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JEE Advanced 2021

Question Paper 2

EXAM TIME : 2.30 AM TO 5.30 PM

Key & Solutions

304, Kasetty Heights, Ayyappa Society, Madhapur, Hyderabad - 500081

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17	V.V.KARTHIKEYA SAI	210310313498
28	P.CHETHAN MANOGNA SAI	210310065447
39	S.HARSHA VARMA	210310253154
47	TERLI TULSI RAM	210310460065
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62	CHAITANYA AGGARWAL	210310103371
63	A.V JAYA CHAITANYA	210310381513
65	MIDATHANA PRANAY	210310130463
69	CH.MANASWITA REDDY	210310176015
74	G SAI PRADHYUMNA	210310195935
75	DHRUV GARG	210310194137
79	SAVARAM DIVAKAR SAI	210310194862
82	BHOGI SAI SATHWIK	210310221202
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PHYSICS

Max. Marks: 60

SECTION-1(Maximum Marks: 24)

One or More Type

- This section contains SIX (06) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s)
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks : 0 If unanswered;
Negative Marks : −2 In all other cases.

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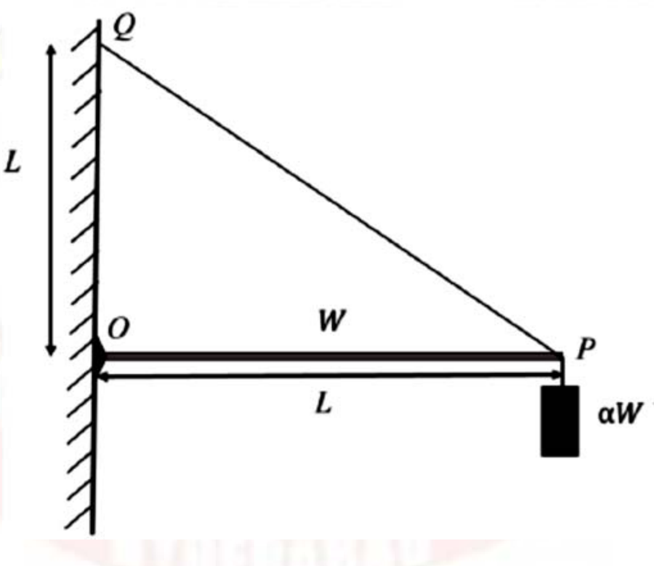
1. One end of a horizontal uniform beam of weight W and length L is hinged on a vertical wall at point O and its other end is supported by a light inextensible rope. The other end of the rope is fixed at point Q, at a height L above the hinge at point O. A block of weight αW is attached at the point P of the beam, as shown in the figure (not to scale). The rope can sustain a maximum tension of $\left(2\sqrt{2}\right)W$. Which of the following statement(s) is(are) correct?



A) The vertical component of reaction force at O does **not** depend on α

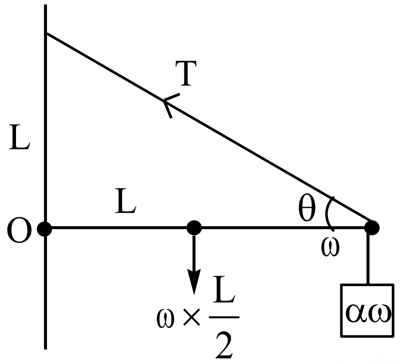
B) The horizontal component of reaction force at O is equal to W for $\alpha = 0.5$

C) The tension in the rope is 2W for $\alpha = 0.5$

D) The rope breaks if $\alpha > 1.5$

Ans. ABD

Sol.



$$\frac{T}{\sqrt{2}}L = \omega\frac{L}{2} + \alpha\omega L$$

$$T = \sqrt{2}\omega\left(\alpha + \frac{1}{2}\right). \text{For } \alpha = \frac{1}{2} T = \omega\sqrt{2}$$

$$\frac{T}{\sqrt{2}} + R_Y = \omega + \alpha\omega \Rightarrow R_Y = \frac{\omega}{2}$$

$$R_x = \frac{T}{\sqrt{2}} = \omega\left(\alpha + \frac{1}{2}\right)$$

For $\alpha = \frac{1}{2} \Rightarrow R_X = \omega$

For $\alpha > 1.5 \Rightarrow T > 2\sqrt{2}\omega$

2. A source, approaching with speed u towards the open end of a stationary pipe of length L, is emitting a sound of frequency f_S. The farther end of the pipe is closed. The speed of sound in air is υ and f_0 is the fundamental frequency of the pipe. For which of the following combination(s) of u and f_S, will the sound reaching the pipe lead to a resonance ?

A) $u = 0.8\upsilon$ and $f_S = f_0$

B) $u = 0.8\upsilon$ and $f_S = 2f_0$

C) $u = 0.8\upsilon$ and $f_S = 0.5f_0$

D) $u = 0.5\upsilon$ and $f_S = 1.5f_0$

Ans. **AD**

Sol. $f_0 = \frac{V}{\lambda}$ and $\lambda = 4L \Rightarrow f_0 = \frac{V}{4L}$

$$\lambda_{app} = \frac{v-u}{f_s} . \lambda_{res} = \frac{V}{(n_{odd})f_0}$$

for resonance ,

$$\lambda_{app} = \lambda \Rightarrow \lambda_{res} = \frac{V-u}{f_s} = 4L$$

But $u = 0.8v \Rightarrow \lambda_{app} = \frac{0.2V}{f_S} = \frac{V}{5f_S}$

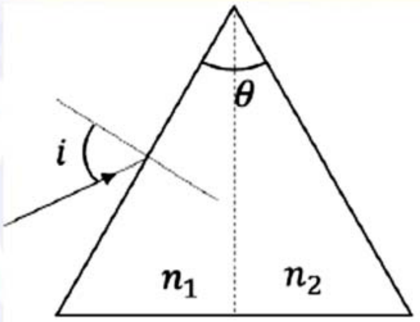
For $f_S = f_0 \Rightarrow \lambda_{app} = \frac{V}{5f_0} = \lambda_{res}$

For $f_s = \frac{f_0}{2} \Rightarrow \lambda_{app} = \frac{V}{10f_0} \neq \lambda_{res}$

For $f_s = \frac{f_0}{2} \Rightarrow \lambda_{app} = \frac{2V}{5f_s} \neq \lambda_{res}$

For $u = \frac{V}{2}$ and $f_s = \frac{3f_0}{2}$ $\Rightarrow$ $\lambda_{app} = \frac{V}{2f_s} = \frac{V}{3f_0} = \lambda_{res}$

3. For a prism of prism angle $\theta = 60^0$, the refractive indices of the left half and the right half are, respectively, n_1 and n_2 $(n_2 \geq n_1)$ as shown in the figure. The angle of incidence *i* is chosen such that the incident light rays will have minimum deviation if $n_1 = n_2 = n = 1.5$. For the case of unequal refractive indices, $n_1 = n$ and $n_2 = n + \Delta n$ (where $\Delta n << n$), the angle of emergence $e = i + \Delta e$. Which of the following statement(s) is(are) correct ?



A) The value of Δe (in radians) is greater than that of Δn

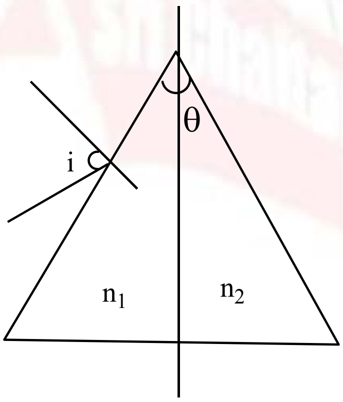
B) Δe is proportional to Δn

C) Δe is lies between 2.0 and 3.0 mill radians, if $\Delta n = 2.8 \times 10^{-3}$

D) Δe is lies between 1.0 and 1.6 mill radians, if $\Delta n = 2.8 \times 10^{-3}$

Ans. BC

Sol. $i = \frac{A+D}{2}$



$\frac{3}{2} = \frac{\sin\frac{A+D}{2}}{\sin\frac{A}{2}}$ $\sin\left(\frac{A+D}{2}\right) = \frac{3}{4}$

$i_1 + i_2 = A + D$ and $r_1 - r_2 = \frac{A}{2}$

as $n = \frac{\sin i}{\sin r} \Rightarrow \sin e = \frac{n + \Delta n}{2} \quad \Rightarrow \quad \sin(i + \Delta e) = \frac{n + \Delta n}{2}$

$\Rightarrow \sin i(\cos \Delta e) + \cos i \sin(\Delta e) = \frac{n + \Delta n}{2}$

$\Rightarrow \quad \frac{n}{2} \times 1 + \frac{\sqrt{3}n}{2}(\Delta e) = \frac{n}{2} + \frac{\Delta n}{2}$

$\Delta e = (\Delta n)\frac{2}{\sqrt{7}}$

For $\Delta n = 2.8 \times 10^{-3}$ and $n = \frac{3}{2}$

$\Delta e = 2.15 \times 10^{-3}$

4. A physical quantity $\vec{S}$ is defined as $\vec{S} = (\vec{E} \times \vec{B}) / \mu_0$, where $\vec{E}$ is electric field, $\vec{B}$ is magnetic field and μ_0 is the permeability of free space. The dimensions of $\vec{S}$ are the same as the dimensions of which of the following quantity (ies) ?

A) $\frac{\text{Energy}}{\text{Charge} \times \text{Current}}$ B) $\frac{\text{Force}}{\text{Length} \times \text{Time}}$

C) $\frac{\text{Energy}}{\text{Volume}}$ D) $\frac{\text{Power}}{\text{Area}}$

Ans. BD

Sol. $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} =$

$[S] = \frac{[E][B]}{[\mu_0]} = \frac{[CB][B]}{[\mu_0]} = \left[\frac{B^2}{\mu_0}\right][C]$

$\frac{B^2}{\mu_0}$ has dimensions of energy density

$\therefore \quad [S] = \frac{[\text{energy}[\text{speed}]]}{[\text{volume}]} = \left(\frac{ML^2T^{-2}}{(L^3)(LT^{-1})}\right) = MT^{-3}$

5. A heavy nucleus N, at rest, undergoes fission $N \rightarrow P + Q$, where P and Q are two lighter nuclei. Let $\delta = M_N - M_P - M_Q$, where M_P, M_Q and M_N are the masses of P, Q and N, respectively. E_P and E_Q are the kinetic energies of P and Q, respectively. The speeds of P and Q are υ_P and υ_Q, respectively. If c is the speed of light, which of the following statement(s) is(are) correct?

A) $E_P + E_Q = c^2\delta$

B) $E_P = \left(\dfrac{M_P}{M_P + M_Q}\right)c^2\delta$

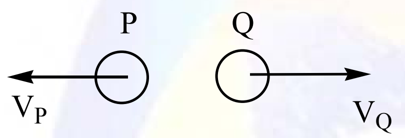
C) $\dfrac{\upsilon_P}{\upsilon_Q} = \dfrac{M_Q}{M_P}$

D) The magnitude of momentum for P as well as Q is $C\sqrt{2\mu\delta}$, where

$\mu = \dfrac{M_P M_Q}{(M_P + M_Q)}$

Ans. **ACD**

Sol. $N \rightarrow P + Q$



$\delta = M_N - M_P - M_Q \quad Q = \delta c^2$

$\therefore \quad E_P + E_Q = \delta C^2$

Since there are no external forces acting on the system. Momentum has to be conserved.

$P_P = P_Q = P$

$M_P V_P = M_Q V_Q$

$\dfrac{V_P}{V_Q} = \dfrac{M_Q}{M_P}$

$K.E = \dfrac{P^2}{2M} \qquad \therefore \dfrac{P^2}{2M_P} + \dfrac{P^2}{2M_Q} = \delta c^2$

$P^2 = \dfrac{2M_P M_Q . \delta c^2}{M_P + M_Q}$

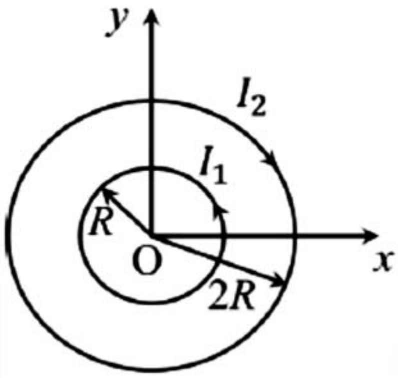
$P = \sqrt{2\mu\delta c^2}$

$P = c\sqrt{2\mu\delta}$

Where $\mu = \dfrac{M_P M_Q}{M_P + M_Q}$

6. Two concentric circular loops, one of radius R and the other of radius 2R, lie in the xy-plane with the origin as their common centre, as shown in the figure. The smaller

loop carries current I_1 in the anti-clockwise direction and the larger loop carries current I_2 is the clockwise direction, with $I_2 > 2I_1$. $\vec{B}$ (x, y) denotes the magnetic field at a point (x, y) in the xy-plane. Which of the following statement(s) is(are) correct?



