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IIT Academy., India

JEE Main 2020

09 Jan 2020, Slot - 2

(2.30 PM - 5.30 PM)

Question Paper

Solutions

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PHYSICS

(SINGLE CORRECT ANSWER TYPE)

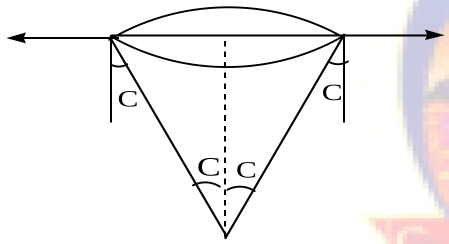
This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.
Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. There is a small source of light at some depth below the surface water (refractive index $= \frac{4}{3}$) in a tank of large cross sectional surface area. Neglecting any reflection from the bottom and absorption by water, percentage of light that emerges out of surface is (nearly): [use the fact that surface area of a spherical cap of height and radius of curvature r is $2\pi rh$]

1) 17% 2) 50% 3) 34% 4) 21%

KEY: 1

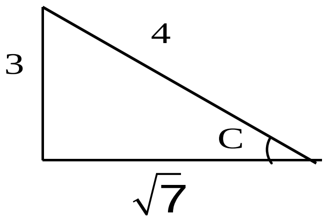
Sol:



When angle of incidence is equal to critical angle then light ray grazes the water surface. If angle of incidence is greater than critical angle light reflected in water only

$\sin c = \frac{1}{\mu} = \frac{3}{4}$

Light source emits the light in three dimensional solid angle subtended by this portion is



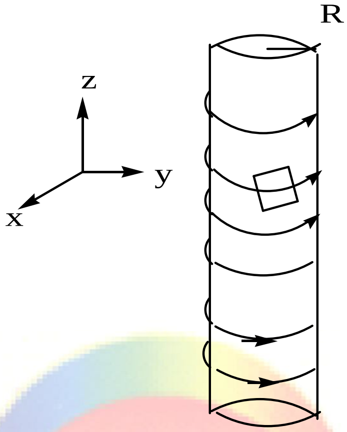
$$\Omega = 2\pi(1-\cos c) \qquad = 2\pi\left(1-\frac{\sqrt{7}}{4}\right)$$

For solid angle 4π ----------100% light comes out

For solid angle Ω.........?

$$x = \frac{\Omega \times 100}{4\pi} = \frac{2\pi\left(1-\frac{\sqrt{7}}{4}\right)}{4\pi}\times 100 = \frac{1-\frac{\sqrt{7}}{4}}{2}\times 100 \quad = 17\%$$

2. An electron gun is placed inside a long solenoid of radius R on its axis. The solenoid has n turns/ length and carries a current I. The electron gun shoots an electron along the radius of the solenoid with speed v. If the electron does not hit the surface of the solenoid, maximum possible value of v is (all symbols have their standard meaning):



1) $\frac{e\mu_0 nIR}{4m}$ 2) $\frac{2e\mu_0 nIR}{m}$ 3) $\frac{e\mu_0 nIR}{m}$ 4) $\frac{e\mu_0 nIR}{2m}$

KEY: 4

Sol: electron does not hit the cylinder if it describes the circular path of radius $r = \frac{R}{2}$

We known that $r = \frac{mv}{Bq}$ $\frac{R}{2} = \frac{mv}{Bq}$ $B = \mu_0 ni$ $\Rightarrow U = \frac{BqR}{2m} = \frac{\mu_0 nieR}{2m}$

3. An electron of mass m and magnitude of charge $|e|$ initially at rest accelerated by a constant electric field E. The rate of change of de-Broglie wavelength of this electron at time t ignoring relativistic effects is:

1) $-\frac{h}{|e|E\sqrt{t}}$ 2) $-\frac{h}{|e|Et}$ 3) $-\frac{|e|Et}{h}$ 4) $\frac{-h}{|e|Et^2}$

KEY: 4

Sol: $F = Eq$ $Ma = E.e$ $a = \frac{Ee}{m}$ $v = u + at$ $v = 0 + \frac{Ee}{m}t$

$\Rightarrow \frac{h}{mv} = \frac{h}{M.\frac{Ee}{m}t}$ $\frac{d\lambda}{dt} = \frac{-h}{Eet^2}$ $\left[\frac{d}{dt}\left(\frac{1}{t}\right) = \frac{-1}{t^2}\right]$

4. A small circular loop of conducting wire has radius a and carries current I. It is placed in a uniform magnetic field B perpendicular to its plane such that when rotated slightly about its diameter and simple harmonic motion of time period T. If the mass of the loop is m then

1) $T=\sqrt{\frac{\pi m}{IB}}$ 2) $T = \sqrt{\frac{\pi m}{2IB}}$ 3) $T = \sqrt{\frac{2m}{IB}}$ 4) $T = \sqrt{\frac{2\pi m}{IB}}$

KEY: 4

Sol: $\bar{\tau} = \bar{M} \times \bar{B}$

The deflecting torque $\bar{\tau}_d = I\alpha$, Restoring torque $\bar{\tau}_r = MB\sin\theta$

At equilibrium $\bar{\tau}_d = \bar{\tau}_r$

$I\alpha = -MB\sin\theta$ $\quad I\alpha = -MB\theta$ $\quad \alpha = -\frac{MB}{I}\theta$ $\quad \omega = \sqrt{\frac{MB}{I}}$

$T = 2\pi\sqrt{\frac{MB}{I}} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{MB}}$

Where $M = nIA$ (Magnetic moment)

$= 1 \times I \times \pi R^2$ $\quad I = \frac{MR^2}{2}$ $\quad T = 2\pi\sqrt{\frac{\frac{MR^2}{2}}{I.\pi R^2}}$

$= \sqrt{\frac{4\pi^2.MR^2}{2.I.\pi R^2}}$ $\quad = \sqrt{\frac{2\pi m}{IB}}$

5. A plane electromagnetic wave is propagating along the direction $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$, with its polarization along the direction $\hat{k}$. The correct form of the magnetic field of the wave would be (here B_0 is an appropriate constant)

1) $B_0\hat{k}\cos\left(\omega t - k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$ 2) $B_0\frac{\hat{i}+\hat{j}}{\sqrt{2}}\cos\left(\omega t - k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

3) $B_0\frac{\hat{i}-\hat{j}}{\sqrt{2}}\cos\left(\omega t - k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$ 4) $B_0\frac{\hat{j}-\hat{i}}{\sqrt{2}}\cos\left(\omega t - k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

KEY: 3

Sol: Generally $\bar{B} = B_0.\sin(wt - kx)$ x-direction of propagation

Here direction of propagation is along $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

Direction of polarization is $\hat{k}$

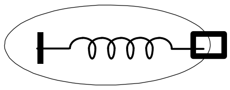
$\hat{E} \times \hat{B} = \frac{\hat{i}+\hat{j}}{\sqrt{2}}$ $\quad \hat{K} \times \hat{B} = \frac{\hat{i}+\hat{j}}{\sqrt{2}}$ $\quad \Rightarrow \hat{B} = \frac{\hat{i}-\hat{j}}{\sqrt{2}}$

6. A spring mass system (mass m , spring constant k and natural length l) rests in equilibrium on a horizontal disc. The free end of the spring is fixed at the centre of the disc. If the disc together with spring mass system, rotates about it's axis with an angular velocity $\omega, (k >> m\omega^2)$ the relative change in the length of the qpring is best given by the option

1) $\frac{2m\omega^2}{k}$ 2) $\frac{m\omega^2}{k}$ 3) $\sqrt{\frac{2}{3}}\left(\frac{m\omega^2}{k}\right)$ 4) $\frac{m\omega^2}{3k}$

KEY: 2

Sol:



Spring force=centrifugal force $kx = m(l+x)\omega^2$

$kx - mx\omega^2 = ml\omega^2$ $\quad x = \frac{ml\omega^2}{k - m\omega^2}$ $\quad x = \frac{ml\omega^2}{k}$

Relative change is $\frac{x}{L} = \frac{m\omega^2}{K}$

7. A particle starts from the origin at $t=0$ with an initial velocity of $3.0\hat{i}$ m/s and moves in the $x-y$ plane with a constant acceleration $(6.0\hat{i} + 4.0\hat{j})m/s^2$. The x-coordinate of the particle at the instnt when its –coordinate is $32m$ is D meters. The value of D is:

1) 50 2) 32 3) 60 4) 40

KEY: 3

Sol: $u_x = 6m/s, a_x = 6m/s^2, a_y = 4m/s^2$

$x = u_x t + \frac{1}{2}a_x t^2$ $\quad y = u_g t + \frac{1}{2}a_y t^2$ $\quad 32 = 0 + \frac{1}{2}\times 4\times t^2$

$t^2 = 16$ $\quad t = 4s$ $\quad x = u_x t + \frac{1}{2}a_x t^2$ $\quad x = 3\times 4 + \frac{1}{2}\times 6\times 16$

$= 12 + 48 = 60m$

8. The energy required to ionize a hydrogen like ion in its ground state is 9 rydbergs. What is the wavelength of the radiation emitted when the electron In this ion jumps from the second excited state to the ground state?

