

Sol: Let the emf of cell is ε internal resistance is 'r' and potential gradient is x.

Only cell connected

$\varepsilon = 560x$

After connecting the resistor_____(1)

$$\frac{\varepsilon \times 10}{10 + r} = 500x$$

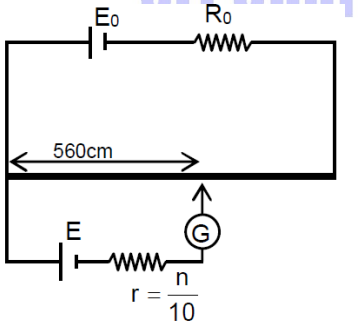
From (1) and (2)

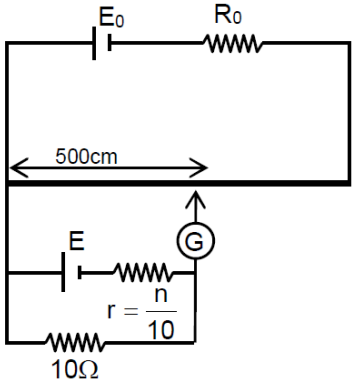
$$\frac{560 \times 10}{10 + r} = 500s$$

$56 = 50 + 5r$

$r = \frac{6}{5} = 1.2\Omega$

$n = 12$





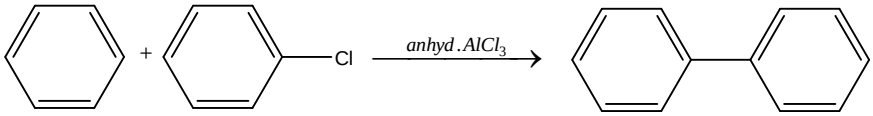
CHEMISTRY

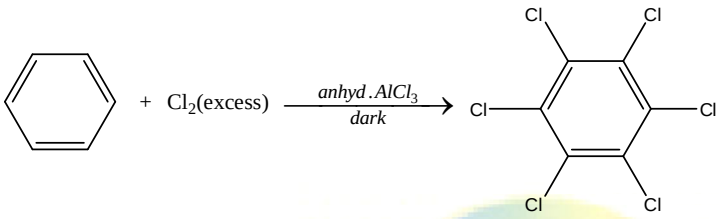
(SINGLE CORRECT ANSWER TYPE)

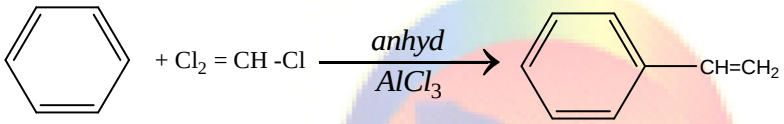
This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

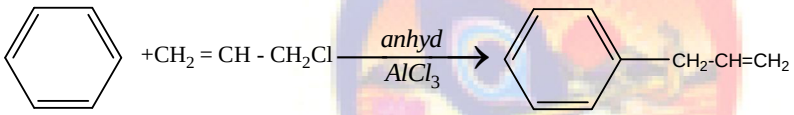
Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

26. Consider the following reactions:

(A) 

(B) 

(C) 

(D) 

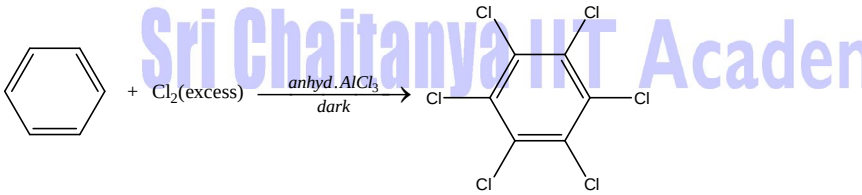
Which of these reactions are possible?

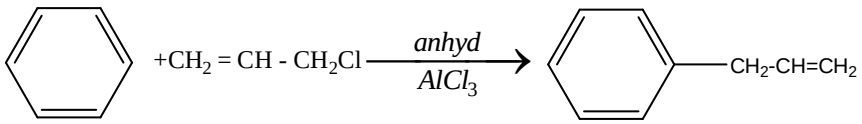
1) (A) and (D) 2) (A) and (B)

3) (B), (C) and (D) 4) (B) and (D)

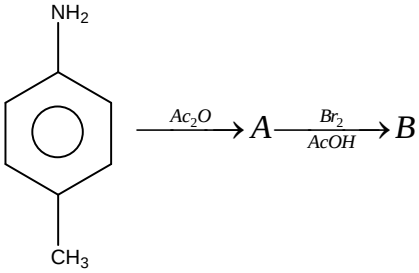
Ans: 4

Sol:

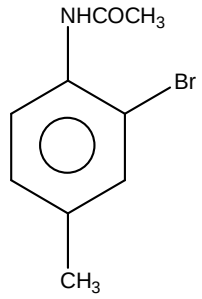


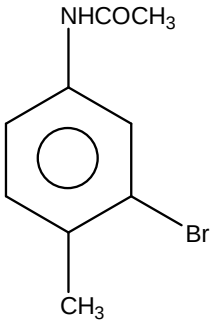


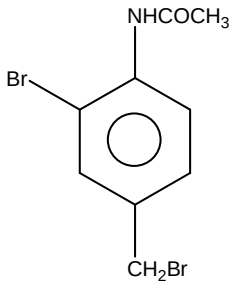
27. In the following reaction sequence,

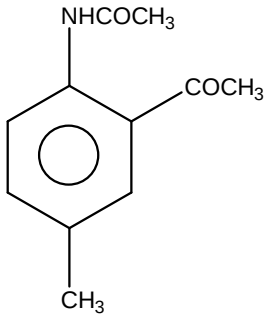


The major product B is :

1) 

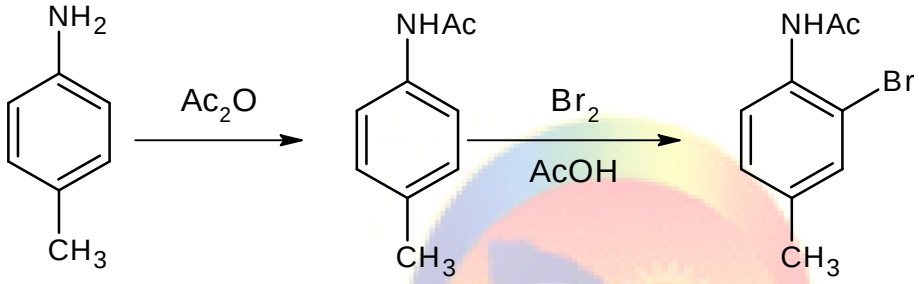
2) 

3) 

4) 

Ans: 1

Sol:



NHAc is more activating than CH_3 group.