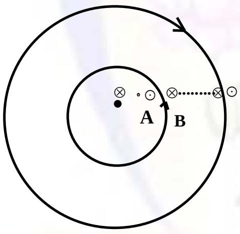
A) $\vec{B}(x,y)$ is perpendicular to the xy-plane at any point in the plane

B) $\left|\vec{B}(x,y)\right|$ depends on x and y only through the radial distance $r = \sqrt{x^2 + y^2}$

C) $\left|\vec{B}(x,y)\right|$ is non-zero at all points for $r < R$

D) $\vec{B}(x,y)$ points normally outward from the xy-plane for all the points between the two loops

Ans. **AB**



Sol.

$B_{center} = \frac{\mu_0 i_1}{2R} \odot \frac{\mu_0 i_2}{4R} \otimes, i_2 > 2i_1 \text{so} B_{center} \text{is} \otimes$

direction A $\vec{B}$ changes as we cross any wire

Due to radial symmetry B depends only on r.

consider point A just inside inner loop and B just outside it field at A due to inner loop is outward so there must be a null point somewhere between O&A (for r <R)

Also field between the loops is in ward field just outside the outer loop is outward and at far a was points it is inwards. So there is another null point outside the outer loop two (for r > 2R)

SECTION-2(Maximum Marks: 12)

Paragraph with Numerical

- This section contains THREE (03) question stems.
- There are TWO (02) questions corresponding to each question stem.
- The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

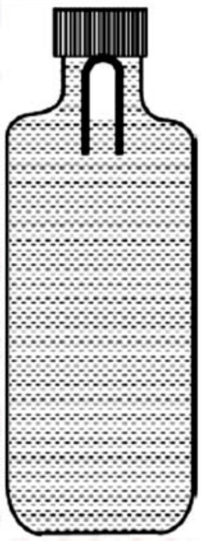
Full Marks : +2 If ONLY the correct numerical value is entered at the designated place;
Zero Marks : 0 In all other cases.

Question Stem for Question Nos. 7 and 8

Question Stem

A soft plastic bottle, filled with water of density 1 gm/cc, carries an inverted glass test-tube with some air (ideal gas) trapped as shown in the figure. The test-tube has a mass of 5 gm, and it is made of a thick glass of density 2.5 gm/cc. Initially the bottle is sealed at atmospheric pressure $p_0 = 10^5\,\text{Pa}$ so that the volume of the trapped air is $v_0 = 3.3\,\text{cc}$. When the bottle is squeezed from outside at constant temperature, the pressure inside rises and the volume of the trapped air reduces. It is found that the test tube begins to sink at pressure $p_0 + \Delta p$ without changing its orientation. At this pressure, the volume of the trapped air is $v_0 - \Delta v$.

Let $\Delta v = X\,\text{cc}$ and $\Delta p = Y \times 10^3\,\text{Pa}$.



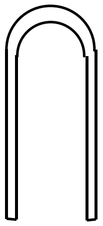
7. The value of X is _____.

Ans. 0.30

8. The value of Y is _____.

Ans. 10.00

Sol. 7&8



mass test tube = 5g

volume $=\dfrac{5g}{\dfrac{2.5g}{cc}}=2cc$

Buoyant force = 2g

ignoring mass of air trapped, minimum value of volume of air trapped for glass tube to float

$=\dfrac{W-B}{S_{water}}=3cc$

$V_{in}=3.3cc\,|\,V_{fin}=3cc\,|\,\Delta V=0.3cc=X$

$P_{in}=P_0\ P_{fin}=P_0+\Delta P$

isothermal condition, hence

$P_{in}V_{in}=P_{fin}V_{fin}$

$(3.3)P_0=3(P_0+\Delta P) \quad \Rightarrow \quad P_0+\Delta P=1.1P_0$

$\Delta P=0.1P_0=10^4\,Pa \quad \therefore \quad Y=10$

Question Stem for Question Nos. 9 and 10

Question Stem

A pendulum consists of a bob of mass m = 0.1 kg and a massless inextensible string of length L= 1.0m. It is suspended from a fixed point at height H = 0.9 m above a frictionless horizontal floor. Initially, the bob of the pendulum is lying on the floor at rest vertically below the point of suspension. A horizontal impulse P = 0.2 kg-m/s is imparted to the bob at some instant. After the bob slides for some distance, the string becomes taut and the bob lifts off the floor. The magnitude of the angular momentum of the pendulum about the point of suspension just before the bob lifts off is J kg- m^2/s. The kinetic energy of the pendulum just after the lift-off is K Joules.

9. The value of J is _____.

Ans. 0.18

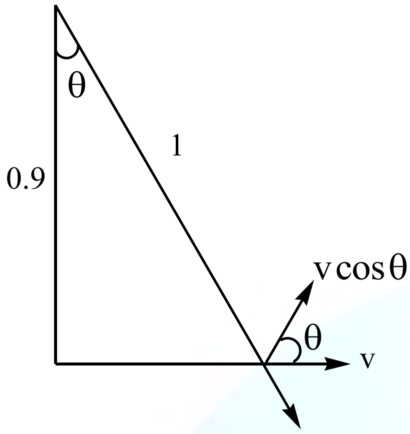
10. The value of K is _____.

Ans. 0.16

Sol. 9&10

$P=mv\Rightarrow v=\dfrac{0.2}{0.1}=2m^{-1}$

$\left|\overline{L}\right| = J = mvH = (0.9)(0.2) = 0.18$



$v\cos\theta = 1.8ms^{-1}$

$\therefore \quad K.E = \frac{1}{2}m(1.8)^2 = 0.162$

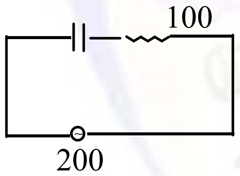
Question Stem for Question Nos. 11 and 12

Question Stem

In a circuit, a metal filament lamp is connected in series with a capacitor of capacitance $C\mu F$ across a 200 V, 50 Hz supply. The power consumed by the lamp is 500 W while the voltage drop across it is 100 V. Assume that there is no inductive load in the circuit. Take *rms* values of the voltages. The magnitude of the phase-angle (in degrees) between the current and the supply voltage is φ. Assume, $\pi\sqrt{3} \approx 5$.

11. The value of C is _____.

Ans. 100



Sol.

$v_0^2 = v_R^2 + v_C^2$

$(200)^2 = (100)^2 + v_C^2$

$v_C = 100\sqrt{3}$

$P = (v)_R \, i_R$

$i(100) = 500$

$i = 5$

$i \times x_c = v_c$

$5\frac{1}{2\pi(50)C} = 100\sqrt{3}, \qquad C = \frac{5}{(100\pi)100\sqrt{3}}$

$C = \frac{5}{10^4\pi\sqrt{3}}$

$C = \frac{1}{10^4}, \qquad C = 100\mu F$

12. The value of φ is _____.

Ans. **60**

Sol. $\tan\phi = \frac{v_C}{v_R} = \frac{100\sqrt{3}}{100}$ $\phi = 60^0$

SECTION-3(Maximum Marks: 12)

Paragraph with Single Answer Type

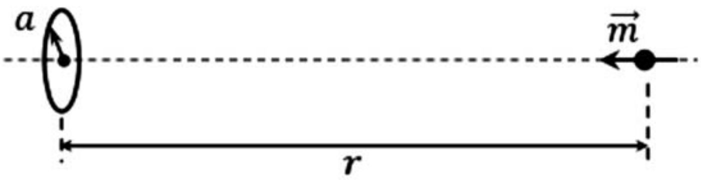
- This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO (02) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

Paragraph

A special metal S conducts electricity without any resistance. A closed wire loop, made of S, does not allow any change in flux through itself by inducing a suitable current to generate a compensating flux. The induced current in the loop cannot decay due to its zero resistance. This current gives rise to a magnetic moment which in turn repels the source of magnetic field or flux. Consider such a loop, of radius a, with its centre at the origin. A magnetic dipole of moment m is brought along the axis of this loop from infinity to a point at distance r(>>a) from the centre of the loop with its north pole always facing the loop, as shown in the figure below.

The magnitude of magnetic field of a dipole m, at a point on its axis at distance r, is $\frac{\mu_0}{2\pi}\frac{m}{r^3}$, where μ_0 is the permeability of free space. The magnitude of the force between two magnetic dipoles with moments, m_1 and m_2, separated by a distance r on the common axis, with their north poles facing each other, is $\frac{k\,m_1 m_2}{r^4}$, where k is a constant of appropriate dimensions. The direction of this force is along the line joining the two dipoles.



13. When the dipole m is placed at a distance r from the centre of the loop (as shown in the figure), the current induced in the loop will be proportional to

A) m/r^3 B) m^2/r^2 C) m/r^2 D) m^2/r

Ans. A

Sol. Suppose self-inductance the loop = L

Then $\left(\frac{\mu_0 m}{2\pi r^3}.\pi a^2\right)-Li=0$ .... (1)

$\Rightarrow i \propto \left(\frac{m}{r^3}\right)$

14. The work done in bringing the dipole from infinity to a distance r from the canter of the loop by the given process is proportional to

A) m/r^5 B) m^2/r^5 C) m^2/r^6 D) m^2/r^7

Ans. C

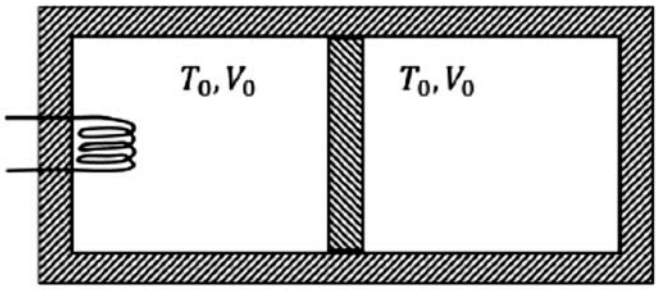
Sol. $W=\int_{r=\infty}^{r} Fdr$ Also $F=\frac{Km.m_{ind}}{r^n}$

$M_{ind}=\pi a^2 i=\pi a^2.\left(\frac{\mu_0 m}{2\pi r^3}\right)\frac{\pi a^2}{L}$ (from equal)

$\therefore W=\int \frac{K_m}{r^n}.\frac{\left(\pi a^2\right)^2 \mu_0}{2\pi L}.\frac{m}{r^3}.dr \propto \int_{r=\infty}^{r}\frac{m^2}{r^7}dr \propto \frac{m^2}{r^6}$

Paragraph

A thermally insulating cylinder has a thermally insulating and frictionless movable partition in the middle, as shown in the figure below. On each side of the partition, there is one mole of an ideal gas, with specific heat at constant volume, $C_V=2R$. Here, R is the gas constant. Initially, each side has a volume V_0 and temperature T_0. The left side has an electric heater, which is turned on at very low power to transfer heat Q to the gas on the left side. As a result the partition moves slowly towards the right reducing the right side volume to $V_0/2$. Consequently, the gas temperatures on the left and the right sides become T_L and T_R, respectively. Ignore the changes in the temperatures of the cylinder, heater and the partition. N'/



15. The value of $\frac{T_R}{T_0}$ is

A) $\sqrt{2}$ B) $\sqrt{3}$ C) 2 D) 3

Ans. A

16. The value of $\frac{Q}{RT_0}$ is

A) $4(2\sqrt{2}+1)$ B) $4(2\sqrt{2}-1)$ C) $(5\sqrt{2}+1)$ D) $(5\sqrt{2}-1)$

Ans. B

Sol. 15&16

Right chamber is undergoing adiabatic process.

