Sri Chaitanya IIT Academy., India.

AP, TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

A right Choice for the Real Aspirant

ICON Central Office, Madhapur–Hyderabad

PHYSICS

SECTION-1 (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the nuermical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will bee evaluated according to the following marking scheme :

Full Marks : +3 **ONLY** if the correct numerical value is entered ;

Partial Marks : 0 In all other cases.

1. Two spherical stars *A* and *B* have densities ρ_A and ρ_B, respectively. *A* and *B* have the same radius, and their masses M_A and M_B are related by $M_B = 2M_A$. Due to an interaction process, star *A* loses some of its mass, so that its radius is halved, while its spherical shape is retained, and its density remains ρ_A. The entire mass lost by *A* is deposited as a thick spherical shell on *B* with the density of the shell being ρ_A. If v_A and v_B are the escape velocities from A and B after the interaction process, the ratio $\frac{v_B}{v_A} = \sqrt{\frac{10n}{15^{1/3}}}$. The value of *n* is _________.

Ans. 2.3

Sol. $V_C = \sqrt{\frac{2Gm}{R}}$

mass of star A after the process = $\frac{m}{8}$

mass of star B after the process $= 2m_A + \frac{7m_4}{8}$

$= \frac{23m_A}{8}$

$$V_A = \sqrt{\frac{2Gm_A}{8\frac{R}{2}}} = \sqrt{\frac{Gm_A}{2R}}$$

$$V_B = \sqrt{\frac{G23m_A 2}{8\left(\frac{15}{8}\right)^{\frac{1}{3}} R}} = \sqrt{\frac{23Gm}{215^{\frac{1}{3}} R}}$$

$$\frac{V_B}{V_A} = \sqrt{\frac{23}{15^{\frac{1}{3}}}}\ 10n = 23\ n = 2.3$$

2. The minimum kinetic energy needed by an alpha particle to cause the nuclear reaction ${}^{16}_{7}N + {}^{4}_{2}He \rightarrow {}^{1}_{1}H + {}^{19}_{8}O$ in a laboratory frame is *n* (in MeV). Assume that ${}^{16}_{7}N$ is at rest in the laboratory frame. The masses of ${}^{16}_{7}N + {}^{4}_{2}He, {}^{1}_{1}H$ and ${}^{19}_{8}O$ can be taken to be 16.006 u, 4.003 u, 1.008 u and 19.003 u, respectively, where $1u = 930 MeVc^{-2}$. The value of *n* is ____________.

Ans. 2.32

Sol. $\frac{1}{2}\mu V_{rel}^2 = |2|$

$$|Q| = \left(m_0 + m_4 + m_N - m_\mu\right)C^2$$

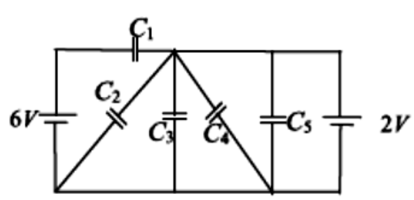
$$= 0.002 \times 930\,MeV$$

$$= 1.86 MeV$$

$$\frac{1}{2}\frac{4m16m}{20m}V_{rel}^2 = 1.86$$

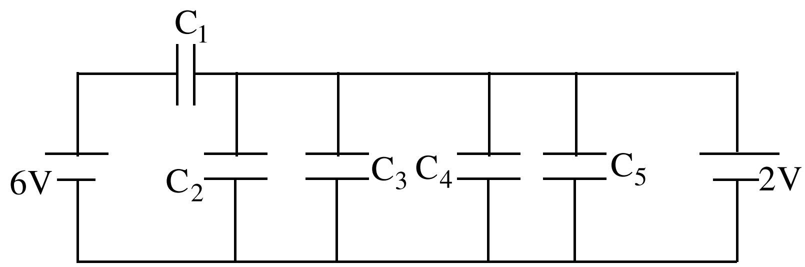
$$\frac{4}{5}K.\in_\alpha = 1.86$$

3. In the following circuit $C_1 = 12\mu F, C_2 = C_3 = 4\mu F$ and $C_4 = C_5 = 2\mu F$. The charge stored in C_3 is ____________ μC.



Ans. 8

Sol.

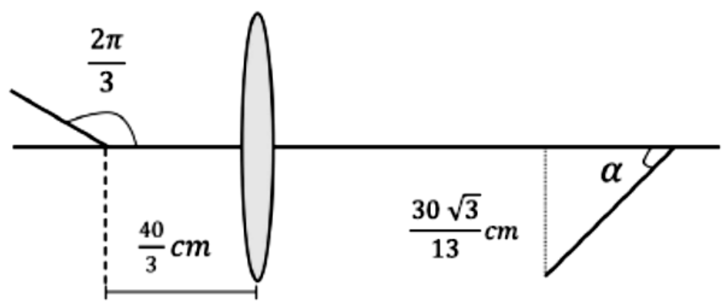


Potential difference across C_3 is 2V.

$$\therefore \frac{Q}{C_3} = 2$$

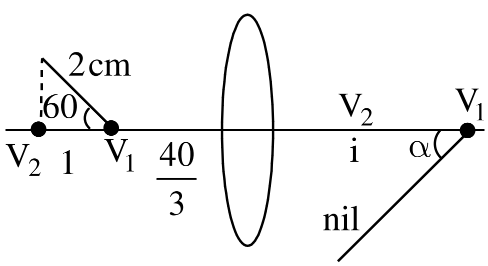
$$Q = 8\mu C$$

4. A rod of length 2 cm makes an angle $\frac{2\pi}{3}$ rad with the principal axis of a thin convex lens. The lens has a focal length of 10 *cm* and is placed at a distance of $\frac{40}{3}$ *cm* from the object as shown in the figure. The height of the image is $\frac{30\sqrt{3}}{13}$ cm and the angle made by its with respect to the principal axis is α rad. The value of α is $\frac{\pi}{n}$ rad, where n is ____________________



Ans. 6

Sol.



$$\frac{1}{V} - \frac{1}{V} = \frac{1}{f} \qquad U = -\frac{40}{3}$$

$$\frac{1}{V}+\frac{3}{40}=\frac{1}{10}$$

$$\frac{1}{V}=\frac{1}{10}-\frac{3}{40}$$

V = 40

$$V=-\left(\frac{40}{3}+1\right)=-\frac{43}{3}$$

$$\frac{1}{V}+\frac{3}{43}=\frac{1}{10}$$

$$\frac{1}{V}=\frac{1}{10}-\frac{3}{43}$$

$$V=\frac{430}{13}$$

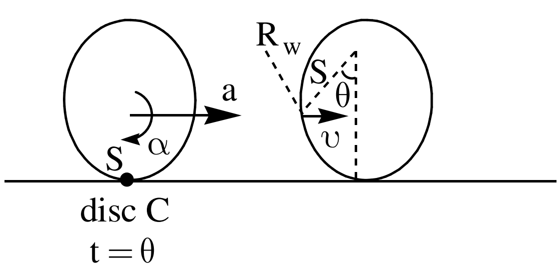
$$\tan\alpha=\frac{h_i}{V_2-V_1}=\frac{\frac{30\sqrt{3}}{13}}{\frac{90}{13}}=\frac{1}{\sqrt{3}}$$

$$\therefore X=\frac{\pi}{6}$$

$$\therefore n=6$$

5. At time t = 0, a disk of radius 1 *m* starts to roll without slipping on a horizontal plane with an angular acceleration of $\alpha=\frac{2}{3}\text{rad s}^{-2}$. A small stone is stuck to the disk. At t = 0, it is at the contact point of the disk and plane. Later, at time $t=\sqrt{\pi}$ s, the stone detaches itself and flies off tangentially from the disk. The maximum height (in *m*) reached by the stone measured from the plane is $\frac{1}{2}+\frac{x}{10}$. The value of *x* is _____. [Take $g=10\text{m s}^{-2}$.]

Ans. 0.52



Sol.

Angle rotated in $\sqrt{\pi}$ xc $\text{is}(\theta)=\frac{1}{\not{2}}\frac{\not{2}}{3}\pi$

$=\frac{\pi}{3}$

$W=\alpha t=\frac{2}{3}\sqrt{\pi}.\text{Rad / sec}$

$V=Rw=\frac{2}{3}\sqrt{\pi}\,\text{m / sec}$

Vertical component of velocity of stone = $\frac{2}{3}\sqrt{\pi}\sin 60$

Maximum height = $\sqrt{\frac{\pi}{3}}$

From point of projection = $\frac{V_v^2}{2g}$

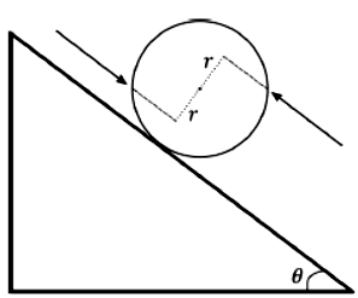
$=\frac{\pi}{60}$

Maximum height from the plane = $R(1-\cos\theta)+\frac{\pi}{60}$

$=\frac{1}{2}+\frac{\pi}{60}$

$\therefore x=\frac{\pi}{6}=0.52$

6. A solid sphere of mass 1 kg and radius 1 *m* rolls without slipping on a fixed inclined plane with an angle of inclination $\theta=30^0$ from the horizontal. Two forces of magnitude 1 *N* each, parallel to the incline, act on the sphere, both at distance r = 0.5 *m* from the center of the sphere, as shown in the figure. The acceleration of the sphere down the plane is ______________ ms^{-2} .(Take $g=10\,ms^{-2}$.)



Ans. 2.85-2.86

Sol. Torque due to the

couple is = $1\times 2(0.5)$

= 1 Nm

Torque about instantaneous axis of rotation

$I_{ioR}\alpha = mg\sin 3OR - T_C$

$=1\times 10\times \frac{1}{2}\times 1-1$

$\frac{7}{5}MR^2\alpha = 4$

$\alpha = \frac{20}{7}$

$a_{cm} = R\alpha$

$a_{cm}\frac{20}{7} = 2.857$

$\therefore \quad a_{cm} 2.85-2.86$

7. Consider an LC circuit, with inductance L = 0.1 *H* and capacitance $C=10^{-3}F$, kept on a plane. The area of the circuit is $1\,m^2$. It is placed in a constant magnetic filed of strength B_0 which is perpendicular to the plane of the circuit. At time *t* = 0, the magnetic field strength starts increasing linearly as $B=B_0+\beta t$ with $\beta = 0.04Ts^{-1}$. The maximum magnitude of the current in the circuit is _______ *mA*.

Ans. 4

Sol. **emf =** $\frac{AdB}{dt}$

$\varepsilon = B$

writing kirchoff's loop law

$\varepsilon - \frac{q}{c} = \frac{Ldi}{dt}$

$\frac{d^2q}{dt^2} = \frac{(q - C\in)}{LC}$

$\Rightarrow q - C\in = q_0\sin(\omega t+\phi)$

Applying boundary conditions

t = 0, q = 0, i = 0

$\phi = \frac{\pi}{2}$

$q_0 = -C \in$

$q = C \in (1 - \cos\omega t)$

$i = C \in \omega \sin\omega t$

$i_{max} = C \in \omega$

$= C \in \frac{1}{\sqrt{LC}}$

$\therefore \quad i_{max} = 4mA$

8. A projectile is fired from horizontal ground with speed v and projection angle θ. When the acceleration due to gravity is g, the range of the projectile is d. If at the highest point in its trajectory, the projectile enters a different region where the effective acceleration due to gravity is $g' = \frac{g}{0.81}$, then the new range is d' = nd. The value of n is _________.

Ans. 0.95

Sol. $d = \frac{U^2 \sin 2\theta}{g}$

$d^{|} = \frac{d}{2} + (u\sin\theta)\sqrt{\frac{2h}{g}(0.81)}$

$d^{|} = \frac{d}{2} + u\sin\theta\sqrt{\frac{2}{g}(0.81)\frac{u^2\sin^2\theta}{2g}}$

$= \frac{d}{2} + (0.9)\frac{u^2 \sin\theta\cos\theta}{g}$

$= \frac{d}{2} + 0.9\frac{d}{2} \qquad d^{|} = 0.95d$

SECTION-2 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evalueated according to the following marking scheme :

Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct ;
Partial Marks : +1 If two or more options are correct but **ONLY** two options are chosen, and it is a correct option ;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : –2 In all other cases.

9. A medium having dielectric constant K>1 fills the space between the plates of a parallel plate capacitor. The plates have large area, and the distance between them is d. The capacitor is connected to a battery of voltage V, as shown in Figure (a). Now, both the plates are moved by a distance of $\frac{d}{2}$ from their original positions, as shown in Figure (b).

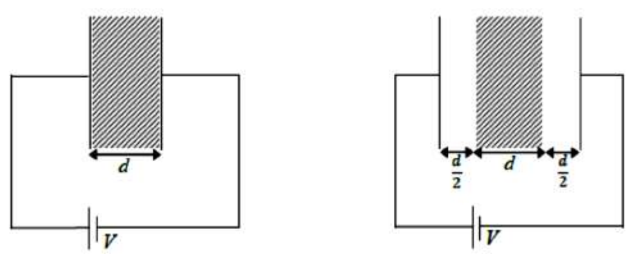


Figure (a) Figure (b)

In the process of going from the configuration depicted in Figure (a) to that in Figure (b), which of the following statement(s) is (are) correct?

A) The electric field inside the dielectric material is reduced by a factor of 2K.

B) The capacitance is decreased by a factor of $\frac{1}{K+1}$.

C) The voltage between the capacitor plates is increased by a factor of (K +1).

D) The work done in the process **DOES NOT** depend on the presence of the dielectric material

Ans. B

Sol. $C_i = \frac{K\varepsilon_0 A}{d}$

$\frac{1}{C_f} = \frac{d}{K\varepsilon_0 A} + \frac{d}{\varepsilon_0 A}$

$C_f = \frac{K\varepsilon_0 A}{d(K+1)}$

$\frac{C_f}{C_i} = \frac{1}{K+1}$

Voltage across the plates remain constant.