28. Which of the following statement is correct?

1) Gluconic acid can form cyclic (acetal/hemiacetal) structure

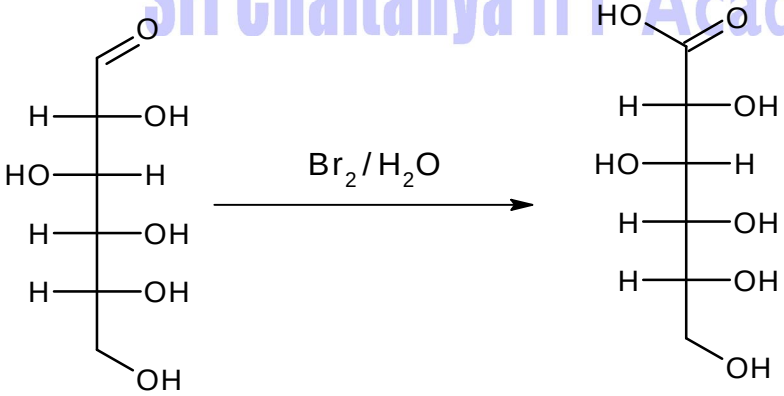
2) Gluconic acid is a partial oxidation product of glucose

3) Gluconic acid is obtained by oxidation of glucose with HNO_3

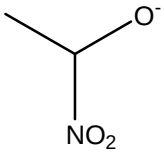
4) Gluconic acid is a dicarboxylic acid

Ans: 2

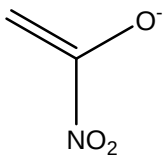
Sol: Partial oxidation of glucose with Bromine water gives gluconic acid



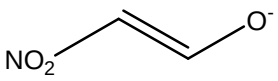
29. The correct order of stability for the following alkoxide is:



(A)



(B)



(C)

1) C > B > A 2) B > C > A 3) B > A > C 4) C > A > B

Ans: 1

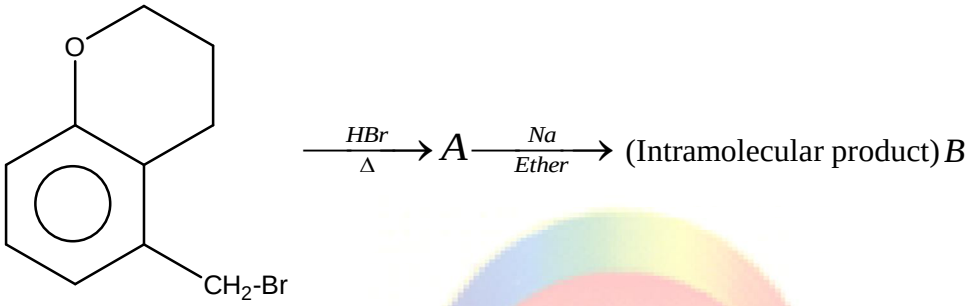
Sol: C is stabilized by –R effect of NO_2 group.

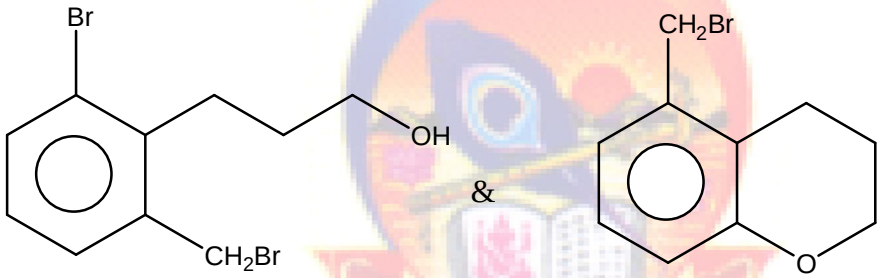
B is stabilized by –I effect of NO_2 and resonance with vinyl group.

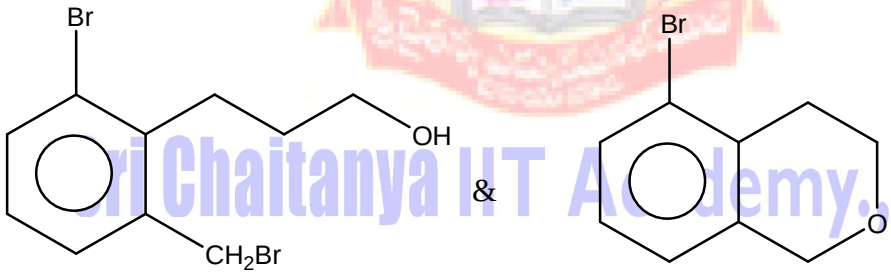
A is stabilized only by –I effect of NO_2 group.

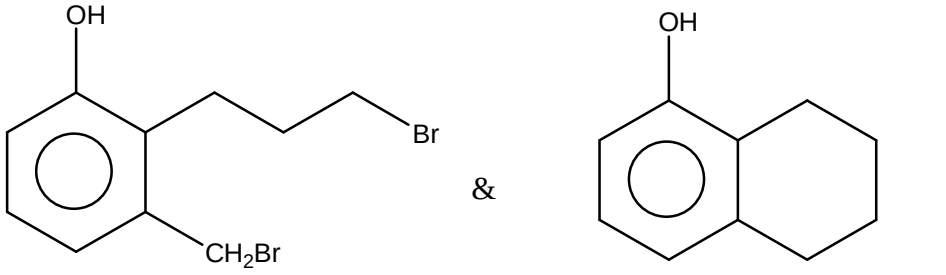
So the stability order is C>B>A

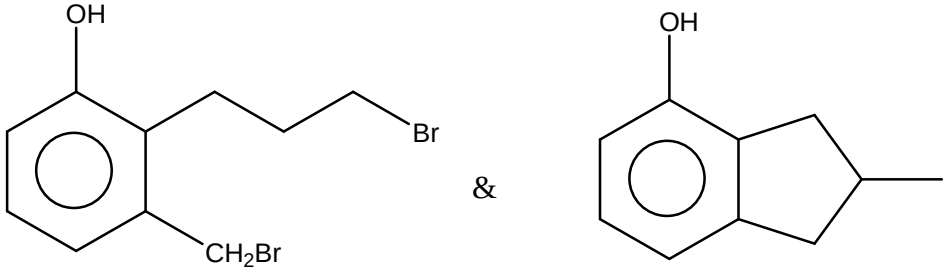
30. In the following reaction sequence, structures of A and B, respectively will be:



1) 

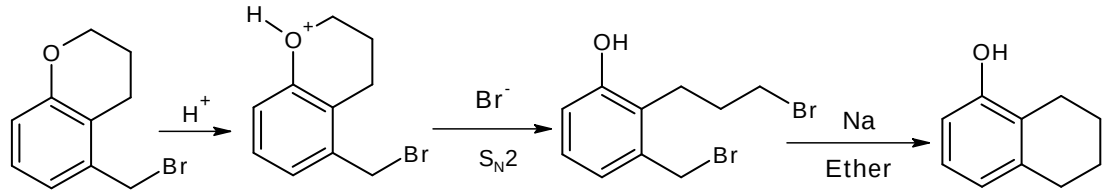
2) 

3) 

4) 

Ans: 1

Sol:



31. The number of possible optical isomers for the complexes Ma_2b_2 with sp^3 and dsp^2 hybridized metal atom, respectively, is:

Note: A and B are unidentate neutral and unidentate monoanionic ligands, respectively

1) 0 and 1 2) 2 and 2 3) 0 and 0 4) 0 and 2

Ans: 3

Sol: A tetrahedral complex with monodentate ligands can exhibit optical isomerism only of all four ligands are different. A square planar complex never exhibits optical isomerism.