$\gamma = \frac{3}{2}$ $T_0 V_0^{\frac{3}{2}-1} = T_R\left(\frac{V_0}{2}\right)^{\frac{3}{2}-1}$

$T_R = \sqrt{2}\,T_0$ $\therefore \frac{T_R}{T_0} = \sqrt{2}$

$P_f\left(\frac{3V_0}{2}\right) = 1RT_L$

$P_f\left(\frac{V_0}{2}\right) = 1RT_L$ $\therefore T_L = 3T_R$

$T_L = 3\sqrt{2}T_6$

$Q = (dU_L) + (dU)_R$

$Q = nC_v dT_L + nC_v dT_R$

$=1(2R)(3\sqrt{2}T_0 - T_0) + 1(2R).(\sqrt{2}T_6 - T_0) = 2R(4\sqrt{2}-2)T_6$

$Q = 4RT_6(2\sqrt{2}-1)$

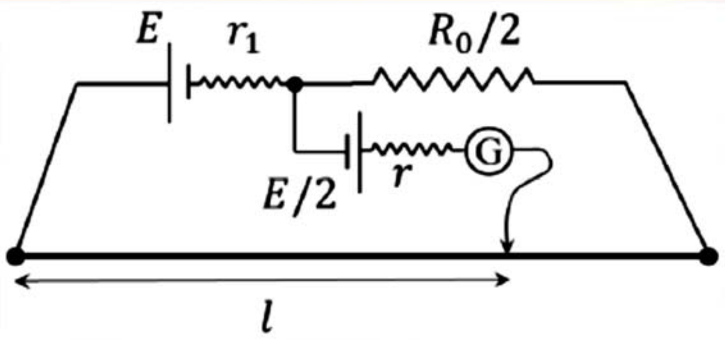
$\frac{Q}{RT_6} = 4(2\sqrt{2}-1)$

SECTION-4(Maximum Marks: 12)
Non-Negative Integer Answer Type

- This section contains THREE (03) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If ONLY the correct integer is entered;
Zero Marks : 0 In all other cases.

17. In order to measure the internal resistance r_1 of a cell of emf E, a meter bridge of wire resistance $R_0 = 50\Omega$, a resistance $R_0/2$, another cell of emf *E/2* (internal resistance *r*) and a galvanometer G are used in a circuit, as shown in the figure. If the null point is found at $l = 72\,cm$, then the value of $r_1 =$ __Ω.



Ans. 3

Sol. i in primary circuit $= \dfrac{E}{R_0 + \dfrac{R_0}{2} + r_1}$

P.D between the points where the secondary cell is connected is

$= i\left(\dfrac{R_0}{2} + \dfrac{28R_0}{100}\right) \quad \therefore \quad \dfrac{E}{2} = \dfrac{E}{\left(r_1 + \dfrac{3R_0}{2}\right)}\left(\dfrac{78R_0}{100}\right)$

$r_1 + \dfrac{3R_0}{2} = \dfrac{156R_0}{100}$

$r_1 = \dfrac{156 \times \cancel{50}}{\cancel{100}\,2} - \dfrac{3}{\cancel{2}} \times \cancel{50}\,25$

$r_1 = 78 - 75$

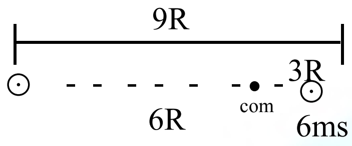
$r_1 = 3$

18. The distance between two stars of masses $3M_S$ and $6M_S$ is 9R. Here R is the mean distance between the centres of the Earth and the Sun, and M_S is the mass of the Sun. The two stars orbit around their common centre of mass in circular orbits with period nT, where T is the period of Earth's revolution around the Sun.
The value of n is ____.

Ans. **9**

Sol. $\frac{G(3Ms)6Ms}{(9R)^2} = (6Ms)3R\omega^2$

$\omega = \sqrt{\frac{GMs}{81R^3}}\ T' = \frac{2\pi}{\omega}$



$T' = 2\pi\sqrt{\frac{81R^3}{GM}}$

$T' = 9T$.

19. In a photoemission experiment, the maximum kinetic energies of photoelectrons from metals P, Q and R are E_P, E_Q and E_R, respectively, and they are related by $E_P = 2E_Q = 2E_R$. In this experiment, the same source of monochromatic light is used for metals P and Q while a different source of monochromatic light is used for the metal R. The work functions for metals P, Q and R are 4.0 eV, 4.5 eV and 5.5 eV, respectively. The energy of the incident photon used for metal R, in eV, is ____.

Ans. **6**

Sol. $E_P = h\nu_1 - 4$

$E_Q = h\nu_1 - 4.5$

$E_R = h\nu_2 - 5.5$

$E_P = 2E_Q = 2E_R$

$h\nu_1 - 4 = 2(h\nu_1 - 4.5)$

$9 - 4 = h\nu_1$

$E_Q = E_R$

$5 - 4.5 = h\nu_2 = 5.5$

$\Rightarrow h\nu_2 = 6\,eV$

$E_{incident\ photon} = 6\,eV$

CHEMISTRY **Max. Marks: 60**

SECTION-1(Maximum Marks: 24)

One or More Type

- This section contains SIX (06) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s)
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks : 0 If unanswered;
Negative Marks : −2 In all other cases.

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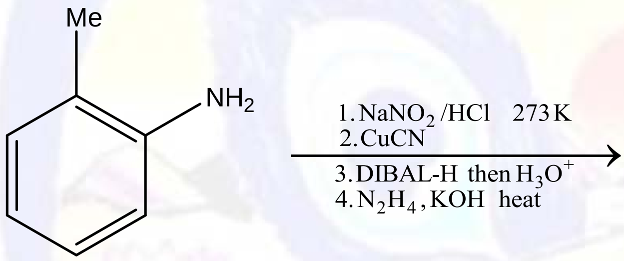
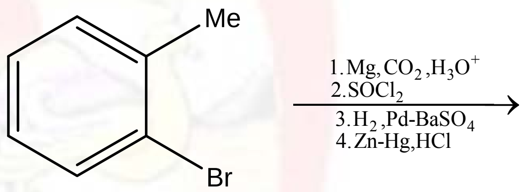
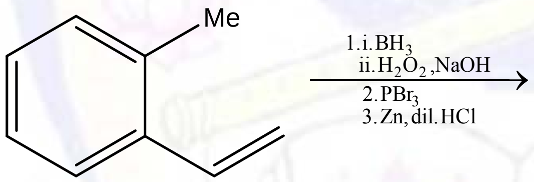
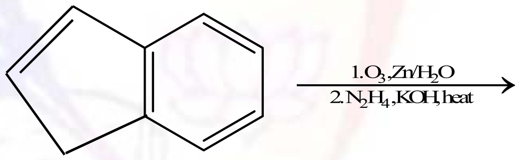
1. The reaction sequence(s) that would lead to *o*-xylene as the major product is(are)

A) 

B) 

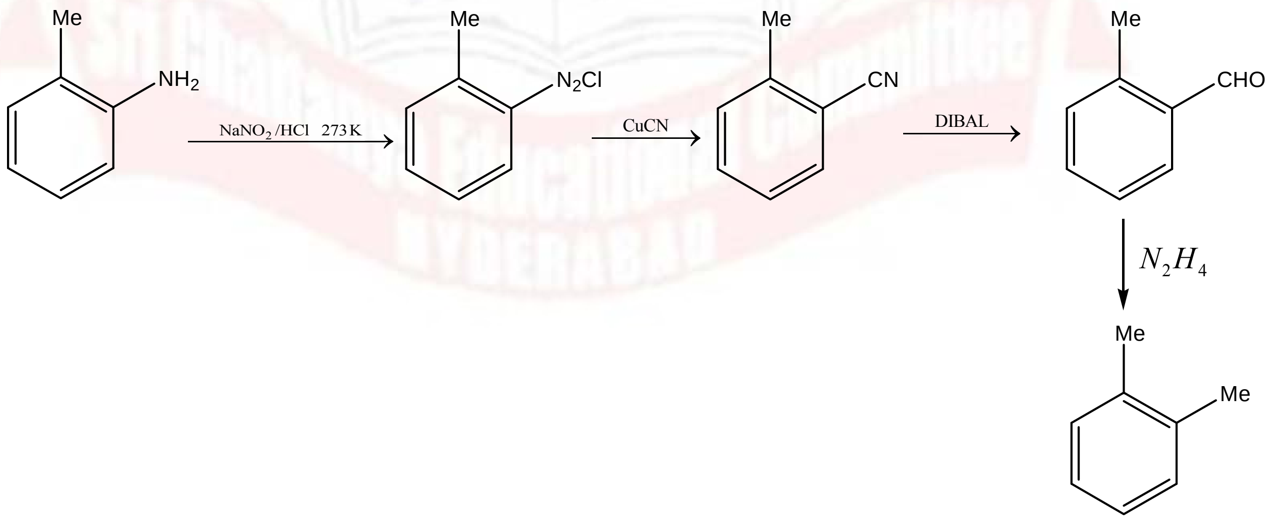
C) 

D) 

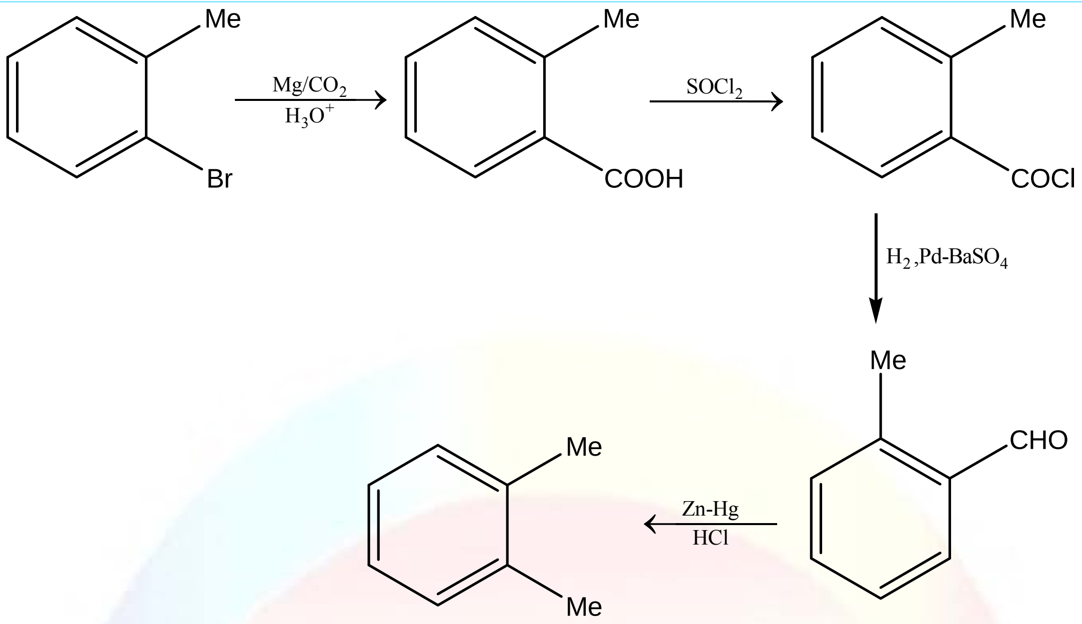
Ans : AB

Sol :

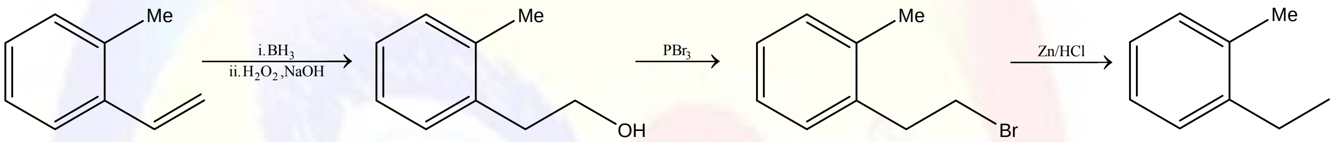
A)



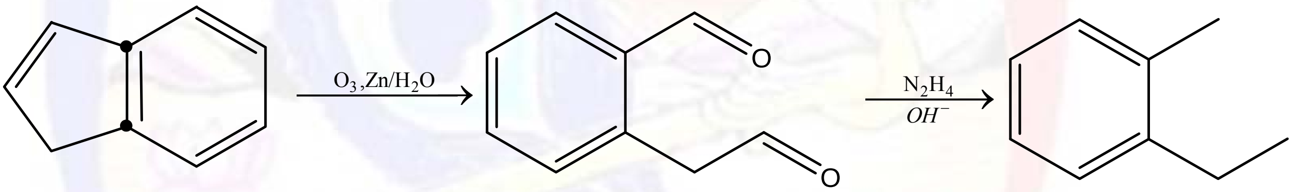
B)