$E_i = \frac{V}{d}$

$E_f d + \frac{E_f d}{K} = V$

$E_f = \frac{KV}{(K+1)d}$

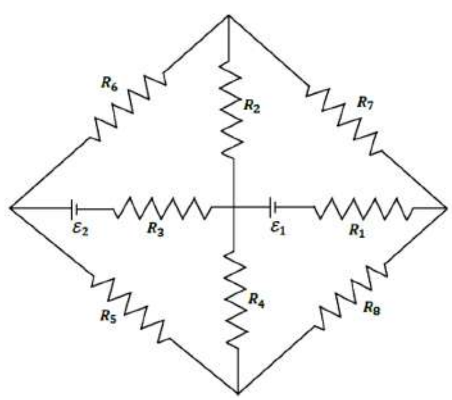
$W_b = \Delta CV^2 = -\frac{KC}{K+1}V^2$

$\Delta V_i = \frac{1}{2}\Delta CV^2 = -\frac{1}{2}\frac{K}{K+1}CV^2$

$\therefore$ Work done by external agent if moved slowly = $\frac{1}{2}\frac{KC}{K+1}V^2$

$\therefore$ B is correct option

10. The figure shows a circuit having eight resistances of 1Ω each, labelled R_1 to R_8, and two ideal batteries with voltages $\varepsilon_1 = 12V$ and $\varepsilon_2 = 6V$.



Which of the following statement(s) is (are) correct?

A) The magnitude of current flowing through R_1 is 7.2 A.

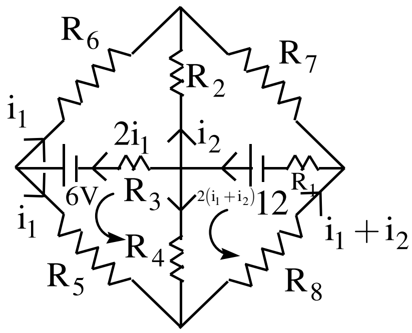
B) The magnitude of current flowing through R_2 is 1.2 A.

C) The magnitude of current flowing through R_3 is 4.8 A.

D) The magnitude of current flowing through R_5 is 2.4 A.

Ans. **ABCD**

Sol.



Let current R_i is $2(i_1+i_2)$ due to symmetry currents R_2R_4 has to be same and let it equal to i_2.

So current in $R_3 = 2i_1$.

Current in R_5, R_6 is i_1 each.

Current R_7, R_8 be i_1+i_2 each.

Kirchoff's law for the left loop.

$$6-2i_1(1)-i_1(1)+i_2=0$$

$$6=3i_1-i_2 \qquad \text{..............} \ (1)$$

Kirchoff's law for right loop.

$$12-2(i_1+i_2)1-i_2(1)-(i_1+i_2)1=0$$

$$12=3i_1+4i_2 \qquad \text{..............} \ (2)$$

Solving for (1) & (2)

$$i_2=\frac{6}{5}=1.2$$

$$i_1=2.4$$

Let current through

$$R_1=2(i_1+i_2)=7.2A$$

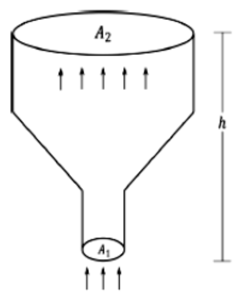
Current through R_2 is $i_2 = 1.2$

Current through R_3 is $2i_1 = 4.8$

Current through R_5 is $i_1 = 2.4$

$\therefore$Correct options are ABCD

11. An ideal gas of density $\rho = 0.2 \text{kg m}^{-3}$ enters a chimney of height h at the rate of $\alpha = 0.8 \text{kg s}^{-1}$ from its lower end, and escapes through the upper end as shown in the figure. The cross-sectional area of the lower end is $A_1 = 0.1 \text{m}^2$ and the upper end is $A_2 = 0.4 \text{m}^2$. The pressure and the temperature of the gas at the lower end are 600 Pa and 300 K, respectively, while its temperature at the upper end is 150 K. The chimney is heat insulated so that the gas undergoes adiabatic expansion. Take g = 10 ms^{-2} and the ratio of specific heats of the gas $\gamma = 2$. Ignore atmospheric pressure.



Which of the following statement(s) is (are) correct?

A) The pressure of the gas at the upper end of the chimney is 300 Pa.

B) The velocity of the gas at the lower end of the chimney is 40 ms^{-1} and at the upper end is 20 ms^{-1}.

C) The height of the chimney is 590 *m*.

D) The density of the gas at the upper end is 0.05 kg m^{-3}

Ans. B

Sol. $TV^{\gamma-1} = C$

$$\frac{T}{\rho^{\gamma-1}} = C$$

$$\frac{300}{0.2} = \frac{150}{\rho}$$

$$\rho_{top} = 0.1$$

$$\frac{dm}{dt} = \text{Constant}$$

$0.8 = 0.2\ (0.1)\ V_\ell$

$V_\ell = 40 m/sec$

$0.8 = 0.1\ 0.4\ V_t$

$V_t = 20 m/s$

Applying Energy Conservation.

$$H + gh + \frac{V^2}{2} = \text{Constant}$$

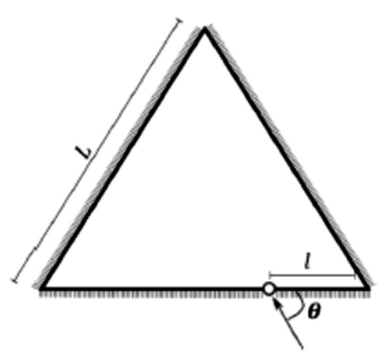
H is enthalpy per unit mass.

$$\frac{2(P_1V_1 - P_2V_2)}{\frac{dm}{dt}} + \frac{V_1^2}{2} - \frac{V_2^2}{2} = gh$$

$$\frac{2(600\times4 - 150\times8)}{0.8} + \frac{40^2}{2} - \frac{20^2}{2} = 10\ h$$

$3600 = 10h$ $\qquad$ $h = 360\ m$

12. Three plane mirrors form an equilateral triangle with each side of length *L*. There is a small hole at a distance $l > 0$ from one of the corners as shown in the figure. A ray of light is passed through the hole at an angle θ and can only come out through the same hole. The cross section of the mirror configuration and the ray of light lie on the same plane



Which of the following statement(s) is (are) correct?

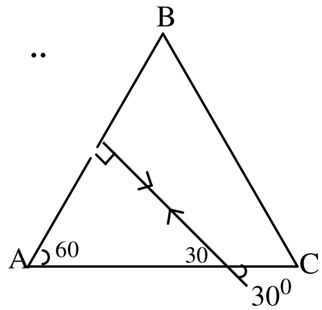
A) The ray of light will come out for $\theta = 30^0$, for $0 < l < L$.

B) There is an angle for $l = \frac{L}{2}$ at which the ray of light will come out after two refections.

C) The ray of light will **NEVER** come out for $\theta = 60^0$, and $l = \frac{L}{3}$.

D) The ray of light will come out for $\theta = 60^0$, and $0 < l < \frac{L}{2}$ after six reflections.

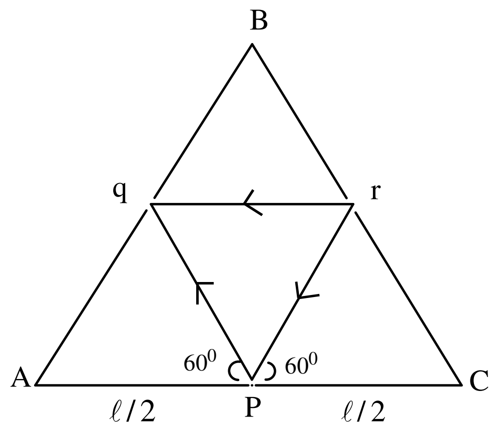
Ans. AB



Sol. a)

for $\theta = 30^0$ angle of incidence on AB is 90^0

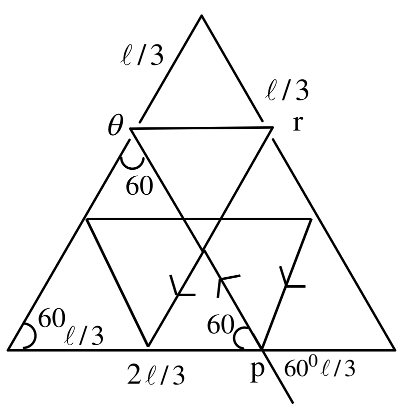
$\therefore$ It will retrace its path and come out



b)

for $\ell = L/2$ if the light ray comes out after 2 reflections then the angle is 60^0

c) for $\theta = 60$ $\ell = \ell/3$

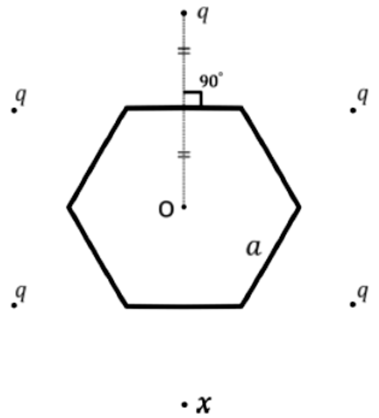


so the light ray comes out after 5 reflections

d) After 5 reflections it retraces comes out of the slit.

Options A and B are correct.

13. Six charges are placed around a regular hexagon of side length a as shown in the figure. Five of them have charge q, and the remaining one has charge x. The perpendicular from each charge to the nearest hexagon side passes through the center O of the hexagon and is bisected by the side.



Which of the following statement(s) is (are) correct in SI units?

A) When x = q, the magnitude of the electric field at 0 is zero.

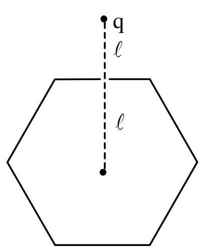
B) When x = -q, the magnitude of the electric field at 0 is $\frac{q}{6\pi \in_0 a^2}$

C) When x = 2q, the potential at 0 is $\frac{q}{4\sqrt{3}\pi \in_0 a}$

D) When x = -3q, the potential at 0 is $-\frac{3q}{4\sqrt{3}\pi \in_0 a}$

Ans. ABC

Sol.



$\ell = \frac{\sqrt{3}}{2}a$

distance of charge from 0 is $2\ell = \sqrt{3}a$

a) if x = q by symmetry electric field at θ is zero

b) if x = -q

$$E = 2\frac{q}{4\pi\varepsilon_0\left(\sqrt{3}a\right)^2} = \frac{q}{6\pi\varepsilon_0 a^2}$$

c) if x = 2q $V = K\sum \frac{q_i}{r_i}$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{7q}{\sqrt{3}a}$$

d) if x = -3 q

$$V = \frac{1}{4\pi\varepsilon_0}\frac{2q}{\sqrt{3}a}$$

$\therefore$ option A, B, C are correct

14. The binding energy of nucleons in a nucleus can be affected by the pairwise Coulomb repulsion. Assume that all nucleons are uniformly distributed inside the nucleus. Let the binding energy of a proton be E_b^p and the binding energy of a neutron be E_b^n in the nucleus.

Which of the following statement (s) is (are) correct?

A) $E_b^p - E_b^n$ is proportional to Z(Z-1) where Z is the atomic number of the nucleus.

B) $E_b^p - E_b^n$ is proportional to $A^{-\frac{1}{3}}$ where A is the mass number of the nucleus.

C) $E_b^p - E_b^n$ is positive.

D) E_b^p increases if the nucleus undergoes a beta decay emitting a positron.

Ans. **ABD**

Sol. a) $E_b^p - E_b^n$ depends only on coulomb repulsion only as nuclear forces are same for both each proton has coulomb repulsion with (z-1) protons there are Z such type of protons.

$\therefore E_b^p - E_b^n \propto z(z-1)$

b) $r = r_0 A^{-1/3}$.

Energy due to coulomb repulsion is inversely proportional to r.

$$\therefore \quad E_b^p - E_b^n \alpha \frac{1}{r} \alpha A^{-y_3}$$

c) binding energy of neutron is more therefore $E_b^p - E_b^n$ is negative

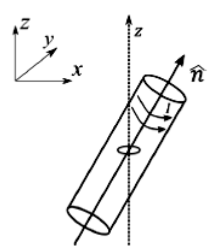
d) Beta decay with positron emission number neutrons increases. So coulomb repulsion decreases increasing the binding energy.

SECTION-3 (Maximum Marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists : **List-I** and **List-II.**
- **List-I** has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Five** entries (P), (Q), (R), (S) and (T).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;
Zero Marks : 0 If none of the options is chosed (i.e. the question is unanswered);
Negative Marks : –1 In all other cases.

15. A small circular loop of area A and resistance R is fixed on a horizontal xy-plane with the center of the loop always on the axis $\hat{n}$ of a long solenoid. The solenoid has m turns per unit length and carries current I counter clockwise as shown in the figure. The magnetic field due to the solenoid is in $\hat{n}$ direction. List-I gives time dependences of $\hat{n}$ in terms of a constant angular frequency ω. List-II gives the torques experienced by the circular loop at time $t = \frac{\pi}{6\omega}$. Let $\alpha = \frac{A^2\mu_0^2 m^2 I^2 \omega}{2R}$.



List-I		List-II	
I)	$\frac{1}{\sqrt{2}}(\sin\omega t\hat{j} + \cos\omega t\hat{k})$	P)	0
II)	$\frac{1}{\sqrt{2}}(\sin\omega t\hat{i} + \cos\omega t\hat{j})$	Q)	$-\frac{\alpha}{4}\hat{i}$
III)	$\frac{1}{\sqrt{2}}(\sin\omega t\hat{i} + \cos\omega t\hat{k})$	R)	$\frac{3\alpha}{4}\hat{i}$
IV)	$\frac{1}{\sqrt{2}}(\cos\omega t\hat{j} + \sin\omega t\hat{k})$	S)	$\frac{\alpha}{4}\hat{j}$
		T)	$-\frac{3\alpha}{4}\hat{i}$

Which one of the following options is correct?