1) $24.2\,nm$ 2) $35.8nm$ 3) $8.6nm$ 4) $11.4nm$

KEY: 4

Sol: Rhydberg energy=13.6 eV, So, ionization energy=$(13.6z^2)eV = 9\times 13.6eV$

$\Rightarrow Z = 3$ $\quad \frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$\frac{1}{\lambda} = 1.09\times 10^2 \times 9\left[\frac{1}{1} - \frac{1}{3^2}\right]$ $\quad \lambda = 11.4nm$

9. Planet A has mass M and radius R. Planet B has half the mass and half the radius of planet A. If the escape velocities from the planets A and B are u_A and u_B respectively, then $\frac{u_A}{u_B} = \frac{n}{4}$. The value of n is

1) 3 2) 2 3) 1 4) 4

KEY: 4

Sol: $V_e = \sqrt{\frac{2GM}{R}}$ $\quad \frac{V_A}{V_B} = \sqrt{\frac{M_A}{M_B} \times \frac{R_B}{R_A}}$

$\frac{V_A}{V_B} = \sqrt{\frac{M}{\frac{M}{2}} \times \frac{\frac{R}{2}}{R}}$ $\quad V_A = V_B$

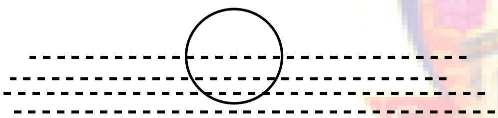
$\frac{V_A}{V_B} = \frac{n}{4}(Given)$ $\quad \Rightarrow n = 4$

10. A small spherical droplet of density d is floating exactly half immersed in a liquid of density ρ and surface tension T. The radius of the droplet is (take note that the surface tension applies an upward force on the droplet)

1) $r = \sqrt{\frac{2T}{3(d+\rho)g}}$ 2) $r = \sqrt{\frac{T}{(d+\rho)g}}$ 3) $r = \sqrt{\frac{T}{(d-\rho)g}}$ 4) $r = \sqrt{\frac{3T}{(2d-\rho)g}}$

KEY: 4

Sol:



Weight is balanced by Buoyancy force &surface tension forces $\Rightarrow mg = F_B + F_{sT}$

$\Rightarrow mg = V_n \rho g + T \times 2\pi R$ $\quad vdg = \frac{v}{2}\rho g + T.2\pi R$

$V\left(d - \frac{\rho}{2}\right)g = T \times 2\pi R$ $\quad \frac{4}{3}\pi R^3\left(\frac{2d-\rho}{2}\right)g = T \times 2\pi R$ $\quad R = \sqrt{\frac{3T}{(2d-\rho)g}}$

11. Two steel wires having same length are suspended from a ceiling under the same load. If the ratio of their energy stored per unit volume is $1:4$, the ratio of their diameters is

1) $\sqrt{2}:1$ 2) $1:\sqrt{2}$ 3) $2:1$ 4) $1:2$

KEY: 1

Sol: $E = \frac{U}{v} = \frac{1}{2} \times stress \times strain$

$E = \frac{1}{2} \times \frac{(stress)^2}{Y}$ $\quad \left(Y = \frac{stress}{strain}\right)$ $\quad E = \frac{1}{2} \times \frac{\left(\frac{F}{A}\right)^2}{Y} = \frac{F^2}{2A^2Y}$

$\frac{E_1}{E_2} = \frac{d_2^4}{d_1^4}$ $\quad \therefore A = \frac{\pi d^2}{4}$ $\quad \frac{1}{4} = \left(\frac{d_2}{d_1}\right)^4$ $\quad \left(\frac{d_1}{d_2}\right)^4 = \frac{4}{1}$ $\quad \frac{d_1}{d_2} = \frac{\sqrt{2}}{1}$

12. Two gases-argon(atomic radius 0.07nm, atomic weight 40) and xenon (atomic radius $0.1nm$, atomic weight 140) have the same number density and are at the same temperature. The ratio of their respective mean free times is closed to

1) 3.67 2) 4.67 3) 1.83 4) 2.3

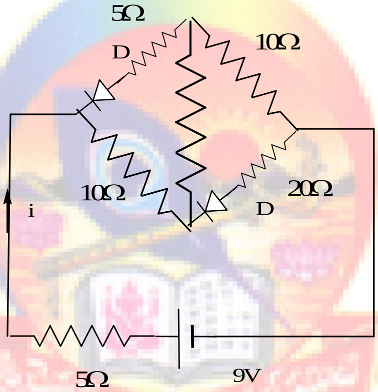
KEY: 1

Sol: $\lambda = \frac{1}{\sqrt{2}\pi n_v d^2}$ $\tau = \frac{\lambda}{v} = \frac{1}{\sqrt{2}\pi n_v d^2 v}$ $\tau = \frac{1}{\sqrt{2}\pi n_v d^2} \times \sqrt{\frac{M}{3RT}}$ $\frac{\tau_1}{\tau_2} = \sqrt{\frac{M_1}{M_2}} \times \frac{d_2^2}{d_1^2}$

$\frac{\tau_{Av}}{\tau_{XR}} = \sqrt{\frac{40}{140}} \times \left(\frac{0.1}{0.07}\right)^2 = \sqrt{\frac{2}{7}} \times \frac{1}{0.49}$

=1.09

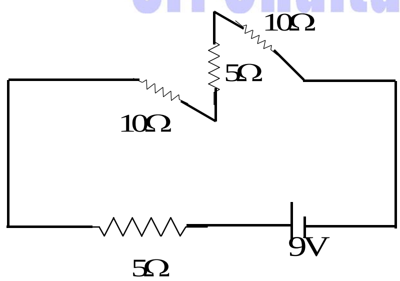
13. The current i in the network is:



1) $0.6\,A$ 2) $0\,A$ 3) $0.2\,A$ 4) $0.3\,A$

KEY: 4

Sol: Both diodes are in reverse bias conditions. Hence diodes do not allow current



$i = \frac{v}{R_{eff}} = \frac{9}{5+10+5+10}$ $i = \frac{9}{30} = 0.3A$

14. For the four sets of three measured physical quantities as given below. Which of the following options is correct?

(i) $A_1 = 24.36, B_1 = 0.0724, C_1 = 256.2$

(ii) $A_2 = 24.44, B_2 = 16.082, C_2 = 240.2$

(iii) $A_3 = 25.2, B_3 = 19.2812, C_3 = 236.183$

(iv) $A_4 = 25, B_4 = 236.191, C_4 = 19.5$

1) $A_4 + B_4 + C_4 < A_1 + B_1 + C_1 < A_3 + B_3 + C_3 < A_2 + B_2 + C_2$
2) $A_1 + B_1 + C_1 < A_2 + B_2 + C_2 < A_3 + B_3 + C_3 < A_4 + B_4 + C_4$
3) $A_4 + B_4 + C_4 < A_1 + B_1 + C_1 = A_2 + B_2 + C_2 = A_3 + B_3 + C_3$
4) $A_1 + B_1 + C_1 = A_2 + B_2 + C_2 = A_3 + B_3 + C_3 = A_4 + B_4 + C_4$

KEY: ADD

Sol: $A_1 + B_1 + C_1 = 24.36 + 0.0724 + 256.2$ $= 280.6324$

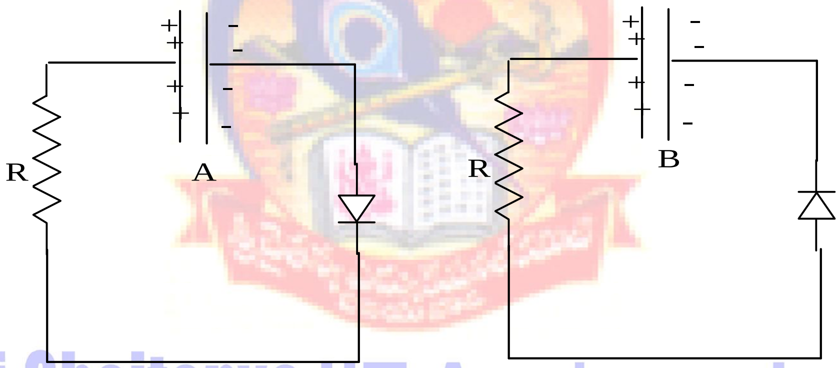
$= 280.6s$ (Using significant figure)

$A_2 + B_2 + C_2 = 24.44 + 16.082 + 240.2$ =280.722 =280.7

$A_3 + B_3 + C_3 = 25.2 + 19.2812 + 236.183$ =280.6642 =280.7

$A_4 + B_4 + C_4 = 25 + 236.191 + 19.5$ =280.691 =280.7

15. Two identical capacitors A and B, charged to the same potential $5V$ are connected in two different circuits as shown below at time $t = 0$. If the charge on capacitors A and B at time $t = CR$ is Q_A and Q_B respectively, then (here e is the base of natural logarithm)

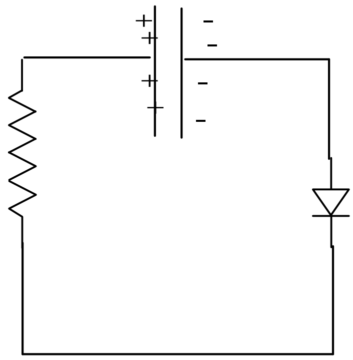


1) $Q_A = \frac{CV}{2}, Q_B = \frac{VC}{e}$ 2) $Q_A = VC, Q_B = CV$

3) $Q_A = VC, Q_B = \frac{VC}{e}$ 4) $Q_A = \frac{VC}{e}, Q_B = \frac{CV}{2}$

KEY: 3

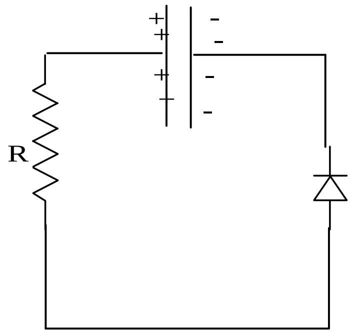
Sol: For A



Diode is in reverse bias condition

No current flows $\Rightarrow Q_A = CV$

For B



Capacitor discharging

$q = q_0 e^{-\frac{t}{cR}}$ $\quad$ $q = CV.e^{-\frac{t}{cR}}$ $\quad$ At t=CR

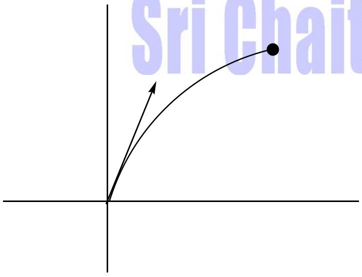
$q = CV.e^{-1}$ $\quad$ $= \frac{CV}{e}$

16. A particle of mass m is projected with a speed u from the ground at angle $\theta = \frac{\pi}{3}$ w.r.t. horizontal ($x-axis$). When it has reached its maximum height, it collides completely inelastically with another particle of the same mass and velocity $u\hat{i}$. The horizontal distance covered by the combined mass before reaching the ground is

1) $\frac{3\sqrt{3}}{8}\frac{u^2}{g}$ $\quad$ 2) $\frac{5}{8}\frac{u^2}{g}$ $\quad$ 3) $\frac{3\sqrt{2}}{4}\frac{u^2}{g}$ $\quad$ 4) $2\sqrt{2}\frac{u^2}{g}$

