Sri Chaitanya IIT Academy., India.

AP, TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

A right Choice for the Real Aspirant

ICON Central Office, Madhapur–Hyderabad

PHYSICS

SECTION-1 (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER ranging from 0 TO 9, BOTH INCLUSIVE.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct integer is entered;

Zero Marks : 0 If the question is unanswered;

Negative Marks : −1 In all other cases.

1. A particle of mass 1kg is subjected to a force which depends on the position as $\vec{F} = -k\left(x\hat{i} + y\hat{j}\right) kg\, m\, s^{-2}$ with $k = 1kg\, s^{-2}$. At time t=0 ,the particle's position $\vec{r} = \left(\frac{1}{\sqrt{2}}\hat{i} + \sqrt{2}\hat{j}\right)m$ and its velocity $\vec{v} = \left(-\sqrt{2}\hat{i} + \sqrt{2}\hat{j} + \frac{2}{\pi}\hat{k}\right)m\, s^{-1}$.Let v_x *and* v_y denote the *x* and the y components of the particle's velocity, respectively. **Ignore gravity**. When z=0.5m, the value of $\left(x v_y - y v_x\right) is$ ______ $m^2 s^{-1}$.

Ans. 3

Sol. **METHOD -1**

The problem can be seen as superposition of two independent SHMs in x and y directions

$$u_x = -\sqrt{2}, x_0 = \frac{1}{\sqrt{2}} \text{ and } u_y = \sqrt{2}, y_0 = \sqrt{2}$$

Time taken in z direction with

$$F_x = \frac{0.5}{\frac{2}{\pi}} = \frac{\pi}{4} see$$

Time period $T = 2\pi\sqrt{\frac{M}{K}} = 2\pi\sec$

So time elapsed = $\frac{T}{8} \equiv \frac{\pi}{4}$ rad rotation of phasor.

In y direction

$w\sqrt{A^2 - y_0^2} = u_y$

$\Rightarrow A = 2 \Rightarrow y_0 = \frac{A}{\sqrt{2}}$

So after $\frac{T}{8}\sec$, $v_y = 0$ and $y = A = 2$ (extreme position)

In x direction

$w\sqrt{A^2 - x_0^2} = u_x$

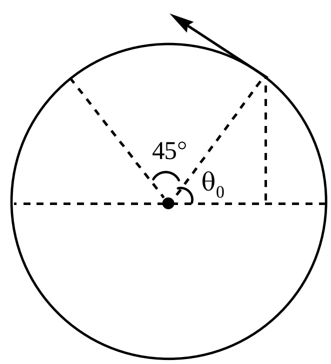
$\Rightarrow A = \frac{\sqrt{5}}{\sqrt{2}} \Rightarrow x_0 = \frac{A}{\sqrt{5}}$

$\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$

Final angle with x axis

$= \cos^{-1}\left(\frac{1}{\sqrt{5}}\right) + \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

So final $v_x = \frac{-3}{2} m/s,\ x = \frac{-1}{2}$



So $xv_y - yv_x = 0 - 2\left(-\frac{3}{2}\right) = 3$

Ans: 3

METHOD -2

$$\bar{F} = m\left(a_x\hat{i} + a_y\hat{j}\right) = -k\left(x\hat{i} + y\hat{j}\right)$$

$$ma_x = -kx \qquad ma_y = -ky$$

$$a_x = \frac{-k}{m}x \qquad a_y = \frac{-k}{m}y$$

$$m = 1k_y,\ K = 1N/M,\ W = \sqrt{\frac{K}{m}} = 1rad/s$$

$$\int V_x dv_x = -\int Kx\,dx$$

$$\frac{V_x^2}{2} = \frac{-x^2}{2} + C_1\text{----------(1)}$$

at t = 0sec

$$V_x = -\sqrt{2}m/s,\ x = \frac{1}{\sqrt{2}}m$$

$$\Rightarrow C_1 = \frac{5}{4}$$

From 1:

$$V_x^2 = \sqrt{\left(\frac{\sqrt{5}}{2}\right)^2 - x^2}$$

$$V_x^2 = \sqrt{a_1^2 - x^2}: \qquad a_1 = \sqrt{\frac{5}{2}}$$

$$x = a_1\sin\left(wt + \phi_1\right)$$

$$V_x = a_1 w\cos\left(wt + \phi_1\right)$$

At t = 0sec

$$x = \frac{1}{\sqrt{2}} = v_a = -\sqrt{2}$$

$$\Rightarrow \sin\phi_1 = \frac{1}{\sqrt{5}}, \cos\phi = \frac{-2}{\sqrt{5}}$$

In the question particle moves along z-axis with constant speed $\frac{2}{\pi}m/s$

$$Z = \frac{2}{\pi}t$$

If z=0.5 $\Rightarrow t = \frac{\pi}{7}sec$

$$x = \sqrt{\frac{5}{2}}\sin\left(\frac{\pi}{4}\times\phi_1\right) = -\frac{1}{2}m$$

$$V_x = \sqrt{\frac{5}{2}}\cos\left(\frac{\pi}{4}\times\phi_1\right) = -\frac{3}{2}m/s$$

And

$$\int V_y dy = -\int k_y dy$$

$$\frac{V_y^2}{2} = \frac{-y^2}{2} + C_2 \text{ ------(1)}$$

At t = 0 sec

$$V_y = 2, y = \sqrt{2}$$

$$C_2 = 2$$

$$V_y^2 = \sqrt{(2)^2 - y^2}$$

$$V_y^2 = \sqrt{a_2^2 - y^2} : a_2 = 2$$

$$y = a_2\sin(wt + \phi_2)$$

$$V_y = a_2 w\cos(wt + \phi_2)$$

at t = 0 sec

$y=\sqrt{2},\ V_y=\sqrt{2}$

$\sin\phi_2\frac{1}{\sqrt{2}},\cos\phi_2=\frac{1}{\sqrt{2}}$

$\phi_2=\frac{\pi}{4}$

In the question particle moves along z-axis with constant speed $\frac{2}{\pi}m/s$

$Z=\frac{2}{\pi}t$

If $z=0.5\Rightarrow t=\frac{\pi}{4}\sec$

At $t=t/4\sec$

$y.2\sin\left(\frac{\pi}{4}+\frac{\pi}{9}\right)=2\,m$

$V_y.2\cos\left(\frac{\pi}{7}+\frac{\pi}{4}\right)=0\,m/s$

$\left(xV_y-yV_x\right)=3$

2. In a radioactive decay chain reaction , $^{230}_{90}Th$ nucleus decays into $^{214}_{84}Po$ nucleus. The ratio of the number of α to number of β^- particles emitted in this process is_________.

Ans. 2

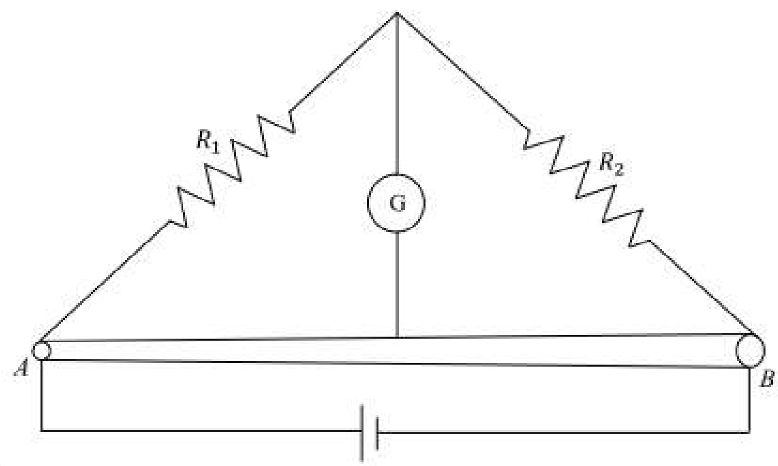
Sol. $^{230}_{90}Th\longrightarrow {}^{214}_{84}P_0+x\cdot{}^{4}_{2}He+y\cdot{}^{0}_{-}e+y.\overline{\nu}$

$230-214=x\times4$ …….(1) $\Rightarrow x=4$

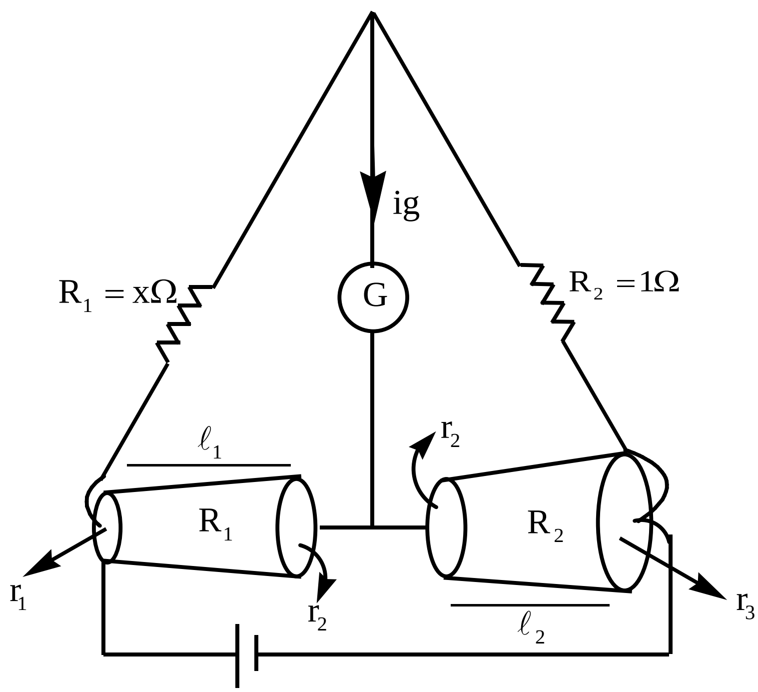
$90-84=x\cdot2-y$ …….(2) $\Rightarrow y=2$

$\therefore$ ratio $=\frac{x}{y}=2$

3. Two resistances $R_1 = X\Omega$ and $R_2 = 1\Omega$ are connected to a wire AB of uniform resistivity, as shown in the figure. The radius of the wire varies linearly along its axis from 0.2mm at A to 1 mm at B. A galvanometer(G) connected to the center of the wire,50 cm from each end along its axis, shows zero deflection when A and B are connected to a battery. The value of X is__________.