A) $I \to Q, II \to P, III \to S, IV \to T$ B) $I \to S, II \to T, III \to Q, IV \to P$

C) $I \to Q, II \to P, III \to S, IV \to R$ D) $I \to T, II \to Q, III \to P, IV \to R$

Ans. C

Sol. $\phi = \overrightarrow{B}.\overrightarrow{A}$

$$\varepsilon = \frac{-d\phi}{dt}$$

$$i = \frac{\varepsilon}{R}$$

$$T. = i\overrightarrow{A}.\times\overrightarrow{B}.$$

$$T = \frac{1}{R}\frac{d\phi}{dt}.\left(\overrightarrow{A}\times\overrightarrow{B}\right)$$

$$\therefore A)\ i = \frac{1}{R}\frac{\mu_0 miA\omega\sin\omega t}{\sqrt{2}}$$

$$i = \frac{\mu_0 miA\omega\sin\omega t}{\sqrt{2}R}$$

$$\overrightarrow{A}\times\overrightarrow{B} = A\widehat{K}\times\mu_0 mi\left(\frac{1}{\sqrt{2}}\sin\omega t + \hat{j} + \cos\omega t\widehat{k}\right)$$

$$= -\frac{\mu_0 m_{iA}}{\sqrt{2}}\sin\omega t\ \hat{i}$$

$$\therefore \quad T = -\frac{\mu_0^2 m^2 A^2 i^2\omega}{2R}\sin^2\omega t\ \hat{i}.$$

$$\text{for } t = \frac{\pi}{6\omega}\ T = -\frac{\alpha}{4}\hat{i}$$

b) $\phi = 0$

$\therefore \varepsilon = 0$

c) $i = \dfrac{\mu_0 miA\omega}{\sqrt{2}R}\sin\omega t \quad \overrightarrow{A}\times\overrightarrow{B} = \dfrac{A\mu_0 miA\sin\omega t}{\sqrt{2}}j$

$$\therefore \varepsilon = \frac{A^2\mu_0^2 m^2 i^2\omega}{2R}\sin^2\omega t\hat{j}$$

$t = \frac{\pi}{4} \varepsilon = \frac{\alpha}{4}\hat{j}$

d) $i = \frac{-\mu_0 miA\omega \cos\omega t}{\sqrt{2}}$

$\overrightarrow{A} \times \overrightarrow{B} = A\hat{k} \times \frac{-\mu_0 mi}{\sqrt{2}}\left(\cos\omega t\hat{j} + \sin\omega t\hat{k}\right)$

$= \frac{\mu_0 miA}{\sqrt{2}}\cos\omega t\hat{i}$

$\therefore \quad T = i\overline{A} \times \overline{B}$

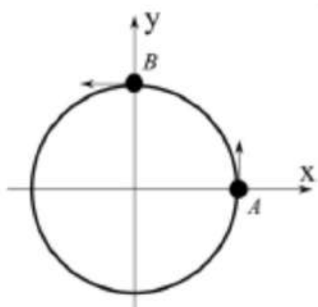
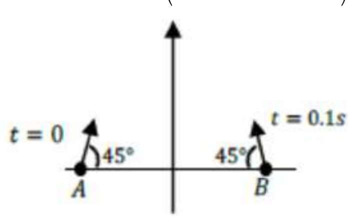
$t = \frac{\mu_0^2 m^2 i^2 A^2 \omega}{2R}\cos^2 \omega t\ \hat{i}$

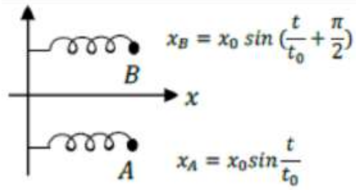
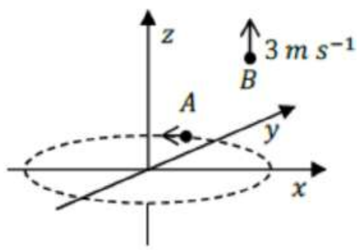
$\therefore \quad$ at $\quad t = \frac{\pi}{6\omega}$

$\tau = \frac{3\alpha}{4}\hat{i}$

$\therefore$ Option C is correct.

16. List I describes four systems, each with two particles A and B in relative motion as shown in figures. List II gives possible magnitudes of their relative velocities (in $m\ s^{-1}$) at time $t=\frac{\pi}{3}$ s.

	List-I		List-II
I)	A and B are moving on a horizontal circle of radius $1m$ with uniform angular speed $\omega = 1 rad\ s^{-1}$. The initial angular positions of A and B at time $t=0$ are $\theta=0$ and $\theta=\frac{\pi}{2}$, respectively. 	P)	$\frac{\sqrt{3}+1}{2}$
II)	Projectiles A and B are fired (in the same vertical plane) at t = 0 and t = 0.1 s respectively, with the same speed $v=\frac{5\pi}{\sqrt{2}} m\ s^{-1}$ and at 45^0 from the horizontal plane. The initial separation between A and B is large enough so that they do not collide. $\left(g=10m\ s^{-2}\right)$. 	Q)	$\frac{\left(\sqrt{3}-1\right)}{\sqrt{2}}$
III)	Two harmonic oscillators A and B moving in the x direction according to $x_A = x_0 \sin\frac{t}{t_0}$ and $x_B = x_0 \sin\left(\frac{t}{t_0}+\frac{\pi}{2}\right)$ respectively, starting from t = 0. Take $x_0 = 1m,\ t_0 = 1s$	R)	$\sqrt{10}$

<table>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>IV)</td><td>Particle A is rotating in a horizontal circular path of radius 1 m on the xy plane, with constant angular speed $\omega = 1 \text{rad s}^{-1}$. Particle B is moving up at a constant speed 3m s^{-1} in the vertical direction as shown in the figure. (Ignore gravity.)
</td><td>S)</td><td>$\sqrt{2}$</td></tr>
<tr><td></td><td></td><td>T)</td><td>$\sqrt{25\pi^2 + 1}$</td></tr>
</table>

A) $I \to R, II \to T, III \to P, IV \to S$

B) $I \to S, II \to P, III \to Q, IV \to R$

C) $I \to S, II \to T, III \to P, IV \to R$

D) $I \to T, II \to P, III \to R, IV \to S$

Ans. C

Sol. I) for option I.

Particle A & B are always perpendicular.

$\therefore \left|\vec{V}_A - \vec{V}_B\right| = \sqrt{1^2 + 1^2 - 2\cos 90}$

$= \sqrt{2}$

II) $V_{xr} = 2V\cos 45$

$= 5\pi$

$V_{yr} = gt = 1$

$\therefore V_r = \sqrt{(5\pi)^2 + 1} = \sqrt{1 + 25\pi^2}$

III) $V_1 = \frac{x_0}{t_0}\cos\frac{t}{t_0}$

$V_2 = \frac{-x_0}{t_0}\sin\frac{t}{t_0}$

$t = \frac{\pi}{3}\sec$

$V_1 = \frac{1}{2}$

$V_2 = -\frac{\sqrt{3}}{2}$

$V_r = \frac{\sqrt{3}+1}{2}$

(IV) $V_r = \sqrt{1^2 + 3^2}$

= $\sqrt{10}$

$\therefore$ Option C is correct answer.

17. List I describes thermodynamic processes in four different systems. List II gives the magnitudes (either exactly or as a close approximation) of possible changes in the internal energy of the system due to the process.

List-I		List-II	
I)	10^{-3} kg of water at $100\,^\circ C$ is converted to steam at the same temperature, at a pressure of 10^5 Pa. The volume of the system changes from $10^{-6}\,m^3$ to $10^{-3}\,m^3$ in the process. Latent heat of water = 2250 kJ/kg.	P)	2kJ
II)	0.2 moles of a rigid diatomic ideal gas with volume V at temperature 500 K undergoes an isobaric expansion to volume 3 V. Assume R = 8.0 $J\ mol^{-1} K^{-1}$.	Q)	7 kJ
III)	One mole of a monatomic ideal gas is compressed adiabatically from volume $V = \frac{1}{3} m^3$ and pressure 2 kPa to volume $\frac{V}{8}$.	R)	4kJ
IV)	Three moles of a diatomic ideal gas whose molecules can vibrates, is given 9 kJ of heat and undergoes isobaric expansion.	S)	5kJ
		T)	3 kJ

Which one of the following options is correct ?

A) $I \to T, II \to R, III \to S, IV \to Q$

B) $I \to S, II \to P, III \to T, IV \to P$

C) $I \to P, II \to R, III \to T, IV \to Q$

D) $I \to Q, II \to R, III \to S, IV \to T$

Ans. C

Sol. (i) $dU = dQ - dW$

$= 2250\ KJ/kg \times 10^{-3} - 10^{5}\left(10^{-3} - 10^{-6}\right)$

$\cong 2250 - 100$

= 2150

= 2.15 KJ

(ii) $dU = \frac{5}{2}nR\Delta T$

$= \frac{5}{2}\times 0.2\times 8\times 1000$

$dU = 4\ KJ$

(iii) $dU = \frac{3}{2}\left(\frac{1}{24}64 - \frac{2}{3}\right)$

= 3 KJ

(iv) dQ = 9 KJ

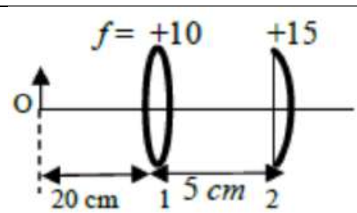
$\frac{dQ}{dU} = \frac{C_P}{C_V}$

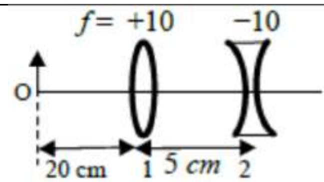
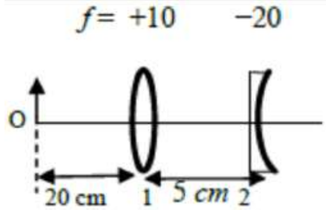
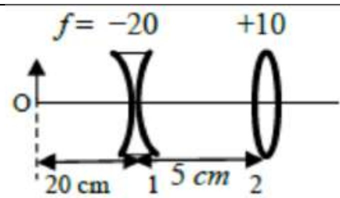
$dU = \frac{dQ}{\gamma}$ $\qquad \gamma = \frac{9}{7}$

dU = 7 KJ

$\therefore$ Option C is correct.

18. List I contains four combinations of two lenses (1 and 2) whose focal lengths (in cm) are indicated in the figures. In all cases, the object is placed 20 cm from the first lens on the left, and the distance between the two lenses is 5 cm. List II contains the positions of the final images.

List-I		List-II	
(I)		P)	Final image is formed at 7.5 cm on the right side of lens 2.

(II)		Q)	Final image is formed at 60.0 cm on the right side of lens 2.
(III)		R)	Final image is formed at 30.0 cm on the left side of lens 2.
(IV)		S)	Final image is formed at 6.0 cm on the right side of lens 2.
		T)	Final image is formed at 30.0 cm on the right side of lens 2.

Which one of the following options is correct ?

A) $(I)\to P;(II)\to R;(III)\to Q;(IV)\to T$
B) $(I)\to Q;(II)\to P;(III)\to T;(IV)\to S$
C) $(I)\to P;(II)\to T;(III)\to R;(IV)\to Q$
D) $(I)\to T;(II)\to S;(III)\to Q;(IV)\to R$

Ans. A

Sol. **(i)** $\frac{1}{V_1}+\frac{1}{20}=\frac{1}{10}$

$V_1=20$

$V_2=+15$

$\frac{1}{V}-\frac{1}{15}=\frac{1}{15}$

$V=7.5$

(ii) $V_1=20$

$V_2=15$

$\frac{1}{V}-\frac{1}{15}=-\frac{1}{10}$

$\frac{1}{V}=\frac{1}{15}-\frac{1}{10}$

$V=20$

(III) $V_1=20$

$V_2 = 15$

$\frac{1}{V} = \frac{1}{15} = -\frac{1}{20}$

$\frac{1}{V} = \frac{1}{15} - \frac{1}{20}$

iv) $\frac{1}{V_1} + \frac{1}{20} = -\frac{1}{20}$

$V_1 = -10$

$V_2 = -15$

$\frac{1}{V_1} + \frac{1}{15} = \frac{1}{10}$

$V_f = 30$

$V\& = \frac{1}{10} - \frac{1}{15}$

Option A is correct

CHEMISTRY

SECTION-1 (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the nuermical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will bee evaluated according to the following marking scheme :

Full Marks : +3 **ONLY** if the correct numerical value is entered ;

Partial Marks : 0 In all other cases.

1. 2 mol of Hg(g) is combusted in a fixed volume bomb calorimeter with excess of O_2 at 298 K and 1 atm into HgO(s). During the reaction, temperature increases from 298.0 K to 312.8 K. If heat capacity of the bomb calorimeter and enthalpy of formation of Hg(g) are 20.00 kJ K^{-1} and 61.32 kJ mol^{-1} at 298 K, respectively, the calculated standard molar enthalpy of formation of HgO(s) at 298 K is X kJ mol^{-1}. The value of $|X|$ is _____.

[Given : Gas constant R = 8.3 J $K^{-1}mol^{-1}$]

Ans. 90.39

Sol. $Hg_{(\ell)} \rightarrow \cancel{Hg}_{(g)}$ $\qquad \Delta H_1 = +61.32 \text{ KJ/mol}$

$$\cancel{Hg}_{(g)} + \frac{1}{2}O_{2(g)} \rightarrow HgO_{(s)} \quad \Delta H_2 = \frac{-20\times14.8}{2} - \frac{3}{2}\times8.3\times298 = -151.7 \text{ KJ / mol}$$

$$Hg_{(\ell)} + \frac{1}{2}O_{2(g)} \rightarrow HgO_{(s)} \qquad \Delta_f H_{(HgO),S} = \Delta H_1 + \Delta H_2$$

$$\Delta_f H_{(HgO),S} = -151.71 + 61.32$$

$$= -90.39 \text{ KJ/mol}$$

2. The reduction potential $\left(E^0, \text{in V}\right)$ of $MnO_4^-(aq)/Mn(S)$ is _____.

[Given : $E^0_{\left(MnO_4^-(aq)/MnO_2(s)\right)} = 1.68\ V; E^0_{\left(MnO_2(s)/Mn^{2+}(aq)\right)} = 1.21V; E^0_{\left(Mn^{2+}(aq)/Mn(s)\right)} = -1.03V$]

Ans. 0.77

Sol. $MnO^-_{4(aq)} \xrightarrow{1.68V} MnO_{2(s)} \xrightarrow{1.21V} Mn^{+2}_{(aq)} \xrightarrow{-1.03V} Mn_{(s)}$

$$E_{MnO_4^-/Mn_{(s)}} = \frac{3(1.68)+2(1.21)+2(-1.03)}{7}$$

$$= \frac{5.4}{7} = 0.771V$$

3. A solution is prepared by mixing 0.01 mol each of $H_2CO_3, NaHCO_3, Na_2CO_3$ and NaOH in 100 mL of water. pH of the resulting solution is _____.