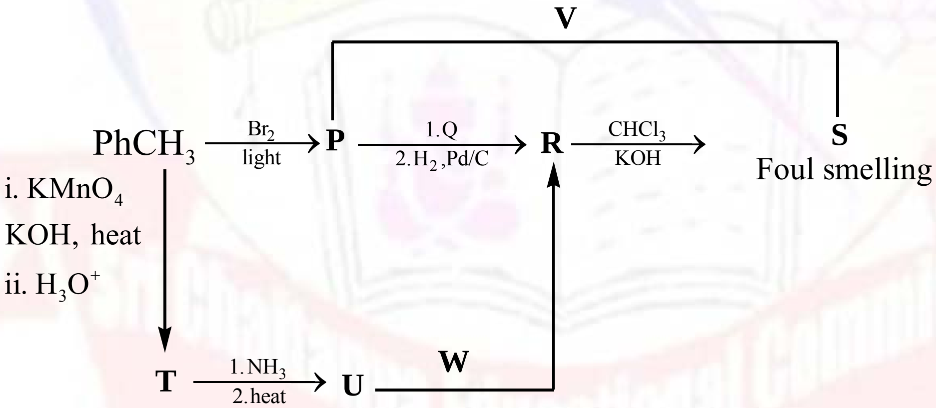
C)



D)



2. Correct option(s) for the following sequence of reactions is(are)



A) **Q** = KNO_2, **W** = $LiAlH_4$

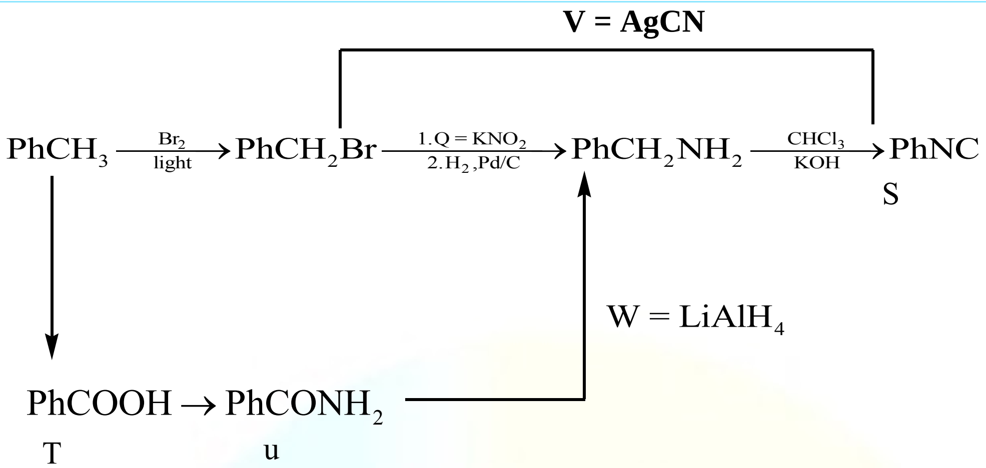
B) **R** = benzenamine, **V** = KCN

C) **Q** = $AgNO_2$, **R** = phenylmethanamine

D) **W** = $LiAlH_4$, **V** = AgCN

Ans : AD (As per standard ref) or CD (as per NCERT)

Sol :



Note : As per the basic NCERT treatment, Q may be taken as $AgNO_2$. But as per mechanism, $PhCH_2Br$ shown preferably react by S_N1 mechanism. As Ag^+ facilitate S_N1 mechanism more, KNO_2 should give more nitro products

3. For the following reaction

$$2\mathbf{X}+\mathbf{Y}\xrightarrow{k}\mathbf{P}$$

the rate of reaction is $\frac{d[\mathbf{P}]}{dt}=k[\mathbf{X}]$. Two moles of X are mixed with one mole of Y to make 1.0 L of solution. At 50 s, 0.5 mole of Y is left in the reaction mixture. The correct statement(s) about the reaction is(are)

(Use: ln 2 = 0.693)

A) The rate constant, k, of the reaction is $13.86\times10^{-4}\,s^{-1}$.

B) Half-life of **X** is 50 s

C) At 50 s, $-\frac{d[\mathbf{X}]}{dt}=13.86\times10^{-3}\,\text{mol L}^{-1}\text{s}^{-1}$.

D) At 100 s, $-\frac{d[\mathbf{Y}]}{dt}=3.46\times10^{-3}\,\text{mol L}^{-1}\text{s}^{-1}$.

Ans : BCD

Sol : $2\mathbf{X}+\mathbf{Y}\xrightarrow{k}\mathbf{P}$

t = 0 2 mol 1 ml 0

t = 50 s(2 – 1) mol 0.5 mol 0.5 mol

$$k=\frac{1}{t}\ln\frac{a_0}{a_t}=\frac{1}{50}\ln\frac{2}{1}$$

$$=\frac{1}{50}\times0.693$$

$$=0.01386\,s^{-1}$$

$$13.86\times10^{-3}$$

$$\text{After} 100s,\ k = \frac{1}{100}\ln\frac{1}{a_t}$$

$$\frac{1}{50}\ln 2 = \frac{1}{100}\ln\frac{1}{a_t}$$

$$\therefore a_t = 0.25$$

$$\frac{d[p]}{dt} = -\frac{d[y]}{dt} = (13.86\times10^{-3})(0.25)$$

$$= 3.46\times10^{-3}$$

4. Some standard electrode potentials at 298 K are given below:

Pb^{2+}/Pb -0.13 V

Ni^{2+}/Ni -0.24 V

Cd^{2+}/Cd -0.40 V

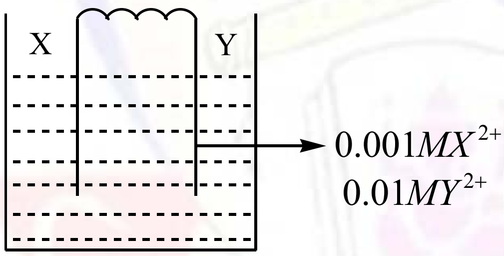
Fe^{2+}/Fe -0.44 V

To a solution containing 0.001 M of $\mathbf{X}^{2+}$ and 0.1 M of $\mathbf{Y}^{2+}$, the metal rods **X** and **Y** are inserted (at 298 K) and connected by a conducting wire. This resulted in dissolution of **X**. The correct combination(s) of **X** and **Y**, respectively, is(are)
(Given: Gas constant, $R = 8.314\ J\ K^{-1}mol^{-1}$, Faraday constant, $F = 96500\ C\ mol^{-1}$)

A) Cd and Ni B) Cd and Fe C) Ni and Pb D) Ni and Fe

Ans : ABC

Sol :



$$E_{Pb^{2+}/Pb} = E^0_{Pb^{2+}/Pb} - 0.03\log10^3$$

= - 0.13 – 0.03 × 3

= 0.13 – 0.09

= - 0.22 V

$$E^1_{Pb^{2+}/Pb} - 0.13 - 0.03\log10$$

= - 0.13 – 0.03

= - 0.16 V

$$E_{Ni^{2+}/Ni} = -0.24 - 0.09 = -0.33V$$

$$E^1_{Ni^{2+}/Ni} = -0.24 - 0.03 = -0.27$$

$$E_{Cd^{2+}/Cd} = -0.40 - 0.09 = -0.49V$$

$E^{1}_{Cd^{2+}/Cd} = -0.40 - 0.03 = -0.43V$

$E_{Fe^{2+}/Fe} = -0.40 - 0.09 = -0.53V$

$E^{1}_{Fe^{2+}/Fe} = -0.44 - 0.03 = -0.47V$

$\Delta G = -nFE_{cell}$

$\Delta G\ of\ A = -ve$

$\Delta G\ of\ B = -ve$

$\Delta G\ of\ C = -ve$

$\Delta G\ of\ D = +ve$

5. The pair(s) of complexes where in both exhibit tetrahedral geometry is(are)
(Note: py = pyridine
Given: Atomic numbers of Fe, Co, Ni and Cu are 26, 27, 28 and 29, respectively)

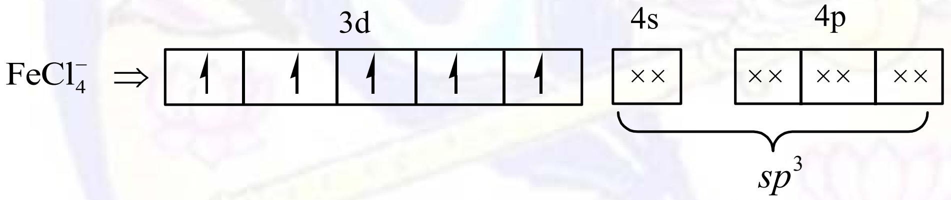
A) $[FeCl_4]^{-}$ and $[Fe(CO)_4]^{2-}$ B) $[Co(CO)_4]^{-}$ and $[CoCl_4]^{2-}$

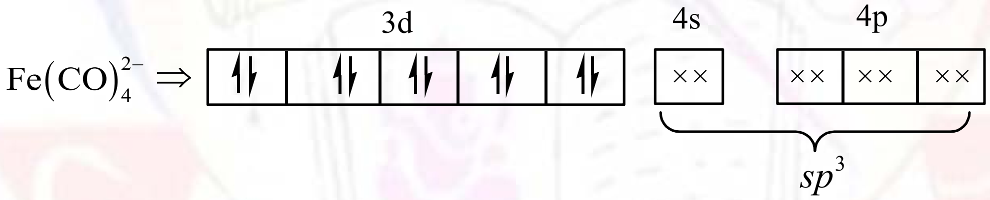
C) $[Ni(CO)_4]$ and $[Ni(CN)_4]^{2-}$ D) $[Cu(py)_4]^{+}$ and $[Cu(CN)_4]^{3-}$

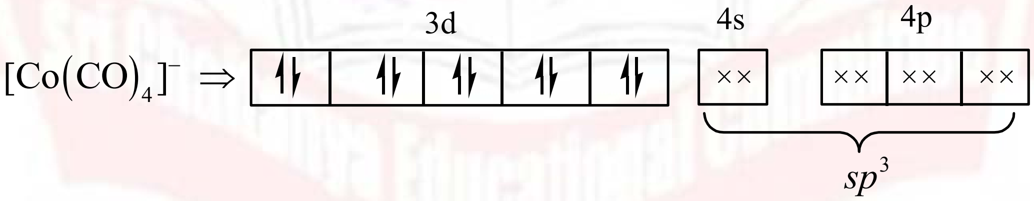
Ans : ABD

Sol : A) $FeCl_4^{-}$

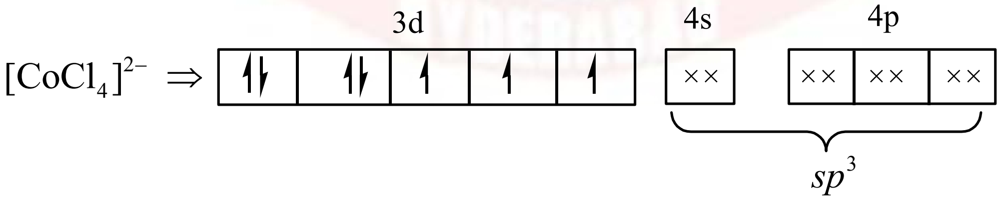
$Fe^{2+} = 3d^6 4s^0$

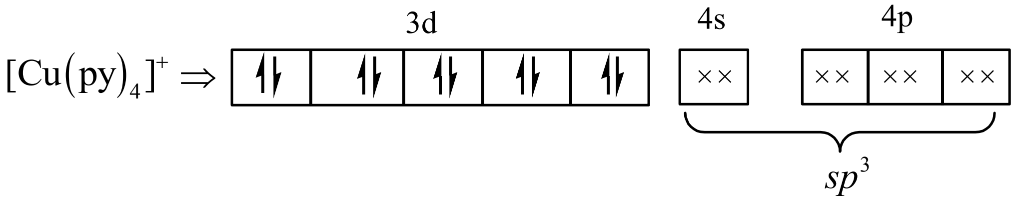




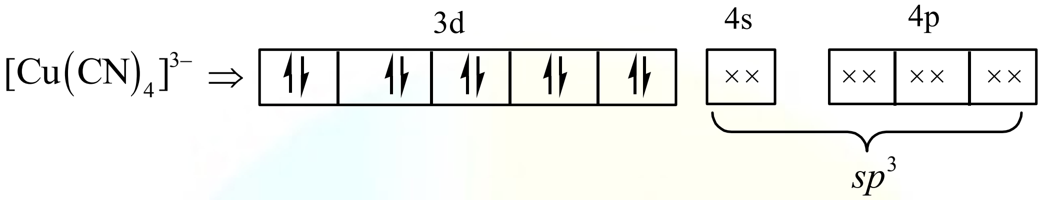


B)





D)



6. The correct statement(s) related to oxoacids of phosphorous is(are)

A) Upon heating, H_3PO_3 undergoes disproportionation reaction to produce H_3PO_4 and PH_3.