Ans. 5



Sol.

$R'_1 = \dfrac{\rho \ell_1}{\pi r_1 r_2}$; $R'_2 = \dfrac{\rho \ell_2}{\pi r_2 r_3}$; $\ell_1 = \ell_2 = 0.5\text{m}$

$r_3 = 1\text{mm}$; $r_1 = 0.2\text{mm}$

If ig = 0

Then $\dfrac{R_1}{R'_1} = \dfrac{R_2}{R'_2}$

$$\Rightarrow \frac{x}{\dfrac{\rho \ell_1}{\pi r_1 r_3}} = \frac{1}{\dfrac{\rho \ell_2}{\pi r_2 r_3}} \Rightarrow x r_1 = r_3$$

$$x = \frac{1}{0.2} = 5\ \Omega.$$

4. In a particular system of units, a physical quantity can be expressed in terms of the electric charge e, electron mass m_e, Planck's constant h, and Coulomb's constant $k = \dfrac{1}{4\pi \epsilon_0}$, where ϵ_0 is the permittivity of vacuum. In terms of these physical constants, the dimension of the magnetic field is $[B] = [e]^{\alpha}[m_e]^{\beta}[h]^{\gamma}[k]^{\delta}$. The value of $\alpha + \beta + \gamma + \delta$ *is* _______.

Ans. **4**

Sol. $[B] = [e]^{\alpha}[me]^{\beta}[h]^{\gamma}[k]^{\delta}$

$$[M\ \ I^{-1}\ \ T^{-2}] = [IT]^{\alpha}[M]^{\beta}[ML^2T^{-1}]^{\gamma}[ML^3T^{-4}I^{-2}]^{\gamma}$$

From above equation

$\beta + \gamma + \delta = 1$ ....(1)

$\alpha - 2\delta = -1$ ....(2)

$2\gamma + 3\delta = 0$ ....(3)

$\alpha - \gamma - 4\delta = -2$.........(4)

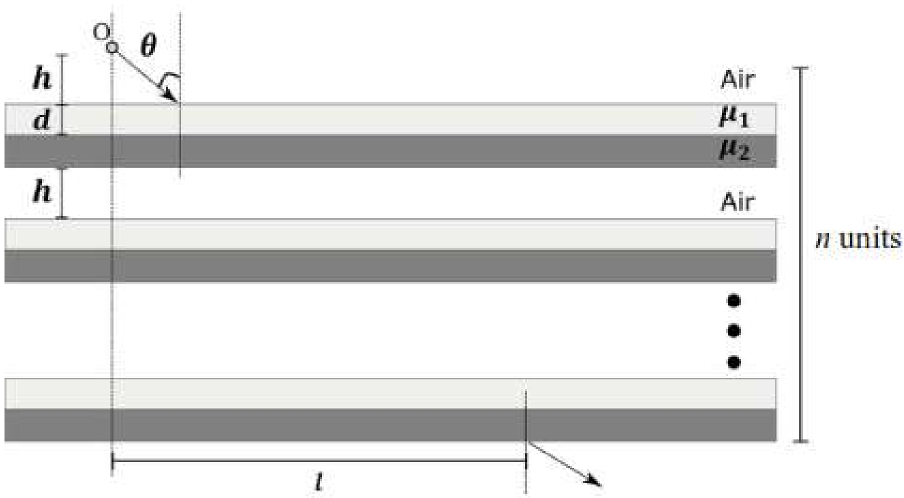
from (2), (3) and (4) equations

$\alpha = 3$

from (1)

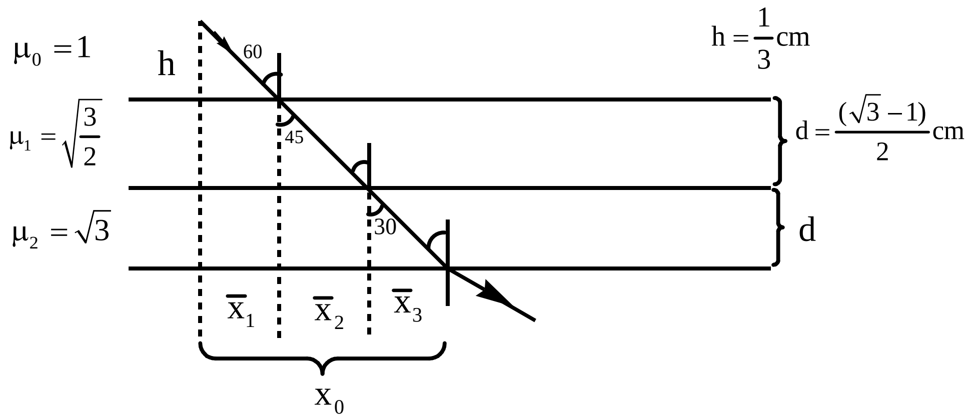
$\alpha + \beta + \gamma + \delta = 1 + 3 = 4$

5. Consider a configuration of n identical units, each consisting of three layers. The first layer is a column of air of height $h = \frac{1}{3} cm,$ and the second and third layers are of equal thickness $d = \frac{\sqrt{3}-1}{2} cm,$ and refractive indices $\mu_1 = \sqrt{\frac{3}{2}}$ and $\mu_2 = \sqrt{3}$,respectively. A light source O is placed on the top of the first unit, as shown in the figure. A ray of light from O is incident on the second layer of the first unit at an angle of $\theta = 60^0$ to the normal. For a specific value of n, the ray of light emerges from the bottom of the configuration at a distance $l = \frac{8}{\sqrt{3}} cm,$ as shown in the figure .The value of n is____________.



Ans. 4

Sol.



For one composite slab $x_0 = x_1 + x_2 + x_3$

By using snell's law

$x_1 = \frac{1}{\sqrt{3}}, \ x_2 = \frac{(\sqrt{3}-1)}{2}; \ x_3 = \frac{(\sqrt{3}-1)}{2}\frac{1}{\sqrt{3}}$

$\Rightarrow \ x_0 = x_1 + x_2 + x_3 = \frac{2}{\sqrt{3}}$

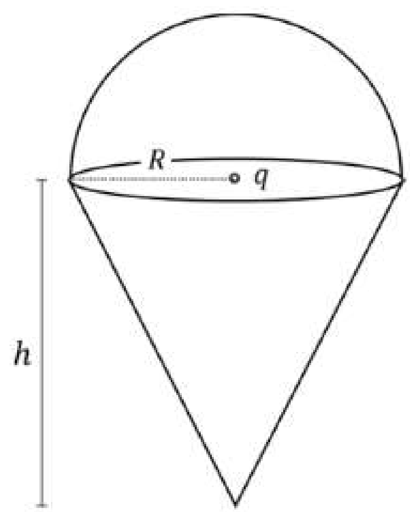
$\therefore$ for n composite slabs $x^1 = n.x_0$

Given that $x^1 = nx_0 = \frac{8}{\sqrt{3}}$ cm

$\Rightarrow \ n = 4$.

6. A charge q is surrounded by a closed surface consisting of an inverted cone of height h and base radius R, and a hemisphere of radius R as shown in the figure. The electric flux through the conical surface is

$\frac{nq}{6 \in_0}(in\,SI\,units)$ The value of n is______________.



Ans. 3

Sol. $\varnothing_{total} = \varnothing_{Hemisphere} + \varnothing_{core} = \frac{q}{\in_0}$

Here $\varnothing_H = \varnothing_C = \frac{q}{2\in_0} = \frac{nq}{6\in_0}$

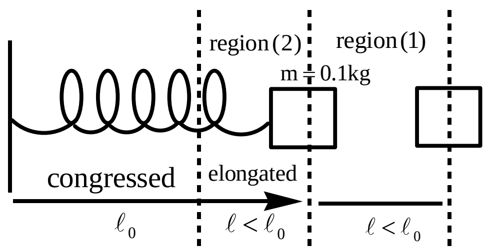
n = 3

7. On a frictionless horizontal plane, a bob of mass m=0.1kg is attached to a spring with natural length l_0=0.1m. the spring constant is $k_1 = 0.009 Nm^{-1}$ when the length of the spring $l > l_0$ and is $k_2 = 0.016 Nm^{-1}$ when $l < l_0$. Initially the bob is released from $l = 0.15m$. Assume that Hooke's law remains valid throughout the motion. If the time period of the full oscillation is $T=(n\pi)s$, then the integer closest to n is________.