32. The bond order and magnetic characteristics of CN^- are :

1) 3, diamagnetic 2) $2\frac{1}{2}$, diamagnetic

3) $2\frac{1}{2}$, paramagnetic 4) 3, paramagnetic

Ans: 1

Sol: As cyanide ion (14 e system) is isoelectronic with N2, it has a bond order of 3 and is diamagnetic.

33. The equation that is incorrect is:

1) $\left(\Lambda_m^o\right)_{NaBr}-\left(\Lambda_m^o\right)_{NaCl}=\left(\Lambda_m^o\right)_{KBr}-\left(\Lambda_m^o\right)_{KCl}$ 2) $\left(\Lambda_m^o\right)_{KCl}-\left(\Lambda_m^o\right)_{NaCl}=\left(\Lambda_m^o\right)_{KBr}-\left(\Lambda_m^o\right)_{NaBr}$

3) $\left(\Lambda_m^o\right)_{NaBr}-\left(\Lambda_m^o\right)_{NaI}=\left(\Lambda_m^o\right)_{KBr}-\left(\Lambda_m^o\right)_{NaBr}$ 4) $\left(\Lambda_m^o\right)_{H_2O}=\left(\Lambda_m^o\right)_{HCl}+\left(\Lambda_m^o\right)_{NaOH}-\left(\Lambda_m^o\right)_{NaCl}$

Ans: 3

Sol: According to Kohlrausch's law correct expression is

$\left(\Lambda_m^o\right)_{NaBr}-\left(\Lambda_m^o\right)_{NaI}=\left(\Lambda_m^o\right)_{KBr}-\left(\Lambda_m^o\right)_{NaBr}$

The other statements are correct

34. In the following reactions, products (A) and (B), respectively, are:

$\underset{Hot\&conc}{NaOH}+Cl_2\rightarrow A+side\ \ products$

$Ca\underset{dry}{(OH)_2}+Cl_2\rightarrow B+side\ \ products$

1) $NaClO_3$ and $Ca(OCl)_2$ 2) $NaOCl$ and $Ca(ClO_3)_2$

3) $NaOCl$ and $Ca(OCl)_2$ 4) $NaClO_3$ and $Ca(ClO_3)_2$

Ans: 1

Sol:

$3Cl_2(g)+6NaOH\xrightarrow{\Delta}5NaCl+NaClO_3+3H_2O$

$2Cl_2(g)+2Ca(OH)_2\longrightarrow Ca(OCl)_2+CaCl_2+2H_2$

35. Two open beakers one containing a solvent and the other containing a mixture of that solvent with a non volatile solute are together sealed in a container. Over time:
1) the volume of the solution increases and the volume of the solvent decreases
2) the volume of the solution does not change and the volume of the solvent decreases
3) the volume of the solution and the solvent does not change
4) the volume of the solution decreases and the volume of the solvent increases

Ans: 1

Sol: The vapour pressure of beaker (II) is less than that of (I) and hence solvent from (I) will evaporate and condense into (II) to make the vapour pressure of baker (II) increase.

36. The refining method used when the metal and the impurities have low and high melting temperatures, respectively, is:
1) Zone refining
2) Vapor phase refining
3) Distillation
4) Liquation

Ans: 4

Sol: Liquation is used to refine metals with low melting points having impurities of high melting points.

37. Among statements (a)-(d), the correct ones are:
(a) Decomposition of hydrogen peroxide gives dioxygen.
(b) Like hydrogen peroxide, compounds, such as $KClO_3$, $Pb(NO_3)_2$ and $NaNO_3$ when heated liberate dioxygen.
(c) 2-Ethylantharaquinone is useful for the industrial preparation of hydrogen peroxide
(d) Hydrogen peroxide is used for the manufacture of sodium perborate.
1) (a), (c) and (d) only
2) (a), (b) and (c) only
3) (a) and (c) only
4) (a), (b), (c) and (d)

Ans: 4

Sol: (a) $H_2O_2 \rightarrow 2H_2O + O_2$

(b) $KClO_3 \xrightarrow[MnO_2]{\Delta} KCl + \frac{3}{2}O_2$

$Pb(NO_3)_2 \xrightarrow{\Delta} PbO + 2NO_2 + \frac{1}{2}O_2$

$NaNO_3 \xrightarrow{\Delta} NaNO_2 + \frac{1}{2}O_2$

(c) $2-ethylanthraquinol \underset{H_2/Pd}{\overset{O_2(air)}{\rightleftharpoons}} 2-ethylanthraquinone + H_2O_2$

(d) $2H_3BO_3 + 2NaOH + 2H_2O_2 \rightarrow Na_2\left[B_2(O_2)(OH)_4\right] + 4H_2O$

All statements are correct

38. The redox reaction among the following is:

1) combination of dinitrogen with dioxygen at 2000K

2) reaction $[Co(H_2O)_6]Cl_3$ with $AgNO_3$

3) formation of ozone from atmospheric oxygen in the presence of sunlight

4) reaction of H_2SO_4 with NaOH

Ans: 1

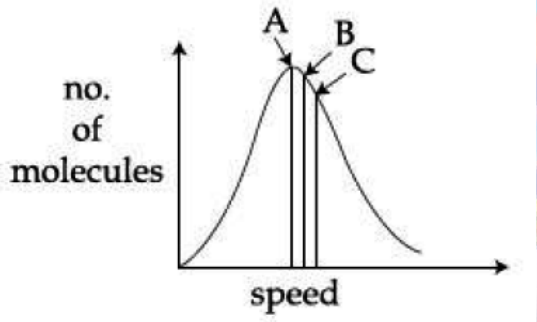
Sol: $N_2 + O_2 \rightarrow 2NO$

$3O_2 \rightarrow 2O_3$

$2NaOH + H_2SO_4 \rightarrow Na_2SO_4 + 2H_2O$

$AgNO_3 + NaCl \rightarrow NaNO_3 + AgCl$

39. Identify the correct labels of A,B and C in the following graph from the options given below:



Root mean square speed (V_{rms}); most probable speed (V_{mp}); Average speed (V_{av})

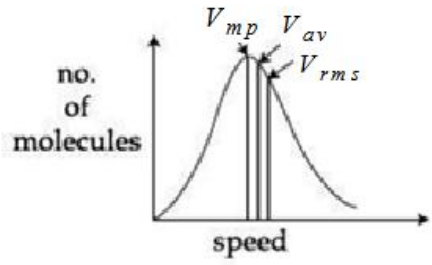
1) $A-V_{mp}; B-V_{rms}; C-V_{av}$ 2) $A-V_{av}; B-V_{rms}; C-V_{mp}$

3) $A-V_{rms}; B-V_{mp}; C-V_{av}$ 4) $A-V_{mp}; B-V_{av}; C-V_{rms}$

Ans: 4

Sol: i) The fraction of molecules with very low or very high speeds is very small. Mariman fraction of molecules have speed near to the most probable speed V_{mp}

ii) The fraction of molecules having speeds greater than minimum goes on increasing with increase in sped. It reaches to a maximum value and then begins to decrease.