KEY: 1

Sol:



Acc to L.C, L.M $\frac{mu}{2} + mu = 2mv^1$ $\quad$ $v^1 = \frac{3u}{4}$

Range of combined mass $R^1 = v^1\sqrt{\frac{2H}{g}}$ $= \frac{3U}{4}\sqrt{\frac{2.H}{g}}$

$= \frac{3U}{4}.\sqrt{\frac{2.u^2\sin^2 60}{3.2g}}$ $= \frac{3U}{4}.\sqrt{\frac{2.u^2.3}{4.g.2g}}$

17. A wire length L and mass per unit length $6.0\times10^{-3}kgm^{-1}$ is put under tension of $540N$. Two consecutive frequencies that it resonates at are: $420Hz$ and $490Hz$. Then L in meters is

1) $1.1m$ $\quad$ 2) $2.1m$ $\quad$ 3) $5.1m$ $\quad$ 4) $8.1m$

KEY: 2

Sol: The difference in the frequencies of two successive harmonics is equal to fundamental frequency $n = 490 - 420 \quad = 70\,Hz$

$$n = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \qquad 70 = \frac{1}{2\times L}\sqrt{\frac{540}{6\times10^{-3}}} \qquad L = \frac{1}{140}\times 300 \qquad = 2.1\text{m}$$

18. A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b\left(\frac{x}{L}\right)^2$, where a and b are constants and $0 \le x \le L$. The value of x for the centre of mass of the rod is at

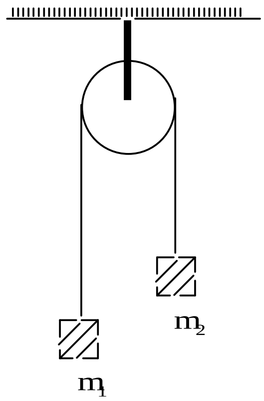
1) $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$ 2) $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$ 3) $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$ 4) $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$

KEY: 2

Sol: $$\frac{dm}{dx} = a + b\frac{x^2}{L^2} \qquad dm = \left(a + b\frac{x^2}{x^2}\right)dx \qquad x_{cm} = \frac{\int xdm}{\int dm} = \frac{\int_0^L \left(ax + \frac{bx^3}{L^2}\right)dx}{\int_0^L \left(a + \frac{bx^2}{L^2}\right)dx} \qquad x_{cm} = \frac{\frac{ax^2}{2} + \frac{bx^4}{4L^2}}{ax + \frac{bx^3}{3L^2}}$$

$$x_{cm} = \frac{\frac{aL^2}{L} + \frac{bL^4}{4L^2}}{aL + \frac{bL^3}{3L^2}} \qquad x_{cm} = \frac{\frac{2aL^2 + bL^2}{4}}{\frac{3aL + bL}{3}} = \frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$$

19. A uniformly thick wheel with moment of inertia I and radius R is free to rotate about its centre of mass (see fig) . A massless string is wrapped over its rim and two blocks of masses m_1 and m_2 $(m_1 > m_2)$ are attached to the ends of the string. The system is released from rest. The angular speed of the wheel when m_1 descents by a distance h is

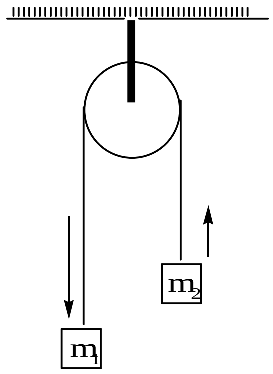


1) $\left[\frac{2(m_1+m_2)gh}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}$ 2) $\left[\frac{m_1-m_2}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}gh$

3) $\left[\frac{m_1+m_2}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}gh$ 4) $\left[\frac{2(m_1-m_2)gh}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}$

KEY: 4

Sol:



Acc to work energy theorem $w = \Delta KE$

$m_1gh - m_2gh = \frac{1}{2}(m_1+m_2)v^2 + \frac{1}{2}I\omega^2$ $\quad (m_1-m_2)gh = \frac{1}{2}(m_1+m_2)R^2\omega^2 + \frac{1}{2}I\omega^2 \ (v = R\omega)$

$(m_1-m_2)gh = \omega^2\left(\frac{(m_1+m_2)R^2+I}{2}\right)$ $\quad \omega = \left(\frac{2(m_1-m_2)gh}{(m_1+m_2)R^2+I}\right)^{1/2}$

20. In LC circuit the inductance $L = 40\,\text{mH}$ and capacitance $C = 100\,\mu F$. If a voltage $V(t) = 10\sin(314t)$ is applied to the circuit, the current in the circuit is given as

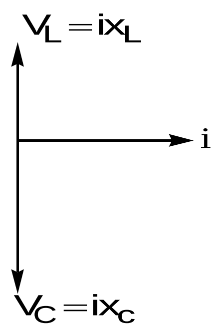
1) $5.2\cos 314t$ 2) $0.52\cos 314t$ 3) $10\cos 314t$ 4) $0.52\sin 314t$

KEY: 2

Sol: $X_L = wL$ $= 314\times 40\times 10^{-3}$ $= 12560\times 10^{-3}$ $= 12.56\Omega$

$X_C = \frac{1}{wc} = \frac{1}{314\times 100\times 10^{-6}}$ $= \frac{1000}{314}$ $= 31.84\Omega$

Phaser diagram



$V_{max} = i_{max}(X_C - X_L)$ $\quad 10 = i_{max}(31.84 - 12.56)$

$i_{max} = \frac{10}{19.28} = 0.52$ $\quad X_C > X_L$

Current leads the voltage by $\frac{\pi}{2}$

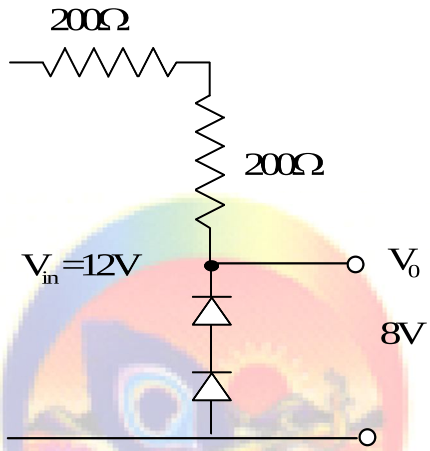
$i = i_{max}\sin\left(wt + \frac{\pi}{2}\right)$ $= 0.52\sin\left(314t + \frac{\pi}{2}\right)$ $= 0.52\cos(314t)$

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value(in decimal notation, truncated/rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

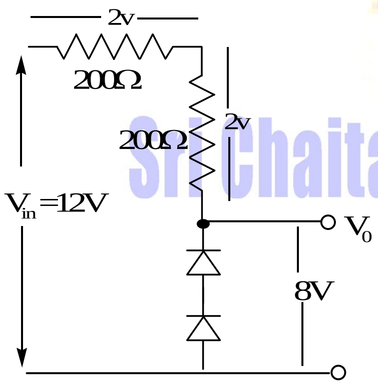
Marking scheme: +4 for correct answer , 0 if not attempted and 0 in all other cases.

21. The circuit shows below is working as a $8V\ dc$ regulated voltage source. When $12V$ is used as input, the power dissipated (in mW)in each diode is; (considering both zener diodes are identical)____



KEY: 12.00

Sol:



Current in the circuit $i=\frac{4}{400}=\frac{1}{100}A$, So, power dissipated in each diode is $p=vi$

$=4\times\frac{1}{100}$ $\quad=40\times10^{-3}w$ $\quad=40mw$

22. Starting at temperature $300\,K$, one mole of an ideal I diatomic gas $(\gamma=1.4)$ is first compressed adiabalically from volume V_1 to $V_2=\frac{V_1}{16}$. It is then allowed to expand isobarically to volume $2V_2$. If all the processes are the quasi-static then the final temperature of the gas (in ^0K) is (to the nearest integer)___

KEY: 1816 to 1820

Sol: During adiabatic process

$TV^{\gamma-1} = cons\tan t$ $\quad T_1 v_1^{\gamma-1} = T_2 . V_2^{\gamma-1}$ $\quad T_2 = T_1\left(\frac{v_1}{v_2}\right)^{\gamma-1}$

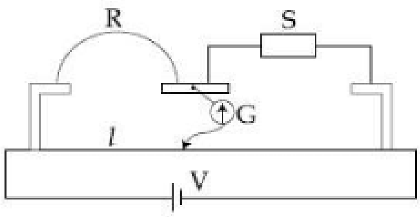
$= 300(16)^{\frac{7}{5}-1}$ $\quad = T_2 = 300\left(2^4\right)^{2/5}$ $\quad = T_2 = 300 \times 2^{4\times\frac{2}{5}}$

During isobaric process

$v = \frac{nRT}{P}$ $\quad v \alpha T$ $\quad \frac{v_2}{v_3} = \frac{T_2}{T_3}$

$\frac{v_2}{2v_2} = \frac{300 \times 2^{4\times\frac{2}{5}}}{T_3}$ $\quad T_3 = 2 \times 300 \times 2^{8/5}$ $\quad$ =1818.85 $\quad$ =1819

23. In a meter bridge experiment S is a standard resistance. R is a resistance wire. It is found that balancing length is $l = 25cm$. If R is replaced by a wire of half length and half diameter that of R of same material, then the balancing distance l'(in cm) will now be_______



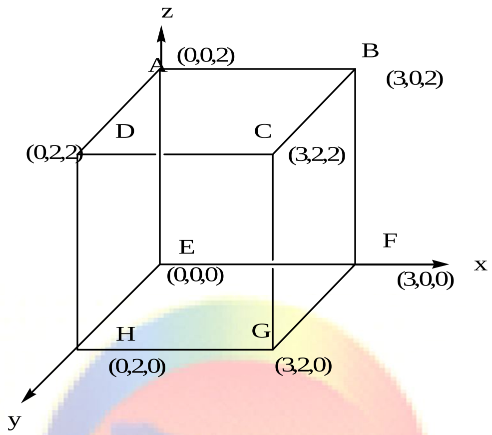
KEY: 40.00

Sol: $\frac{R}{S} = \frac{L}{100-L} = \frac{25}{75}$ $\quad R = \frac{1}{3}.s$ $\quad R = \frac{\rho L}{A} - \frac{\rho L}{\pi r^2}$