Ans. 6

Sol. given that region (1) $\ell > \ell_0$ spring constant is $k_1 = 0.009\,N/m$

region (2) $\ell > \ell_0$ spring constant is $k_2 = 0.016\,N/m$



$$T = T_1 + T_2 = \pi\sqrt{\frac{m}{k_1}} + \pi\sqrt{\frac{m}{k_2}}$$

$$= \pi\sqrt{\frac{0.1}{0.009}} + \pi\sqrt{\frac{0.1}{0.016}}$$

$$= \pi\sqrt{\frac{1}{0.09}} + \pi\sqrt{\frac{1}{0.16}}$$

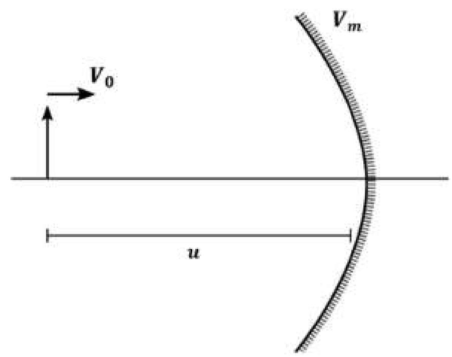
$$= \pi\left(\frac{1}{0.3} + \frac{1}{0.4}\right)$$

$$= (5.83)\pi$$

$$\approx 6\pi$$

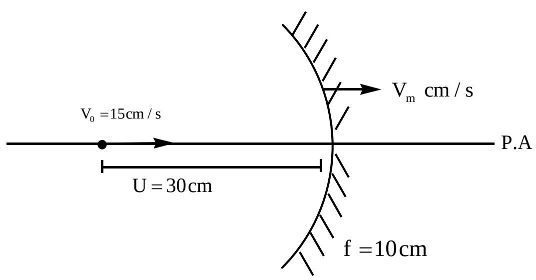
$\therefore$ n = 6.

8. An object and a concave mirror of focal length f = 10 cm both move along the principal axis of the mirror with constant speeds. The object moves with speed $V_0 = 15\,\text{cm s}^{-1}$ towards the mirror with respect to a laboratory frame. The distance between the object and the mirror at a given moment is denoted by u. When u = 30 cm, the speed of the mirror V_m is such that the image is instantaneously at rest with respect to the laboratory frame, and the object forms a real image. The magnitude of V_m is __________ cm s^{-1}.



Ans. 3

Sol.



We can solve by using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ formula

Magnification $m = -\frac{1}{2}$

$V_0 m^2 + V_i = (1 + m^2) V_m$

Given that $V_i = 0$

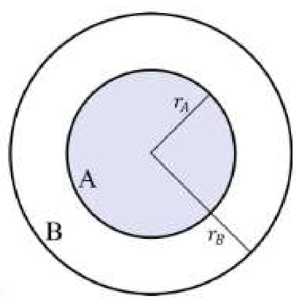
$\Rightarrow \quad 15\frac{1}{4} + 0 = \left(1 + \frac{1}{4}\right) \cdot V_m \qquad V_m = 3\,\text{cm/s}$

SECTION-2 (Maximum Marks : 24)

- This section contains **NINE (09)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct ;
Partial Marks : +1 If two or more options are correct but **ONLY** two options are chosen, and it is a correct option ;
Zero Marks : 0 If unanswered;
Negative Marks : –2 In all other cases.

9. In the figure, the inner (shaded) region A represents a sphere of radius $r_A = 1$, within which the electrostatic charge density varies with the radial distance **r** from the center as $\rho_A = kr$, where k is positive. In the spherical shell B of outer radius r_B, the electrostatic charge density varies as $\rho_B = \frac{2k}{r}$. Assume that dimensions are taken care of. All physical quantities are in their SI units.



Which of the following statement(s) is(are) correct?

(A) If $r_B = \sqrt{\frac{3}{2}}$, then the electric field is zero everywhere outside B.

(B) If $r_B = \frac{3}{2}$, then the electric potential just outside B is $\frac{k}{\epsilon_0}$

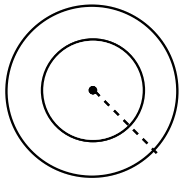
(C) If $r_B = 2$, then the total charge of the configuration is $15\pi k$.

(D) If $r_B = \frac{5}{2}$, then the magnitude of the electric field just outside B is $\frac{13\pi k}{\epsilon_0}$.

Ans. B

Sol. Appling Gauss law for just out side the shell,

R.H.S for total charge



$$=\int_0^1 kr\,4\pi r^2 dr + \int_1^{r_B} \frac{2k}{r} 4\pi r^2 dr$$

$$=\frac{4\pi k}{4}(1-0)+\frac{8\pi k}{2}(r_B^2-1)$$

$$=k(4\pi r_B^2-3\pi) \quad \text{.......... (1)}$$

Option A: if $r_B=\frac{\sqrt{3}}{2}$ then q = 0 and E = 0

Option B: if $r_B=\frac{3}{2}$ then $q=6\pi k$ and $V=\frac{q}{4\pi\in_0 r_B}$

$$\Rightarrow V=\frac{k}{\in_0}$$

Option C: if $r_B=2$ then $q=13\pi k$

Option D: if $r_B=\frac{5}{2}$ then $q=22\pi k$ and $E=\frac{q}{4\pi\in_0 r_B^2}$

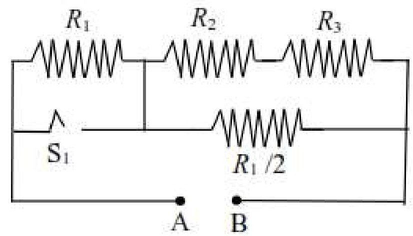
$$\Rightarrow E=\frac{22\pi k}{4\pi\in_0\left(\frac{25}{4}\right)}=\frac{22\pi k}{25\in_0}$$

Answer is B

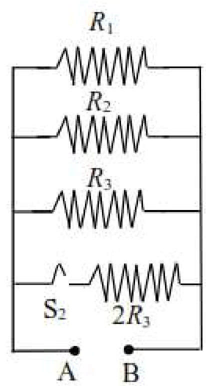
10. In Circuit-1 and Circuit-2 shown in the figures, $R_1 = 1\Omega$, $R_2 = 2\Omega$ and $R_3 = 3\Omega$.

P_1 and P_2 are the power dissipations in Circuit-1 and Circuit-2 when the switches S_1 and S_2 are in open conditions, respectively.

Q_1 and Q_2 are the power dissipations in Circuit-1 and Circuit-2 when the switches S_1 and S_2 are in closed conditions, respectively.



Circuit-1



Circuit-2

Which of the following statement(s) is(are) correct?

(A) When a voltage source of 6V is connected across A and B in both circuits, $P_1 < P_2$.

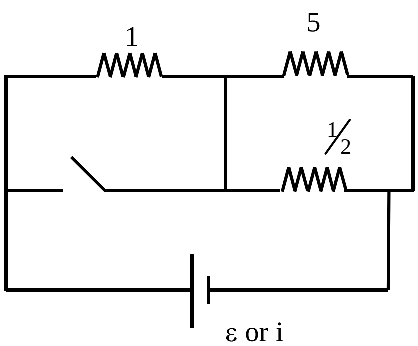
(B) When a constant current source of 2 Amp is connected across A and B in both circuits, $P_1 > P_2$.

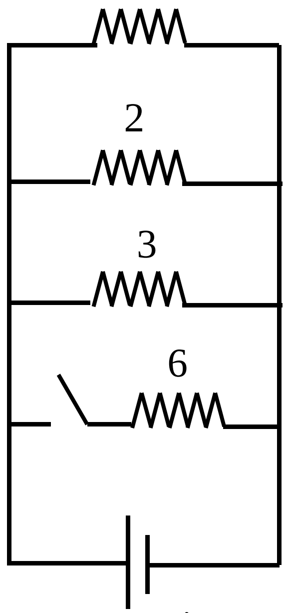
(C) When a voltage source of 6V is connected across A and B in Circuit -1 . $Q_1 > P_1$.

(D) When a constant current source of 2 Amp is connected across A and B in both circuits, $Q_2 < Q_1$.

Ans. ABC

Sol.





Option A: Circuit – 1, $R_{eff\,1} = 1 + \frac{5/2}{5+\frac{1}{2}} = \frac{16}{11}\,\Omega$

Circuit-2, $R_{eff\,2} = \frac{6}{2+6+3} = \frac{6}{11}\,\Omega$

As $R_{eff\,1} > R_{eff\,2} \Rightarrow P_1 < P_2$

Option B: Using same logic as option A

$i^2 R_{eff\,1} > i^2 R_{eff\,2} \Rightarrow P_1 > P_2$

Option C: Circuit-1, if switch is closed

$R_{eff\,1}$ changes to $\frac{5}{11}\Omega$

$\Rightarrow Q_1 > P_1$

Option D: Circuit-2 if switch is closed

$$R_{eff\,2} = \frac{6\times\frac{6}{11}}{6+\frac{6}{11}} = \frac{1}{2}\,\Omega$$

So now $R_{eff\,1} < R_{eff\,2} \Rightarrow Q_1 < Q_2$

Correct answer : A, B, C

11. A bubble has surface tension S. The ideal gas inside the bubble has ratio of specific heats $\gamma = \frac{5}{3}$. The bubble is exposed to the atmosphere and it always retains its spherical shape. When the atmospheric pressure is P_{a1}, the radius of the bubble is found to be r_1 and the temperature of the enclosed gas is T_1. When the atmospheric pressure is P_{a2}, the radius of the bubble and the temperature of the enclosed gas are r_2 and T_2, respectively.

Which of the following statement(s) is(are) correct?

(A) If the surface of the bubble is a perfect heat insulator, then $\left(\frac{r_1}{r_2}\right)^5 = \frac{P_{a2}+\frac{2S}{r_2}}{P_{a1}+\frac{2S}{r_1}}$

(B) If the surface of the bubble is a perfect heat insulator, then the total internal energy of the bubble including its surface energy does not change with the external atmospheric pressure.