[Given : pK_{a1} and pK_{a2} of H_2CO_3 are 6.37 and10.32 respectively ; log 2 = 0.30]

Ans. 10.02

Sol. 0.01 mole each of $H_2CO_3, NaHCO_3, Na_2CO_3$ and NaOH taken in 100 ml water.

$$H_2CO_3 + NaOH \longrightarrow NaHCO_3 + H_2O$$

0.01   0.01

–   –   0.01 moles

$$\left[HCO_3^-\right] = \frac{0.01}{100}, \left[CO_3^{-2}\right] = \frac{0.01}{100}$$

$$p^H = p^{Ka_2} + \ell og\frac{\left[CO_3^{-2}\right]}{\left[HCO_3^-\right]} = 10.32 + \ell og\frac{1}{2}$$

= 10.02

4. The treatment of an aqueous solution of 3.74 g of $Cu(NO_3)_2$ with excess KI results in a brown solution along with the formation of a precipitate. Passing H_2S through this brown solution gives another precipitate **X**. The amount of **X** (in g) is _____.

[Given : Atomic mass of H =1, N = 14, O = 16, S = 32, K = 39, Cu = 63, I = 127]

Ans. 0.32

Sol. $2Cu(NO_3)_2 + 4KI \rightarrow Cu_2I_2 + 2KI_3 + 2KNO_3$

$KI_3 + H_2S \rightarrow KI + 2HI + S$

$I_2 + H_2S \rightarrow 2HI + S$

$Mol.wt\ of\ Cu(NO_3)_2 = 187$

Another precipitate formed is sulphur

$Cu(NO_3)_2 + KI \rightarrow CuI + I + 2KNO_3$

187 127

3.74 ?

3.74 gm gives ----- 2.54 g of I_2

2.54 gm of I_2 gives – 32

$$\frac{2.54 \times 32}{254} = 0.32\ gm$$

5. Dissolving 1.24 g of white phosphorous in boiling NaOH solution in an inert atmosphere gives a gas **Q.** The amount of $CuSO_4$ (in g) required to completely consume the gas **Q** is _____.

[Given : Atomic mass of H = 1, O = 16, Na = 23, P = 31,S = 32, Cu = 63]

Ans. 2.38

Sol. $P_4 + 3NaOH + 3H_2O \rightarrow PH_3 + 3NaHPO_2$

124 34

$3CuSO_4 + 2PH_3 \rightarrow Cu_3P_2 + 3H_2SO_4$

$477 \; + \; 68 \rightarrow 251$

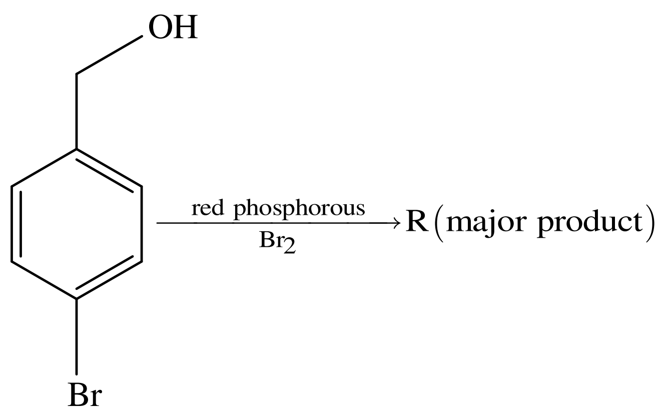
$124\, P_4 \quad gives \quad 34\, gm\, of\; PH_3$

$1.24 \quad gives \quad 0.34\, gm\, of\; PH_3$

$68\, gm\, of\; PH_3\, consume\; 477\, gm\, of\; CuSO_4$

$6.34\, g \rightarrow 2.385$

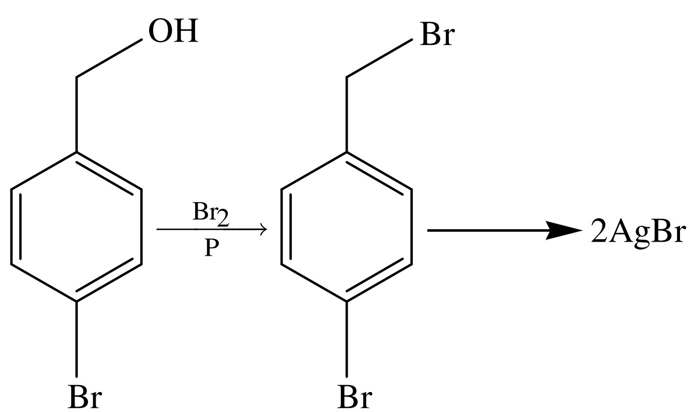
6. Consider the following reaction.



On estimation of bromine in 1.00 g of **R** using Carius method, the amount of AgBr formed (in g) is _____.

[Given : Atomic mass of H = 1, C = 12, O = 16, P = 31, Br = 80, Ag = 108]

Ans. 1.50



Sol. (R)

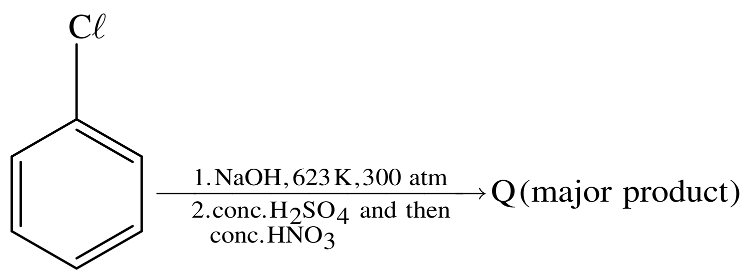
No. of moles of R $= \dfrac{1}{250}$

$\therefore$ No. of moles of AgBr $= \dfrac{2}{250}$

$W_{AgBr} = \dfrac{2}{250} \times 188$

= 1.50 g

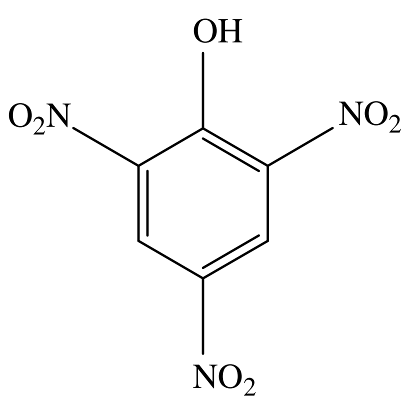
7. The weight percentage of hydrogen in **Q**, formed in the following reaction sequence, _____.



[Given : Atomic mass H = 1, C = 12, N = 14, O = 16. S = 32, Cl = 35]

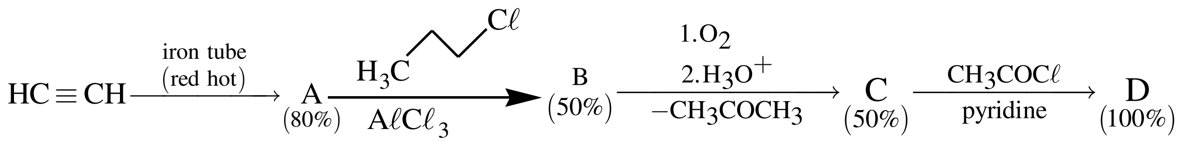
Ans. **1.31**

Sol.



$\% \text{ H} = \dfrac{3}{229} \times 100 = 1.31$

8. If the reaction sequence given below is carried out with 15 moles of acetylene, the amount of the product **D** formed (in g) is _____.

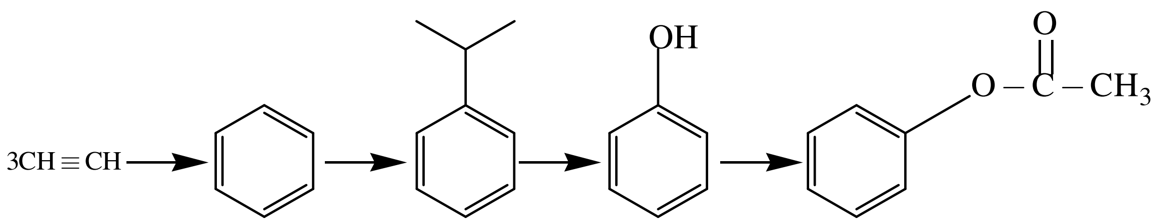


The yields of **A, B, C** and **D** are given in parentheses.

[Given : Atomic mass of H = 1, C = 12, O = 16, Cl = 35)

Ans. 136

Sol.



$15\,mol$ $\quad 5\times0.8$ $\quad 5\times0.8\times0.5$ $\quad 5\times0.8\times0.5\times0.5$ $\quad W = 5\times0.8\times0.5\times0.5\times136$

SECTION-2 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evalueated according to the following marking scheme :

Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct ;
Partial Marks : +1 If two or more options are correct but **ONLY** two options are chosen, and it is a correct option ;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : –2 In all other cases.

9. For diatomic molecules, the correct statements(s) about the molecular orbitals formed by the overlap of two $2p_z$ orbitals is(are)

A) σ orbital has a total of two nodal planes.

B) σ^* orbital has one node in the xz-plane containing the molecular axis.

C) π orbital has one node in the plane which is perpendicular to the molecular axis and goes through the center of the molecule.

D) π^* orbital has one node in the xy-plane containing the molecular axis.

Ans. AD

Sol.

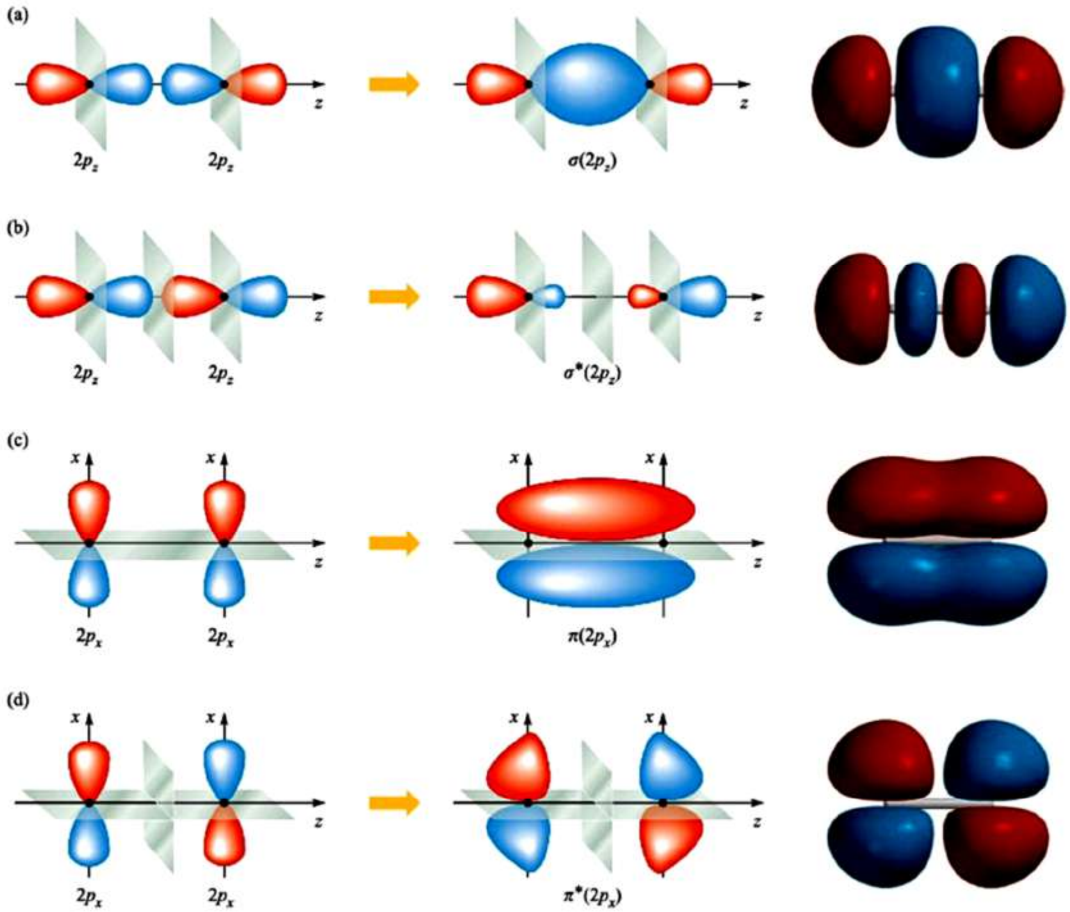


Fig. 2.7 The overlap of two $2p$ atomic orbitals for which the atomic nuclei are defined to lie on the z axis: (a) direct overlap along the z axis gives a $\sigma_g(2p_z)$ MO (bonding); (b) the formation of the $\sigma_u{}^*(2p_z)$ MO (antibonding); (c) sideways overlap of two $2p_x$ atomic orbitals gives a $\pi_u(2p_x)$ MO (bonding); (d) the formation of $\pi_g{}^*(2p_x)$ MO (antibonding). Atomic nuclei are marked in black and nodal planes in grey. The diagrams on the right-hand side are more realistic representations of the MOs and have been generated computationally using Spartan '04, ©Wavefunction Inc. 2003.

10. The correct option(s) related to adsorption processes is(are)

A) Chemisorption results in a unimolecular layer

B) The enthalpy change during physisorption is in the range of 100 to 140 kJ mol^{-1}.

C) Chemisorption is an endothermic process.

D) Lowering the temperature favors physisorption processes.

Ans. **AD**

Sol. (A) Chemisorption is unilayer process.

(B) $\Delta H_{physisorption}$ is less than 100 KJ/mol.

(C) Chemisorption can be exothermic.

(D) Physisorption is lowered by increasing temperature

11. The electrochemical extraction of aluminium from bauxite ore involves

A) the reaction of $A\ell_2O_3$ with coke (C) at a temperature > 2500^0C.

B) the neutralisation of aluminate solution by passing CO_2 gas to precipitate hydrated alumina $(A\ell_2O_3.3H_2O)$

C) the dissolution of $A\ell_2O_3$ in hot aqueous NaOH.

D) the electrolysis of $A\ell_2O_3$ mixed with $Na_3A\ell F_6$ to give $A\ell$ and CO_2.