$R^1 = \frac{\rho . \frac{L}{2}}{\pi\left(\frac{r}{2}\right)^2} = 2R$ $\quad \frac{2R}{S} = \frac{L^1}{100-L^1}$ $\quad \frac{2.S/3}{S} = \frac{L^1}{100-L^1}$

$\frac{2}{3} = \frac{L^1}{100-L^1}$ $\quad \frac{2}{3} = \frac{L^1}{100-L^1}$ $\quad$ =40 cm

24. An electric field $\vec{E} = 4x\hat{i} - \left(y^2+1\right)\hat{j}N/C$ passes through the box shown in figure. The flux of the electric field through surfaces $ABCD$ and $BCGF$ are marked as ϕ_1 *and* ϕ_{11} respectively .The difference between $\left(\phi_1 - \phi_{11}\right)$ is $\left(\text{in Nm}^2/C\right)$____



KEY: -48.00

Sol: $\bar{E} = 4x\hat{i} - \left(y^2+1\right)\hat{j}.N/c$

Area of ABCD is $ds_{ABCD} = lb\,\hat{k}$ $= 3\times 2\hat{k}$ $= 6\hat{k}$

$\phi_1 = \bar{E}.d\bar{s}_{ABCD}$ =0

Area of BCGF $d\bar{s}_{BCGF} = 2\times 2\hat{i} = 4\hat{i}$ $\phi_2 = \bar{E}.d\bar{s}_{BCGF}$ $= \left[4(3)\hat{i} - \left(y^2+1\right)\hat{j}\right].4\hat{i}$

$\phi_2 = 12\times 4$ $= 48Nm^2/C$ $\Rightarrow \phi_1 - \phi_2 = -48.00\,Nm^2/C$

25. In a Young's double slit experiment 15 fringes are observed on a small portion of the screen when light of wavelength 500nm is used. Ten fringes are observed on the same section of the screen when another light source of wavelength λ is used. Then the value of λ is (in nm)____

KEY: 750.00

Sol: $Y = \dfrac{n\lambda D}{d}$ $y_1 = y_2$ $n_1\lambda_1 = n_2\lambda_2$

$15\times 500 = 10\times\lambda_2$ $\lambda_2 = 750nm$

CHEMISTRY

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

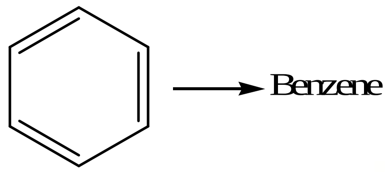
Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

26. The number of SP^2 hybrid orbitals in a molecule of benzene is

1) 6   2) 24   3) 12   4) 18

KEY: 4

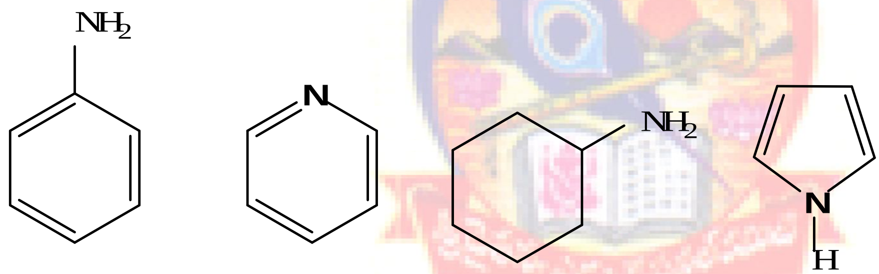
Sol:



In benzene all ‘6’ carbons are sp^2 hybridized.

No of hybrid orbitals are $6\times 3\,sp^2 \Rightarrow 18sp^2$

27. The decreasing order of basicity of the following amines is



(I)   (II)   (III)   (IV)

1) $(III)>(I)>(II)>(IV)$ 2) $(II)>(III)>(IV)>(I)$

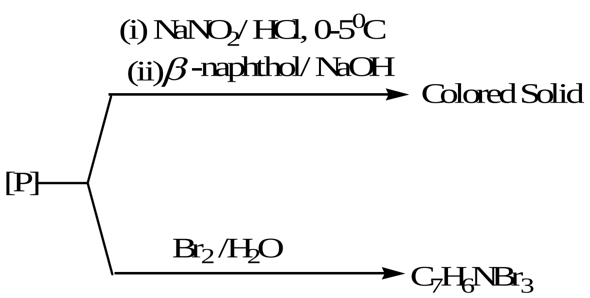
3) $(III)>(II)>(I)>(IV)$ 4) $(I)>(III)>(IV)>(II)$

KEY: 3

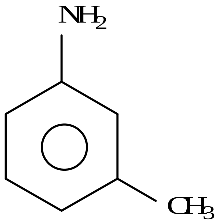
Sol: In Pyrrole lone pair involved in aromatization, so least basic of all. In Aniline, lone pair is in conjugation, hence less basic than others. sp^2 ‘N’ is less basic then sp^3’N’.

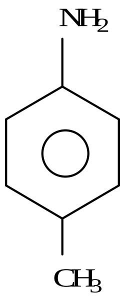
$\therefore$ Order of basicity is Cyclohexylamine >pyridine>Aniline>pyrrole

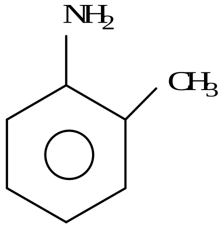
28. Consider the following reactions,

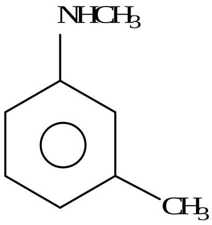


The compound [P] is

1) 

2) 

3) 

4) 

KEY: 1

Sol: Coloured solid upon azo coupling reaction indicates aromatic 1^0 amine

$\Rightarrow$ Tribromo product indicates that ortho & para positions are free

$\Rightarrow$ Hence m-methyl aniline is the compound

29. Which of the following has the shortest $C-CI$ bond ?

1) $CI-CH=CH_2$ 2) $CI-CH=CH-CH_3$ 3) $CI-CH=CH-OCH_3$ 4) $CI-CH=CH-NO_2$

KEY: 4

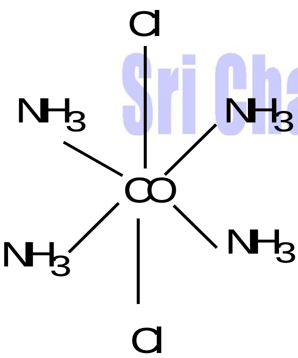
Sol: More the extended conjugation, less will be the bond length

30. The isomer(s) of $\left[Co(NH_3)_4Cl_2\right]$ that has/ have a $CI-Co-CI$ angle of 90^0, is / are

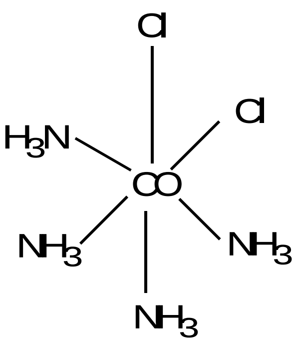
1) meridional and trans
2) trans only
3) cis and trans
4) cis only

KEY: 4

Sol:



Trans=180^0



cis=90^0

31. Amongst the following , the form of water with the lowest ionic conductance at 298K is

1) distilled water 2) sea water

3) water from a well 4) saline water used for intravenous injection

KEY: 1

Sol: Distilled water is free from ions. So it is bad conductor

32. A,B and C are three biomolecules. The results of the tests performed on them are given below

	Molisch's Test	Barfoed Test	Biurent Test
A	Positive	Negative	Negative
B	Positive	Positive	Negative
C	Negative	Negative	Positive

A ,B and C are respectively

1) A=Lactose, B=Fructose, C=Alanine

2) A=Glucose, B=Fructose, C=Albumin

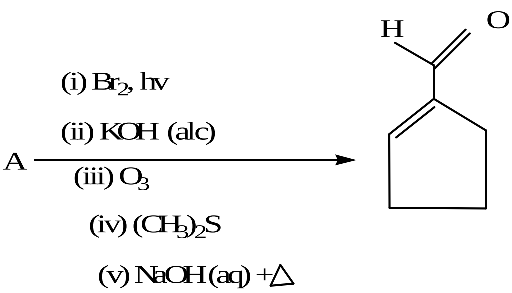
3) A=Lactose, B=Glucose, C=Albumin

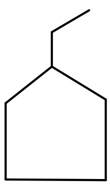
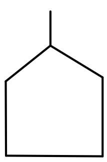
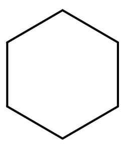
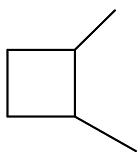
4) A=Lactose, B=Glucose, C=Alanine

KEY: 3

Sol: Barfoed test is given by Monosaccharides. Molisch's test is given by carbohydrates. Biuret test, also known as piotrowski's test, is for detecting peptide bond in proteins

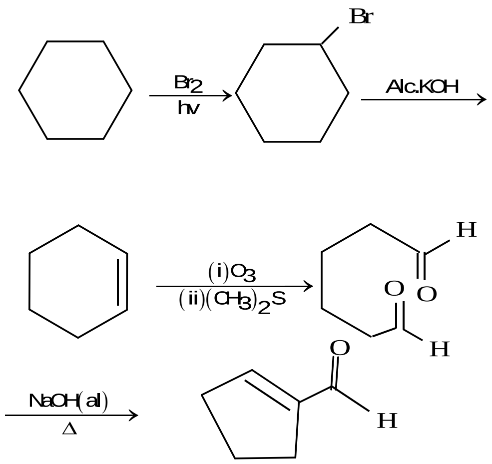
33. In the following reaction A is



1)  2)  3)  4) 

KEY: 3

Sol:



34. 5g of zinc is treated separately with an excess of

a) dilute hydrochloric acid and

b) aqueous sodium hydroxide

The ratio of the volumes of H_2 evolved in these two reactions is

1) 1:1 2) 1:2 3) 2:1 4) 1:4

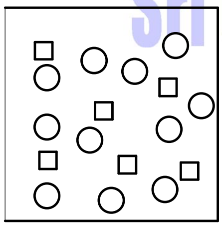
KEY: 1

Sol: $zn + 2HCl \Rightarrow ZnCl_2 + H_2 \uparrow$

$zn + 2NaOH \Rightarrow Na_2ZnO_2 + H_2$

1 Mole of 'Zn" reacts with equal moles of $HCl, NaOH$ separately.No of ratio of $H_2 = 1:1$

35. In the figure shown below reactant A (represented by square) is in equilibrium with product B (represented by circle). The equilibrium constant is



1) 4 2) 2 3) 1 4) 8

KEY: 2

Sol: Equilibrium constant $Kc = \frac{[products]}{[Re\,actions]}$

$\Rightarrow \frac{11}{2} \approx 2$

36. Biochemical Oxygen Demand (BOD) is the amount of oxygen required(in ppm):

1) by anaerobic bacteria to breakdown inorganic waste present in a water body

2) for the photochemical breakdown of waste present in 1 m^3 volume of a water body

3) for sustaining life in a water body

4) by bacteria to break-down organic waste in a certain volume of a water sample

KEY: 4

Sol: Definition of biological oxygen demand $[BOD]$

37. The first and second ionization enthalpies of a metal are 496 and 4560kJ mol^{-1} respectively. How many moles of HCl and H_2SO_4, respectively, will be needed to react completely with 1 mole of the metal hydroxide?