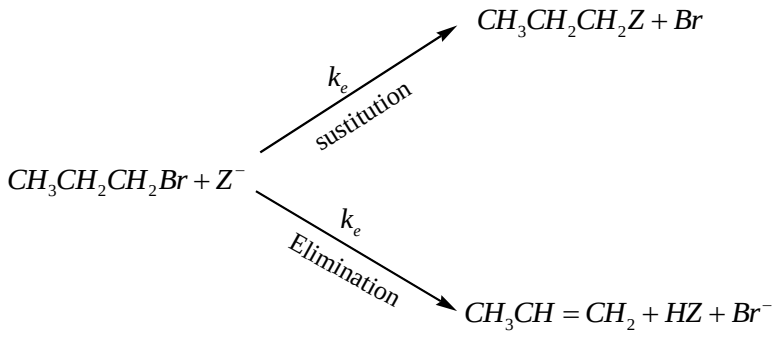


$\Rightarrow A-V_{mp}$

$\Rightarrow B-V_{av}$

$\Rightarrow C-V_{rms}$

40. For the following reaction



where,

$$Z^- = CH_3CH_2O^-(A) \text{ or } H_3C - \underset{\displaystyle CH_3}{\overset{\displaystyle CH_3}{\overset{|}{\underset{|}{C}}}} - O^-(B).$$

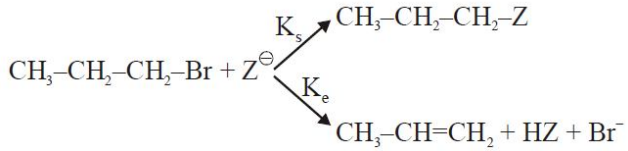
K_s and k_e, are, respectively, the rate constants for substitution and elimination and $\mu = \frac{k_s}{k_e}$,

the correct option is

1) $\mu_B > \mu_A$ *and* $k_e(B) > k_e(A)$
2) $\mu_B > \mu_A$ *and* $k_e(A) > k_e(B)$
3) $\mu_A > \mu_B$ *and* $k_e(B) > k_e(A)$
4) $\mu_A > \mu_B$ *and* $k_e(A) > k_e(B)$

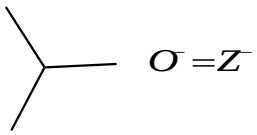
Ans: 2

Sol:



(A) $CH_3 - CH_2 - O = Z^- \rightarrow$

(B)



(B) with more steric crowding forms elimination product compared to substitution

$K_e(B) > K_e(A)$

$$\mu_B = \frac{K_s(B)}{K_e(A)} < \mu_A = \frac{K_s(A)}{K_e(A)}$$

41. For the reaction

$2H_2(g) + 2NO(g) \rightarrow N_2(g) + 2H_2O(g)$

the observed rate expression is, rate $= k_f[NO]^2[H_2]$. The rate expression for the reverse reaction is:

1) $k_b[N_2][H_2O]^2/[H_2]$ 2) $k_b[N_2][H_2O]^2$

3) $k_b[N_2][H_2O]$ 4) $k_b[N_2][H_2O]^2/[NO]$

Ans: 1

Sol: $K_{eq}=\frac{k_f}{k_b}=\frac{[N_2][H_2O]^2}{[H_2]^2[NO]^2}$

At equilibrium $r_f = r_b$

Hence, rate expression for reverse reaction $= k_b\frac{[N_2][H_2O]^2}{[H_2]}$

42. The ammonia (NH_3) released on quantitative reaction of 0.6 g urea (NH_2CONH_2) with sodium hydroxide (NaOH) can be neutralized by:

1) 100 mL of 0.2 N HCl 2) 100 mL of 0.1 N HCl

3) 200 mL of 0.4 N HCl 4) 200 mL of 0.2 N HCl

Ans: 1

Sol: 2 × mole of Urea ≡ mole of NH_3 .....(1)

Mole of NH_3 = mole of HCl .....(2)

$\therefore$ mole of HCl = 0.02 mole

43. Within each pair of elements F & Cl, S & Se, and Li & Na, respectively, the element that release more energy upon an electron gain are:

1) Cl, Se and Na 2) F, Se and Na 3) F, S and Li 4) Cl, S and Li

Ans: 4

Sol: (i) Electron affinity of second period p-block element is less than third period p-block element due to small size of second period p-block element.

E.A. order : F < Cl

(ii) Down the group electron affinity decreases due to size increases.

EA. Order : S > Se

Li > Na

44. A chromatography column, packed with silica gel as stationary phase was used to separate a mixture of compounds consisting of (A) benzanilide (B) aniline and (C) acetophenone. When the column is eluted with a mixture of solvent hexane: ethyl acetate (20:80), the sequence of obtained compounds is:

1) (B), (A) and (C) 2) (C), (A) and (B)

3) (B), (C) and (A) 4) (A), (B) and (C)

Ans: 2

Sol:

(A) Benzanilide $\rightarrow Ph-NH-\overset{\overset{O}{\|}}{C}-Ph(\mu=2.71D)$

(B) Aniline $\rightarrow Ph-NH_2(\mu=1.59\,D)$

(C)Acetophenone $\rightarrow Ph-\overset{\overset{O}{\|}}{C}-CH_3 \quad (\mu=3.05\,D)$

Dipole moment : C > A > B

Hence the sequence of obtained compounds is (C), (A) and (B)

45. Among the statement (a)-(d), the incorrect ones are:

(a) Octahedral Co(III) complexes with strong field ligands have very high magnetic moments

(b) When $\Delta_0 < P$, the d-electron configuration of Co(III) in an octahedral complex is $t_{eg}^4\, e_g^2$

(c) Wave length of light absorbed by $\left[Co(en)_3\right]^{3+}$ is lower than that of $\left[CoF_6\right]^{3-}$

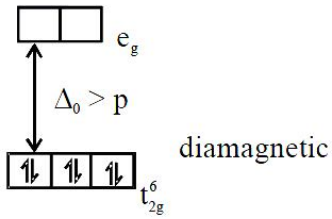
(d) If the Δ_0 for an octahedral complex of Co(III) is 18,000 cm^{-1}, the Δ_t for its tetrahedral complex with the same ligand will be 16,000 cm^{-1}

1) (a) and (d) only 2) (a) and (b) only

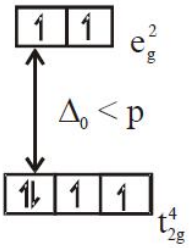
3) (b) and (c) only 4) (c) and (d) only

Ans: 1

Sol: (a) Co^{+3} (with strong field ligands)



(b) If $\Delta_0 < p$;