B) While H_3PO_3 can act as reducing agent, H_3PO_4 cannot.

C) H_3PO_3 is a monobasic acid.

D) The H atom of P–H bond in H_3PO_3 is not ionizable in water.

Ans : ABD

Sol : A) $4H_3PO_3 \rightarrow 3H_3PO_4 + PH_3$

B) H_3PO_3 is a reducing acid as it has P – H bond

C) H_3PO_3 is a dibasic acid

D) $H_3PO_3 + H_2O \rightleftharpoons H_2PO_3^- + H_3O^+$

$H_2PO_3^- + H_2O \rightleftharpoons HPO_3^{2-} + H_3O^+$

SECTION-2(Maximum Marks: 12)

<u>Paragraph with Numerical</u>

- This section contains THREE (03) question stems.
- There are TWO (02) questions corresponding to each question stem.
- The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +2 If ONLY the correct numerical value is entered at the designated place;
Zero Marks : 0 In all other cases.

Question Stem for Question Nos. 7 and 8

At 298 K, the limiting molar conductivity of a weak monobasic acid is $4\times10^2 S\ cm^2 mol^{-1}$. At 298 K, for an aqueous solution of the acid the degree of dissociation is α and the molar conductivity is $\mathbf{y} \times 10^2 S\ cm^2\ mol^{-1}$. At 298 K, upon

20 times dilution with water, the molar conductivity of the solution becomes $3\mathbf{y}\times 10^{2}\,\text{S cm}^{2}\ \text{mol}^{-1}$.

7. The value of α is ___.

Ans : 0.22

Sol : $\wedge^{\circ} = 4\times 10^{2}\, S\, cm^{2}\, mol^{-1}$

For concentration C,

$$\alpha = \frac{y\times 10^{2}}{4\times 10^{2}} = \frac{y}{4}$$

$$k = \frac{C.\alpha^{2}}{1-\alpha} = \frac{C\left(\frac{y}{4}\right)^{2}}{1-\frac{y}{4}}$$

For concentration $\frac{C}{20}$,

$$k = \frac{\frac{C}{20}.\left(\frac{3y}{4}\right)^{2}}{1-\frac{3y}{4}}$$

$$\Rightarrow y = \frac{44}{51} = 0.86$$

$$\alpha = \frac{11}{51} = 0.21 to 0.22$$

8. The value of **y** is ___.

Ans : 0.86

Sol : $\wedge^{\circ} = 4\times 10^{2}\, S\, cm^{2}\, mol^{-1}$

For concentration C,

$$\alpha = \frac{y\times 10^{2}}{4\times 10^{2}} = \frac{y}{4}$$

$$k = \frac{C.\alpha^{2}}{1-\alpha} = \frac{C\left(\frac{y}{4}\right)^{2}}{1-\frac{y}{4}}$$

For concentration $\frac{C}{20}$,

$$k = \frac{\frac{C}{20}.\left(\frac{3y}{4}\right)^{2}}{1-\frac{3y}{4}} \qquad \Rightarrow y = \frac{44}{51} = 0.86$$

$$\alpha = \frac{11}{51} = 0.21 to 0.22$$

Question Stem for Question Nos. 9 and 10

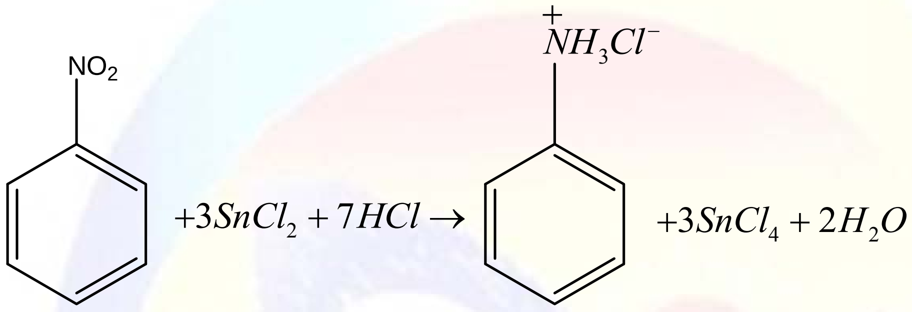
Reaction of x g of Sn with HCl quantitatively produced a salt. Entire amount of the salt reacted with y g of nitrobenzene in the presence of required amount of HCl to produce 1.29 g of an organic salt (quantitatively).

(Use Molar masses $\left(\text{in g mol}^{-1}\right)$ of H, C, N, O, Cl and Sn as 1, 12, 14, 16, 35 and 119, respectively).

9. The value of **x** is ___.

Ans : 3.57

Sol : $Sn + 2HCl \rightarrow SnCl_2 + H_2$



1.29 g

$n_{PhNH_3+Cl^-} = \dfrac{1.29}{129} = 0.01$

$\therefore\ n_{SnCl_2} = 0.03$

$\therefore\ n_{Sn} = 0.03$

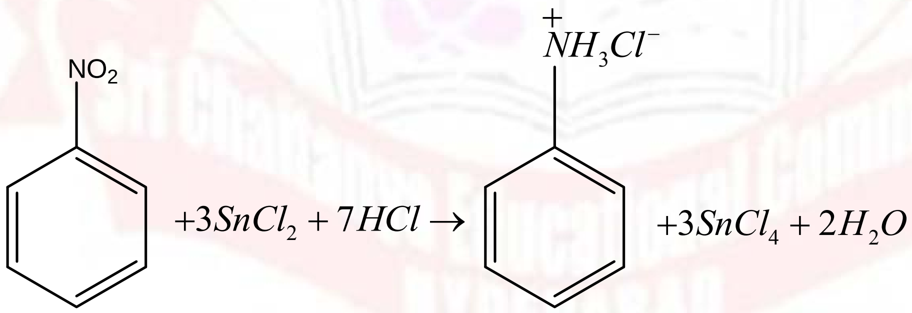
x = 0.03 × 114 = 3.57

y = 0.01 × 123 = 1.23

10. The value of **y** is ___.

Ans : 1.23

Sol : $S_n + 2HCl \rightarrow SnCl_2 + H_2$



1.29 g

$n_{PhNH_3+Cl^-} = \dfrac{1.29}{129} = 0.01$

$\therefore\ n_{SnCl_2} = 0.03$

$\therefore\ n_{Sn} = 0.03$

x = 0.03 × 114 = 3.57

y = 0.01 × 123 = 1.23

Question Stem for Question Nos. 11 and 12

A sample (5.6 g) containing iron is completely dissolved in cold dilute HCl to prepare a 250 mL of solution. Titration of 25.0 mL of this solution requires 12.5 mL of 0.03 M $KMnO_4$ solution to reach the end point. Number of moles of Fe^{2+} present in 250 mL solution is $\mathbf{x} \times 10^{-2}$ (consider complete dissolution of $FeCl_2$). The amount of iron present in the sample is **y%** by weight.

(Assume: $KMnO_4$ reacts only with Fe^{2+} in the solution

Use: Molar mass of iron as $56\ g\ mol^{-1}$)

11. The value of **x** is ___.

Ans : 1.875

Sol : $ngl\ of\ Fe^{2+} = nge\ KMnO_4$

$= 12.5 \times 10^{-3} \times 0.03 \times 5$

$= 1.875 \times 10^{-3}$

$\therefore n_{Fe^{2+}} in\ 250\ of\ SA_n$

$= 1.875 \times 10^{-3} \times 10$

$= 1.875 \times 10^{-2}$

$\Rightarrow x = 1.875$

$W_{Fe} = 1.875 \times 10^{-2} \times 56$

$= 1.05\ g$

$Fe = \frac{1.05}{5.6} \times 100 = 18.75$

12. The value of **y** is ___.

Ans : 18.75

Sol : $ngl\ of\ Fe^{2+} = nge\ KMnO_4$

$= 12.5 \times 10^{-3} \times 0.03 \times 5$

$= 1.875 \times 10^{-3}$

$\therefore n_{Fe^{2+}} in\ 250\ of\ SA_n$

$= 1.875 \times 10^{-3} \times 10$

$= 1.875 \times 10^{-2}$ $\Rightarrow x = 1.875$

$W_{Fe} = 1.875 \times 10^{-2} \times 56$

$= 1.05\ g$

$Fe = \frac{1.05}{5.6} \times 100 = 18.75$

SECTION-3(Maximum Marks: 12)

Paragraph with Single Answer Type

- This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO (02) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

Paragraph-1:

The amount of energy required to break a bond is same as the amount of energy released when the same bond is formed. In gaseous state, the energy required for homolytic cleavage of a bond is called Bond Dissociation Energy (BDE) or Bond Strength. BDE is affected by s-character of the bond and the stability of the radicals formed. Shorter bonds are typically stronger bonds. BDEs for some bonds are given below:

$$H_3C-H(g) \rightarrow H_3C^{\bullet}(g)+H^{\bullet}(g) \quad \Delta H° = 105\,\text{Kcal mol}^{-1}$$

$$Cl-Cl(g) \rightarrow Cl^{\bullet}(g)+Cl^{\bullet}(g) \quad \Delta H° = 58\,\text{Kcal mol}^{-1}$$

$$H_3C-Cl(g) \rightarrow H_3C^{\bullet}(g)+Cl^{\bullet}(g) \quad \Delta H° = 85\,\text{Kcal mol}^{-1}$$

$$H-Cl(g) \rightarrow H^{\bullet}(g)+Cl^{\bullet}(g) \quad \Delta H° = 103\,\text{Kcal mol}^{-1}$$

13. Correct match of the **C–H** bonds (shown in bold) in Column **J** with their BDE in Column **K** is

Column **J** Molecule	Column **K** BDE (kcal mol^{-1})
P) $\mathbf{H-C}H(CH_3)_2$	i) 132
Q) $\mathbf{H-C}H_2Ph$	ii) 110
R) $\mathbf{H-C}H=CH_2$	iii) 95
S) $\mathbf{H-C}\equiv CH$	iv) 88

A) P – iii, Q – iv, R – ii, S – i
B) P – i, Q – ii, R – iii, S – iv
C) P – iii, Q – ii, R – i, S – iv
D) P – ii, Q – i, R – iv, S – iii

Ans : A

Sol : Q < P < R < S

14. For the following reaction

$$CH_4(g)+Cl_2(g)\xrightarrow{\text{light}}CH_3Cl(g)+HCl(g)$$

the correct statement is

A) Initiation step is exothermic with $\Delta H^\circ = -58$ kcal mol^{-1}

B) Propagation step involving $^\bullet CH_3$ formation is exothermic with $\Delta H^\circ = -2$ kcal mol^{-1}

C) Propagation step involving CH_3Cl formation is endothermic with $\Delta H^\circ = +27$ kcal mol^{-1}.

D) The reaction is exothermic with $\Delta H^\circ = -25$ kcal mol^{-1}.

Ans : D

Sol : $CH_4 + Cl_2 \rightarrow CH_3Cl + HCl$

105 58 85 103

$\Delta H = 105 + 58 - (85 + 103) \quad = -25\,k\,cal\,mol^{-1}$

Paragraph-2:

The reaction of $K_3[Fe(CN)_6]$ with freshly prepared $FeSO_4$ solution produces a dark blue precipitate called Turnbull's blue. Reaction of $K_4[Fe(CN)_6]$ with the $FeSO_4$ solution in complete absence of air produces a white precipitate X, which turns blue in air. Mixing the $FeSO_4$ solution with $NaNO_3$, followed by a slow addition of concentrated H_2SO_4 through the side of the test tube produces a brown ring.