Ans. BCD

Sol. BCD all are correct statements

12. The treatment of galena with HNO_3 produces a gas that is

A) paramagnetic B) bent in geometry C) an acidic oxide D) colorless

Ans. AD or ABC

Sol. If dil HNO_3 is used the gas is NO

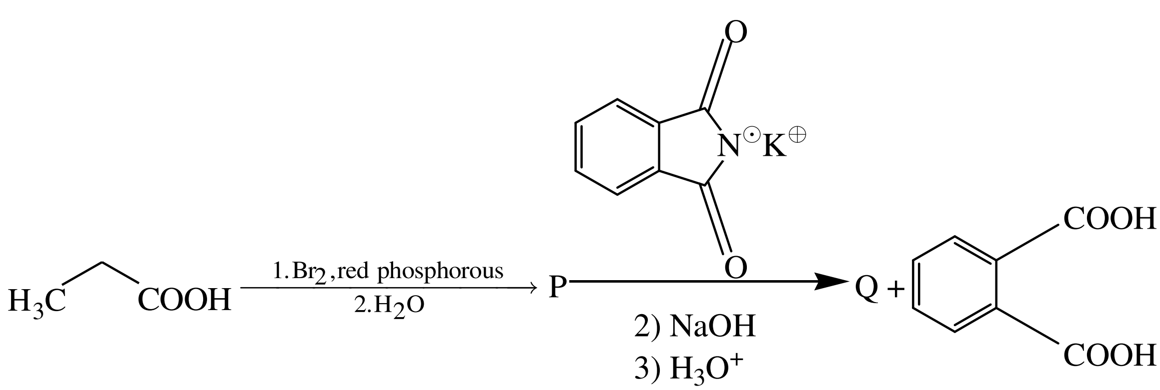
Then answer is AD

If Conc HNO_3 is used the gas is NO_2

Then answer is ABC

The concentration is not mentioned. So generally it is dissolved in dil HNO_3. So answer must be AD

13. Considering the reaction sequence given below, the correct statement(s) is(are)



A) **P** can be reduced to a primary alcohol using $NaBH_4$.

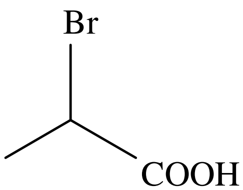
B) Treating **P** with conc. NH_4OH solution followed by acidification gives **Q**.

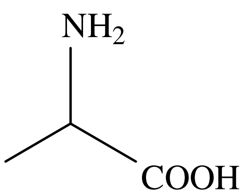
C) Treating **Q** with a solution of $NaNO_2$ in aq. HCl liberates N_2.

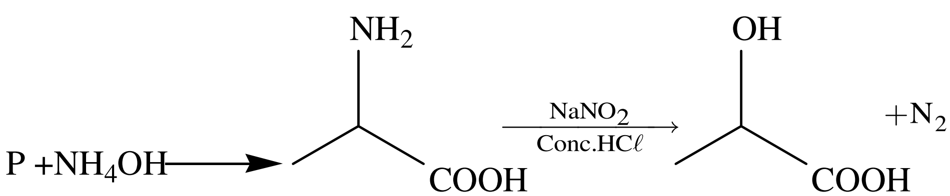
D) **P** is more acidic than CH_3CH_2COOH.

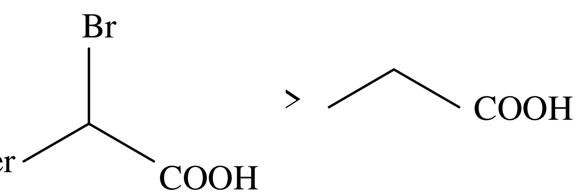
Ans. BCD

Sol.

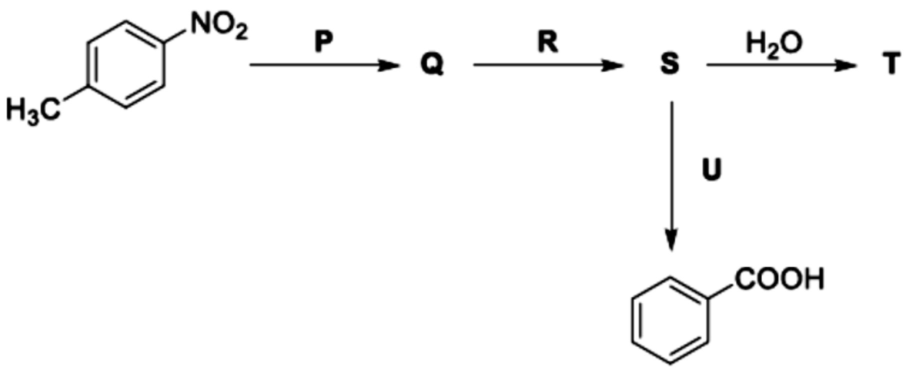
P = 

Q = 

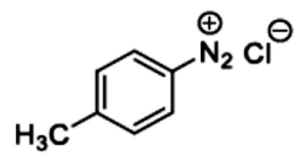
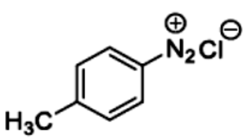
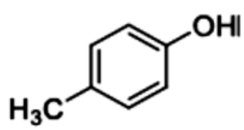
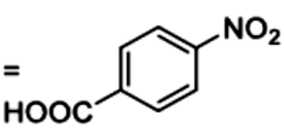
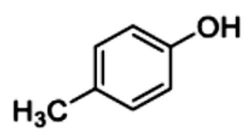


Acidic Character 

14. Considering the following reaction sequence.

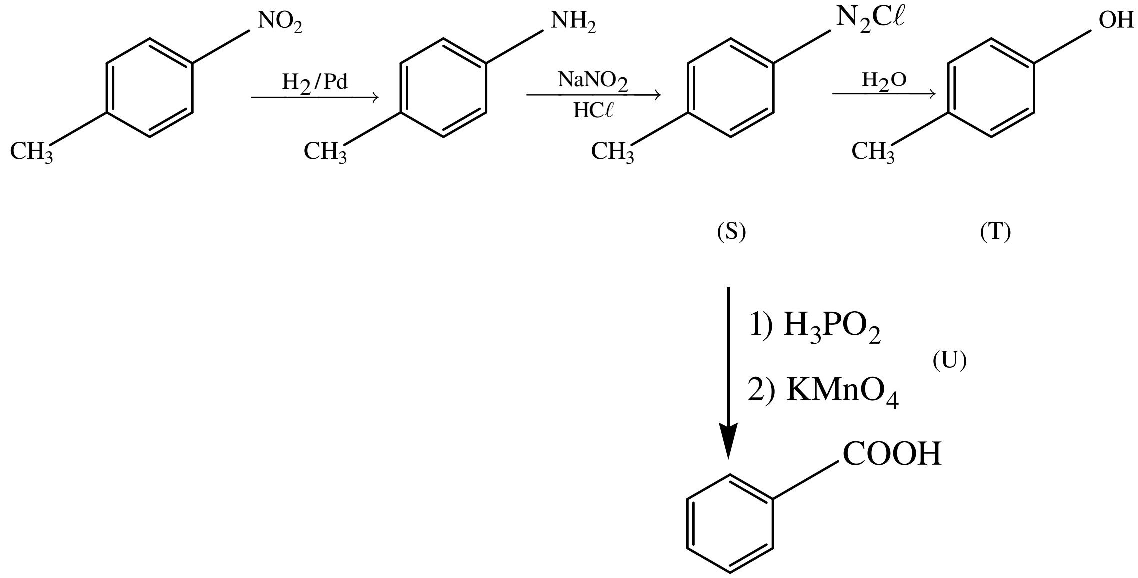


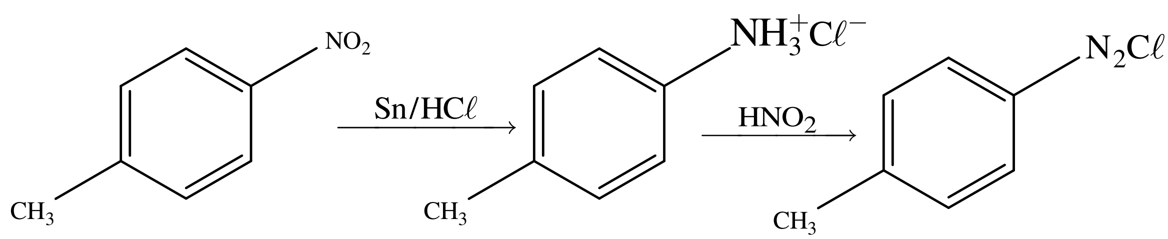
the correct option(s) is(are)

A) $P = H_2 / Pd, ethanol$	$R = NaNO_2 / HC\ell$	$U = 1.\ H_3PO_2$ $2.\ KMnO_4 - KOH, heat$
B) $P = Sn / HC\ell$	$R = HNO_2$	S = 
C) S = 	T = 	$U = 1.\ CH_3CH_2OH$ $2.\ KMnO_4 - KOH, heat$
D) Q = 	$R = H_2 / Pd, ethanol$	T = 

Ans. **ABC**

Sol.





U can also be (1. EtOH, 2. $KMnO_4$, KOH, Δ)

SECTION-3 (Maximum Marks : 12)

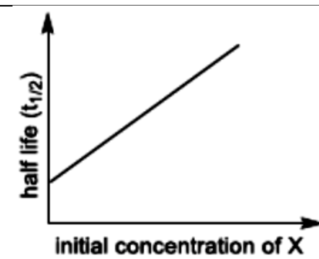
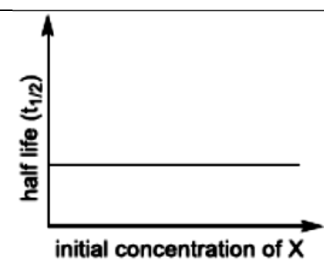
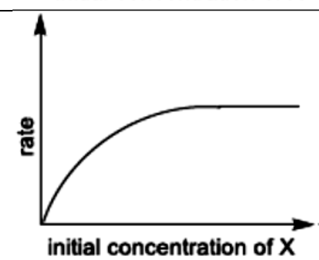
- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists : **List-I** and **List-II.**
- **List-I** has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Five** entries (P), (Q), (R), (S) and (T).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme :

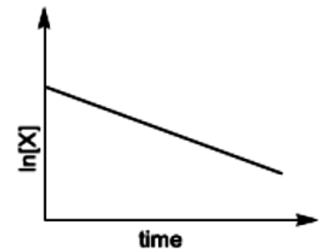
Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;

Zero Marks : 0 If none of the options is chosed (i.e. the question is unanswered);

Negative Marks : –1 In all other cases.

15. Match the rate expressions in LIST-I for the decomposition of X with the corresponding profiles provided in LIST-II. Xs and k are constants having appropriate units.

	LIST-I		LIST-II
I)	$\text{rate} = \frac{k[X]}{X_s + [X]}$ under all possible initial concentrations of X.	P)	
II)	$\text{rate} = \frac{k[X]}{X_s + [X]}$ where initial concentrations of X are much less than X_s.	Q)	
III)	$\text{rate} = \frac{k[X]}{X_s + [X]}$ Where initial concentrations of X are much higher than X_s.	R)	

IV)	$\text{rate} = \dfrac{k[X]^2}{X_s + [X]}$ where initial concentration of X is much higher than X_s.	S)	
		T)	

The correct option is :

(A) (I) $\rightarrow$ (P), (II) $\rightarrow$ (Q), (III) $\rightarrow$ (S), (IV) $\rightarrow$ (T)

(B) (I) $\rightarrow$ (R), (II) $\rightarrow$ (S), (III) $\rightarrow$ (S), (IV) $\rightarrow$ (T)

(C) (I) $\rightarrow$ (P), (II) $\rightarrow$ (Q), (III) $\rightarrow$ (Q), (IV) $\rightarrow$ (R)

(D) (I) $\rightarrow$ (R), (II) $\rightarrow$ (S), (III) $\rightarrow$ (Q), (IV) $\rightarrow$ (R)

Ans. **A**

Sol. Given rate = $\dfrac{K[X]}{K_S + [X]}$

For (II) [X] < < X_S

$\therefore \text{rate} = \dfrac{K}{K_S}[X]$

First order reaction

Half life is independent on initial concentration of [X].

For (III) [x] > > X_s

Rate = $\dfrac{K}{K_S}$ = Constant

Zero order reaction

$\therefore$Concentration of 'X' linearly decreases with time.

For (IV) rate = $\dfrac{K[X]^2}{K_S + [X]}$

$[X]_0 >>> K_S$

Rate = K[X]

It is first order reaction.

$\ell n[X]$ linearly decreases with time.

16. LIST-I contains compounds and LIST-II contains reactions.

LIST-I		LIST-II	
I)	H_2O_2	P)	$Mg(HCO_3)_2 + Ca(OH)_2 \rightarrow$
II)	$Mg(OH)_2$	Q)	$BaO_2 + H_2SO_4 \rightarrow$
III)	$BaC\ell_2$	R)	$Ca(OH)_2 + MgC\ell_2 \rightarrow$
IV)	$CaCO_3$	S)	$BaO_2 + HC\ell \rightarrow$
		T)	$Ca(HCO_3)_2 + Ca(OH)_2 \rightarrow$

Match each compound in LIST-I with its formation reaction(s) in LIST-II, and choose the correct option.

(A) (I)$\rightarrow$(Q), (II)$\rightarrow$(P), (III)$\rightarrow$(S), (IV)$\rightarrow$(R)
(B) (I)$\rightarrow$(T), (II)$\rightarrow$(P), (III)$\rightarrow$(Q), (IV)$\rightarrow$(R)
(C) (I)$\rightarrow$(T), (II)$\rightarrow$(R), (III)$\rightarrow$(Q), (IV)$\rightarrow$(P)
(D) (I)$\rightarrow$(Q), (II)$\rightarrow$(R), (III)$\rightarrow$(S), (IV)$\rightarrow$(P)

Ans. **D**

Sol. $BaO_2 + H_2SO_4 \rightarrow H_2O_2$ $\quad I \rightarrow Q$

$MgCl_2 + Ca(OH)_2 \rightarrow Mg(OH)_2 + CaCl_2$ $\quad II \rightarrow R$

$BaO_2 + 2HCl \rightarrow BaCl_2 + H_2O_2$ $\quad III \rightarrow S$

$Mg(HCO_3)_2 + Ca(OH)_2 \rightarrow CaCO_3 + MgCO_3 + 2H_2O$ $\quad IV \rightarrow P$

17. LIST-I contains metal species and LIST-II contains their properties.

LIST-I		LIST-II	
I)	$\left[Cr(CN)_6\right]^{4-}$	P)	t_{2g} orbital contain 4 electrons
II)	$\left[RuC\ell_6\right]^{2-}$	Q)	μ (spin-only) = 4.9 BM
III)	$\left[Cr(H_2O)_6\right]^{2+}$	R)	low spin complex ion
IV)	$\left[Fe(H_2O)_6\right]^{2+}$	S)	Metal ion in 4+oxidation state
		T)	d^4 species

[Given : Atomic number of Cr = 24, Ru = 44, Fe = 26]

Match each metal species in LIST-I with their properties in LIST-II, and choose the correct option.