(C) If the surface of the bubble is a perfect heat conductor and the change in atmospheric temperature is negligible, then $\left(\frac{r_1}{r_2}\right)^3 = \frac{P_{a2}+\frac{4S}{r_2}}{P_{a1}+\frac{4S}{r_1}}$

(D) If the surface of the bubble is a perfect heat insulator, then $\left(\frac{T_2}{T_1}\right)^{\frac{5}{2}} = \frac{P_{a2}+\frac{4S}{r_2}}{P_{a1}+\frac{4S}{r_1}}$

Ans. **CD**

Sol. $\frac{PV}{T}$ and PV^{γ} should be constant for an a diabatic change

$$\left(P_{\alpha_1}+\frac{4S}{r_1}\right)\left(\frac{4\pi}{3}r_1^3\right)^{5/3}=\left(P_{a_2}+\frac{4S}{r_2}\right)\left(\frac{4\pi}{3}r_2^3\right)^{5/3}$$

$$\left(\frac{r_1}{r_2}\right)^3=\left(\frac{P_{a_2}+\frac{4S}{r_2}}{P_{a_1}+\frac{4S}{r_1}}\right) \qquad \text{............ (1)}$$

Also for isothermal case PV = constant

$$\Rightarrow\left(\frac{r_1}{r_2}\right)^3=\left(\frac{P_{a_2}+\frac{4S}{r_2}}{P_{a_1}+\frac{4S}{r_1}}\right)$$

Taking bubble shell as system (excluding gas)

Work done by all external agents = ΔU_{bubble}

$$W_{\text{inside gas}}+W_{\text{outside gas}}=\Delta U_{\text{bubble}}$$

$$-\Delta U_{\text{inside gas}}+W_{\text{outside gas}}=\Delta U_{\text{bubble}}$$

$$W_{\text{outside gas}}=\Delta U_{\text{inside gas}}+\Delta U_{\text{bubble}}$$

Also (1) implies $P^{1-\gamma}T^{\gamma}=\text{costant}$

$$\Rightarrow P^2\propto T^5\Rightarrow P\propto T^{5/2}$$

So $$\left(\frac{T_2}{T_1}\right)^{5/2}=\left(\frac{P_{a_2}+\frac{4s}{r_2}}{P_{a_1}+\frac{4S}{r_1}}\right)$$

So Answer is C, D

12. A disk of radius R with uniform positive charge density σ is placed on the xy plane with its center at the origin. The Coulomb potential along the z – axis is

$$V(z)=\frac{\sigma}{2\epsilon_0}\left(\sqrt{R^2+z^2}-z\right).$$

A particle of positive charge q is placed initially at rest at a point on the z axis with $z=z_0$ and $z_0>0$. In addition to the Coulomb force, the particle experiences a vertical force $\vec{F}=-c\hat{k}$ with $c>0$. Let $\beta=\frac{2c\epsilon_0}{q\sigma}$. Which of the following statement(s) is(are) correct?

(A) For $\beta=\frac{1}{4}$ and $z_0=\frac{25}{7}R$, the particle reaches the origin.

(B) For $\beta=\frac{1}{4}$ and $z_0=\frac{3}{7}R$, the particle reaches the origin.

(C) For $\beta=\frac{1}{4}$ and $z_0=\frac{R}{\sqrt{3}}$, the particle returns back to $z=z_0$

(D) For $\beta>1$ and $z_0>0$, the particle always reaches the origin.

Ans. ACD

Sol. $E=\frac{\sigma}{2\epsilon_0}\left(1-\frac{z}{\sqrt{R^2+z^2}}\right)$

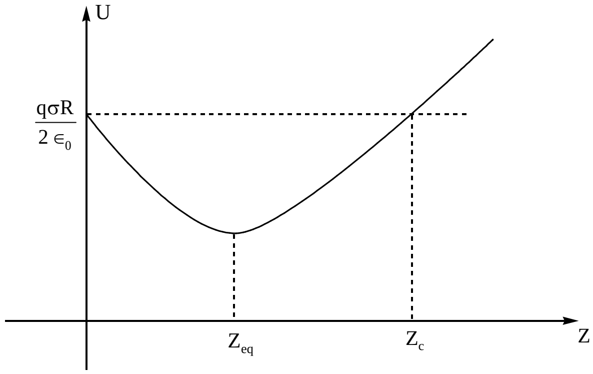
E is never zero $\Rightarrow$ V is monotonous function

$E_{max}=\frac{\sigma}{2\epsilon_0}$ and $V_{max}=\frac{\sigma R}{2\epsilon_0}$ occur at Z = 0.

$U_{net}=\frac{q\sigma}{2\epsilon_0}\left(\sqrt{R^2+Z^2}-Z+\beta Z\right)$

We need to find Z_c

Where $U(Z_c)=U(0)$



$$\Rightarrow \sqrt{R^2 + Z_C^2} - Z_C + \beta Z_C = R \qquad \ldots\ldots(1)$$

If particle is released between 0 and Z_c it will not reach origin. But if $Z_0 > Z_c$, it will cross origin so, for each option, we have to check $LHS > RHS$ for equation (1)

Option A: $\beta = \frac{1}{4}, Z_0 = \frac{25}{7}R$

$$\frac{RHS}{R} = \sqrt{1 + \frac{625}{49}} - \frac{3}{4}\frac{25}{7}$$

$$\approx \frac{26}{7} - \frac{\left(3/4\right)25}{7} > 1$$

$$\Rightarrow Z_0 > Z_C$$

Option B: $\beta = \frac{1}{4}, \qquad Z_0 = \frac{3R}{7}$

$$\frac{RHS}{R} = \sqrt{1 + \frac{9}{49}} - \frac{3}{4}\left(\frac{3}{7}\right) < 1$$

$$\Rightarrow Z_0 < Z_C$$

Option C: $\beta = \frac{1}{4} \qquad Z_0 = \frac{R}{\sqrt{3}}$

$$\frac{RHS}{R} = \sqrt{1 + \frac{1}{3}} - \frac{3}{4}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{\sqrt{2}}{3} - \frac{\sqrt{3}}{4} < 1$$

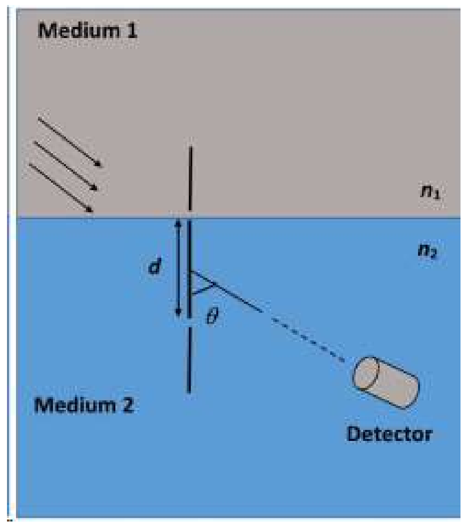
$\Rightarrow$ $Z_0 < Z_C$ and particle is trapped in the potential well, so it oscillates

Option D: for $\beta > 1$, RHS of equation 1 > R

$\Rightarrow$ particle always reaches origin

Correct Options are A, C, D

13. A double slit setup is shown in the figure. One of the slits is in medium 2 of refractive index n_2. The other slit is at the interface of this medium with another medium 1 of refractive index $n_1 (\neq n_2)$. The line joining the slits is perpendicular to the interface and the distance between the slits is d. The slit widths are much smaller than d. A monochromatic parallel beam of light is incident on the slits from medium 1. A detector is placed in medium 2 at a large distance from the slits, and at an angle θ from the line joining them, so that θ equals the angle of refraction of the beam. Consider two approximately parallel rays from the slits received by the detector.



Which of following statement(s) is(are) correct?

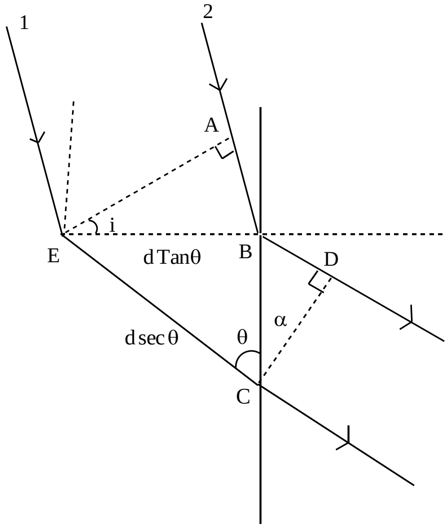
(A) The phase difference between the two rays is independent of d.

(B) The two rays interfere constructively at the detector.

(C) The phase difference between the two rays depends on n_1 but is independent of n_2.

(D) The phase difference between the two rays vanishes only for certain values of d and the angle of incidence of the beam, with θ being the corresponding angle of refraction.

Ans. AB



Sol.

AB, EC and BD

Are extra geometric paths

$\phi_1 - \phi_2 = \frac{2\pi}{\lambda}\left(n_1 AB + n_2 BD - n_2 EC\right)$

$= \frac{2\pi\alpha}{\lambda}\left(n_1 Tan\theta \sin i + n_2 \cos\theta - n_2 \sec\theta\right)$ {snell's law gives $n_1 \sin i = n_2 \sin\theta$ }

$= \frac{2\pi d n_2}{\lambda}\left(Tan\theta \sin\theta + \cos\theta - \sec\theta\right)$

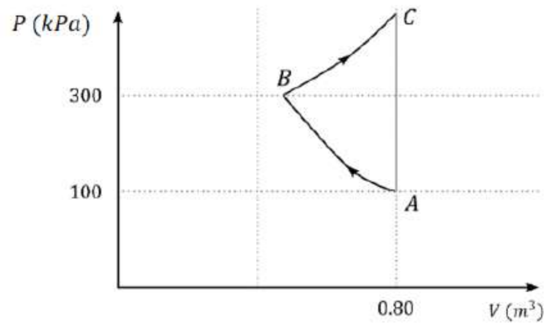
$= \frac{2\pi d n_2}{\lambda}\left(\frac{\sin^2\theta + \cos^2\theta - 1}{\cos\theta}\right)$

= 0

So, phase difference is independent of d.