15. Precipitate **X** is

A) $Fe_4[Fe(CN)_6]_3$ B) $Fe[Fe(CN)_6]$ C) $K_2Fe[Fe(CN)_6]$ D) $KFe[Fe(CN)_6]$

Ans : C

Sol : $K_2Fe[Fe(CN)_6] + FeSO_4$

$\rightarrow \underset{\text{white}}{K_2Fe[Fe(CN)_6]} \downarrow + K_2SO_4$

16. Among the following, the brown ring is due to the formation of

A) $[Fe(NO)_2(SO_4)_2]^{2-}$ B) $[Fe(NO)_2(H_2O)_4]^{3+}$

C) $[Fe(NO)_4(SO_4)_2]$ D) $[Fe(NO)(H_2O)_5]^{2+}$

Ans : D

Sol : $Fe^{2+} + 5H_2O + NO \rightarrow \underset{\text{brown ring}}{\left[Fe(H_2O)_5(NO)\right]^{2+}}$

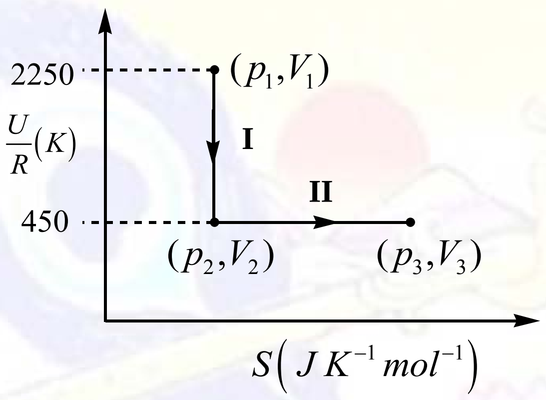
SECTION-4(Maximum Marks: 12)

Non-Negative Integer Answer Type

- This section contains THREE (03) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If ONLY the correct integer is entered;
Zero Marks : 0 In all other cases.

17. One mole of an ideal gas at 900 K, undergoes two reversible processes, **I** followed by **II**, as shown below. If the work done by the gas in the two processes are same, the value of $\ln\frac{V_3}{V_2}$ is ____.



(U: internal energy, S: entropy, p: pressure, V: volume, R: gas constant)

(Given: molar heat capacity at constant volume, $C_{V,m}$ of the gas is $\frac{5}{2}R$)

Ans : 10

Sol : $\frac{\Delta u}{R}=1800 \Rightarrow \Delta u = 1800\times R = W$

$1800R = nRT\ln\frac{V_3}{V_2}$

For process I

Q = 0

$\Delta u = W$ = 1800 R

For process II

$\Delta u = 0$

$u = W = nRT\ln\frac{V_3}{V_2}$

$T_1 = 900K$

$\Delta u = nC_v \Delta T \qquad -1800R = 1 \times \frac{5R}{2}(T - 900)$

T = 180 K

$1 \times 180 \ln \frac{V_3}{V_2} = 1800 \qquad \ln \frac{V_3}{V_2} = 10$

18. Consider a helium (He) atom that absorbs a photon of wavelength 330 nm. The change in the velocity (in cm s^{-1}) of He atom after the photon absorption is ___.

(Assume: Momentum is conserved when photon is absorbed.

Use: Planck constant $= 6.6 \times 10^{-34}$ **J** s, Avogadro number $= 6 \times 10^{23} \text{mol}^{-1}$, Molar mass of He $= 4 \text{ g mol}^{-1}$)

Ans : 30

Sol : $V = \frac{h}{m\lambda} = \frac{6.6 \times 10^{-34} \times 6 \times 10^{23}}{4 \times 10^{-3} \times 330 \times 10^{-}} cms^{-1}$

19. Ozonolysis of ClO_2 produces an oxide of chlorine. The average oxidation state of chlorine in this oxide is ___.

Ans : 6

Sol : $ClO_2 + O_3 \rightarrow ClO_3 + O_2$

$\therefore$ Oxidation state of Cl in ClO_3 is

MATHEMATICS

Max. Marks: 60

SECTION-1(Maximum Marks: 24)

One or More Type

- This section contains SIX (06) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s)
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks : 0 If unanswered;
Negative Marks : −2 In all other cases.

---

1. Let

$S_1=\{(i,j,k):i,j,k\in\{1,2,....,10\}\}$,

$S_2=\{(i,j):1\le i<j+2\le 10, i,j\in\{1,2,.....,10\}\}$,

$S_3=\{(i,j,k,l):1\le i<j<k<l, i,j,k,l\in\{1,2,....,10\}\}$

and

$S_4=\{(i,j,k,l):i,j,k$ and l are distinct elements in $\{1,2,....,10\}\}$.

If the total number elements in the set S_r is $n_r, r=1,2,3,4,$ then which of the following statements is(are) TRUE?

A) $n_1=1000$ B) $n_2=44$ C) $n_3=220$ D) $\frac{n_4}{12}=420$

Ans: ABD

Sol: $n_1=10\times10\times10$

$n_2=8+8+7+6+5+4+3+2+1=8+\frac{8\times9}{2}=44$

$[i=1, j=1,2,3,......8];[i=2, j=1,2,3,......8];[i=3, j=2,3,....8],...$ and so on

$n_3={}^{10}C_4=\frac{10\times9\times8\times7}{24}=210$ (Select 4 numbers and arrange in increasing order)

$n_4={}^{10}P_4=10\times9\times8\times7\Rightarrow\frac{n_4}{12}=420$

2. Consider a triangle PQR having sides of lengths p,q and r opposite to the angles P,Q and R, respectively. Then which of the following statements is(are) TRUE?

A) $\cos P \ge 1-\frac{p^2}{2qr}$

B) $\cos R \ge \left(\frac{q-r}{p+q}\right)\cos P+\left(\frac{p-r}{p+q}\right)\cos Q$

C) $\frac{q+r}{p}<2\frac{\sqrt{\sin Q \sin R}}{\sin P}$

D) If $p<q$ and $p<r$, then $\cos Q>\frac{p}{r}$ and $\cos R>\frac{p}{q}$

Ans: AB

Sol: A) $\cos P=\frac{q^2+r^2}{2qr}-\frac{p^2}{2qr}\ge 1-\frac{p^2}{2qr}$

B) $p+q>r \Rightarrow (r\cos Q+q\cos R)+(p\cos R+r\cos P)>(p\cos Q+q\cos P)$

C) $\frac{q+r}{P}=\frac{\sin Q+\sin R}{\sin P}\ge 2\frac{\sqrt{\sin Q.\sin R}}{\sin P}$

D) $\cos Q>\frac{q}{r}\Rightarrow \angle Q$ is acute $\Rightarrow \angle R$ is acute (similarly)

$\cos Q>\frac{q}{r}\Rightarrow \frac{p^2-q^2+r^2}{2pr}>\frac{q}{r}\Rightarrow p^2+r^2-q^2>2p^2\Rightarrow p^2+q^2<r^2$

$\angle R$ is obtuse(contradiction)

3. Let $f:\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\to\mathbb{R}$ be a continuous function such that

$f(0)=1$ and $\int_0^{\pi/3} f(t)dt=0$

Then which of the following statements is(are) TRUE?

A) The equation $f(x)-3\cos 3x=0$ has at least one solution in $\left(0,\frac{\pi}{3}\right)$

B) The equation $f(x)-3\sin 3x=-\frac{6}{\pi}$ has at least one solution in $\left(0,\frac{\pi}{3}\right)$

C) $\lim_{x\to 0}\frac{x\int_0^x f(t)dt}{1-e^{x^2}}=-1$

D) $\lim_{x\to 0}\frac{\sin x\int_0^x f(t)dt}{x^2}=-1$

Ans: ABC

Sol: $f(0)=1, \int_0^{\pi/3} f(t)dt = 0$

A) Let $g(x)=\int_0^x f(x)dx - \sin 3x$

$g(0)=0=g(\pi/3) \Rightarrow g'(x)=0$ has atleast one solution in $\left(0,\frac{\pi}{3}\right)$

B) Let $g(x)=\int_0^x f(t)dt + \cos 3x + \frac{6x}{\pi}$

$g(0)=1=g(\pi/3)$

$\Rightarrow g'(x)=0$ has atleast one solution in $(0,\pi/3)$

C) $\lim_{x\to 0}\frac{x\int_0^x f(t)dt}{1-e^{x^2}} = \lim_{x\to 0}\frac{\int_0^x f(t)dt + xf(x)}{-2xe^{x^2}}$

$= \lim_{x\to 0}\frac{f(x)+xf'(x)+f(x)}{-2\left[e^{x^2}+2x^2ex^2\right]} = -1$

D) $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)\left(\frac{\int_0^x f(t)dt}{x}\right) = 1\times 1 = +1$

4. For any real numbers α and β, let $y_{\alpha,\beta}(x), x\in\mathbb{R}$, be the solution of the differential equation $\frac{dy}{dx}+\alpha y = xe^{\beta x}, y(1)=1$.

Let $S=\{y_{\alpha,\beta}(x):\alpha,\beta\in\mathbb{R}\}$. Then which of the following functions belong(s) to the set S?

A) $f(x)=\frac{x^2}{2}e^{-x}+\left(e-\frac{1}{2}\right)e^{-x}$ B) $f(x)=-\frac{x^2}{2}e^{-x}+\left(e+\frac{1}{2}\right)e^{-x}$

C) $f(x)=\frac{e^x}{2}\left(x-\frac{1}{2}\right)+\left(e-\frac{e^2}{4}\right)e^{-x}$ D) $f(x)=\frac{e^x}{2}\left(\frac{1}{2}-x\right)+\left(e+\frac{e^2}{4}\right)e^{-x}$

Ans: AC

Sol: $ye^{\alpha x} = \int x\, e^{(\alpha+\beta)x}dx$

If $\alpha+\beta=0$, then $ye^{\alpha x}=\frac{x^2}{2}+c$

$(1,1)\Rightarrow e^{\alpha}=\frac{1}{2}+c\Rightarrow c=e^{\alpha}-\frac{1}{2}$

$\therefore y=\left(\frac{x^2}{2}+e^{\alpha}-\frac{1}{2}\right)e^{-\alpha x}$

Put $\alpha=1\Rightarrow y=\left(\frac{x^2}{2}+e-\frac{1}{2}\right)e^{-2}$ (A is correct)

If $\alpha+\beta=0$, then $ye^{\alpha x}=\frac{xe^{(\alpha+\beta)x}}{\alpha+\beta}-\frac{e^{(\alpha+\beta)x}}{(\alpha+\beta)^2}+c$

Put $\alpha=\beta=1$

$y=\frac{x}{2}e^x-\frac{e^x}{4}+c$

$(1,1)\Rightarrow y=\frac{e^x}{2}\left(x-\frac{1}{2}\right)+e^{-x}\left(e-\frac{e^2}{4}\right)$ (C is corret)

5. Let O be the origin and $\overrightarrow{OA}=2\hat{i}+2\hat{j}+\hat{k},\overrightarrow{OB}=\hat{i}-2\hat{j}+2\hat{k}$ and $\overrightarrow{OC}=\frac{1}{2}\left(\overrightarrow{OB}-\lambda\overrightarrow{OA}\right)$ for some $\lambda>0$. If $\left|\overrightarrow{OB}\times\overrightarrow{OC}\right|=\frac{9}{2}$, then which of the following statements is(are) True?

A) Projection of $\overrightarrow{OC}$ on $\overrightarrow{OA}$ is $-\frac{3}{2}$

B) Area of the triangle OAB is $\frac{9}{2}$

C) Area of the triangle ABC is $\frac{9}{2}$

D) The acute angle between the diagonals of the parallelogram with adjacent sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$ is $\frac{\pi}{3}$

Ans: ABC

Sol: $\overrightarrow{OC}=\frac{1}{2}\left[(1-2\lambda)\hat{i}-(2)(1+\lambda)\hat{j}+(2-\lambda)\hat{k}\right]$

$$\left|\overrightarrow{OB}\times\overrightarrow{OC}\right|=\frac{9}{2}=\frac{1}{2}\left\|\begin{matrix}\hat{i}&\hat{j}&\hat{k}\\1&-2&2\\1-2\lambda&-2-2\lambda&2-\lambda\end{matrix}\right\|=\frac{1}{2}\left\|\begin{matrix}\hat{i}&\hat{j}&\hat{k}\\1&-2&2\\2\lambda-1&2\lambda+2&\lambda-2\end{matrix}\right\|$$

$=\frac{1}{2}\left|-6\lambda\hat{i}+3\lambda\hat{j}+(6\lambda)\hat{k}\right|$

$\Rightarrow 9=\sqrt{36+9+36}\,|\lambda|=\lambda=1$

$\therefore \overline{OC} = \frac{1}{2}\left(-\hat{i} - 4\hat{j} + \hat{k}\right)$

A) $\frac{\overline{OC}.\overline{OA}}{OA} = \frac{-2-8+1}{2.3} = \frac{-3}{2}$

B) $ar\left(\Delta OAB\right) = \frac{1}{2}\left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{matrix}\right\| = \frac{1}{2}\left|-6\hat{i} - 3\hat{j} - 6\hat{k}\right| = \frac{9}{2}$

C) $ar\left(ABC\right) = \frac{1}{2}\left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & -1 \\ \frac{5}{2} & \frac{8}{2} & \frac{1}{2} \end{matrix}\right\| = \frac{1}{4}\left|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & -1 \\ 5 & 8 & 1 \end{matrix}\right|$

$= \frac{1}{4}\left|12\hat{i} - 6\hat{j} - 12\hat{k}\right| = \frac{1}{2}\left|6\hat{i} - 3\hat{j} - 6\hat{k}\right| = \frac{9}{2}$

D) $\vec{d_1} = \overline{OA} + \overline{OC}, \vec{d_2} = \overline{OA} - \overline{OC}$ $\quad \cos\theta = \frac{\vec{d_1}\vec{d_2}}{\left|\vec{d_2}\right|.\left|\vec{d_2}\right|} = \frac{9 - \frac{9}{2}}{\frac{3}{2}\sqrt{2}.\frac{3\sqrt{10}}{2}} = \sqrt{\frac{2}{5}}$

6. Let E denote the parabola $y^2 = 8x$. Let $P = \left(-2, 4\right)$, and let Q and Q' be two distinct points on E such that the lines PQ and PQ' are tangents to E. Let F be the focus of E. Then which of the following statements is(are) TRUE?