(A) (I) $\rightarrow$ (R,T), (II) $\rightarrow$ (P,S), (III) $\rightarrow$ (Q,T), (IV) $\rightarrow$ (P,Q)
(B) (I) $\rightarrow$ (R,S), (II) $\rightarrow$ (P,T), (III) $\rightarrow$ (P,Q), (IV) $\rightarrow$ (Q,T)
(C) (I) $\rightarrow$ (P,R), (II) $\rightarrow$ (R,S), (III) $\rightarrow$ (R,T), (IV) $\rightarrow$ (P,T)
(D) (I) $\rightarrow$ (Q,T), (II) $\rightarrow$ (S,T), (III) $\rightarrow$ (P,T), (IV) $\rightarrow$ (Q,R)

Ans. A

Sol. I) $\left[Cr_2(CN)_6\right]^{4-}$ Since CN^- is strong ligand and Cr is in + 2 Oxidation state with d^4 configuration, in complex it will have $t_{2g}^4 eg^0$. It is a low spin complex. — I) RT

II) $\left[RuCl_6\right]^{2-}$ is low spin complex, metal ion in + 4 oxidation state — II) PS

III) $\left[Cr(H_2O)_6\right]^{2+}$ Cr is in + 2 oxidation state H_2O weak ligand. Magnetic moment 4.9 BM. It is d^4 species. — III) QT

IV) $\left[Fe(H_2O)_6\right]^{2+}$ Fe is + 2 oxidation state having d^6 configuration. In complex it have $t_{2g}^4 eg^2$. — IV) PQ

18. Match the compounds in LIST-I with the observation in LIST-II, and choose the correct option.

LIST-I	LIST-II

I)	Aniline	P)	Sodium fusion extract of the compound on boiling with $FeSO_4$, followed by acidification with conc. H_2SO_4, gives Prussian blue color
II)	o-Cresol	Q)	Sodium fusion extract of the compound on treatment with sodium nitroprusside gives blood red color.
III)	Cysteine	R)	Addition of the compound to a saturated solution of $NaHCO_3$ results in effervescence.
IV)	Caprolactam	S)	The compound reacts with bromine water to give a white precipitate.
		T)	Treating the compound with neutral $FeCl_3$ solution produces violet color.

(A) (I)$\rightarrow$(P,Q), (II)$\rightarrow$(S), (III)$\rightarrow$(Q,R), (IV)$\rightarrow$(P)

(B) (I)$\rightarrow$(P), (II)$\rightarrow$(R,S), (III)$\rightarrow$(R), (IV)$\rightarrow$(Q,S)

(C) (I)$\rightarrow$(Q,S), (II)$\rightarrow$(P,T), (III)$\rightarrow$(P), (IV)$\rightarrow$(S)

(D) (I)$\rightarrow$(P,S), (II)$\rightarrow$(T), (III)$\rightarrow$(Q,R), (IV)$\rightarrow$(P)

Ans. **D**

Sol. Aniline has both C & N $\Rightarrow$ P,S

o – Cresol is a phenol $\Rightarrow$ T

Cystein has S, N and C & COOH $\Rightarrow$ Q,R

Caprolactam has C and N $\Rightarrow$ P

MATHEMATICS

SECTION-1 (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the nuermical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will bee evaluated according to the following marking scheme :

Full Marks : +3 **ONLY** if the correct numerical value is entered ;
Partial Marks : 0 In all other cases.

1. Considering only the principal values of the inverse trigonometric functions, the value of

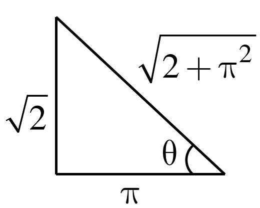
$$\frac{3}{2}\cos^{-1}\sqrt{\frac{2}{2+\pi^2}}+\frac{1}{4}\sin^{-1}\frac{2\sqrt{2}\pi}{2+\pi^2}+\tan^{-1}\frac{\sqrt{2}}{\pi}$$

is ________.

Ans. **2.35 to 2.36**

Sol. Given expression, $\frac{3}{2}\cos^{-1}\sqrt{\frac{2}{2+\pi^2}}+\frac{1}{4}\sin^{-1}\frac{2\sqrt{2}\pi}{2+\pi^2}+\tan^{-1}\frac{\sqrt{2}}{\pi}$

Let $\tan^{-1}\frac{\sqrt{2}}{\pi}=\theta$



Then given expression becomes,

$$\frac{3}{2}\left(\frac{\pi}{2}-\theta\right)+\frac{1}{4}.2\theta+\theta$$

$$\frac{3\pi}{4}-\frac{3\theta}{2}+\frac{\theta}{2}+\theta \qquad =\frac{3\pi}{4}=0.75\times 3.14\cong 2.355$$

2. Let α be a positive real number. Let $f:\mathbb{R}\to\mathbb{R}$ and $g:(\alpha,\infty)\to\mathbb{R}$ be the functions defined by

$$f(x)=\sin\left(\frac{\pi x}{12}\right) \text{ and } g(x)=\frac{2\ell og_e\left(\sqrt{x}-\sqrt{\alpha}\right)}{\ell og_e\left(e^{\sqrt{x}}-e^{\sqrt{\alpha}}\right)}.$$

Then the value of $\lim\limits_{x\to\alpha^+} f(g(x))$ is ________.

Ans. 0.50

Sol. $G.E = f\left(\underset{x\to\alpha^+}{Lt}\, g(x)\right)$

$$= f\left(\underset{x\to\alpha^+}{Lt}\frac{2\log_e\left(\sqrt{x}-\sqrt{\alpha}\right)}{\log_e\left(e^{\sqrt{x}}-e^{\sqrt{\alpha}}\right)}\right)$$

By L-Hospital rule

$$= f\left(\underset{x\to\alpha^+}{Lt}\frac{2\frac{1}{\sqrt{x}-\sqrt{\alpha}}\cdot\frac{1}{2\sqrt{x}}}{\left(\frac{1}{e^{\sqrt{x}}-e^{\sqrt{\alpha}}}\right)e^{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}}\right)$$

$$= f\left(\frac{2}{e^{\sqrt{\alpha}}}\cdot\underset{x\to\alpha^+}{Lt}\frac{e^{\sqrt{x}}-e^{\sqrt{\alpha}}}{\sqrt{x}-\sqrt{\alpha}}\right)$$

$$\left(\because \text{ we know that } \underset{x\to 0}{Lt}\frac{e^x-1}{x}=1\right)$$

= f(2 × 1) = f(2)

$$=\sin\left(\frac{\pi(2)}{12}\right)=\frac{1}{2}$$

3. In a study about a pandemic, data of 900 persons was collected. It was found that

190 persons had symptom of fever,

220 persons had symptom of cough,

220 persons had symptom of breathing problem,

330 persons had symptom of fever or cough or both,

350 persons had symptom of cough or breathing problem or both,

340 persons had symptom of fever or breathing problem or both,

30 persons had all three symptoms (fever, cough and breathing problem).

If a person is chosen randomly from these 900 persons, then the probability that the person has at most one symptom is ________.

Ans. 0.80

Sol. Let, f, c, b be set of persons representing symptoms of fever, cough, breathing problem respectively

Given

$n(f)=190$ $n(c)=220$ $n(b)=220$ $n(c\cup f)=330$ $n(c\cup b)=350$

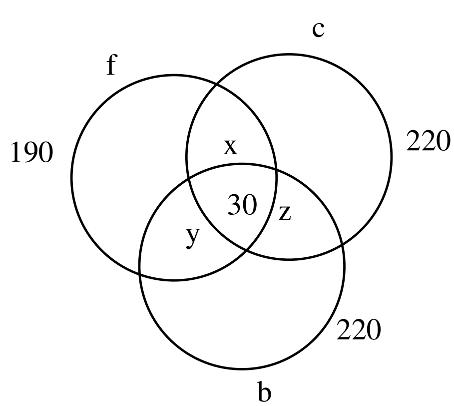
$n(f\cup b)=340$ $n(c\cap f\cap b)=30$ $n(s)=900$

Let

$n(f\cap c\cap \bar{b})=x$

$n(f\cap b\cap \bar{c})=y$

$n(c\cap b\cap \bar{f})=z$



$n(f\cap c)=n(f)+n(c)-n(f\cup c)=x+30$

= 190 + 220 – 330 = 80

$x+30=80$

$\Rightarrow x=50$

$n(f\cap b)=y+30=n(f)+n(b)-n(f\cup b)$

= 190 + 220 – 340 = 70

$\Rightarrow y=40$

$n(c\cap b)=z+30=n(c)+n(b)-n(c\cup b)$

= 220 + 220 – 350 = 90

$\Rightarrow z=60$

Required probability = 1 – P (a person suffering from atleast two diseases)

$=1-\frac{(x+y+z+30)}{n(s)}$

$=1-\frac{180}{900}$

$=\frac{4}{5}=0.80$

4. Let z be a complex number with non-zero imaginary part. If

$$\frac{2+3z+4z^2}{2-3z+4z^2}$$

is a real number, then the value of $|z|^2$ ________.

Ans. 0.50

Sol. $\frac{2+3Z+4Z^2}{2-3Z+4Z^2}=\frac{K}{1}(real)$

$\Rightarrow\frac{2Z^2+1}{Z}=\frac{3}{2}\left(\frac{K+1}{K-1}\right)$

$\Rightarrow\frac{2Z^2+1}{Z} is\ real$

$\Rightarrow 2Z + \frac{1}{Z} is\, real$

$\Rightarrow I.P = 0$

$\Rightarrow x^2 + y^2 = \frac{1}{2} (\because\ y \neq 0)$

$\Rightarrow |Z|^2 = \frac{1}{2}$

5. Let $\bar{z}$ denote the complex conjugate of a complex number z and let $i = \sqrt{-1}$. In the set of complex numbers, the number of distinct roots of the equation

$$\bar{z} - z^2 = i\left(\bar{z} + z^2\right)$$

is ________.

Ans. 4

Sol. Given equation $\bar{Z} = Z^2 = i\left(\bar{Z} + Z^2\right)$

$\frac{\bar{Z} - Z^2}{\bar{Z} + Z^2} = i$

By applying componendo and devidendo on both sides

$\frac{2\bar{Z}}{\left(-2Z^2\right)} = \frac{i+1}{i-1}$

$\Rightarrow \bar{Z} = \frac{i+1}{i-1} Z^2$

Applying Modulus on both sides

$|\bar{Z}| = \frac{|i+1|}{|i-1|} |Z^2|$

$\Rightarrow |\bar{Z}| = |Z^2|$

$\Rightarrow |\bar{Z}| = 0\, or\, |Z| = 1$

If $|Z| = 0 \Rightarrow Z = 0$

If $|\bar{Z}| = 1$

$\Rightarrow \bar{Z} = \frac{1}{Z}$

$\Rightarrow Z^3 = \frac{1+i}{1-i}$

$Z^3 = -i$

$Z = (-i)^{1/3}$

Here we get 3 solution

Total we have 4 solutions

$\Rightarrow$ Number of solutions = 4

M – II

$\frac{\bar{Z} - Z^2}{\bar{Z} + Z^2} = i$

$\frac{\bar{Z} + Z^2}{\bar{Z} - Z^2} = \frac{1}{i}$

$\frac{2\bar{Z}}{2Z^2} = \frac{1+i}{1-i}$

$\Rightarrow \; Z^2 = \frac{(1-i)}{1+i} \times \bar{Z}$

$\Rightarrow \; Z^2 = (-i)(\bar{Z})$

Let $Z = x + iy$

$(x+iy)^2 = -i\{x - iy\}$

$\{x^2 - y\} + i\{2xy\} = -y - xi$

$x^2 - y^2 = -y$ ______ (1)

$2xy = -x$ ______ (2)

$\Rightarrow x = 0 (or)\, y = -\frac{1}{2}$

$x = 0 \Rightarrow y^2 = y$ $\qquad$ $y = -\frac{1}{2} \Rightarrow x^2 = \frac{3}{4}$

$\Rightarrow\ y = 0\, or\; y = 1 \qquad\qquad x = +\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$

$\Rightarrow$ Solutions are $0, i, \frac{\sqrt{3}-i}{2}, \frac{-\sqrt{3}-i}{2}$

$\Rightarrow$ Number of solutions = 4

6. Let $\ell_1, \ell_2, ...\ell_{100}$ be consecutive terms of an arithmetic progression with common difference d_1, and let $w_1, w_2, ...w_{100}$ be consecutive terms of another arithmetic progression with common difference d_2, where $d_1 d_2 = 10$. For each i = 1, 2, ..., 100, let R_i be a rectangle with length ℓ_i, width w_i and area A_i. If $A_{51} - A_{50} = 1000$, then the value of $A_{100} - A_{90}$ is ________.

Ans. 18900

Sol. Given $l_1, l_2, ...l_{100}$ are consecutive terms of an A.P. with common difference d_1 we know that, $l_n = l_1 + (n-1)d_1$

Similarly : $w_n = w_1 + (n-1)d_2$

Also given : $A_{51} - A_{50} = 1000$

$\Rightarrow l_{51}.W_{51} - l_{50}.W_{50} = 1000$

$\Rightarrow (l_1 + 50d_1)(w_1 + 50d_2) - (l_1 + 49d_1)(w_1 + 49d_2) = 1000$

$\Rightarrow \left(l_1 w_1 + 50(d_1 w_1 + d_2 l_1) + 50^2 d_1 d_2\right) - \left(l_1 w_1 + 49(d_1 w_1 + d_2 l_1) + 49^2 d_1 d_2\right) = 1000$

$\Rightarrow\ (d_1 w_1 + d_2 l_1) + (50^2 - 49^2) d_1 d_2 = 1000$

$\Rightarrow\ d_1 w_1 + d_2 l_1 + (50+49)(50-49)(10) = 1000 \qquad (\because d_1 d_2 = 10(given))$

$\Rightarrow d_1 w_1 + d_2 l_1 = 10$ _______(1)

Required to find, $A_{100} - A_{90}$

$= l_{100}.w_{100} - l_{90}.w_{90}$

$= (l_1 + 99d_1)(w_1 + 99d_2) - (l_1 + 89d_1)(w_1 + 89d_2)$

$= 10(d_1 w_1 + d_2 w_2)(99^2 - 89^2) d_1 d_2$

= 10 (10) + (99 + 89) (10)(10) $\left(\because from(1) \& d_1 d_2 = 10(given)\right)$

= 100 (189)

= 18900

7. The number of 4–digit integers in the closed interval [2022, 4482] formed by using the digits 0, 2, 3, 4, 6, 7 is ________.