(c)Splitting power of ethylenediamine (en) is greater than fluoride (F^-) ligand therefore more energy absorbed by $\left[Co(en)_3\right]^{3+}$ as compared to $\left[CoF_6\right]^{3-}$

So wave length of light absorbed by $\left[Co(en)_3\right]^{3+}$ is lower than that of $\left[CoF_6\right]^{3-}$

(d) $\Delta_t = \frac{4}{9}\Delta_0$

So if $\Delta_0 = 18,000\,cm^{-1}$

$\Delta_t = \frac{4}{9} \times 18000 = 8000\,cm^{-1}$

Statement (a) and (d) are incorrect

(NUMERICAL VALUE TYPE)

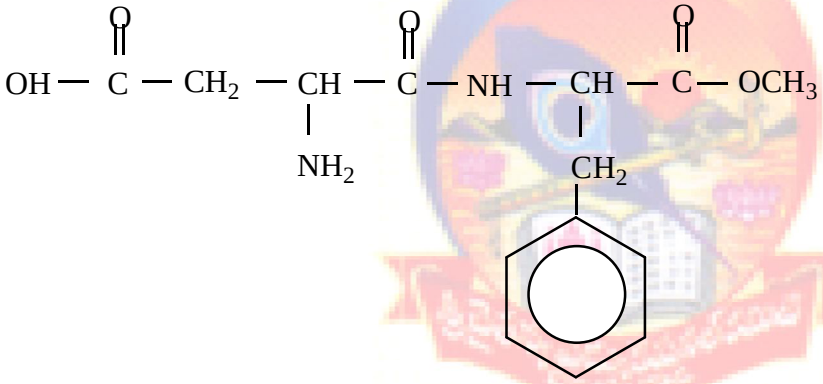
This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer , 0 if not attempted and 0 in all other cases.

46. The Number of sp^2 hybrid carbons present in "Aspartame" is ____

Ans: 9

Sol:



47. 3g of acetic acid is added to 250 mL of 0.1 M HCl and the solution made up to 500 mL.

To 20 mL of this solution $\frac{1}{2}$ mL of 5M NaOH is added. The pH of this solution is [

Given pKa of acetic acid = 7.75, molar mass of acetic acid = 60 g/mol,

log 3 = 0.4771]

Neglect any changes in volume.

Ans: 5.22 to 5.24

Sol: m mole of acidic acid in 20 mL = 2

M mole of HCl in 20 mL = 1

M mole of NaOH = 2.5

$CH_3COOH + NaOH\,(remaining) \rightarrow CH_3COONa + water$

2	3/2	0	0
0.5	0	3/2	–

$$pH = PKa + \log\frac{3/2}{2} \quad = 4.74 + \log 3 \quad = 4.74 + 0.48 = 5.22$$

48. The flocculation value of HCl for arsenic sulphide sol is 30 m mol L^{-1}. If H_2SO_4 is used for the flocculation of arsenic sulphide, the amount, in grams, of H_2SO_4 in 250 ml required for the above purpose is _____

(molecular mass of H_2SO_4 = 98 g/mol)

Ans: 0.36 to 0.38

Sol: For 1L sol 30 m mol of HCl is required

$\therefore$ For 1L sol 15m mol H_2SO_4 is required

For 250 mL of sol $\frac{15}{4}\times 10^{-3} m mol\, H_2SO_4 \equiv 0.3675 g$

49. Consider the following reactions:

$$NaCl + K_2Cr_2O_7(s) + \underset{conc}{H_2SO_4} \rightarrow (A) + side\ \ products$$

$$(A) + NaOH \rightarrow (B) + Side\ products$$

$$(B) + \underset{(dilute)}{H_2SO_4} + H_2O_2 \rightarrow (C) + Side\ \ products$$

The sum of the total number of atoms in one molecule each of (A), (B) and (C0 is _____

Ans: 18.00

Sol: (A) = CrO_2Cl_2

(B) = Na_2CrO_4

(C) = CrO_5

50. The standard heat of formation $\left(\Delta_f H^0_{298}\right)$ of ethane (in kJ/mol), if the heat of combustion of ethane, hydrogen and graphite are -1560, -393.5 and -286 kJ/mol, respectively is ___

Ans: -192.5 / -85

Sol: $2C(graphite) + 3H_2(g) \rightarrow C_2H_6(g)$

$$\Delta_r H(C_2H_6) = 2\Delta H_{com}(C_{graphite}) + 3\Delta H_{comb}(H_2) - \Delta H_{comb}(C_2H_6)$$

$$= -(286\times 2) - (393.5\times 3) - (-1560)$$

$$= -572 - 1180.5 + 1560 = -192.5\ kJ.mole$$

MATHEMATICS

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

51. Let $A=\left[a_{ij}\right]$ *and* $B=\left[b_{ij}\right]$ be two 3×3 real matrices such that $b_{ij}=(3)^{(i+j-2)}a_{ji}$, where $i,j=1,2,3$. If the determinant of A is

1) $\frac{1}{9}$ 2) 3 3) $\frac{1}{81}$ 4) $\frac{1}{3}$

Ans : 1

Sol : $\begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = \begin{vmatrix} 3^0a_{11} & 3^1a_{12} & 3^2a_{13} \\ 3^1a_{21} & 3^2a_{22} & 3^3a_{23} \\ 3^2a_{31} & 3^3a_{32} & 3^4a_{33} \end{vmatrix} \Rightarrow 81=3^3.3.3^2\,|A| \Rightarrow |A|=\frac{1}{9}$

52. The locus of the mid-points of the perpendiculars drawn from points on the line, x=2y to the line x = y is:

1) $2x-3y=0$ 2) $5x-7y=0$ 3) $3x-2y=0$ 4) $7x-5y=0$

Ans : 2

Sol : Slope of PQ $=\frac{k-\alpha}{h-2\alpha}=-1$ $\Rightarrow k-\alpha=-h+2\alpha$

$\Rightarrow \alpha=\frac{h+k}{3}$ ..... (1)

Also $2h=2\alpha+\beta$ $2k=\alpha+\beta$

$\Rightarrow 2h=\alpha+2k$ $\Rightarrow \alpha=2h-2k$ ..... (2)

From (1) and (2)

$\frac{h+k}{3}=2(h-k)$

So locus is $6x-6y=x+y \Rightarrow 5x=7y$

53. Let the tangents dran from the origin to the circle, $x^2+y^2-8x-4y+16=0$ touch it at the points A and B. The $(AB)^2$ is equal to:

1) $\frac{64}{5}$ 2) $\frac{56}{5}$ 3) $\frac{52}{5}$ 4) $\frac{32}{5}$

Ans : 1

Sol : $L=\sqrt{S_1}=\sqrt{16}=4$

$R=\sqrt{16+4-16}=2$

Length of Chord of contact $=\frac{2LR}{\sqrt{L^2+R^2}}=\frac{2\times4\times2}{\sqrt{16+4}}=\frac{16}{\sqrt{20}}$

Square of length of chord of contact $=\frac{64}{5}$

54. Let A,B,C and D be four non-empty sets. The contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$, then $A \subseteq C$" is :