A) The triangle PFQ is a right- angled triangle

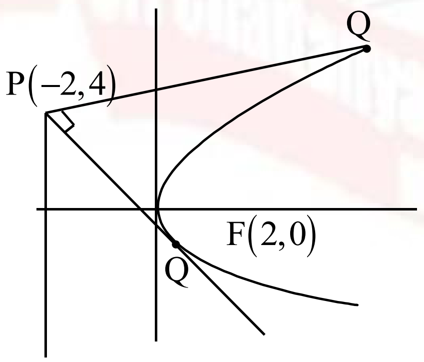
B) The triangle QPQ' is a right – angled triangle

C) The distance between P and F is $5\sqrt{2}$

D) F lies on the line joining Q and Q'

Ans: ABD

Sol: $y^2 = 8x; p\left(-2, 4\right)$ lies on directrix



A) Portion of tangent between POC and directrix substneds 90°at focus

B) Tangents at ends of focal chords are perpendicular

C) $\sqrt{4+4} = 4\sqrt{2}$

D) QQ' is a focal chord.

SECTION-2(Maximum Marks: 12)

Paragraph with Numerical

- This section contains THREE (03) question stems.
- There are TWO (02) questions corresponding to each question stem.
- The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +2 If ONLY the correct numerical value is entered at the designated place;
Zero Marks : 0 In all other cases.

Question Stem for Question Nos. 7 and 8

Consider the region $R = \{(x,y) \in \mathbb{R} \times \mathbb{R} : x \geq 0 \text{ and } y^2 \leq 4 - x\}$. Let F be the family of all circles that are contained in R and have centers on the x-axis. Let C be the circle that has largest radius among the circle in F. Let (α, β) be a point where the circle C meets the curve $y^2 = 4 - x$.

7. The radius of the circle C is ________.

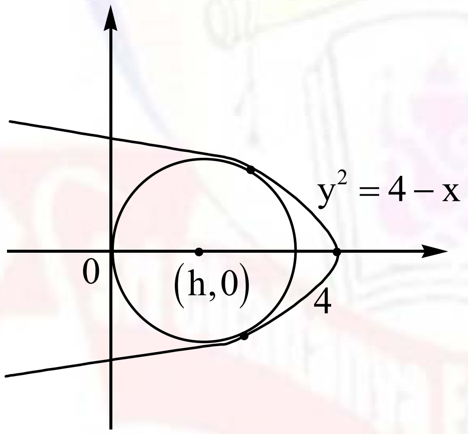
Ans: 1.5

8. The value of α is __________.

Ans: 2

Sol(7&8Q):

$x \geq 0, y^2 \leq 4 - x$



Let equation of circle be $(x-h)^2 + y^2 = h^2$

Solving with $y^2 = 4 - x$

$x^2 - 2hx + 4 - x = 0$

$\Rightarrow x^2 - x(2h+1) + 4 = 0$ .....(1)

For touching, D = 0

$\Rightarrow (2h+1)^2 = 16$

$\Rightarrow 2h+1=4 \qquad \Rightarrow h=\frac{3}{2}$

Putting $h=\frac{3}{2}$ in (1)

$x^2-4x+4=0 \qquad \Rightarrow x=2$

So $\alpha=2$

Question Stem for Question Nos. 9 and 10

Let $f_1:(0,\infty)\to\mathbb{R}$ and $f_2:(0,\infty)\to\mathbb{R}$ be defined by

$$f_1(x)=\int_0^x \prod_{j=1}^{21}(t-j)^j\,dt, \qquad x>0$$

and

$$f_2(x)=98(x-1)^{50}-600(x-1)^{49}+2450, x>0,$$

where, for any positive integer n and real numbers $a_1,a_2,......,a_n,\prod_{i=1}^{n}a_i$ denotes the product of $a_1,a_2,.....,a_n$. Let m_i and n_i, respectively, denote the number of points of local minima and the number of points of local maxima of function $f_i, i=1,2,$ in the interval $(0,\infty)$

9. The value of $2m_1+3n_1+m_1n_1$ is_______.

Ans: 57

10. The value of $6m_2+4n_2+8m_2n_2$ is _________.

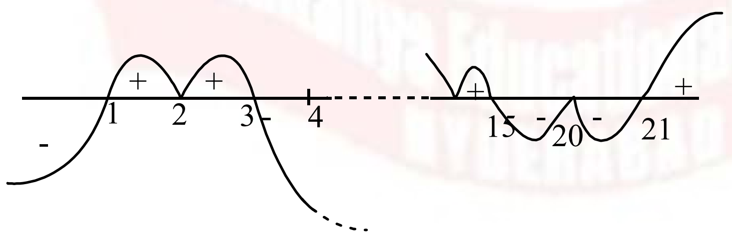
Ans: 6

Sol (9&10Q):

$$f_1(x)=\int_0^x (t-1)^1(t-2)^2(t-3)^3(t-4)^4......(t-21)^{21}\,dt$$

$$f_1^1(x)=(x-1)(x-2)^2(x-3)^3(x-4)^4....(x-21)^{21}$$

Plotting wavycurve of $f_1^1(x)$



So for $x=4k+1,(k\in W)f_1^1(x)$ changes sign from –ve to +ve

for $x=4k+3,(k\in W)f_1^1(x)$ changes sign from +ve to –ve

So m_1 = no.of local minima = 6

n_1 = no.of local maxima = 5

(Q9) $2m_1 + 3n_1 + m_1 n_1 = 12 + 15 + 30 = 57$

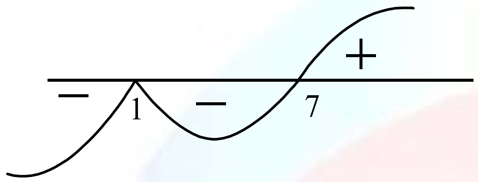
$f_2(x) = 98(x-1)^{50} - 600(x-1)^{49} + 2450$

$f_2^1(x) = 98 \times 50(x-1)^{49} - 600 \times 49(x-1)^{48}$

$= 98 \times 50(x-1)^{48}(x-1-6)$

$= 98 \times 50(x-1)^{48}(x-7)$

Wavy curve of $f_2'(x)$ is



Clearly $m_2 = 1, n_2 = 0$

(10) $6m_2 + 4n_2 + 8m_2 n_2 = 6 + 0 + 0 = 6$

Question Stem for Question Nos. 11 and 12

Let $g_i : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \to \mathbb{R}, i = 1,2$, and $f : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \to \mathbb{R}$ be functions such that

$g_1(x) = 1, g_2(x) = |4x - \pi|$ and $f(x) = \sin^2 x$, for all $x \in \left[\frac{\pi}{8}, \frac{3\pi}{8}\right]$

Define

$$S_i = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} f(x).g_i(x)\,dx, i = 1,2$$

11. The value of $\frac{16s_1}{\pi}$ is ________.

Ans: 2

12. The value of $\frac{48s_2}{\pi^2}$ is ________.

Ans: 1.5

Sol(11&12Q):

$$S_1 = \int_{\frac{\pi}{8}}^{3\frac{\pi}{8}} \sin^2 x.1.dx \quad = \int_{\frac{\pi}{8}}^{3\frac{\pi}{8}} \frac{1-\cos 2x}{2} dx$$

$$= \frac{x}{2} - \frac{\sin 2x}{4}\Big|_{\pi/8}^{3\pi/8} \quad = \frac{\pi}{8} - \frac{1}{4}\left(\sin\frac{3\pi}{4} - \sin\frac{\pi}{4}\right) \qquad = \frac{\pi}{8}$$

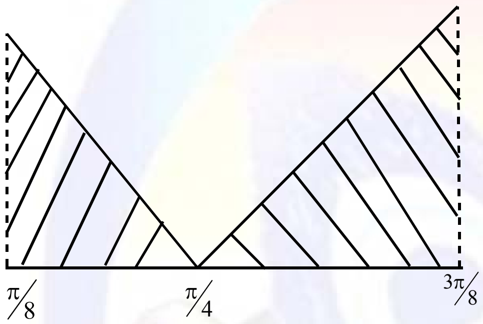
(Q11) $\frac{16S_1}{\pi}=2$

$S_2=\int_{\pi/8}^{3\pi/8}|4x-\pi|.\sin^2 x\,dx$

Using King's Property

$S_2=\int_{\pi/8}^{3\pi/8}|\pi-4x|.\cos^2 xdx$

Adding $2S_2=\int_{\pi/8}^{3\pi/8}|\pi-4x|dx$



$\Rightarrow 2S_2=$ area under graph $=2\times\frac{1}{2}\times\frac{\pi}{8}\times\frac{\pi}{2}=\frac{\pi^2}{16}$

(Q12) $\Rightarrow \frac{48S_2}{\pi^2}=\frac{24}{16}=1.5$

SECTION-3(Maximum Marks: 12)

Paragraph with Single Answer Type

- This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO (02) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

Paragraph-1:

Let $M=\{(x,y)\in\mathbb{R}\times\mathbb{R}:x^2+y^2\le r^2\}$,

where $r>0$. Consider the geometric progression $a_n=\frac{1}{2^{n-1}}, n=1,2,3,.....$ Let $S_0=0$ and, for $n\ge1$, let S_n denote the sum of the first n terms of this progression. For $n\ge1$, let C_n denote the circle with center $\left(S_{n-1},0\right)$ and radius a_n, and D_n denote the circle with center $\left(S_{n-1},S_{n-1}\right)$ and radius a_n.

13. Consider M with $r=\frac{1025}{513}$. Let k be the number of all those circles C_n that are inside M. Let l be the maximum possible number of circles among these k circles such that no two circles intersect. Then

A) $k+2l=22$ B) $2k+l=26$ C) $2k+3l=34$ D) $3k+2l=40$

Ans: D

14. Consider M with $r=\frac{\left(2^{199}-1\right)\sqrt{2}}{2^{198}}$. The number of all those circles D_n that are inside M is

A) 198 B) 199 C) 200 D) 201

Ans: B

Sol(13&14Q):

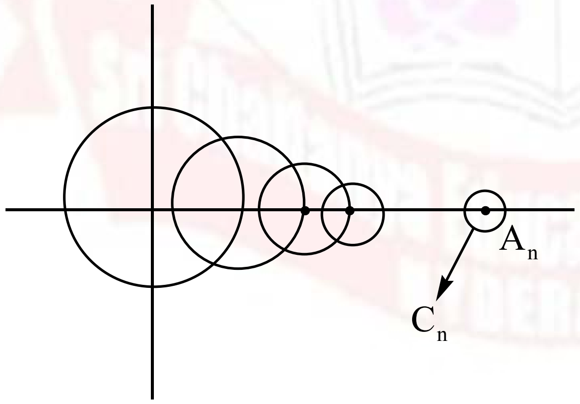
$$S_n=\sum_{n=1}^{n}a_n=\frac{1}{2^0}+\frac{1}{2^1}+\frac{1}{2^2}+.....+\frac{1}{2^{n-1}}$$

$$=\frac{1-\frac{1}{2^n}}{1-\frac{1}{2}}=\frac{2^n-1}{2^{n-1}}$$

For C_n, centre of circle is $\left(\frac{2^{n-1}-1}{2^{n-2}},0\right)$

Radius of circle $=\frac{1}{2^{n-1}}$

Plotting circles,



Finding bigger x intercept of C_n

$$=\frac{2^{n-1}-1}{2^{n-2}}+\frac{1}{2^{n-1}}=\frac{2^n-2+1}{2^{n-1}}=\frac{2^n-1}{2^{n-1}}$$

For C_n to be inside M for $r = \frac{1025}{513} = \frac{2^{10}+1}{2^9+1}$

$$\frac{2^n-1}{2^{n-1}} \le \frac{2^{10}+1}{2^9+1}$$

$$\Rightarrow 2 - \frac{1}{2^{n-1}} \le \frac{2^{10}+1}{2^9+1} \qquad \Rightarrow \frac{1}{2^{n-1}} \ge \frac{1}{2^9+1}$$

$$\Rightarrow 2^{n-1} \le 2^9+1 \Rightarrow n-1 \le 9 \Rightarrow n \le 10$$

Hence no. Of circles possible = 10

For non intersecting pair of circles we need to choose alternating circles. Hence maximum 5 circles can be chosen.