Ans: AB

14. In the given P-V diagram, a monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is first compressed adiabatically from state A to state B. Then it expands isothermally from state B to state C. [Given: $\left(\frac{1}{3}\right)^{0.6} \simeq 0.5$, $\ln 2 \simeq 0.7$].



Which of the following statement(s) is(are) correct?

(A) The magnitude of the total work done in the process $A \to B \to C$ is 144 kJ.

(B) The magnitude of the work done in the process $B \to C$ is 84 kJ.

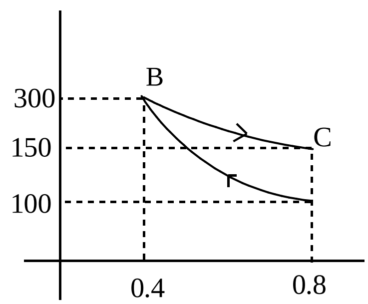
(C) The magnitude of the work done in the process $A \to B$ is 60 kJ

(D) The magnitude of the work done in the process $C \to A$ is zero.

Ans. BCD

Sol. Correct PV diagram should be as shown.

Diagram given in the question is wrong.



$$W_{AB} = -nC_V(T_f - T_i) = \frac{-3}{2}\Delta(PV) = -60\,KJ$$

$$W_{BC} = nRT\,\ell n\left(\frac{V_f}{V_i}\right) = 300(0.4)(0.7) = 84\,KJ$$

$$W_{A \to B \to C} = 24\,KJ$$

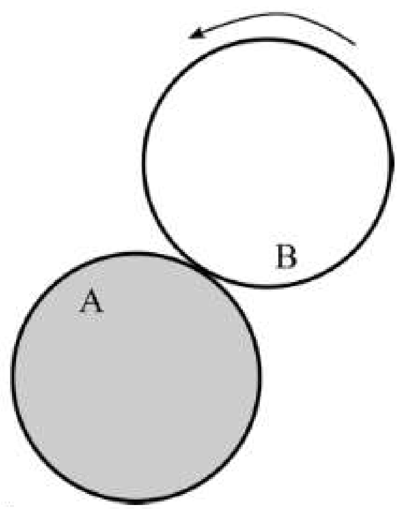
Answer is B, C, D

SECTION-3 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

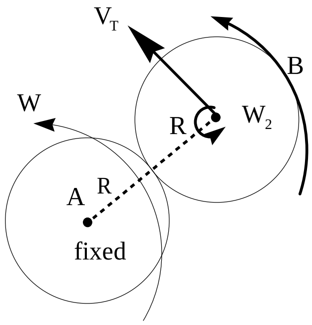
Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

15. A flat surface of a thin uniform disk A of radius R is glued to a horizontal table. Another thin uniform disk B of mass M and with the same radius R rolls without slipping on the circumference of A, as shown in the figure. A flat surface of B also lies on the plane of the table. The center of mass of B has fixed angular speed ω about the vertical axis passing through the center of A. The angular momentum of B is $nM\omega R^2$ with respect to the center of A. Which of the following is the value of n?



(A) 2 (B) 5 (C) $\frac{7}{2}$ (D) $\frac{9}{2}$

Ans. **B**



Sol.

Given that $2RW = RW_2$

$W_2 = 2W$

$\therefore$ Angular momentum of B w.r.t A is

$= \overline{L}_S + \overline{L}_O$

$= \dfrac{mR^2}{2} W_2 + mV_T 2R$

$= \dfrac{mR^2}{2}(2W) + m(2R)^2 W$

$= 5mR^2 W$

n = 5

Option B is correct

16. When light of a given wavelength is incident on a metallic surface, the minimum potential needed to stop the emitted photoelectrons is 6.0 V. This potential drops to 0.6V if another source with wavelength four times that of the first one and intensity half of the first one is used. What are the wavelength of the first source and the work function of the metal, respectively?

[Take $\dfrac{hc}{e} = 1.24 \times 10^{-6}\, J\, m\, C^{-1}$.]

(A) $1.72 \times 10^{-7}\, m, 1.20\, eV$

(B) $1.72 \times 10^{-7}\, m, 5.60\, eV$

(C) $3.78 \times 10^{-7}\, m, 5.60\, eV$

(D) $3.78 \times 10^{-7}\, m, 1.20\, eV$

Ans. A

Sol. $\frac{hc}{\lambda}-\phi=\frac{1}{2}mv_1^2=6ev$ ....(1)

$\frac{hc}{4\lambda}-\phi=\frac{1}{2}mv_2^2=0.6v$ ....(2)

From (1) and (2)

$\frac{hc}{\lambda}-\frac{hc}{4\lambda}=5.4\text{ ev}$

$\frac{hc}{\lambda}=\frac{4}{3}\times(5.4)ev\Rightarrow\frac{hc}{\lambda}=7.2ev$

$\Rightarrow\frac{1}{\lambda}=\frac{7.2v}{\left(\frac{hc}{e}\right)}=\frac{7.2}{1.24}\times10^{6}\Rightarrow\lambda=\frac{1.24\times10^{-6}}{7.2}$

$\lambda=1.72\times10^{-7}\text{ m}$

From (1)

$\phi=\frac{hc}{\lambda}-\frac{1}{2}mv_1^2=\frac{hc}{\lambda}-6ev=(7.2-6)ev$

$= 1.2\text{ ev}$

Option (A) is correct

17. Area of the cross-section of a wire is measured using a screw gauge. The pitch of the main scale is 0.5 mm. The circular scale has 100 divisions and for one full rotation of the circular scale, the main scale shifts by two divisions. The measured readings are listed below.

Measurement condition	Main scale reading	Circular scale reading
Two arms of gauge touching each other without wire	0 division	4 divisions
Attempt-1 : With wire	4 divisions	20 divisions
Attempt-2: With wire	4 divisions	16 divisions

What are the diameter and cross-sectional area of the wire measured using the screw gauge?

(A) $2.22 \pm 0.02\,\text{mm},\ \pi(1.23 \pm 0.02)\,\text{mm}^2$

(B) $2.22 \pm 0.01\text{mm},\ \pi(1.23 \pm 0.01)\,\text{mm}^2$

(C) $2.14 \pm 0.02\,\text{mm},\ \pi(1.14 \pm 0.02)\,\text{mm}^2$

(D) $2.14 \pm 0.01\text{mm},\ \pi(1.14 \pm 0.01)\,\text{mm}^2$

Ans. D

Sol. L. C of the instrument = $\dfrac{2 \times 0.5\,\text{mm}}{100} = 0.01\text{mm}$

error in the instrument in the

$\therefore$ +ve error = $+4 \times \text{L.C} = 0.04$ mm

$R_1 = 4 \times (0.5)\,\text{mm} + 20 \times \text{L.C} = 2 + 0.20$

$R_2 = 4 \times (0.5)\,\text{mm} + 16 \times \text{L.C} = 2 + 0.16$

True reading $R_1^1 = R_1 - (+\text{ve error}) = 2 + 0.16$

$R_2^1 = R_2 - (+\text{ve error}) = 2 + 0.12$

$$\Rightarrow R_0 = \frac{R_1^1 + R_2^1}{2} = \frac{2.16 + 2.12}{2} = 2.14$$

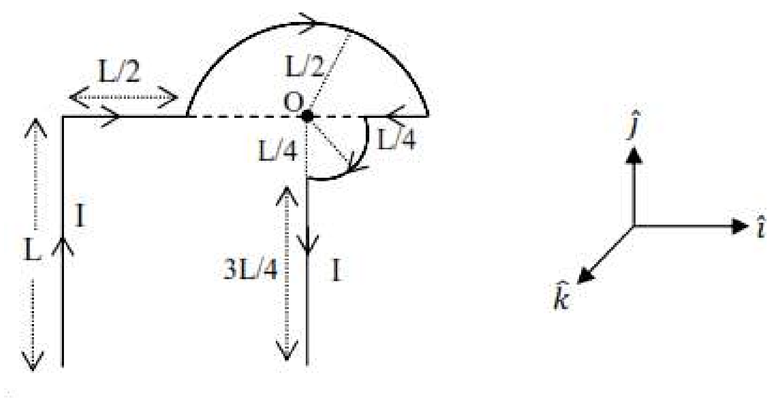
$\therefore$ diameter of the wire = $2.14 \pm 0.01\text{mm}$

$$\therefore \text{ Area } = \frac{\pi d^2}{4} = \left(\frac{\pi(2.14)^2}{4} \pm 0.01\right)\text{mm}^2$$

$$= (\pi \cdot (1.14) \pm 0.01)\text{mm}^2$$

Option (D) in correct

18. Which one of the following options represents the magnetic field $\vec{B}$ at O due to the current flowing in the given wire segments lying on the xy plane?