1) If $A \subseteq C$,then $B \subset A$ or $D \subset B$ 2) If $A \not\subseteq C$,then $A \subseteq B$ and $B \subseteq D$

3) If $A \not\subseteq C$,then $A \not\subseteq B$ and $B \subseteq D$ 4) If $A \not\subseteq C$,then $A \not\subseteq B$ or $B \not\subseteq D$

Ans : 4

Sol : Contrapositive of $p \to q$ is $\sim q \to \sim p$

$(A \subseteq B) \wedge (B \subseteq D) \to (A \subseteq C)$

Contrapositive is

$\sim (A \subseteq C) \to \sim (A \subseteq B) \vee \sim (B \subseteq D)$

$(A \not\subseteq C) \to (A \not\subseteq B) \vee (B \not\subseteq D)$

55. Let y=y(x) be the solution curve of the differential equation $(y^2 - x)\frac{dy}{dx} = 1$, satisfying y(0) = 1. This curve intersects the x-axis at a point whose abscissa is:

1) $2-e$ 2) $2+e$ 3) 2 4) -e

Ans : 1

Sol : $\frac{dx}{dy} + x = y^2$

I.F. $= e^{\int i.dy} = e^y$ $x.e^y = \int y^2.e^y.dy$ $= y^2.e^y - \int 2y.e^y.dy$

$\Rightarrow y^2e^y - 2(y.e^y - e^y) + c$ $x.e^y = y^2e^y - 2ye^y + 2e^y + C$

$x = y^2 - 2y + 2 + c.e^{-y}$ $x = 0, y = 1$ $0 = 1 - 2 + 2 + \frac{c}{e}$ $c = -e$

$y = 0, x = 0 - 0 + 2 + (-e)(e^{-0})$ $x = 2 - e$

56. If θ_1 and θ_2 be respectively the smallest and the largest values of $\theta \ln (0, 2\pi) - \{\pi\}$ which satisfy the equation $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$ then $\int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta$ is equal to:

1) $\frac{\pi}{3}$ 2) $\frac{2\pi}{3}$ 3) $\frac{\pi}{9}$ 4) $\frac{\pi}{3} + \frac{1}{6}$

Ans : 1

Sol : $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$ $\frac{2\cos^2\theta}{\sin^2\theta} - \frac{5}{\sin\theta} + 4 = 0$

$2\cos^2\theta - 5\sin\theta + 4\sin^2\theta = 0, \sin\theta \neq 0$ $2\sin^2\theta - 5\sin\theta + 2 = 0$ $(2\sin\theta - 1)(\sin\theta - 2) = 0$

$\sin\theta = \frac{1}{2}$ $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $\therefore \int_{\pi/6}^{5\pi/6} \cos^2 3\theta d\theta = \int_{\pi/6}^{5\pi/6} \frac{1 + \cos 6\theta}{2} d\theta$

$$=\frac{1}{2}\left[\theta+\frac{\sin 6\theta}{6}\right]_{\pi/6}^{5\pi/6}=\frac{1}{2}\left[\frac{5\pi}{6}-\frac{\pi}{6}+\frac{1}{6}(0-0)\right]=\frac{1}{2}.\frac{4\pi}{6}=\frac{\pi}{3}$$

57. If the sum of the first 40 terms of the series, $3+4+8+9+13+14+18+19.....$ is (102)m, then m is equal to:

1) 20 2) 25 3) 10 4) 5

Ans : 1

Sol : $S=3+4+8+9+13+14+18+19.....40$ terms

$S=7+17+27+37+47+.....20$ terms

$$S_{40}=\frac{20}{2}\left[2\times 7+(19)10\right]=10\left[14+190\right]=10\left[2040\right]=(102)(20) \qquad \Rightarrow m=20$$

58. The number of ordered pairs (r,k) for which $6.{}^{35}C_r=\left(k^2-3\right).{}^{36}C_{r+1}$, where k is an integer is:

1) 6 2) 2 3) 3 4) 4

Ans : 4

Sol : $\frac{36}{r+1}\times\frac{35}{C_r}\left(k^2-3\right)=\frac{35}{C_r}$ $\qquad k^2-3=\frac{r+1}{6}\Rightarrow k^2=3+\frac{r+1}{6}$

r 5, 35 r = 5, $k\pm 2$ $r=35, k=\pm 3$ =4

59. The value of α for which $4\alpha\int_{-1}^{2}e^{-\alpha|x|}dx=5$, is:

1) $\log_e 2$ 2) $\log_e\sqrt{2}$ 3) $\log_e\left(\frac{3}{2}\right)$ 4) $\log_e\left(\frac{4}{3}\right)$

Ans : 1

Sol : $4\alpha\left\{\int_{-1}^{0}e^{\alpha x}dx+\int_{0}^{2}e^{-\alpha x}dx\right\}=5$ $\qquad \Rightarrow 4\alpha\left\{\left(\frac{e^{\alpha x}}{\alpha}\right)_{-1}^{0}+\frac{e^{-\alpha x}}{-\alpha}\right\}=5$

$\Rightarrow 4\alpha\left\{\left(\frac{1+e^{-\alpha}}{\alpha}\right)-\left(\frac{e^{-2\alpha}-1}{\alpha}\right)\right\}=5$ $\qquad \Rightarrow 4\left(2-e^{-\alpha}-e^{-2\alpha}\right)=5$ Put $e^{-\alpha}=t$

$\Rightarrow 4t^2+4t-3=0$ $\qquad \Rightarrow (2t+3)(2t-1)=0$ $\qquad \Rightarrow e^{-\alpha}=\frac{1}{2}$ $\qquad \Rightarrow \alpha=\ln 2$

60. Let $f(x)$ be a polynomial of degree 5 such that $x=\pm 1$ are its critical points. If $\lim_{x\to 0}\left(2+\frac{f(x)}{x^3}\right)=4$, then which one of the following is not true?

1) x = 1 is a point of maxima and x = -1 is a point of minimum of f.

2) $f(1)-4f(-1)=4$

3) x=1 is a point of minima and x = -1 is a point of maxima of f.