1) 2 and 0.5 2) 1 and 1 3) 1 and 2 4) 1 and 0.5

KEY: 4

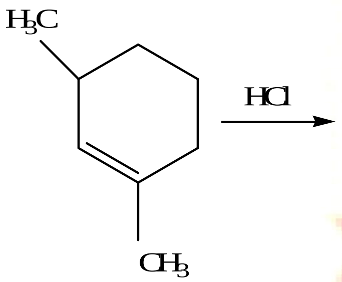
Sol: There is a large gap in 1st and 2nd I.P values indicates Metal contains only one valance electron,So ‘n’ factor for metal is one ‘n’ factor for HCl is one

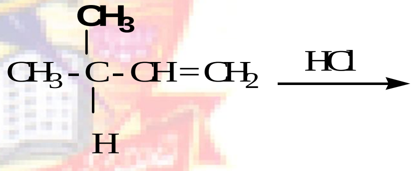
Reaction moles ratio is 1:1

‘n’ factor for H_2SO_4 is ‘2’

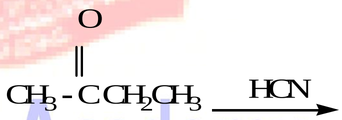
It requires ½ mole only to react metal hydrixide

38. Which of the following reactions will not produce a racemic product?

1) 

2) 

3) $CH_3CH_2CH=CH_2 \xrightarrow{HBr}$

4) 

KEY: 3

Sol: Option ‘1’ gives distereomeric mixture. Option 3 & 4 gives enantiomers. Option ‘2’ gives achiral compound upon rearrangement

39. The solubility product of $Cr(OH)_3$ at $298K$ is 6.0×10^{-31}. The concentration of hydroxide ions in a saturated solution of $Cr(OH)_3$ will be

1) $(4.86\times10^{-29})^{1/4}$ 2) $(18\times10^{-31})^{1/4}$ 3) $(2.22\times10^{-31})^{1/4}$ 4) $(18\times10^{-31})^{1/2}$

KEY: 2

Sol: $Cr(OH)_3 \rightleftharpoons Cr^{+3} + 3(OH^-)$

$(s) \quad (3s)$

$27s^4 = 6\times10^{-31} \qquad (3s)^4 = 18\times10^{-31}$

$(3s) = (18\times10^{-31})^{1/4} \quad [3(OH^-)] = (18\times10^{-31})^{1/4}$

40. Among the statements $(a)-(d)$, the correct ones are

(a) Lithium has the highest hydration enthalphy among the alkali metals
(b) Lithium chloride is insoluble in pyridine
(c) Lithium cannot form ethynide upon its reaction with ethyne
(d) Both lithium and magnesium react slowly with H_2O.

1) (a) and (d) only
2) (a), (b) and (d) only
3) (a), (c) and (d) only
4) (b) and (c) only

KEY: 3

Sol: 'Li' has more covalent character.So it is soluble in pyridine and it has high heat of hydration 'Li' cannot form ethynide with ethyne.
'h' Mg reacts with H_2O gives respective hydroxides.

41. The true statement amongst the following is:

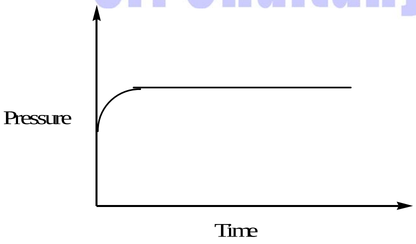
1) Both S and ΔS are not functions of temperature.
2) S is a function of temperature but ΔS is not a function of temperature.
3) Both ΔS and S are functions of temperature.
4) S is not a function of temperature but ΔS is a function of temperature.

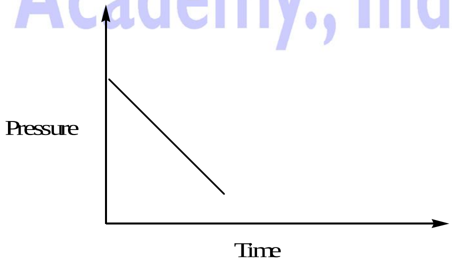
KEY: 3

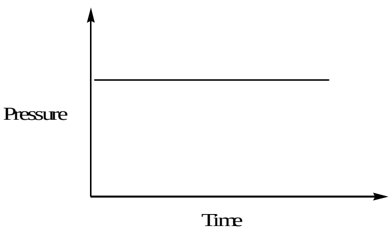
Sol: $\Delta S = \frac{q_{rev}}{T}$ and $\int ds = \int \frac{d\ q_{rev}}{T}$

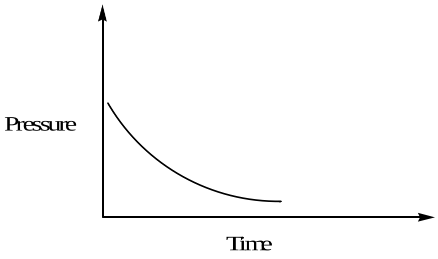
Both S and ΔS are functions of temperature

42. A mixture of gases O_2, H_2 and CO are taken in a closed vessel containing charcoal. The graph that represents the correct behavior of pressure with time is :

1) 

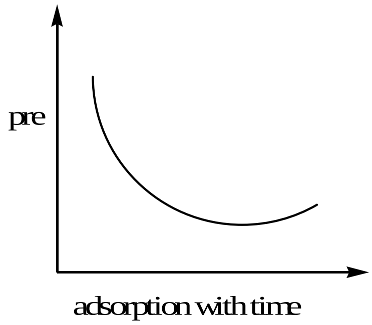
2) 

3) 

4) 

KEY: 4

Sol: Adsorption of gases over activated charcoal decreases with pressure exponentially.



43. Which polymer has 'chiral' monomer (s) ?

1) Nylon 6,6 2) Buna-N 3) PHBV 4) Neoprene

KEY: 3

Sol: Monomer's of Nylon-6,6 are adipic acid & hexa methylene diamine, both are achiral. Monomers of Buna-N are 1,3 butadiene and acrylonitrile,both are achiral. Monomer's of PHBV are β-hydroxy butanoic acid & β-hydroxy valeric acid, both carry chiral centres. Monomers of Neoprene is chloroprene which is achiral

44. The correct order of the spin-only magnetic moments of the following complexes is:

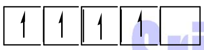
I) $\left[Cr(H_2O)_6\right]Br_2$ II) $Na_4\left[Fe(CN)_6\right]$ III) $Na_3\left[Fe(C_2O_4)_3\right](\Delta_0 > P)$ IV) $(Et_4N)_2[CoCl_4]$

1) $(III)>(I)>(II)>(IV)$ 2) $(III)>(I)>(IV)>(II)$

3) $(I)>(IV)>(III)>(II)$ 4) $(II)\approx(I)>(IV)>(III)$

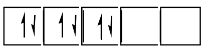
KEY: 4

Sol: $Cr^{+2} = 4s^0 3d^4$



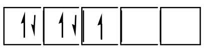
$\mu = 4.88BM$

$Fe^{+2} = 4s^0 3d^6$



$\mu = 0BM$

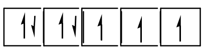
$Fe^{+3} = 4s^0 3d^5$



No of unpaired $e^- = 01$

$\mu = 1.7BM$

$CO^{+2} = 4s^0 3d^7$



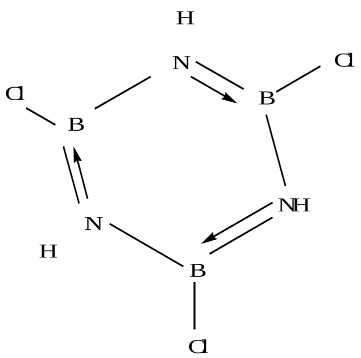
No of unpaired $e^- = 03$

$\mu = 3.8BM$

45. The reaction of $H_3N_3B_3Cl_3\,(A)$ with $LiBH_4$ in tetrahydrofuran gives inorganic benzene (B). Further, the reaction of (A) with (C) leads to $H_3N_3B_3(Me)_3$. Compounds (B) and (C) respectively are:

1) Borazine and $MeMgBr$
2) Boron nitride and $MeBr$
3) Borazine and $MeBr$
4) Diborane and $MeMgBr$

KEY: 1



Sol:

$$2B_3N_3H_3Cl_3 + 6LiBH_4 \rightarrow 2B_3N_3H_6 + 3B_2H_6 + 6LiCl$$

$$B_3N_3H_3Cl_3 + 3CH_3MgBr \rightarrow B_3N_3H_3\left(CH_3\right)_3 + 3MgBrCl$$

Both are nucleiophilic displacement reactions

(NUMERICAL VALUE TYPE)

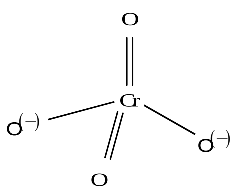
This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value(in decimal notation, truncated/rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer , 0 if not attempted and 0 in all other cases.