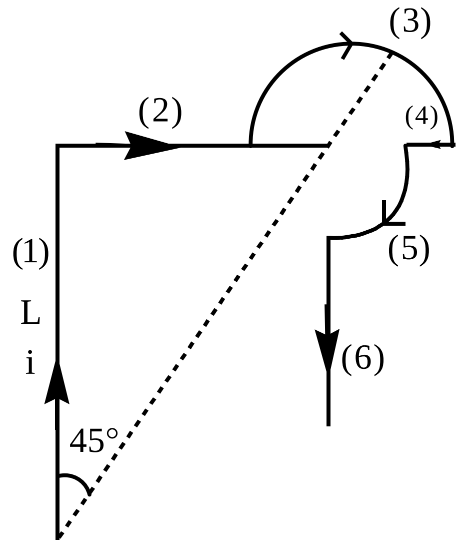
(A) $\vec{B} = \frac{-\mu_0 I}{L}\left(\frac{3}{2} + \frac{1}{4\sqrt{2}\pi}\right)\hat{k}$

(B) $\vec{B} = \frac{-\mu_0 I}{L}\left(\frac{3}{2} + \frac{1}{2\sqrt{2}\pi}\right)\hat{k}$

(C) $\vec{B} = \frac{-\mu_0 I}{L}\left(1 + \frac{1}{4\sqrt{2}\pi}\right)\hat{k}$

(D) $\vec{B} = \frac{-\mu_0 I}{L}\left(1 + \frac{1}{4\pi}\right)\hat{k}$

Ans. C



Sol.

$$B_0 = \frac{\mu_0 0}{4\pi L}\left(\frac{1}{\sqrt{2}}\right)(-\hat{k}) + 0 + \frac{2\mu_0 i}{4\pi L}\cdot\pi(-\hat{k}) + 0 + \frac{4\mu_0 i}{4\pi L}\cdot\frac{\pi}{2}(-\hat{k}) + 0$$

$$= \frac{\mu_0 i}{L}\left(\frac{1}{4\sqrt{2}\pi} + 1\right)\left(-\hat{k}\right)$$

Option (C) is correct

CHEMISTRY

SECTION-1 (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER ranging from 0 TO 9, BOTH INCLUSIVE.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct integer is entered;
Zero Marks : 0 If the question is unanswered;
Negative Marks : −1 In all other cases.

1. Concentration of H_2SO_4 and Na_2SO_4 in a solution is 1 M and 1.8×10^{-2}M, respectively. Molar solubility of $PbSO_4$ in the same solution is $X\times10^{-Y}$M (expressed in scientific notation). The value of Y is ________.

[Given : Solubility product of $PbSO_4(K_{sp})=1.6\times10^{-8}$. For H_2SO_4, K_{a1} is very large and $K_{a2}=1.2\times10^{-2}$]

Ans. 6

Sol. $H_2SO_4 \rightarrow H^+ + HSO_4^-$

$HSO_4^- \rightleftharpoons H^+ + SO_4^{-2}$ $\quad Ka_2 = 1.2\times10^{-2}$

$[Na_2SO_4]=1.8\times10^{-2}M \Rightarrow [SO_4^{-2}]=1.8\times10^{-2}M$

Degree of dissociation of HSO_4^- is lowered.

$HSO_4^- \rightleftharpoons H^+ + SO_4^{-2}$

$1+x \quad (1-x) \quad (1.8\times10^{-2}-x)$

$$\frac{(1-x)(1.8\times10^{-2}-x)}{1+x}=1.2\times10^{-2}$$

$x=0.6\times10^{-2}$

$$\left[Pb^{+2}\right]\left[SO_4^{-2}\right]=1.6\times10^{-8}$$

$$\left[Pb^{+2}\right]=\frac{4}{3}\times10^{-6}$$

2. An aqueous solution is prepared by dissolving 0.1 mol of an ionic salt in 1.8 kg of water at 35^0C. The salt remains 90% dissociated in the solution. The vapour pressure of the solution is 59.724 mm of Hg. Vapor pressure of water at 35^0C is 60.000 mm of Hg. The number of ions present per formula unit of the ionic salt is ________.

Ans. 5

Sol. $$\frac{P^0-P}{P^0}=\frac{n_i}{n_i+N}$$

$$\frac{P^0-P}{P^0}=\frac{n_i}{N}$$ For dilute solutions $n<<N,\ n+N\simeq N$

$$\frac{60-59.724}{60}=\frac{0.1\times i}{\frac{1800}{18}}$$

$$\frac{0.276}{60}=\frac{0.1i}{100}$$

$$27.6=6i\Rightarrow i=4.6$$

$$\alpha=\frac{i-1}{n-1}$$

$$0.9=\frac{4.6-1}{n-1}\qquad\Rightarrow 0.9n-0.9=3.6$$

$$\Rightarrow 0.9n=4.5$$

$$\Rightarrow n=5$$

3. Consider the strong electrolytes Z_mX_n, U_mY_p and V_mX_n. Limiting molar conductivity $(\wedge^0)$ of U_mY_p and V_mX_n are 250 and 440 S cm^2mol^{-1}, respectively.

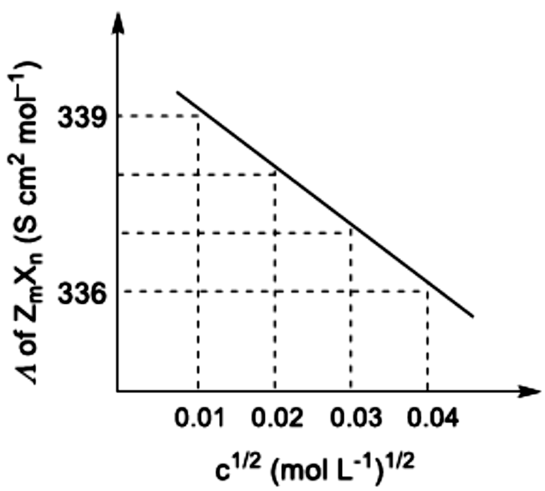
The value of (m + n + p) is ________.

Given :

Ion	Z^{n+}	U^{p+}	V^{n+}	X^{m-}	Y^{m-}
$\lambda^0(Scm^2mol^{-1})$	50.0	25.0	100.0	80.0	100.0

λ^0 is the limiting molar conductivity of ions.

The plot of molar conductivity $(\wedge)$ of Z_mX_n vs $c^{1/2}$ is given below.



Ans. 7

Sol. $\underline{Z_mX_n}$

$\lambda_C = \lambda_0 - b\sqrt{C}$

$-b = \frac{3}{-0.03} \Rightarrow b = 100$

$339 = \lambda_0 - 100 \times 0.01$

$\lambda_0 = 340$

340 = 50 m + 80n …… (1)

$(1)\times(2)$ …… (3)

$\underline{U_mY_p}$

$250 = m \times 25 + P \times 100$ …… (2)

$\underline{V_mX_n}$

$440 = m \times 100 + n \times 80$ …… (3)

$680 = 100\,m + 160\,n$

$\underline{440 = 100m + 80n}$

$240 = 80n$

$\boxed{n = 3}$

Put n value in (1)

$340 = 50m + 80 \times 3$

$\boxed{\Rightarrow m = 2}$

Put m value in (2) $250 = 2 \times 25 + 100P \Rightarrow P = 2$

$\therefore m + n + p = 2 + 3 + 2 = 7$

4. The reaction of Xe and O_2F_2 gives a Xe compound **P**. The number of moles of HF produced by the complete hydrolysis of 1 mol of **P** is_________.

Ans. 2

Sol. $Xe + O_2F_2 \longrightarrow XeF_2 + O_2$

$2XeF_2 + 2H_2O \longrightarrow 2Xe + 4HF + O_2$

5. Thermal decomposition of $AgNO_3$ produces two paramagnetic gases. The total number of electrons present in the antiboding molecular orbitals of the gas that has the higher number of unpaired electrons is________.

Ans. 6

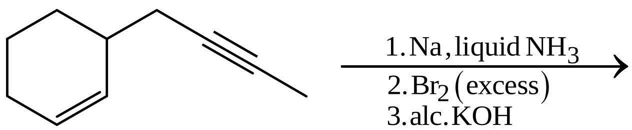
Sol. $AgNO_3 \longrightarrow Ag + NO_2 + O_2$

NO_2 - 1 unpaired electron

O_2 - 2 unpaired electrons

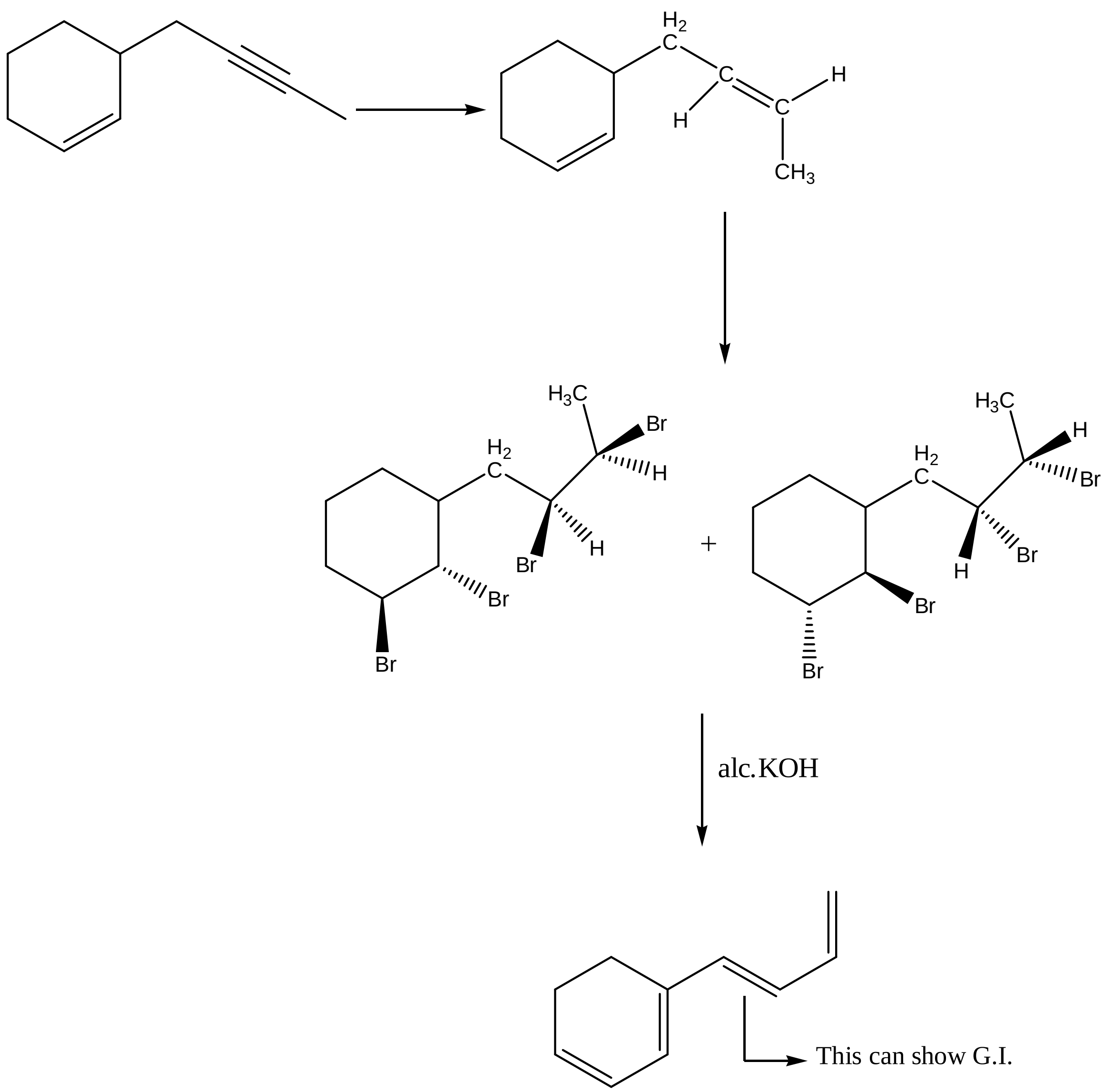
$O_2 : \sigma_{1s^2}\sigma^*_{1s^2}\sigma_{2s^2}\sigma^*_{2s^2}\sigma_{2p_z^2}\left(\pi_{2p_x^2} = \pi_{2p_y^2}\right)\left(\pi^*_{2p_x^1} = \pi^*_{2p_y^1}\right)$