4) f is an odd function

Ans : 3

Sol : $f(x)=ax^5+bx^4+cx^3$

$$\lim_{x\to 0}\left(2+\frac{ax^5+bx^4+cx^3}{x^3}\right)=4\Rightarrow 2+c=4\Rightarrow x=2$$

$f'(x)=5ax^4+4bx^3+6x^2 \qquad =x^2\left(5ax^2+4bx+6\right)$

$f'(1)=0 \qquad \Rightarrow 5a+4b+6=0$

$f'(-1)=0 \qquad \Rightarrow 5a-4b+6=0$

$b=0 \qquad a=-\frac{6}{5}$

$f(x)=\frac{-6}{5}x^5+2x^3 \quad f'(x)=-6x^4+6x^2$

$=6x^2\left(-x^2+1\right) \qquad =-6x^2(x+1)(x-1)$

-1	$+$	$1-$
$1-$		1

Minimal at x = -1

Maxima at x = 1

61. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. If $\lambda=\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ and $\vec{d}=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ then the ordered pair, $(\lambda,\vec{d})$ is equal to:

1) $\left(\frac{3}{2},3\vec{b}\times\vec{c}\right)$ 2) $\left(\frac{3}{2},3\vec{a}\times\vec{c}\right)$ 3) $\left(-\frac{3}{2},3\vec{c}\times\vec{b}\right)$ 4) $\left(-\frac{3}{2},3\vec{a}\times\vec{b}\right)$

Ans : 4

Sol : $\left|\vec{a}+\vec{b}+\vec{c}\right|^2=0 \qquad 3+2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=0$

$\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=\frac{-3}{2} \quad \Rightarrow \lambda=\frac{-3}{2}$

$\vec{d}=\vec{a}\times\vec{b}+\vec{b}\times\left(-\vec{a}-\vec{b}\right)+\left(-\vec{a}-\vec{b}\right)\times\vec{a} \qquad =\vec{a}\times\vec{b}+\vec{a}\times\vec{b}+\vec{a}\times\vec{b} \qquad \vec{d}=3\left(\vec{a}\times\vec{b}\right)$

62. The coefficient of x^7 in the expression $(1+x)^{10}+x(1+x)^9+x^2(1+x)^8+.....+x^{10}$ is-

1) 330 2) 210 3) 420 4) 120

Ans : 1

Sol : $\frac{(1+x)^{10}\left[1-\left(\frac{x}{1+x}\right)^{11}\right]}{\left(1-\frac{x}{1+x}\right)} \qquad \frac{(1+x)^{10}\left[(1+x)^{11}-x^{11}\right]}{(1+x)^{11}\times\frac{1}{(1+x)}} \qquad =(1+x)^{11}-x^{11}$

Coefficient of x^7 is ${}^{11}C_7={}^{11}C_4=330$

63. Let α and β be the roots of the equation $x^2-x-1=0$. If $P_k=(\alpha)^k+(\beta)^k, k\geq 1$,then which one of the following statement is not true?

1) $P_5=P_2.P_3$ 2) $(P_1+P_2+P_3+P_4+P_5)=26$

3) $P_3=P_5-P_4$ 4) $P_5=11$

Ans : 1

Sol : $\alpha^5=5\alpha+3$ $\beta^5=5\beta+3$ $\overline{P_5=5(\alpha+\beta)+6}$ $=5(1)+6$

$P_5=11$ and $P_5=\alpha^2+\beta^2=\alpha+1+\beta+1$

$P_2=3$ and $P_3=\alpha^3+\beta^3=2\alpha+1+2\beta+1=2(1)+2=4$

$P_2\times P_3=12$ and $P_5=11\Rightarrow P_5\neq P_2\times P_3$

64. The value of c in the Lagrange's mean value theorem for the function $f(x)=x^3-4x^2+8x+11$, when $x\in[0,1]$ is:

1) $\frac{4-\sqrt{7}}{3}$ 2) $\frac{4-\sqrt{5}}{3}$ 3) $\frac{2}{3}$ 4) $\frac{\sqrt{7}-2}{3}$

Ans : 1

Sol : $f(x)$ is a polynomial function $\therefore$ It is continuous and differentiable in $[0,1]$

Here $f(0)=11, f(1)=1-4+8+11=16$

$f'(x)=3x^2-8x+8$ $\therefore f'(c)=\frac{f(1)-f(0)}{1-0}=\frac{16-11}{1}=3c^2-8c+8$

$\Rightarrow 3c^2-8c+3=0$ $C=\frac{8\pm 2\sqrt{7}}{6}=\frac{4\pm\sqrt{7}}{3}$ $\therefore c=\frac{4-\sqrt{7}}{3}\in(0,7)$

65. The area(in sq. units) of the region $\{(x,y)\in R^2 \mid 4x^2\leq y\leq 8x+12\}$ is _____

1) $\frac{127}{3}$ 2) $\frac{128}{3}$ 3) $\frac{124}{3}$ 4) $\frac{125}{3}$

Ans : 2

Sol : $4x^2=y$ $y=8x+12$ $4x^2=8x+12$

$x^2-x-3=0$ $x^2-2x-3=0]$ $x^2-3x+x-3=0$

$(x+1)(x+3)=0$ $x=-1$ $A=\int_{-1}^{3}(8x+12-4x^2)\,dx$

$A=\frac{8x^2}{2}+12x-\frac{4x^3}{3}\Big|_{-1}^{3}=(4(9)+36-36)-\left(4-12+\frac{4}{3}\right)=36+8-\frac{4}{3}$

$=44-\frac{4}{3}=\frac{132-4}{3}=\frac{128}{3}$

66. In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3 k$ then k is equal to:

1) $\frac{17}{2}$ 2) 4 3) $\frac{17}{8}$ 4) $\frac{17}{4}$

Ans : 3

Sol : Required probability = when no. machine has fault + when only one machine has fault + when only two machines have fault.

$$={}^5C_0\left(\frac{3}{4}\right)^5+{}^5C_1\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4+{}^5C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$$

$$=\frac{243}{1024}+\frac{405}{1024}+\frac{270}{1024}=\frac{918}{1024}=\frac{459}{512}=\frac{27\times17}{64\times8}$$

$$=\left(\frac{3}{4}\right)^3\times k=\left(\frac{3}{4}\right)^3\times\frac{17}{8} \qquad \therefore k=\frac{17}{8}$$

67. If $3x+4y=12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{9}=1$ for some $a\in R$, then the distance between the foci the of ellipse is:

1) $2\sqrt{5}$ 2) $2\sqrt{2}$ 3) $2\sqrt{7}$ 4) 4

Ans : 3

Sol : $3x+4y=12\sqrt{2} \Rightarrow 4y=-3x+12\sqrt{2} \Rightarrow y=-\frac{3}{4}x+3\sqrt{2}$

Condition of tangency $c^2=a^2m^2+b^2$

$18=a^2.\frac{9}{16}+9 \qquad a^2.\frac{9}{16}=9 \qquad a^2=16 \qquad a=4$

$e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{9}{16}}=\frac{\sqrt{7}}{4} \qquad \therefore ae=\frac{\sqrt{7}}{4}.4=\sqrt{7}$

$\therefore$ focus are $\left(\pm\sqrt{7},0\right)$ $\therefore$ distance between foci $=2\sqrt{7}$

68. If $\frac{3+i\sin\theta}{4-i\cos\theta}, \theta\in[0,2\pi]$, is a real number, then an argument of $\sin\theta+i\cos\theta$ is:

1) $-\tan^{-1}\left(\frac{3}{4}\right)$ 2) $\pi-\tan^{-1}\left(\frac{3}{4}\right)$ 3) $\pi-\tan^{-1}\left(\frac{4}{3}\right)$ 4) $\tan^{-1}\left(\frac{4}{3}\right)$

Ans : 3

Sol : $z=\frac{(3+i\sin\theta)}{(4-i\cos\theta)}\times\frac{(4+i\cos\theta)}{4+i\cos\theta}$

as z is purely real $\Rightarrow 3\cos\theta+4\sin\theta=0\Rightarrow \tan\theta=-\frac{3}{4}$