So. k = 10, l =5

3k + 2l = 40

For D_n centre is $\left(\frac{2^{n-1}-1}{2^{n-2}}, \frac{2^{n-1}-1}{2^{n-2}}\right)$

Radius $= \frac{1}{2^{n-1}}$

Max distance of a point on D_n from origin.

= (Distance of centre from origin) + (radius)

$$= \frac{2^{n-1}-1}{2^{n-2}}\sqrt{2} + \frac{1}{2^{n-1}} = \frac{(2^n-2)\sqrt{2}+1}{2^{n-1}}$$

So for D_n to be inside M with $r = \left(\frac{2^{199}-1}{2^{198}}\right)\sqrt{2}$

$$\frac{(2^n-2)\sqrt{2}+1}{2^{n-1}} \le \frac{(2^{199}-1)\sqrt{2}}{2^{198}}$$

$$\Rightarrow \frac{(2^{n-1}-1)\sqrt{2}}{2^{n-2}} \frac{(2^n-2)\sqrt{2}+1}{2^{n-1}} \le \frac{(2^{199}-1)\sqrt{2}}{2^{198}} \qquad \Rightarrow n < 200$$

Hence 199 circles are possible

Paragraph-II:

Let $\Psi_1 : [0,\infty) \to \mathbb{R}, \Psi_2 : [0,\infty) \to \mathbb{R}, f : [0,\infty) \to \mathbb{R}$ and $g : [0,\infty) \to \mathbb{R}$ be functions such that $f(0) = g(0) = 0$,

$\Psi_1(x) = e^{-x} + x, x \ge 0,$

$\Psi_2(x) = x^2 - 2x - 2e^{-x} + 2, x \ge 0,$

$$f(x) = \int_{-x}^{x} (|t| - t^2) e^{-t^2} dt, x > 0$$

and

$$g(x)=\int_0^{x^2}\sqrt{t}\,e^{-t}dt, x>0.$$

15. Which of the following statements is TRUE?

A) $f\left(\sqrt{\ln 3}\right)+g\left(\sqrt{\ln 3}\right)=\frac{1}{3}$

B) For every $x>1$, there exists an $\alpha\in(1,x)$ such that $\Psi_1(x)=1+\alpha x$

C) For every $x>0$, there exists a $\beta\in(0,x)$ such that $\Psi_2(x)=2x\left(\Psi_1(\beta)-1\right)$

D) f is an increasing function on the interval $\left[0,\frac{3}{2}\right]$

Ans: C

Sol:

$$\Psi_1(x)=e^{-x}+x, x\ge 0, \qquad \Psi_2(x)=x^2-2x-2e^{-x}+2, x\ge 0$$

$$f(x)=\int_{-x}^{x}\left(|t|-t^2\right)e^{-t^2}dt, x>0$$

$$\Rightarrow f(x)=2\int_0^x\left(|t|-t^2\right)e^{-t^2}dt, x>0 \qquad =2\int_0^x\left(t-t^2\right)e^{-t^2}dt \;......(1)$$

$$g(x)=\int_0^{x^2}\sqrt{t}.e^{-t}dt, x>0$$

$$t=z^2 \qquad =\int_0^x z.e^{-z^2}.2z.dz$$

$$=2\int_0^x z^2.e^{-z^2}.dz \qquad =2\int_0^x t^2.e^{-t^2}.dt \qquad .....(2)$$

$$f^1(x)=2\left(x-x^2\right)e^{-x^2}$$

$$=2x(1-x)e^{-x^2}$$

$f\uparrow$ for $x\in(0,1)$

$f\downarrow$ for $x\in(1,\infty)$ (Option D is wrong)

$$\therefore f(x)+g(x)=2\int_0^x t.e^{-t^2}dt$$

$$=\left[-e^{-t^2}\right]_0^x=\left(-e^{-x^2}\right)-(-1)=1-e^{-x^2}$$

$$\therefore f(x)+g(x)=1-e^{-x^2}$$

$\therefore f\left(\sqrt{\ln 3}\right)+g\left(\sqrt{\ln 3}\right)=1-\frac{1}{3}=\frac{2}{3}$

(Option A wrong)

$\Psi_1(x)=e^{-x}+x, x\ge 0$

$\Psi_1^1(x)=1-e^{-x}x\ge 0\Rightarrow \Psi_1(x)$ is increasing

$\Psi(0)=1$

$\Psi_1^1(0)=0$

$\Psi_1''(x)=e^{-x}>0$

$\therefore \Psi_1(x)$ is concave up

Method-I:

$\Psi_2(x)=x^2-2x-2e^{-x}+2, x\ge 0$

$\Psi_2(0)=0$

$\Psi_2^1(x)=2x-2+2.e^{-x}$

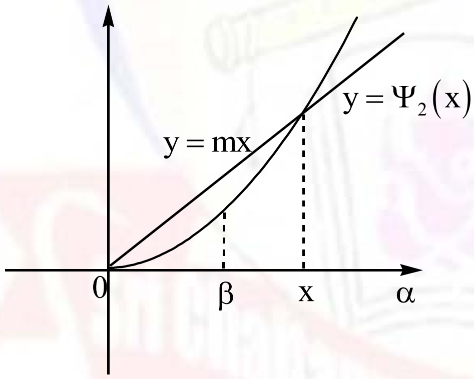
$=2\left(x-1+e^{-x}\right)$

$\Psi_2''(x)=2\left(1-e^{-x}\right)>0$ (concave up)

$\Psi_2(x)=2\left(\Psi_1(\beta)-1\right)x$

$\Psi_2(x)=mx\,(m>0)$

Option C correct



Method-2:

LMVT

$\Psi_2'(\beta)=\frac{\Psi_2(x)-\Psi_2(0)}{x-0}\Rightarrow 2\left(\Psi_1(\beta)-1\right)=\frac{\Psi_2(x)-0}{x-0}\quad\Rightarrow \Psi_2(x)=2\left(\Psi_1(\beta)-1\right)$

16. Which of the following statements is TRUE?

A) $\Psi_1(x)\le 1,$ for all $x>0$

B) $\Psi_2(x)\le 0,$ for all $x>0$

C) $f(x)\ge 1-e^{-x^2}-\frac{2}{3}x^3+\frac{2}{5}x^5,$ for all $x\in\left(0,\frac{1}{2}\right)$

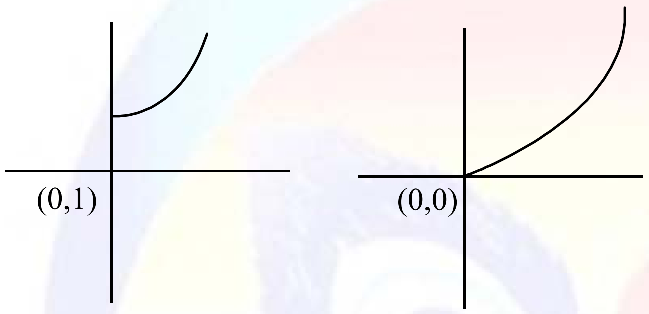
D) $g(x) \le \frac{2}{3}x^3 - \frac{2}{3}x^5 + \frac{1}{7}x^7$, for all $x \in \left(0, \frac{1}{2}\right)$

Ans: D

Sol: $g(x) = 2\int_0^x t^2\left(1 - t^2 + \frac{t^4}{2!}.....\right)dt$

$= 2\int_0^x \left(t^2 - t^4 + \frac{t^6}{2!}.....\right)dt \qquad = 2\left[\frac{t^3}{3} - \frac{t^5}{5} + \frac{t^7}{7.2!}....\right]_0^x dt$

$= 2\left[\frac{x^3}{3} - \frac{x^5}{5} + \frac{x^7}{7.2!}......\right] = \frac{2x^3}{3} - \frac{2}{5}x^5 + \frac{x^7}{7}...... \qquad \le \frac{2}{3}x^3 - \frac{2}{5}x^5 + \frac{x^7}{7}$



$\Psi_1(x) = e^{-x} + x, x \ge 0$

$\Psi_1(x) \ge 1$

Option A wrong $\quad \Psi_2(x) = x^2 - 2x - 2e^{-x} + 2, x \ge 0$

$\Psi_2(x) \ge 0 \qquad \therefore$ Option B wrong also Option C wrong

SECTION-4(Maximum Marks: 12)

Non-Negative Integer Answer Type

- This section contains THREE (03) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If ONLY the correct integer is entered;

Zero Marks : 0 In all other cases.

17. A number is chosen at random from the set $\{1,2,3,.....,2000\}$. Let p be the probability that the chosen number is a multiple of 3 or a multiple of 7. Then the value of 500p is ________.

Ans: 214

Sol: $\{1,2,3,......,2000\}$

$E_1 =$ Event that it is a multiple of 3

$E_2 =$ Event that it is a multiple of 3

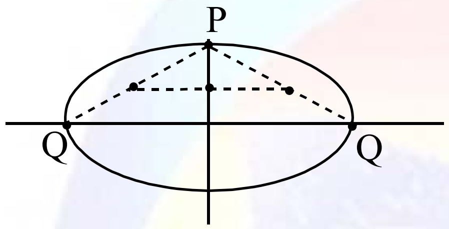
$\therefore P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$$= \frac{666+285-95}{2000} = \frac{856}{2000} \qquad \therefore GE = 500 \times \frac{856}{2000} = \frac{856}{4} = 214$$

18. Let E be the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$. For any three distinct points P,Q and Q' on E, let M(P,Q) be the mid-point of the line segment joining P and Q, and $M(P,Q')$ be the mid-point of the line segment joining P and Q'. Then the maximum possible value of the distance between $M(P,Q)$ and $M(P,Q')$, as P,Q and Q' vary on E, is ______

Ans: 4

Sol: Maximum chord = 2a = 8



Required distance between

$$M(P,Q), M(P,Q') = \frac{1}{2}(8) = 4$$

19. For any real number x, let [x] denote the largest integer less than or equal to x. If $I = \int_0^{10} \left[\sqrt{\frac{10x}{x+1}}\right] dx$, then the value of 9I is ________.

Ans: 182

Sol: $\phi(x) = \frac{10x}{x+1} \Rightarrow \phi'(x) > 0 \Rightarrow \phi \uparrow \qquad \therefore \phi(0) = 0, \phi(10) = \frac{100}{11} = 9.01$

$\sqrt{\frac{10x}{x+1}} \in [0, 3.01] \qquad \frac{10x}{x+1} = 1 \text{ (or) } 4 \text{ (or) } 9$

$9x = 1 \qquad 10x = 4x+4 \qquad 10x = 9x+9$

$x = \frac{1}{9} \qquad x = \frac{2}{3} \qquad x = 9$

$$GI = \int_0^{1/9} 0.dx + \int_{1/9}^{2/3} 1.dx + \int_{2/3}^{9} 2.dx + \int_9^{10} 3.dx = 0 + \left(\frac{2}{3} - \frac{1}{9}\right) + 2\left(9 - \frac{2}{3}\right) + 3(10-9)$$

$$= \frac{5}{9} + \frac{50}{3} + 3 = \frac{5+150+27}{9} = \frac{182}{9} \Rightarrow 9I = 182$$