6. The number of isomeric tetraenes (NOT containing sp-hybridized carbon atoms) that can be formed from the following reaction sequence is _________.



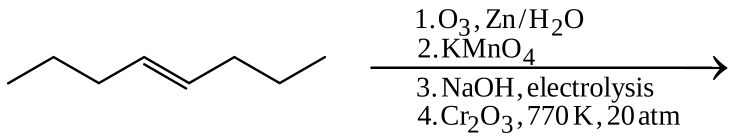
Ans. 2

Sol.



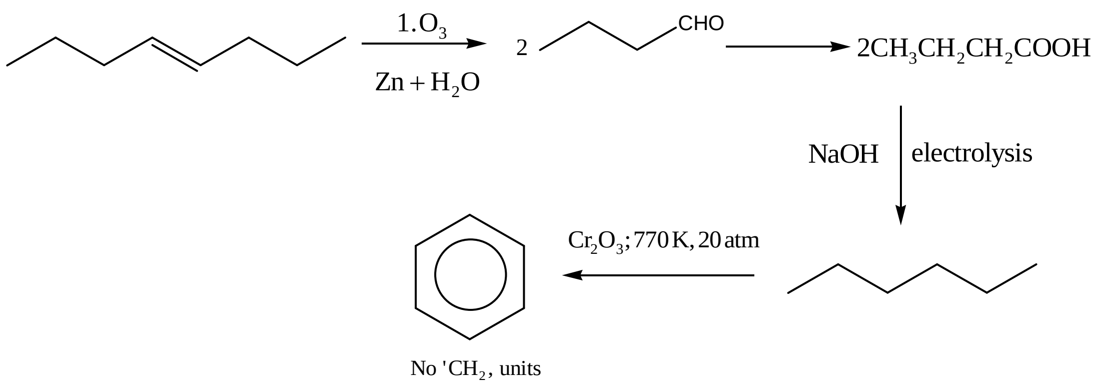
So 2 isomers.

7. The number of $-CH_2-$ (methylene) groups in the product formed from the following reaction sequence is_______.

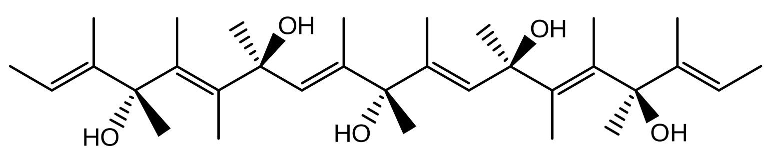


Ans. **0**

Sol. The number of CH_2 groups in product is "Zero"



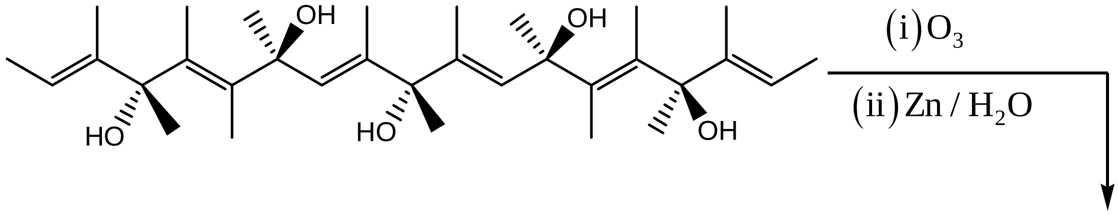
8. The total number of chiral molecules formed from one molecule of P on complete ozonolysis $(O_3, Zn/H_2O)$ is___.

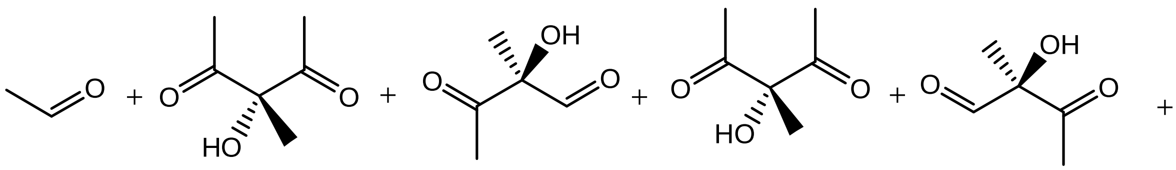


P

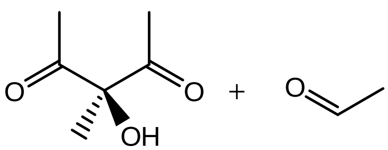
Ans. **2**

Sol.





I II III IV V



VI VII

I, II, IV, VI, VII are achiral molecules.

III, V are chiral molecules.

Ans is 2.

SECTION-2 (Maximum Marks : 24)

- This section contains **NINE (09)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct ;
Partial Marks : +1 If two or more options are correct but **ONLY** two options are chosen, and it is a correct option ;
Zero Marks : 0 If unanswered;
Negative Marks : –2 In all other cases.

9. To check the principle of multiple proportions, a series of pure binary compounds (P_mQ_n) were analyzed and their composition is tabulated below. The correct option(s) is(are)

Compound	**Weight % of P**	**Weight % of Q**
1	**50**	**50**
2	**44.4**	**55.6**
3	**40**	**60**

A) If empirical formula of compound 3 is P_3Q_4, then the empirical formula of compound 2 is P_3Q_5.

B) If empirical formula of compound 3 is P_3Q_2 and atomic weight of element P is 20, then the atomic weight of Q is 45.

C) If empirical formula of compound 2 is PQ, then the empirical formula of the compound 1 is P_5Q_4.

D) If atomic weight of P and Q are 70 and 35, respectively, then the empirical formula of compound 1 is P_2Q.

Ans. **BC**

Sol. Let ‘x’ and ‘y’ be the molecular masses of P and Q.

(B) $\frac{3x}{2y}=\frac{4}{6}$

$\Rightarrow x=20$ $\quad\quad y=45$

(C) $\frac{x}{y}=\frac{4}{5}$

$\frac{P_{50}}{40}\frac{Q_{50}}{50}$ $\quad\Rightarrow$ Empirical formula P_5Q_4

10. The correct option(s) about entropy (S) is(are)

[R = gas constant, F = Faraday constant, T = Temperature]

A) For the reaction, $M(s)+2H^{+}(aq)\rightarrow H_2(g)+M^{2+}(aq)$, if $\frac{dE_{cell}}{dT}=\frac{R}{F}$, then the entropy change of the reaction is R (assume that entropy and internal energy changes are temperature independent).

B) The cell reaction,

$Pt(s)|H_2(g,1\ bar)|H^{+}(aq),0.01M||H^{+}(aq,0.1M)|H_2(g,1\ bar)|Pt(s)$, is an entropy driven process.

C) For racemisation of an optically active compound, $\Delta S>0$.

D) $\Delta S>0$, for $[Ni(H_2O)_6]^{2+}+3\ en\rightarrow[Ni(en)_3]^{2+}+6H_2O$ (where en = ethylenediamine).

Ans. **BCD**

Sol. (A) $\frac{\Delta S}{nF}=\frac{dE_{cell}}{dT}$

$\Delta S=nF\frac{dE_{cell}}{dT}=2\cancel{F}\frac{R}{\cancel{F}}=2R$

(B) Reaction is spontaneous $\Delta G<0$

$\Delta S>0$

(C) For racemisation $\Delta H = 0$ and $\Delta G < 0$

$\Delta S > 0$

(D) No. of particles increases in solution $\Delta S > 0$

11. The compound(s) which react(s) with NH_3 to give boron nitride (BN) is (are)

(A) B (B) B_2H_6 (C) B_2O_3 (D) HBF_4

Ans. ABC

Sol. $* B_2H_6 + NH_3(\text{excess}) \xrightarrow{\text{high temp}} (BN) + H_2$

$* \; B_2O_3 + NH_3 \longrightarrow (BN) + H_2O$

$* \; B + NH_3 \longrightarrow (BN) + H_2$

12. The correct option(s) related to the extraction of iron from its ore in the blast furnace operating in the temperature range 900-1500 K is(are)

(A) Limestone is used to remove silicate impurity.