46. The sum of the total number of bonds between chromium and oxygen atoms in chromate and dichromate ions is ________

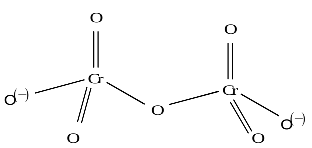
KEY: 12.00

Sol: Chromate CrO_4^{2-}



No of Cr-O bonds are '6'

Dichromate $Cr_2O_7^{2-}$



No of Cr-O bonds is 12

Total 6+12=18

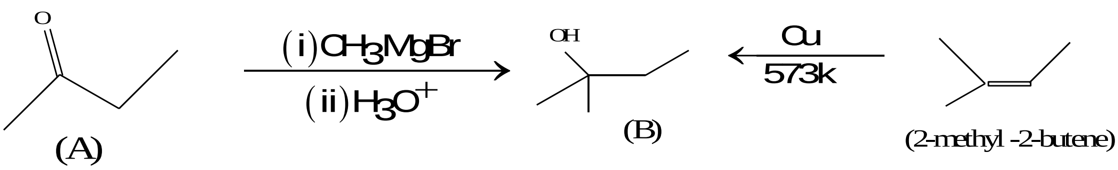
47. Consider the following reactions

$A \xrightarrow[(ii)H_3O^+]{(i)CH_3MgBr} B \xrightarrow[573K]{Cu}$ 2-methyl- 2- butane

The mass percentage of carbon in A is ___________

KEY: 66.65 to 66.70

Sol:



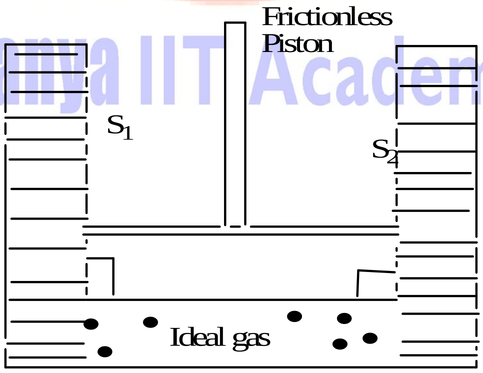
Mol formula of ‘A’ is C_4H_8O

Mol wt of ‘A ‘ is

$(4\times12)+(8\times1)+(1\times16) \quad =48+8+16=72$

$\%of \ \ 'C' = \frac{48}{72}\times100 = 66.66\%$

48. A cylinder containing an ideal gas (0.1 mol of 1.0 dm^3) is in thermal equilibrium with a large volume of 0.5 molal aqueous solution of ethylene glycol at its freezing point. If the stoppers S_1 and S_2 (as shown in the figure) are suddenly withdrawn, the volume of the gas in litres equilibrium is achieved will be________ (Given, $K_f(water)=2.0\,K\,kg\,mol^{-1} R=0.08\,dm^3 atm\,K^{-1}mol^{-1}$)



KEY: 2.17 to 2.23

Sol: For an ideal gas

$PV=nRT$

$P=n/V\times RT$

$=0.1\times0.08\lambda T$

But $\Delta Tf = kf\times m\times i$

$\Delta Tf = 2\times0.5\times1 \qquad =1$

$273 - Ts = 1$

$Ts = 272$

$P = 0.1 \times 0.08 \times 272 \Rightarrow 2.176$

At eqilibrium $P_1V_1 = PV_2$

$2.176 \times 1 = 1 \times V_2$

$V_2 = 2.176 lit$

49. A sample of milk splits after 60 min. at 300 K and after 40 min. At 400 K when the population of lactobacillus acodophilus in it doubles. The activation energy (in KJ/mol) for this process is closest to________ $\left(Given, R = 8.3 mol^{-1}K^{-1}, \ln\left(\frac{2}{3}\right) = 0.4, e^{-3} = 4.0\right)$

KEY: 3.94 to 4.00 or -3.94 to -4.00

Sol: $\log_{10}\left(\frac{t_2}{t_1}\right) = \frac{Ea}{2.303R} \times \left[\frac{1}{T_1} + \frac{1}{T_2}\right]$

$\log 2 = \frac{Ea}{8.3 \times 2.303}\left[\frac{1}{300} - \frac{1}{400}\right]$

$0.3010 = \frac{Ea}{19.11} \times \left[\frac{100}{300 \times 400}\right]$

$0.17 = \frac{Ea}{2.303} \times \left[\frac{100}{400 \times 300}\right]$

$Ea = 0.3194 KJ$

50. 10.30 mg of O_2 is dissolved into a liter of sea water of density 1.03 g/ml. The concentration of O_2 in ppm is ___________

KEY: 10.00

Sol: Wt of O_2

$\Rightarrow 10.3 \times 10^{-3} gr / lit$

$10.3 gr / 10^6 ml$

$Conductivity = \frac{wt\ of\ CO_2}{Wt\ of\ soln} \times 10^6$

$\Rightarrow \frac{10.3}{\left(1.03 \times 10^6\right)} \times 10^6 = 10$

MATHEMATICS

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.
Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

51. Given : $f(x)=\begin{cases} x & ,0\le x<\frac{1}{2} \\ \frac{1}{2} & ,x=\frac{1}{2} \\ 1-x, & \frac{1}{2}<x\le 1 \end{cases}$ and $g(x)=\left(x-\frac{1}{2}\right)^2, x\in R.$ Then the area (in sq. units) of the region bounded by the curves, $y=f(x)$ and $y=g(x)$ between the lines, 2x=1 and $2x=\sqrt{3}$, is:

1) $\frac{1}{2}+\frac{\sqrt{3}}{4}$ 2) $\frac{1}{2}-\frac{\sqrt{3}}{4}$ 3) $\frac{\sqrt{3}}{4}-\frac{3}{4}$ 4) $\frac{1}{3}+\frac{\sqrt{3}}{4}$

KEY: 3

Sol: If $f(x)=\begin{cases} x & 0<x<\frac{1}{2} \\ \frac{1}{2} & x=\frac{1}{2} \\ 1-x & \frac{1}{2}<x<1 \end{cases}$

$g(x)=\left(x-\frac{1}{2}\right)^2$

$$\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}\left[(1-x)-\left(x-\frac{1}{2}\right)^2\right]dx=\frac{\sqrt{3}}{4}-\frac{1}{3}$$

(or)

area of prepizium - $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}\left(x-\frac{1}{2}\right)^2 dx$

$$\frac{1}{2}\left(\frac{1}{2}+1-\frac{\sqrt{3}}{2}\right)\times\left(\frac{\sqrt{3}-1}{2}\right)-\left[\frac{\left(x-\frac{1}{2}\right)^3}{3}\right]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}=\frac{\sqrt{3}}{4}-\frac{1}{3}$$

52. A random variable X has the following probability distribution :

X :	1	2	3	4	5
$P(X)$:	K^2	$2K$	K	$2K$	$5K^2$

Then $P(X>2)$ is equal to

1) $\frac{1}{36}$ 2) $\frac{7}{12}$ 3) $\frac{23}{36}$ 4) $\frac{1}{6}$

KEY: 3

Sol: $6k^2+5k=1$

$k=\frac{1}{6}$ or $k=-1$ $k=-1$ not possible

$p(x>2)=p(x=3)+p(x=4)+p(x=5)$

$k+2k+5k^2=3k+5k^2$ $\frac{23}{36}$

53. If $\int\frac{d\theta}{\cos^2\theta(\tan 2\theta+\sec 2\theta)}=$ $\lambda\tan\theta+2\log_e|f(\theta)|+C$ where C is a constant of integration, then the ordered pair $(\lambda, f(\theta))$ is equal to:

1) $(-1,1+\tan\theta)$ 2) $(1,1+\tan\theta)$ 3) $(1,1-\tan\theta)$ 4) $(-1,1-\tan\theta)$

KEY: 1

Sol: $\int\frac{d\theta}{\cos^2\theta(Tan2\theta+\sec 2\theta)}=\lambda\tan\theta+2\log|f(0)|+C$ then $(\lambda, f(\theta))$

$\int\frac{\sec^2\theta d\theta}{\left(\frac{1+\tan^2\theta}{1-\tan^2\theta}+\frac{2\tan\theta}{1-\tan^2\theta}\right)}=$

Put $Tan\theta=t$

$\int\frac{(1-t^2)dt}{1+t^2+2t}=\int\frac{(1+t)(1-t)dt}{(1+t)^2}$

$\int\frac{1-t}{1+t}dt$ =

$-\int\left(\frac{t+1-2}{(t+1)}\right)dt=-\int 1dt+2\int\frac{dt}{t+1}$

$-t+2\log(t+1)$

$-\tan\theta+2\log|Tan\theta+1|=\lambda\tan\theta+2\log|f(\theta)|$

$\lambda=-1, f(\theta)=1+\tan\theta$

$(-1,1+\tan\theta)$

54. Let a_n be the n^{th} term of a G.P of positive terms. If $\sum_{n=1}^{100} a_{2n+1} = 200$ and $\sum_{n=1}^{100} a_{2n} = 100$, then $\sum_{n=1}^{200} a_{2n}$ is equal to:

1) 175 2) 225 3) 150 4) 300

KEY: 3

Sol: $a_3 + a_5 .... a_{201} = 200 \quad a_2 + a_4 .... a_{200} = 100$

$ar^2 + ar^4 .... ar^{200} = 200, \; ar + ar^3 .... ar^{199} = 100$

$\frac{ar^2(r^{200}-1)}{r^2-1} = 200 \_\_\_\_ (1) \quad \frac{ar(r^{200}-1)}{r^2-1} = 100 \_\_\_\_ (2)$