Ans. 569

Sol. Four digited number is to be formed by using numbers 0, 2, 3, 4, 6, 7 such that it belongs to [2022, 4482]

<u>Case 1</u> (1st digit is 2)

$$\frac{2}{1}\times\frac{}{5}\times\frac{}{6}\times\frac{}{6} + \frac{2}{1}\times\frac{0}{1}\times\frac{}{4}\times\frac{}{6} + \frac{2}{1}\times\frac{0}{1}\times\frac{2}{1}\times\frac{}{5}$$

$\Downarrow$ $(2,3,4,6,7)$ $\Downarrow$ $(3,4,6,7)$ $\Downarrow$ $(2,3,4,6,7)$

$= 5(6)^2 + (4\times 6) + (5)$

= 209

<u>Case 2</u> (1st digit is 3)

$$\frac{3}{1}\times\frac{}{6}\times\frac{}{6}\times\frac{}{6} = 216$$

<u>Case 3</u> (1st digit is 4)

$$\frac{4}{1}\times\frac{}{4}\times\frac{}{6}\times\frac{}{6}$$

$\Downarrow$

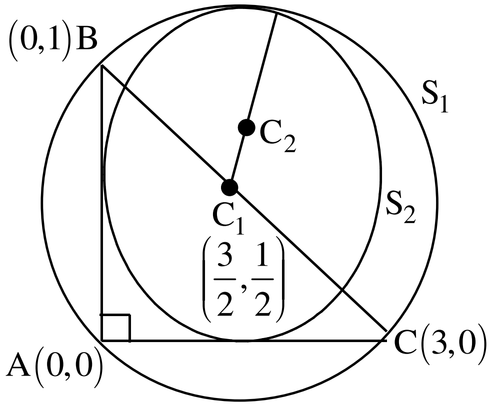
$(0,2,3,4)$

$\therefore$ Total number of such numbers = 209 + 216 + 144 = 569

8. Let ABC be the triangle with AB = 1, AC = 3 and $\angle BAC = \frac{\pi}{2}$. If a circle of radius r > 0 touches the sides AB, AC and also touches internally the circumcircle of the triangle ABC, then the value of r is ________.

Ans. 0.83 to 0.84

Sol. Let S_1 be the circum circle if ΔABC and S_2 be the circle satisfying the given conditions. R be the circumradius of ΔABC. C_1 be the centre of circle S_1 and C_2 be the centre of circle S_2



Assuming AC to be x-axis and AB to be the y-axis with origin at point A $\left(\because \text{ given } \angle BAC = \frac{\pi}{2}\right)$ then

Coordinates of A = (0, 0), B = (0, 1), C = (3, 0). (∵ given AB = 1, AC = 3)

$C_1 = \left(\frac{3}{2}, \frac{1}{2}\right)$ (∵ In a right angled Δle, circumcentre is the midpoint of hypotenuse)

Given S_2 touches AB, AC (ie., x-axis and y-axis)

$\therefore$ Coordinates of $C_2 = (r, r)$ (where r = radius of S_2)

Also given $S_2 \,\&\, S_1$ touches internally.

$\Rightarrow C_1C_2 = R - r$

$\Rightarrow (C_1C_2)^2 = (R-r)^2$ ______(1)

$R = \frac{BC}{2} = \frac{\sqrt{10}}{2}$

$(1) \Rightarrow \left(\frac{3}{2} - r\right)^2 + \left(\frac{1}{2} - r\right)^2 = \left(\frac{\sqrt{10}}{2} - r\right)^2$

$\Rightarrow \frac{10}{4} + 2r^2 - 4r = \frac{10}{4} + r^2 - \sqrt{10}r \quad \Rightarrow r^2 = \left(4 - \sqrt{10}\right)r$

$\Rightarrow r = \left(4 - \sqrt{10}\right) \Rightarrow r = 4 - 3.162 = 0.838 \quad = 0.83$ to 0.84

SECTION-2 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evalueated <u>according to the following marking scheme :</u>

Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct ;
Partial Marks : +1 If two or more options are correct but **ONLY** two options are chosen, and it is a correct option ;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : –2 In all other cases.

9. Consider the equation

$$\int_1^e \frac{(\ell og_e x)^{1/2}}{x\left(a-(\ell og_e x)^{3/2}\right)^2}dx = 1, \quad a \in (-\infty, 0) \cup (1, \infty).$$

Which of the following statements is/are TRUE ?

A) **No** a satisfies the above equation

B) An integer a satisfies the above equation

C) An irrational number a satisfies the above equation

D) More than one a satisfy the above equation

Ans. **CD**

Sol. Put, $\left(\log_e^x\right)^{\frac{3}{2}} = t \Rightarrow \frac{3}{2}\left(\log_e^x\right)^{\frac{1}{2}}\frac{1}{x}dx = dt$

$$\Rightarrow \int_0^1 \frac{2dt}{3(a-t)^2} = 1 \Rightarrow \left[\frac{2}{3}\frac{1}{a-t}\right]_0^1 = 1$$

$$\Rightarrow \frac{2}{3}\left(\left(\frac{1}{a-1}\right)-\frac{1}{a}\right)=1$$

$$\Rightarrow 2=3\left(a^2-a\right)$$

$$\Rightarrow 3a^2-3a-2=0$$

$$\Rightarrow a=\frac{3\pm\sqrt{9+24}}{6}$$

$$\Rightarrow a=\frac{3\pm\sqrt{33}}{6}$$

10. Let $a_1, a_2, a_3, ...$ be an arithmetic progression with $a_1 = 7$ and common difference 8. Let $T_1, T_2, T_3, ...$ be such that $T_1 = 3$ and $T_{n+1} - T_n = a_n$ for $n \geq 1$. Then, which of the following is/are TRUE ?

A) $T_{20} = 1604$

B) $\sum_{k=1}^{20} T_k = 10510$

C) $T_{30} = 3454$

D) $\sum_{k=1}^{30} T_k = 35610$

Ans. **BC**

Sol. $T_{n+1} - T_n = a_n$ for $n \geq 1$

$$\Rightarrow T_{n+1} = 3+\frac{n}{2}\left(14+(n-1)8\right)=4n^2+3n+3$$

$$T_n = 4n^2-5n+4$$

$$T_{20}=1504, T_{30}=3454$$

$$\sum_{k=1}^{20} T_k = 10510, \sum_{k=1}^{30} T_k = 35615$$

11. Let P_1 and P_2 be two planes given by

$$P_1 : 10x + 15y + 12z - 60 = 0,$$

$$P_2 : -2x + 5y + 4z - 20 = 0.$$

Which of the following straight lines can be an edge of some tetrahedron whose two faces lie on P_1 and P_2?

A) $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{5}$

B) $\frac{x-6}{-5} = \frac{y}{2} = \frac{z}{3}$

C) $\frac{x}{-2} = \frac{y-4}{5} = \frac{z}{4}$

D) $\frac{x}{1} = \frac{y-4}{-2} = \frac{z}{3}$

Ans. **ABD**

Sol. $P_1 : 10x + 15y + 12x - 60 = 0$

$P_2 : -2x + 15y + 4z - 20 = 0$

For A : let $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{5} = \alpha \Rightarrow P(1,1,1+5\alpha)$

if P lies an $P_1 \Rightarrow \alpha = \frac{23}{60} \Rightarrow P\left(1,1,\frac{35}{12}\right)$ it does not lies on P_2

For B : let $\frac{x-6}{-5} = \frac{y}{2} = \frac{z}{3} = \beta \Rightarrow \theta(6-5\beta, 2\beta, 3\beta)$

If θ lies on $P_1 \Rightarrow P = 0 \Rightarrow \theta(6,0,0)$ it does not lies on P_2

For C : let $\frac{x}{-2} = \frac{y-4}{5} = \frac{z}{4} = \gamma \Rightarrow R(-2\gamma, 4+5\gamma, 4\gamma)$

If R lies on $P_1 \Rightarrow \gamma = 0 \Rightarrow P(0,4,0)$,but, it lies on P_2

for D : let $\frac{x}{1} = \frac{y-4}{-2} = \frac{z}{3} = \delta \Rightarrow S(\delta, 4-2\delta, 3\delta)$

If S lies on $P_1 \Rightarrow \delta = 0 \Rightarrow S(0,4,0)$, but , it lies on P_2 and given line completely lies on P_2.

12. Let S be the reflection of a point Q with respect to the plane given by

$$\vec{r} = -(t+p)\hat{i} + t\hat{j} + (1+p)\hat{k}$$

where t, p are real parameters and $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along the three positive coordinate axes. If the position vectors of Q and S are $10\hat{i}+15\hat{j}+20\hat{k}$ and $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ respectively, then which of the following is/are TRUE ?

A) $3(\alpha+\beta)=-101$ B) $3(\beta+\gamma)=-71$

C) $3(\gamma+\alpha)=-86$ D) $3(\alpha+\beta+\gamma)=-121$

Ans. ABC

Sol. $\vec{r} = k + p(-i+k) + t(-i+5)$

equation of plane is $\begin{vmatrix} x & 4 & z-1 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{vmatrix} = 0 \qquad \Rightarrow x+y+z=1$

Q(10, 15, 20) s be the reflection of Q

$$\frac{\alpha-10}{1} = \frac{\beta-15}{1} = \frac{\gamma-20}{1} = \frac{-2(10+15+20-1)}{3} = \frac{-88}{3}$$

$$\Rightarrow \alpha = \frac{-58}{3}, \beta = \frac{-43}{4}, \gamma = \frac{-28}{3}$$

13. Consider the parabola $y^2 = 4x$. Let S be the focus of the parabola. A pair of tangents drawn to the parabola from the point $P=(-2,1)$ meet the parabola at P_1 and P_2. Let Q_1 and Q_2 be points on the lines SP_1 and SP_2 respectively such that PQ_1 is perpendicular to SP_1 and PQ_2 is perpendicular to SP_2. Then, which of the following is/are TRUE ?

A) $SQ_1 = 2$ B) $Q_1Q_2 = \frac{3\sqrt{10}}{5}$ C) $PQ_1 = 3$ D) $SQ_2 = 1$

Ans. **BCD**

Sol. Let $P_1(t_1), P_2(t_2)$

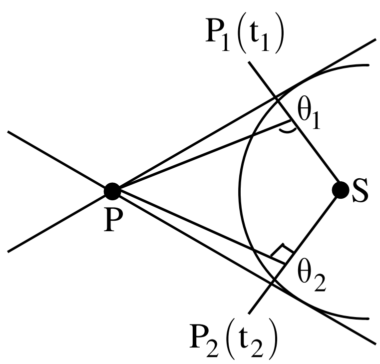
$\therefore P\left(at_1t_2, a(t_1+t_2)\right) = (-2,1)$

$\Rightarrow t_1t_2 = -2, t_1+t_2 = 1$

$\Rightarrow t_1 = 2, t_2 = -1$

$\Rightarrow P_1(4,4), P_2(1,-2)$ equation (1, 0)

$\Rightarrow SP_2$ is reaction of $y^2 = 4x$.



$\Rightarrow$ Equation of SP_2 is x = 1 $\Rightarrow$ equation of $P\theta_2$ is y = 1 $\Rightarrow \theta_2(1_{11})$

$SP_1 = \frac{4}{3} \Rightarrow$ equation of SP_1 IS 4X – 34 = 4 and equation of $P\theta_1$ is 3x + 4y + 2= 0

$\Rightarrow \theta_1\left(\frac{2}{5}, \frac{-4}{5}\right)$

$\therefore SQ_1 = SQ_2 = 1, Q_1Q_2 = \frac{3\sqrt{10}}{5}, PQ_1 = 3$

14. Let $|M|$ denote the determinant of a square matrix M. Let $g:\left[0,\frac{\pi}{2}\right]\to R$ be the function defined by

where $g(\theta)=\sqrt{f(\theta)-1}+\sqrt{f\left(\frac{\pi}{2}-\theta\right)-1}$

$$f(\theta)=\frac{1}{2}\begin{vmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{vmatrix}+\begin{vmatrix}\sin\pi & \cos\left(\theta+\frac{\pi}{4}\right) & \tan\left(\theta-\frac{\pi}{4}\right)\\ \sin\left(\theta-\frac{\pi}{4}\right) & -\cos\frac{\pi}{2} & \log_e\left(\frac{4}{\pi}\right)\\ \cot\left(\theta+\frac{\pi}{4}\right) & \log_e\left(\frac{\pi}{4}\right) & \tan\pi\end{vmatrix}$$

Let p(x) be a quadratic polynomial whose roots are the maximum and minimum values of the function $g(\theta)$, and $p(2)=2-\sqrt{2}$. Then, which of the following is/are TRUE ?

A) $p\left(\frac{3+\sqrt{2}}{4}\right)<0$ B) $p\left(\frac{1+3\sqrt{2}}{4}\right)>0$ C) $p\left(\frac{5\sqrt{2}-1}{4}\right)>0$ D) $p\left(\frac{5-\sqrt{2}}{4}\right)<0$

Ans. **AC**

Sol. $f(\theta)=1+\sin^2\theta$

$g(\theta)=\sin\theta+\cos\theta$ for all $\theta\in\left[0,\frac{\pi}{4}\right]$

$\Rightarrow\quad g(\theta)=\sqrt{2}\sin\left(\theta+\frac{\pi}{4}\right)\in\left[1,\sqrt{2}\right]$

minimum value of $g(\theta)=\sqrt{2}$ minimum value of $g(\theta)$ is 1.