$\arg(\sin\theta+i\cos\theta)=\pi+\tan^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)=\pi-\tan^{-1}\left(-\frac{4}{3}\right)$

69. Let $a_1,a_2,a_3,....$ be a G.P such that $a_1<0$. $a_1+a_2=4$ and $a_3+a_4=16$. If $\sum_{i=1}^{9}a_i=4\lambda$, then λ is equal to:

1) -171 2) -513 3) $\frac{511}{3}$ 4) 171

Ans : 1

Sol : $a_1+a_2=4\Rightarrow a_1+a_1r=4$ .....(i)

$a_3+a_4=16\Rightarrow a_1r^2+a_1r^3=16$ .....(ii)

$\frac{1}{r^2}+\frac{1}{4}\Rightarrow r^2=4$ $r=\pm2$ $r=2, a_1(1+2)=4\Rightarrow a_1=\frac{4}{3}$

$r=-2, a_1(1-2)=4\Rightarrow a_1=-4$

$\sum_{i=1}^{a}a_i=\frac{a_1(r^q-1)}{r-1}=\frac{(-4)\left((-2)^9-1\right)}{-2-1}=\frac{4}{3}(-513)=4\lambda$ $\lambda=-171$

70. Let y=y(x) be a function of x satisfying $y\sqrt{1-x^2}=k-x\sqrt{1-y^2}$ where k is a constant and $y\left(\frac{1}{2}\right)=-\frac{1}{4}$. Then $\frac{dy}{dx}$ at $x=\frac{1}{2}$ is equal to:

1) $\frac{2}{\sqrt{5}}$ 2) $-\frac{\sqrt{5}}{4}$ 3) $-\frac{\sqrt{5}}{2}$ 4) $\frac{\sqrt{5}}{2}$

Ans : 3

Sol : $x=\frac{1}{2}, y=\frac{-1}{4}\Rightarrow xy=\frac{-1}{8}$ $y.\frac{1.(=2x)}{2\sqrt{-x^2}}+y^1.\sqrt{1-x^2}=-\left\{1.\sqrt{1-y^2}+\frac{x.(-2y)}{2\sqrt{1-y^2}}y'\right\}$

$-\frac{xy}{\sqrt{1-x^2}}+y'\sqrt{1-x^2}=-\sqrt{-1-y^2}+\frac{xy.y'}{\sqrt{1-y^2}}$

$y'\left(\sqrt{1-x^2}-\frac{xy}{\sqrt{1-y^2}}\right)=\frac{xy}{\sqrt{1-x^2}}-\sqrt{1-y^2}$

$y'\left(\frac{\sqrt{3}}{2}+\frac{1}{8.\frac{\sqrt{15}}{4}}\right)=\frac{-1}{8\sqrt{\frac{3}{2}}}-\frac{\sqrt{15}}{4}$ $y'\left(\frac{\sqrt{45}+1}{2\sqrt{15}}\right)=\frac{\left(1+\sqrt{45}\right)}{4\sqrt{3}}$ $y'=-\frac{\sqrt{5}}{2}$

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

71. Let $X=\{n\in N:1\le n\le 50\}$. If $A=\{n\in X:n \text{ is a multiple of } 2\}$ and $B=\{n\in X:n \text{ is a multiple of } 7\}$ and number of elements in the smallest subset of X containing both A and B is _____

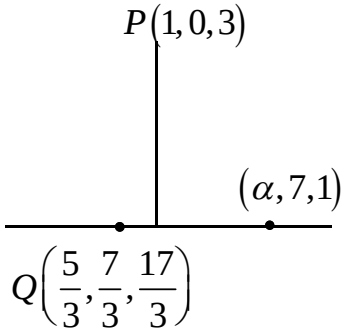
Ans : 29

Sol : $n(A\cup B)=n(A)+n(B)-n(A\cap B)$ = 25 + 7 – 3 = 29

72. If the foot of the perpendicular drawn from the point (1,0,3) on a line passing though $(\alpha,7,1)$ is $\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$, then α is equal to _____

Ans : 4

Sol : Since PQ is perpendicular to L, therefore



$$\left(1-\frac{5}{3}\right)\left(\alpha-\frac{5}{3}\right)+\left(\frac{-7}{3}\right)+\left(3-\frac{17}{3}\right)\left(1-\frac{17}{3}\right)=0$$

$$\Rightarrow\frac{-2\alpha}{3}+\frac{10}{9}-\frac{98}{9}+\frac{112}{9}=0$$

$$\Rightarrow\frac{2\alpha}{3}=\frac{24}{9}\Rightarrow\alpha=4$$

73. If the function f defined on $\left(-\frac{1}{3},\frac{1}{3}\right)$ by $f(x)=\begin{cases}\frac{1}{x}\log_e\left(\frac{1+3x}{1-24}\right), & when\ x\neq 0\\ k, & when\ x=0\end{cases}$ is continuous, then k is equal to _____

Ans : 5

Sol : $\lim\limits_{x\to0}f(x)\lim\limits_{x\to0}\left(\frac{1}{x}\ln\left(\frac{1+3x}{1-2x}\right)\right)=\lim\limits_{x\to0}\left(\frac{\ln(1+3x)}{x}-\frac{\ln(1-2x)}{x}\right)$

$$=\lim_{x\to0}\left(\frac{3\ln(1+3x)}{3x}-\frac{2\ln(1-2x)}{-2x}\right)=3+2=5$$

$\therefore f(x)$ will be continuous if $f(0)=\lim\limits_{x\to0}f(x)$

74. If the system of linear equations,

$x+y+z=6$

$x+2y+3z=10$

$3x+2y+\lambda z=\mu$

has more than two solutions, then $\mu-\lambda^2$ is equal to _____

Ans : 13

Sol : $x+y+z=6$ _____(1)

$x+2y+3z=10$ _____ (2)

$3x+2y+\lambda z=\mu$ _____ (3)

from (1) and (2)

If $z=0\Rightarrow x+y=6$ and $x+2y=10\ \Rightarrow y=4, x=2$

$(2,4,0)$

If $y=0\Rightarrow x+z=6$ and $x+3z=10$

$\Rightarrow z=2$ and $x=4$

Must pass through $(2,4,0)$ and $(4,0,2)$ and $12+2\lambda=\mu$

$12+2\lambda=14\Rightarrow\lambda=1$

so $\mu-\lambda^2=14-1=13$

75. If the mean and variance of eight numbers 3,7,9,12,13,20, x and y be 10 and 25 respectively, then x.y is equal to _____

Ans : 54

Sol : $\frac{3+7+9+12+13+20+x+y}{8}=10$

$x+y=16$ $\qquad \frac{\sum x^2}{n}-\left(\frac{\sum x}{n}\right)^2=25$

$3^2+7^2+9^2+12^2+13^2+20^2+x^2+y^2=1000$

$x^2+y^2=148$ $\qquad xy=54$

Prepared by

Sri Chaitanya Faculty