$(1) \div (2)$

$$\frac{\frac{ar^2(r^{200}-1)}{r^2-1}}{\frac{ar(r^{200}-1)}{r^2-1}} = 2$$

$r = 2$

$(1) + (2)$

$a_2 + a_3 + a_4 .... + a_{201} = 300$

$ar + ar^2 + ar^3 ..... ar^{200} = 300$

$r(a + ar + ar^2 ..... ar^{199}) = 300$

$r(a_1 + a_2 + a_3 .... a_{200}) = 300$

$\sum_{r=1}^{200} a_r = \frac{300}{r} = \frac{300}{2} = 150$

55. If z be a complex number satisfying $|\mathrm{Re}(z)| + |\mathrm{Im}(z)| = 4$, then $|z|$ cannot be:

1) $\sqrt{7}$ 2) $\sqrt{8}$ 3) $\sqrt{10}$ 4) $\sqrt{\frac{17}{2}}$

KEY: 1

Sol: Let $z = x + iy$

$|\mathrm{Re}\, z| + |\mathrm{Im}\, z| = 4$

$|x| + |y| = 4$

$|z| = \sqrt{x^2 + y^2}$

$\sqrt{8} \le |z| \le 4$

$|z| = \sqrt{7}$ is not possible

56. Let $[t]$ denote the greatest integer $\le t$ and $\lim_{x\to 0} x\left[\frac{4}{x}\right] = A$. Then the function,

$f(x) = [x^2]\sin(\pi x)$ is discontinuous, when x is equal to :

1) $\sqrt{A+5}$ 2) $\sqrt{A+21}$ 3) $\sqrt{A+1}$ 4) $\sqrt{A}$

KEY: 3

Sol: $\frac{4}{x} - 1 < \left[\frac{4}{x}\right] \le \frac{4}{x}$

$x > 0\ x\left(\frac{4}{x} - 1\right) < x\left[\frac{4}{x}\right] \le x.\frac{4}{x}$

$\underset{x\to 0}{lt} + 4 - x \le \underset{x\to 0}{lt}\ x\left[\frac{4}{x}\right] \le \underset{x\to 0}{lt}\ 4$

$\underset{x\to 0}{lt}\ x\left[\frac{4}{x}\right] = 4$

We can prove $\underset{x\to 0}{lt}\ x\left[\frac{4}{x}\right] = 4$

$\underset{x\to 0}{lt}\ x\left[\frac{4}{x}\right] = 4$

$f(x) = [x^2]\sin(\pi\, x)$ check the

At i) $\sqrt{A+5} = \sqrt{9} = 3$

ii) $\sqrt{A+21} = 5$

iii) $\sqrt{A+1} = \sqrt{5}$

iv) $\sqrt{A} = 2$

$\sin \pi x = 0$

when

x= $3, 5, 2$

$\underset{x+\sqrt{5}}{lt}[x^2]\sin \pi x$

$\underset{x+\sqrt{5}}{lt} 5\sin \pi x = 5.\sin \pi\sqrt{5}$

$\underset{x+\sqrt{5}}{lt} 4\sin \pi x = 4.\sin \pi\sqrt{5}$

$L.H.L \ne R.H.L$

$x = \sqrt{5}$

57. If $p \to (p \wedge \sim q)$ is false, then the truth values of p and q are respectively:

1) F,T 2) T,T 3) F,F 4) T,F

KEY: 2

Sol:

p	q	~q	$p\wedge\sim q$	$p\to(p\wedge\sim q)$
T	F	T	T	T
F	T	F	F	T
T	T	F	F	F
F	F	T	F	T

58. Let a,b $\in R, a\neq 0$ be such that the equation, $ax^2-2bx+5=0$ has a repeated root α , which is also a root of the equation, $x^2-2bx-10=0$. If β is the other root of this equation, then $\alpha^2+\beta^2$ is equal to:

1) 24 2) 26 3) 25 4) 28

KEY: 3

Sol: $ax^2-2bx+5=0\, roots\, \alpha,\alpha$

$2\alpha=\frac{2b}{a}$, $\alpha^2=\frac{5}{a}$ $\quad x^2-2bx-10=0$

$\alpha=\frac{b}{a}, \frac{b^2}{a^2}=\frac{5}{a}$ $\quad roots\, \alpha,\beta \quad \alpha+\beta=2b$

$b^2 5a$ $\quad \alpha\beta=-10$

α is a root of $x^2-2bx-10=0$

$\alpha^2-2b\alpha-10=0$

$\frac{b^2}{a^2}-2b\frac{b}{a}-10=0$

$\frac{b^2}{a^2}-\frac{2b^2}{a}-10=0$

$\frac{5}{a}-10-10=0 \; a=\frac{1}{4}$

$b^2=5.a \quad =b^2=\frac{5}{4}$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=4b^2+20$

$4.\frac{5}{4}+20=25$

59. Let f and g be differentiable functions on R such that fog is the identity function. If for some a,b$\in R, g'(a)=5$ and $g(a)=b$ then $f'(b)$ is equal to :

1) $\frac{1}{5}$ 2) $\frac{2}{5}$ 3) 5 4) 1

KEY: 1

Sol: $(fog)(x)=I(x) \;\; I(x)=x$

$(fog)(x)=x$

$f(g(x))=x$

$f^1(g(x)),g^1(x)=1$

$f^1(g(b)),g^1(b)=1$

$f^1(a)=\frac{1}{g^1(b)}=\frac{1}{5}$

60. If $x=2\sin\theta-\sin 2\theta$ *and* $y=2\cos\theta-\cos 2\theta$, $\theta\in[0,2\pi]$, *then* $\frac{d^2y}{dx^2}$ *at* $\theta=\pi$ is:

1) $\frac{3}{2}$ 2) $-\frac{3}{4}$ 3) $-\frac{3}{8}$ 4) $\frac{3}{4}$

KEY: ADD

Sol: $x=2\sin\theta-\sin 2\theta$ $y=2\cos\theta-\cos 2\theta$

$\frac{dx}{d\theta}=2\cos\theta-2\cos 2\theta$ $\frac{dy}{d\theta}=-2\sin\theta+2\sin 2\theta$

$\frac{dy}{dx}=\frac{-2\sin\theta+2\sin 2\theta}{2\cos\theta-2\cos 2\theta}=\frac{-\sin\theta+\sin 2\theta}{\cos\theta-\cos 2\theta}$

$\frac{d^2y}{dx^2}=\frac{(\cos\theta-\cos 2\theta)(-\cos\theta+2\cos 2\theta)-(\sin 2\theta-\sin\theta)(-\sin\theta+2\sin 2\theta)}{(\cos\theta-\cos 2\theta)^2}\times\frac{d\theta}{dx}$

$\frac{d^2y}{dx^2(\theta=\pi)}=\frac{(-1-1)(1+2)-(0)}{(-1-1)^2}\times\left(\frac{-1}{4}\right)=\frac{3}{8}$

61. Let a function $f:[0,5]\to R$ be continuous, $f(1)=3$ and F be defined as:

$F(x)=\int_1^x t^2g(t)\,dt$, where $g(t)=\int_1^t f(u)du$ Then for the function F, the point x=1 is:

1) a point of inflection
2) a point of local minima
3) not a critical point
4) a point of local maxima

KEY: 2

Sol: $F^{'}(x)=x^2g(x)$

$f^{''}(x)=2xg(x)+x^2+g^{'}(x)$

$f^{''}(0)=0$

$f^{''}(1)=2g(1)+g^{'}(1)=3>0$

$f^{'}(1)=0$ $f^{''}(1)>0$

Local minima at x=1

max or min $f^{'}(x)=0$

$x^2g(x)=0$

$x=0,\ g(x)=0$

$g(1)=0$

62. If 10 different balls are to be placed in 4 distinct boxes at random, then the probability that two of these boxes contain exactly 2 and 3n ball is :

1) $\frac{965}{2^{11}}$ 2) $\frac{965}{2^{10}}$ 3) $\frac{945}{2^{11}}$ 4) $\frac{945}{2^{10}}$

KEY: 4

Sol:

B_1	B_2	B_3	B_4
2	3	0	5
2	3	1	4
2	3	2	3

$10_{C_\alpha}\times 8_{C_3}\times 5_{C_5}\times 4!+10_{C_2}\times 8_{C_3}\times 5_{C_1}\times 4!+10_{C_2}\times 8_{C_3}\times 5_{C_2}\times \frac{4!}{2!2!}$

$\frac{10_{C_\alpha}\times 8_{C_3}\times 4!\times 17}{2}=2^5\times 3^3\times 5\times 7\times 17$

Probability $=\frac{2^5\times 3^3\times 5\times 7\times 17}{4^{10}}=\frac{3^3\times 5\times 7\times 17}{2^{15}}$

63. The length of the minor axis $(along\ y-axis)$ of an ellipse in the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the line, $x+6y=8$, then its eccentricity is :

1) $\frac{1}{2}\sqrt{\frac{5}{3}}$ 2) $\frac{1}{3}\sqrt{\frac{11}{3}}$ 3) $\sqrt{\frac{5}{6}}$ 4) $\frac{1}{2}\sqrt{\frac{11}{3}}$

KEY: 4

Sol: $\frac{x^2}{a^2}+\frac{3y^2}{4}=1$

$y=mx\pm\sqrt{a^2m^2+b^2}$

$y=\frac{-x}{6}+\frac{8}{6}$

$a^2=16, b^2=\frac{4}{3}$

$m=-\frac{1}{6}$

$a^2m^2+b^2-\frac{16}{9}$

$\frac{a^2}{36}+\frac{4}{3}=\frac{16}{9}$

$a^2=16,\quad b^2=\frac{4}{3}$

$e=\sqrt{\frac{a^2-b^2}{a^2}}=\sqrt{\frac{11}{12}}$

64. If $\frac{dy}{dx}=\frac{xy}{x^2+y^2}; y(1)=1$, then a value of x satisfying y(x)=e is:

1) $\sqrt{2}e$ 2) $\frac{1}{2}\sqrt{3}e$ 3) $\sqrt{3}e$ 4) $\frac{e}{\sqrt{2}}$

KEY: 3

Sol: $\frac{dx}{dy}=\frac{x^2+y^2}{xy}$ put $x=vy$

$\frac{dy}{dx}=v+y\frac{dv}{dy}$

$V+y\frac{dv}{dy}=\frac{v^2+1}{v}$

$V+y\frac{dv}{dy}=V+\frac{1}{v}$

$\int Vdv=\int\frac{dy}{y}$

$\frac{V^2}{2}=\log y+\log C$

$x=1, y=1$

$\frac{1}{2}=\log C$

$\frac{x^2}{2y^2}=\log y+\frac{1}{2}$

$y=e \quad \frac{x^2}{2e^2}=1+\frac{1}{2}$

$\frac{x^2}{2e^2}=\frac{3}{2}$

$x=e\sqrt{3}$

65. If $A=\{x\in R:|x|<2\}$ and

$B=\{x\in R:|x-2|\geq 3\}$; then

1) $A\cap B=(-2,-1)$ 2) $A-B=(-1,2)$ 3) $A\cup B=R-(2,5)$ 4) $B-A=R-(-2,5)$

KEY: 4

Sol: $|x|<2$ $\quad |x-2|\geq 3$

$-2<x<2$ $\quad -3\leq x-2\leq 3$

$A\in(-2,2)$ $\quad -1\leq x\leq 5$

$B\in[-1,5]$

66. Let $a-2b+c=1$

If $f(x)=\begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$ then:

1) $f(50)=1$ 2) $f(-50)=-1$ 3) $f(50)=-501$ 4) $f(-50)=501$

KEY: 1

Sol: $f(x)=\begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$ $\& \, a-2b+C=1$

$R_1 \to R_1-2R_2+R_3$

$f(x)=\begin{vmatrix} a-2b+c & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$

$(x+3)^2-(x+4)(x+2)$

$x^2+6x+9-(x^2+6x+8)=1$ $f(x)=1$

$f(50)=1$

67. The following system of linear equations

$7x+6y-2z=0$

$3x+4y+2z=0$ has

$x-2y-6z=0$

1) no solution

2)infinitely many solutions (x,y,z)satisfying x=2z

3) infinitely many solutions (x,y,z)satisfying y=2z

4) Only the trivial solution.