$\therefore P(x)=a(x-1)\left(x-\sqrt{2}\right)$

But, $P(2)=2-\sqrt{2}=a(1)\left(2-\sqrt{2}\right)\Rightarrow a=1$

$\Rightarrow P(x)=(x-1)\left(x-\sqrt{2}\right)$

SECTION-3 (Maximum Marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists : **List-I** and **List-II.**
- **List-I** has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Five** entries (P), (Q), (R), (S) and (T).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;

Zero Marks : 0 If none of the options is chosed (i.e. the question is unanswered);

Negative Marks : –1 In all other cases.

15. Consider the following lists :

List-I	**List-II**
(I) $\left\{x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$	(P) has two elements
(II) $\left\{x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3}\tan 3x = 1\right\}$	(Q) has three elements
(III) $\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2\cos(2x) = \sqrt{3}\right\}$	(R) has four elements
(IV) $\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$	(S) has five elements
	(T) has six elements

The correct option is :

(A) (I) $\rightarrow$ (P), (II) $\rightarrow$ (S), (III) $\rightarrow$ (P), (IV) $\rightarrow$ (S)

(B) (I) $\rightarrow$ (P), (II) $\rightarrow$ (P), (III) $\rightarrow$ (T), (IV) $\rightarrow$ (R)

(C) (I) $\rightarrow$ (Q), (II) $\rightarrow$ (P), (III) $\rightarrow$ (T), (IV) $\rightarrow$ (S)

(D) (I) $\rightarrow$ (Q), (II) $\rightarrow$ (S), (III) $\rightarrow$ (P), (IV) $\rightarrow$ (R)

Ans. **B**

Sol. For (I) : $\sin x+\cos x=1\Rightarrow\cos\left(x-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$

$\Rightarrow x-\frac{\pi}{4}=2n\pi\pm\frac{\pi}{4}$

$\Rightarrow x=2n\pi(\text{or})2n\pi+\frac{\pi}{2},n\in I$

$\Rightarrow x=0,\frac{\pi}{2}\left[\because x\in\left[-\frac{2\pi}{3},\frac{2\pi}{3}\right]\right]$

For (II) : $\tan 3x=\frac{1}{\sqrt{3}},x\in\left[\frac{-5\pi}{18},\frac{5\pi}{18}\right]$

$3x=n\pi+\frac{\pi}{6}$

$\Rightarrow x=\frac{n\pi}{3}+\frac{\pi}{18},n\in I$

$\therefore x=\frac{\pi}{18},\frac{-5\pi}{18}$

For (III) : $\cos 2x=\frac{\sqrt{3}}{2},x\in\left[\frac{-6\pi}{5},\frac{6\pi}{5}\right]$

$2x=2n\pi\pm\frac{\pi}{6}$

$\Rightarrow x=n\pi\pm\frac{\pi}{2},n\in I$

$x=\pm\frac{\pi}{12},\pi-\frac{\pi}{12},-\pi+\frac{\pi}{12},\pi+\frac{15}{12},-\pi-\frac{\pi}{12}$

For (IV) : $\cos x - \sin x - 1 \Rightarrow \cos\left(x + \frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}}$

$x + \frac{\pi}{4} = 2n\pi \pm \frac{3\pi}{4}$

$x = 2n\pi + \frac{\pi}{2}(\text{or})2n\pi - \pi, n \in I$

$x = \frac{\pi}{2}, \frac{-3\pi}{2}, -\pi, \pi$

16. Two players P_1 and P_2, play a game against each other. In every round of the game, each player rolls a fair die once, where the six faces of the die have six distinct numbers. Let x and y denote the readings on the die rolled by P_1 and P_2, respectively. If x > y, then P_1 scores 5 points and P_2 scores 0 point. If x = y, then each player scores 2 points. If x < y, then P_1 scores 0 point and P_2 scores 5 points. Let X_i and Y_i be the total scores of P_1 and P_2, respectively, after playing the i^{th} round.

List-I	**List-II**
(I) Probability of $(X_2 \geq Y_2)$ is	(P) $\frac{3}{8}$
(II) Probability of $(X_2 > Y_2)$ is	(Q) $\frac{11}{16}$
(III) Probability of $(X_3 = Y_3)$ is	(R) $\frac{5}{16}$
(IV) Probability of $(X_3 > Y_3)$ is	(S) $\frac{355}{864}$
	(T) $\frac{77}{432}$

The correct option is :
(A) (I)$\rightarrow$(Q), (II)$\rightarrow$(R), (III)$\rightarrow$(T), (IV)$\rightarrow$(S)
(B) (I)$\rightarrow$(Q), (II)$\rightarrow$(R), (III)$\rightarrow$(T), (IV)$\rightarrow$(T)
(C) (I)$\rightarrow$(P), (II)$\rightarrow$(R), (III)$\rightarrow$(Q), (IV)$\rightarrow$(S)
(D) (I)$\rightarrow$(P), (II)$\rightarrow$(R), (III)$\rightarrow$(Q), (IV)$\rightarrow$(T)

Ans. A

Sol. $E_1 : win(5\,\text{points})$

$E_2 : lose(0\,\text{points})$

$E_3 : tie(\text{both get same number})(2\,\text{points for each team})$

Events $E_1 \& E_2$ are equally probable with probability $P(E_1)(=P(E_2))$

Let $P(E_3)$ be probability for E_3 to occur.

$P(E_3)=\left(\frac{1}{36}\right)(6)=\frac{1}{6}$

$\because\ P(E_1)+P(E_2)+P(E_3)=1$

$\Rightarrow 2P(E_1)+\frac{1}{6}=1 \Rightarrow P(E_1)=\frac{5}{12}=P(E_2)$

I) Probability of $(X_2 \geq Y_2)$ is

for this to happen the possible cases are

1) P_1 one win, P_2 one win : $\left(\frac{5}{12}\right)\left(\frac{5}{12}\right)(2)=\frac{50}{144}$

2) Two ties : $\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)=\frac{1}{36}$

3) P_1 one win, one tie : $\left(\frac{5}{12}\right)\left(\frac{1}{6}\right)(2)=\frac{10}{72}$

4) P_1 two win : $\left(\frac{5}{12}\right)\left(\frac{5}{12}\right)=\frac{25}{144}$

$\Rightarrow\ P(X_2 \geq Y_2)=\frac{50}{144}+\frac{1}{36}+\frac{10}{72}+\frac{25}{144}=\frac{11}{16}(Q)$

II) Probability of $(X_2 \geq Y_2)$ is for this to happen the possible cases are

1) P_1 one win, one tie : $\left(\frac{5}{12}\right)\left(\frac{1}{6}\right)(2)=\frac{10}{72}$

2) P_1 two ties : $\left(\frac{5}{12}\right)\left(\frac{1}{6}\right)=\frac{25}{144}$

$\Rightarrow\ P(X_2 \geq Y_2)=\frac{10}{72}+\frac{25}{144}=\frac{5}{16}(R)$

III) Probability of $(X_3 = Y_3)$ is

1) P_1 one win, P_2 one win , one tie : $\left(\frac{5}{12}\right)\left(\frac{5}{12}\right)\left(\frac{1}{6}\right)(6)$

2) 3 ties : $\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)$

$\Rightarrow\ P(X_3 = Y_3) = \frac{25}{144} + \frac{1}{266} = \frac{77}{432}(T)$

IV) Probability of $(X_3 > Y_3)$ is p (say)

Probability of $(X_3 > Y_3)$ is q (say)

Equally parabola p = q

$p + q + P(X_3 = Y_3) = 1$

$\Rightarrow 2P + \frac{77}{432} = 1$

$\Rightarrow P(X_3 > Y_3) = \frac{355}{864}(S)$

17. Let p, q, r be nonzero real numbers that are, respectively, the 10th, 100th and 1000th terms of a harmonic progression. Consider the system of linear equations.

$$x + y + z = 1$$

$$10x + 100y + 1000z = 0$$

$$qr\,x + pr\,y + pq\,z = 0$$

List-I	**List-II**
(I) If $\frac{q}{r} = 10$, then the system of linear equations has	(P) $x = 0,\ y = \frac{10}{9}, z = -\frac{1}{9}$ as a solution
(II) If $\frac{p}{r} \neq 100$, then the system of linear equations has	(Q) $x = \frac{10}{9}, y = -\frac{1}{9}, z = 0$ as a solution
(III) If $\frac{p}{q} \neq 10$, then the system of linear equations has	(R) infinitely many solutions
(IV) If $\frac{p}{q} = 10$, then the system of linear Equations has	(S) no solution
	(T) at least one solution

The correct option is :
(A) (I)$\rightarrow$(T), (II)$\rightarrow$(R), (III)$\rightarrow$(S), (IV)$\rightarrow$(T)
(B) (I)$\rightarrow$(Q), (II)$\rightarrow$(S), (III)$\rightarrow$(S), (IV)$\rightarrow$(R)
(C) (I)$\rightarrow$(Q), (II)$\rightarrow$(R), (III)$\rightarrow$(P), (IV)$\rightarrow$(R)
(D) (I)$\rightarrow$(T), (II)$\rightarrow$(S), (III)$\rightarrow$(P), (IV)$\rightarrow$(T)

Ans. B

Sol. p, q, r $\rightarrow$ 10^{th}, 100^{th}, 1000^{th} terms in HP

$\frac{1}{p}, \frac{1}{q}, \frac{1}{r} \rightarrow$ 10^{th}, 100^{th}, 1000^{th} terms in AP

$\frac{1}{p} = a + 9d; \frac{1}{q} = a + 99d; \frac{1}{r} = a + 999d$

I) $\frac{q}{r} = 10 \Rightarrow \frac{a+999d}{a+99d} = 10$

$9a = 9d \qquad \therefore a = d$

$\frac{1}{p} = 10a; \frac{1}{q} = 100a; \frac{1}{r} = 1000a$

$(qr)x + (pr)y + (pq)z = 0$

$\Rightarrow \frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 0$

$\therefore$ 2^{nd} and 3^{rd} equations are same $\Rightarrow$ infinite solutions

$\therefore I \rightarrow PQRT$

II) $\frac{p}{q} = 10 \Rightarrow \frac{a+99d}{a+9d} = 10$

$\Rightarrow a = d$

Similar to (I) $\qquad IV \rightarrow PQRT$

II) $\frac{p}{r} \neq 100$

$\frac{a+999d}{a+9d} \neq 100 \Rightarrow a \neq d$

If $a \neq d$,

$x + y + z = 1$_____(1)

$10x + 100y + 1000z = 0$_____(2)

$(qr)x + (pr)y + (pq)z = 0$_____(3)

(3) $\Rightarrow \frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 0$

$(a+9d)x+(a+99d)y+(a+999d)z=0$

$a+0+9d(y+11z)=0$ _____ (4)

(2) $\Rightarrow$ 10 × 1

$10x+100y+1000z=0$

$*10x(x+\quad y+\quad z=\ 1)$

$0+90y+990z=-10$

9 (y + 11z) = -1

(4) becomes a + d(- 1) = 0

$\Rightarrow a=d$

But $a \neq d$

It is a contradiction

$\therefore$ *If* $(a \neq d)$ there is no solution

Checking options key is B

18. Consider the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$, Let $H(\alpha,0), 0<\alpha<2$, be a point. A straight line drawn through H parallel to the y-axis crosses the ellipse and its auxiliary circle at points E and F respectively, in the first quadrant. The tangent to the ellipse at the point E intersects the positive x-axis at a point G. Suppose the straight line joining F and the origin makes an angle ϕ with the positive x-axis.

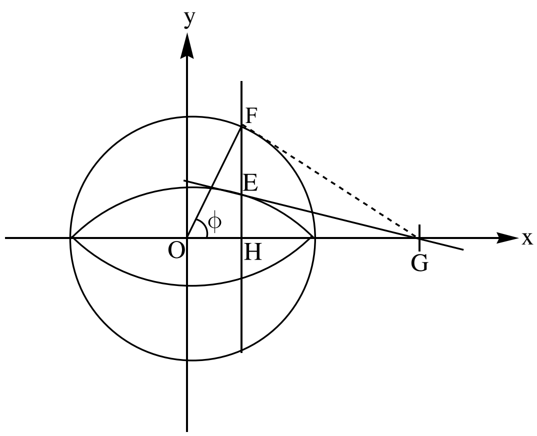
List-I	**List-II**
(I) If $\phi=\frac{\pi}{4}$, then the area of the triangle FGH is	(P) $\frac{(\sqrt{3}-1)^4}{8}$
(II) If $\phi=\frac{\pi}{3}$, then the area of the triangle FGH is	(Q) 1
(III) $\phi=\frac{\pi}{6}$, then the area of the triangle FGH is	(R) $\frac{3}{4}$
(IV) If $\phi=\frac{\pi}{12}$, then the area of the triangle FGH is	(S) $\frac{1}{2\sqrt{3}}$
	(T) $\frac{3\sqrt{3}}{2}$

The correct option is :
(A) (I) → (R), (II) → (S), (III) → (Q), (IV) → (P)
(B) (I) → (R), (II) → (T), (III) → (S), (IV) → (P)
(C) (I) → (Q), (II) → (T), (III) → (S), (IV) → (P)
(D) (I) → (Q), (II) → (S), (III) → (Q), (IV) → (P)

Ans. **C**

Sol. $H(\alpha,0)$,let $F(a\cos\phi, a\sin\phi) \Rightarrow E(a\cos\phi, b\sin\phi)\left(a=2, b=\sqrt{3}\right)$

equation of tangent at E to the ellipse is



$$\frac{x}{a}\cos\phi + \frac{y}{b}\sin\phi = 1 \Rightarrow G\left(\frac{a}{\cos\phi}, 0\right)$$

$$S = [\Delta FGH] = \frac{1}{2}\left(\frac{a}{\cos\phi} - a\cos\phi\right)a\sin\phi$$

$$= \frac{a^2}{2}(\sec\phi - \cos\phi)\sin\phi$$

$$= 2(\sec\phi - \cos\phi)\sin\phi$$

if $\phi = \frac{\pi}{4} \Rightarrow s = 1(Q)$

If $\phi = \frac{\pi}{3} \Rightarrow S = \frac{3\sqrt{3}}{2}(T)$

If $\phi = \frac{\pi}{6} \Rightarrow S = \frac{1}{2\sqrt{3}}(S)$

If $\phi = \frac{\pi}{12} \Rightarrow S = \frac{\left(2-\sqrt{3}\right)^2}{2} = \frac{\left(\sqrt{3}-1\right)^4}{8}(P)$