KEY: 2

Sol: $\begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix}$

$7(-24+4)-6(-18-2)-2(-6-4)$

$-140+120+20=-140+140$

=0

Infinitely many solutions

Let z=k

$7x+6y-2k=0$

$3x+4y+2k=0$

$x-2y-6k=0$

$x+2k=6k$

$3x=6k$

$x=2k$ $\qquad y=-2k$ $\qquad y=-2z$

68. In the expansion of $\left(\frac{x}{\cos\theta}+\frac{1}{x\sin\theta}\right)^{16}$, if l_1 is the least value of the term independent of x when $\frac{\pi}{8}\le\theta\le\frac{\pi}{4}$ and l_2 is the least value of the term independent of x when $\frac{\pi}{16}\le\theta\le\frac{\pi}{8}$, then the ratio $l_2:l_1$ is equal to:

1) 1:16   2) 1:8   3) 8:1   4) 16:1

KEY: 4

Sol: Independent term $\qquad r=\frac{16(1)-0}{1+1}=8$

$^{16}C_8\left(\frac{1}{\cos\theta}\right)^8\left(\frac{1}{\sin\theta}\right)^8 \qquad {}^{16}C_8\frac{1}{(\sin\theta\cos\theta)^8}={}^{16}C_8\frac{2^8}{(\sin 2\theta)^8}$

$L_1=least\left({}^{16}C_8\frac{2^8}{\sin^8 2\theta}\right)=\frac{{}^{16}C_8 2^8}{\max(\sin 2\theta)^8}\qquad\frac{{}^{16}C_8.2^8}{1}$

$L_2=least\,value\left({}^{16}C_8\frac{2^8}{(\sin 2\theta)^8}\right)\qquad {}^{16}C_8\frac{2^8}{\max(\sin 2\theta)^8}$

$^{16}C_8\frac{2^8}{\left(\sin 2\left(\frac{\pi}{8}\right)\right)^8}={}^{16}C_8\frac{2^8}{\left(\frac{1}{\sqrt{2}}\right)^8}={}^{16}C_8.2^4.2^8 \qquad \frac{{}^{16}C_8.2^8.2^4}{{}^{16}C_8.2^8}=2^4=16:1$

69. If $x=\sum_{n=0}^{\infty}(-1)^n\tan^{2n}\theta$ and $\sum_{n=0}^{\infty}\cos^{2n}\theta$ for $0<\theta<\frac{\pi}{4}$, then:

1) $x(1+y)=1$ 2) $y(1+x)=1$ 3) $x(1-y)=1$ 4) $y(1-x)=1$

KEY: 4

Sol: $x=1-(\tan\theta)^2+(\tan\theta)^4.....\infty \qquad x=\frac{1}{1+\tan^2\theta}\,x=\frac{1}{\sec^2\theta}=\cos^2\theta$

$y=(\cos\theta)^0+(\cos\theta)^2+(\cos\theta)^4.....\infty \qquad y=\frac{1}{1-\cos^2\theta}=\frac{1}{\sin^2\theta}$

$\frac{1}{y}=\sin^2\theta \qquad \cos^2\theta+\sin^2\theta=1$

$x+\frac{1}{y}=1 \qquad xy+1=y \qquad xy-y=-1 \qquad y(1-x)=1$

70. If one end of a focal chord AB of the parabola $y^2=8x$ is at $A\left(\frac{1}{2}-2\right)$, then the equation of the tangent to it at B is :

1) $x+2y+8=0$ 2) $2x+y-24=0$ 3) $x-y+8=0$ 4) $2x-y-24=0$

KEY: 3

Sol: $\left(at_1^2\, 2at_1\right)=\left(\frac{1}{2}1-2\right)$

$4a=8\;\; a=2\;\; 2at_1=-2 \qquad t_1=-\frac{1}{2} \qquad t_1t_2=-1 \qquad t_2=2$

$\left(at_1^2,2at_2\right)=(8,8) \qquad y^2=8x \qquad 2y\frac{dy}{dx}=8 \qquad \frac{dy}{dx}=\frac{8}{2y}$

$\frac{dy}{dx}(8,8)=\frac{8}{2(8)}=\frac{1}{2} \quad (y-8)=\frac{1}{2}(x-8) \quad 2y-16=x-8 \quad -8=x-2y \quad x-2y+8=0$

(NUMERICAL VALUE TYPE)

This section contains 5 questions. Each question is numerical value. For each question, enter the correct numerical value(in decimal notation, truncated/rounded-off to second decimal place.(e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

Marking scheme: +4 for correct answer , 0 if not attempted and 0 in all other cases.

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71. If the curves, $x^2-6x+y^2+8=0$ and $x^2-8y+y^2+16-k=0,(k>0)$ touch each other at a point, then the largest value of k is __________

KEY: 36.00

Sol: $C_1C_2=5 \quad r_1=\sqrt{k} \quad r_2=1 \qquad C_1C_2=r_1+r_2 \qquad C_1C_2=|r_1-r_2|$

$5-\sqrt{k}+1 \quad 5=\left|\sqrt{k}-1\right| \qquad 5-1=\sqrt{k} \qquad \sqrt{k}=\pm5+1$

$\sqrt{k}=4 \qquad \sqrt{k}=5+1 \qquad k=16 \quad k-36 \qquad$ Max value of k =36

72. Let $\vec{a},\vec{b}$ *and* $\vec{c}$ be three vectors such that $\left|\vec{a}\right|=\sqrt{3},\left|\vec{b}\right|=5,\vec{b}.\vec{c}=10$ and the angle between $\vec{b}$ *and* $\vec{c}$ *is* $\frac{\pi}{3}$. If $\vec{a}$ is perpendicular to the vector $\vec{b}\times\vec{c}$, then $\left|\vec{a}\times\left(\vec{b}\times\vec{c}\right)\right|$ is equal to __________

KEY: 30.00

Sol: $|a|=\sqrt{3},\left|\bar{b}\right|=5 \qquad \bar{b}.\bar{c}=10$

$\left|\bar{b}\right|\left|\bar{c}\right|\cos\left(\bar{b}\,\bar{c}\right)=10 \qquad 5.\left|\bar{c}\right|\cos\frac{\pi}{3}=10 \qquad \left|\bar{c}\right|=4$

$\left(\bar{a},\bar{b}\times\bar{c}\right)=90^0 \qquad \left|\bar{a}\times\left(\bar{b}\times\bar{c}\right)\right|=\left|\bar{a}\right|\left|\bar{b}\times\bar{c}\right|\sin\left(\bar{a},\bar{b}\times\bar{c}\right) \qquad \sqrt{3}.5.4\sin\frac{\pi}{3}.\sin\left(\frac{\pi}{2}\right)$

$\sqrt{3}.5.4.\frac{\sqrt{3}}{2}$ =30

73. The number of terms common to the two A.P's 3,7,11,….,407 and 2,9,16,….,709 is __________

KEY: 14.00

Sol: $3+(n-1)4=t_n \qquad t_m=2+(m-1)7$

$t_n=4n-1 \qquad t_m=7m-5 \qquad 4n-1=7m-5 \qquad n=\frac{7m-4}{4} \qquad$ Put $m=-4,8$

$n=6,13 \qquad T_6=3+5(4)=23 \qquad T_{13}=3+(13-1)(4)=3+48=51$

$23+(n-1)28\le 407 \qquad 23+28n-28\le 407 \qquad 28n\le 407+5$

$n\le 14.8 \qquad n=14$

74. If $C_r={}^{25}C_r$ and

$C_0+5.C_1+9.C_2+....+(101).C_{25}=2^{25}.K$ then K is equal to________

KEY: 51.00

Sol: $c_o+5.C_1+9.C_2....101C_{25} \qquad \sum_{r=0}^{n=25}(4r+1)C_r$

$4\sum_{r=0}^{n}rC_r+\sum_{r=0}^{n}C_r \qquad 4.n2^{n-1}+2^n \qquad 2^n(2n+1)$

$2^{25}(51) \qquad k=51$

75. If the distance between the plane $23x-10y-2z+48=0$ and the plane containing the lines $\frac{x+1}{2}=\frac{y-3}{4}=\frac{z+1}{3}$ and $\frac{x+3}{2}=\frac{y+2}{6}=\frac{z-1}{\lambda}(\lambda\in R)$ is equal to $\frac{k}{\sqrt{633}}$, then k is equal to ________

KEY: 3.00

Sol: $\begin{vmatrix} x+1 & y-3 & z+1 \\ 2 & 4 & 3 \\ -2 & -5 & 2 \end{vmatrix}=0$

$23x-10y-27+51=0$

$23x-10y-27+48=0$

$\frac{(51-48)}{(23^2)+10^2+2^2}=\frac{k}{\sqrt{633}}$

$3=k$

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