(B) Pig iron obtained from blast furnace contains about 4% carbon.

(C) Coke (C) converts CO_2 to CO.

(D) Exhaust gases consist of NO_2 and CO.

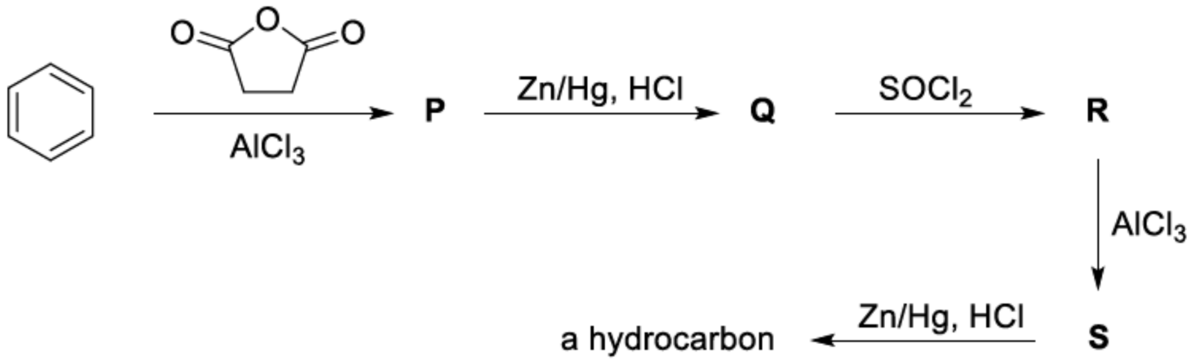
Ans. ABC

Sol. $* CaO + SiO_2 \longrightarrow CaSiO_3$

$* \; CO_2 + C \longrightarrow CO$

* The iron obtained from blast furnace contains about 4% carbon

13. Considering the following reaction sequence, the correct statement(s) is(are)



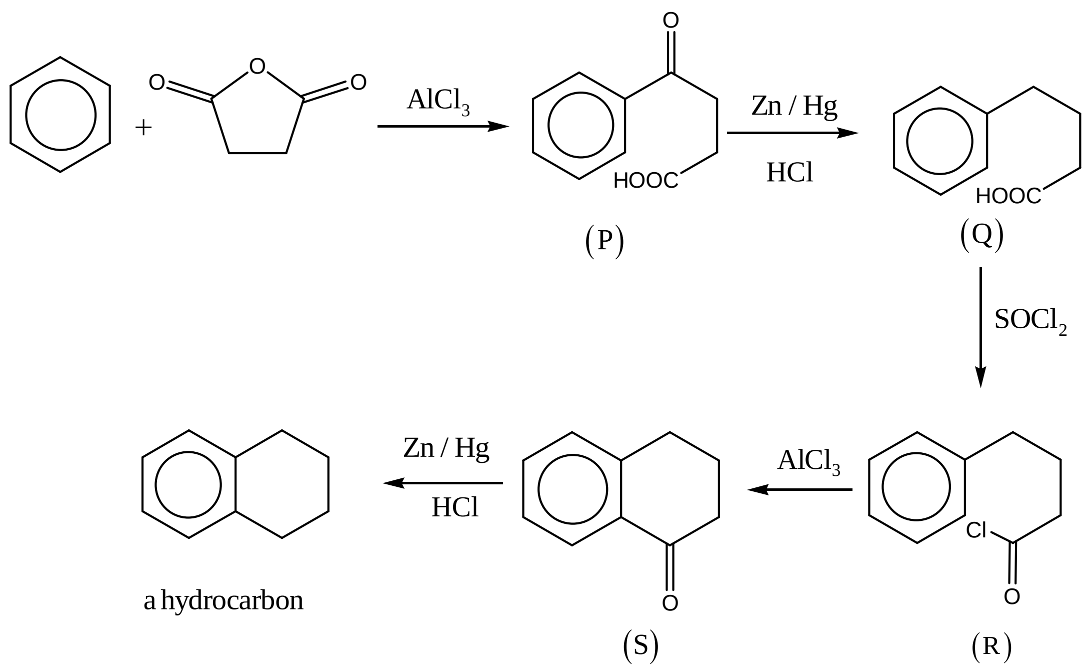
(A) Compounds P and Q are carboxylic acids.

(B) Compound S decolorizes bromine water.

(C) Compounds P and S react with hydroxylamine to give the corresponding oximes.

(D) Compound R reacts with dialkylcadmium to give the corresponding tertiary alcohol.

Ans. AC



Sol.

14. Among the following, the correct statement(s) about polymers is(are)

(A) The polymerization of chloroprene gives natural rubber.

(B) Teflon is prepared from tetrafluoroethene by heating it with persulphate catalyst at high pressures.

(C) PVC are thermoplastic polymers.

(D) Ethene at 350-570 K temperature and 1000-2000 atm pressure in the presence of a peroxide initiator yields high density polythene.

Ans. **BC**

Sol. Directly from NCERT(chapter polymer).

SECTION-3 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −1 In all other cases.

15. Atom X occupies the fcc lattice sites as well as alternate tetrahedral voids of the same lattice. The packing efficiency (in %) of the resultant solid is closest to

A) 25 B) 35 C) 55 D) 75

Ans. B

Sol. 4 from lattice sites and 4 from tetrahedral voids i.e., Interlocked fcc. for which distance of closest approach is given by

$$\frac{\sqrt{3}a}{4}=2r \Rightarrow r=\frac{\sqrt{3}a}{4\times 2}=\frac{\sqrt{3}a}{8}$$

$$pf=\frac{8\times\frac{4}{3}\pi\left(\frac{\sqrt{3}a}{8}\right)^{8}}{a^{3}}$$

$$=\cancel{8}\times\frac{4}{\cancel{8}}\times\frac{1}{8^{2}\cancel{3}}.\cancel{3}\sqrt{3}\times 3.14$$

$$\approx 34\%$$

16. The reaction of $HClO_3$ with HCl gives a paramagnetic gas, which upon reaction with O_3 produces

(A) Cl_2O (B) ClO_2 (C) Cl_2O_6 (D) Cl_2O_7

Ans. C

Sol. $HClO_3 + HCl \longrightarrow ClO_2 + \frac{1}{2}Cl_2 + H_2O$

$2ClO_2 + O_3 \longrightarrow Cl_2O_6 + \frac{1}{2}O_2$

17. The reaction of $Pb(NO_3)_2$ and NaCl in water produces a precipitate that dissolves upon the addition of HCl of appropriate concentration. The dissolution of the precipitate is due to the formation of

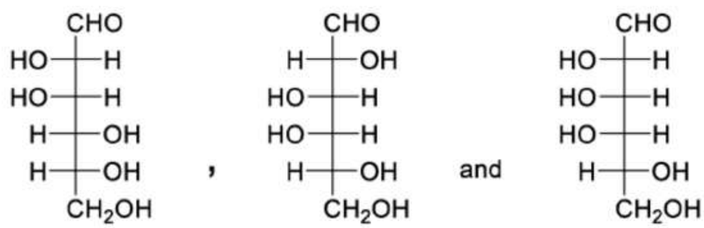
(A) $PbCl_2$ (B) $PbCl_4$ (C) $[PbCl_4]^{2-}$ (D) $[PbCl_6]^{2-}$

Ans. C

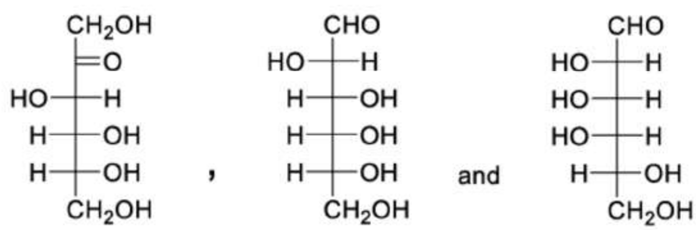
Sol. $Pb^{2+} \xrightarrow{Cl^-} \underset{(White)}{PbCl_2}\downarrow \xrightarrow{Conc\,HCl} [PbCl_4]^{2-}$

18. Treatment of D-glucose with aqueous NaOH results in a mixture of monosaccharides, which are

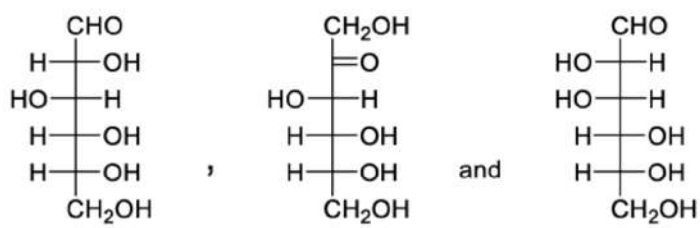
(A)



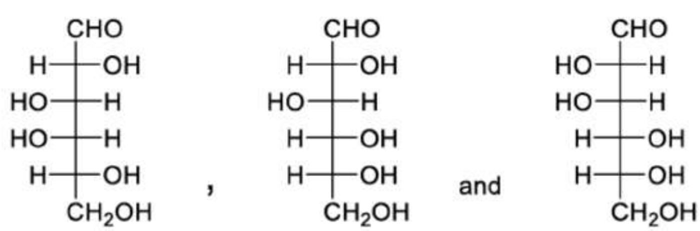
(B)



(C)

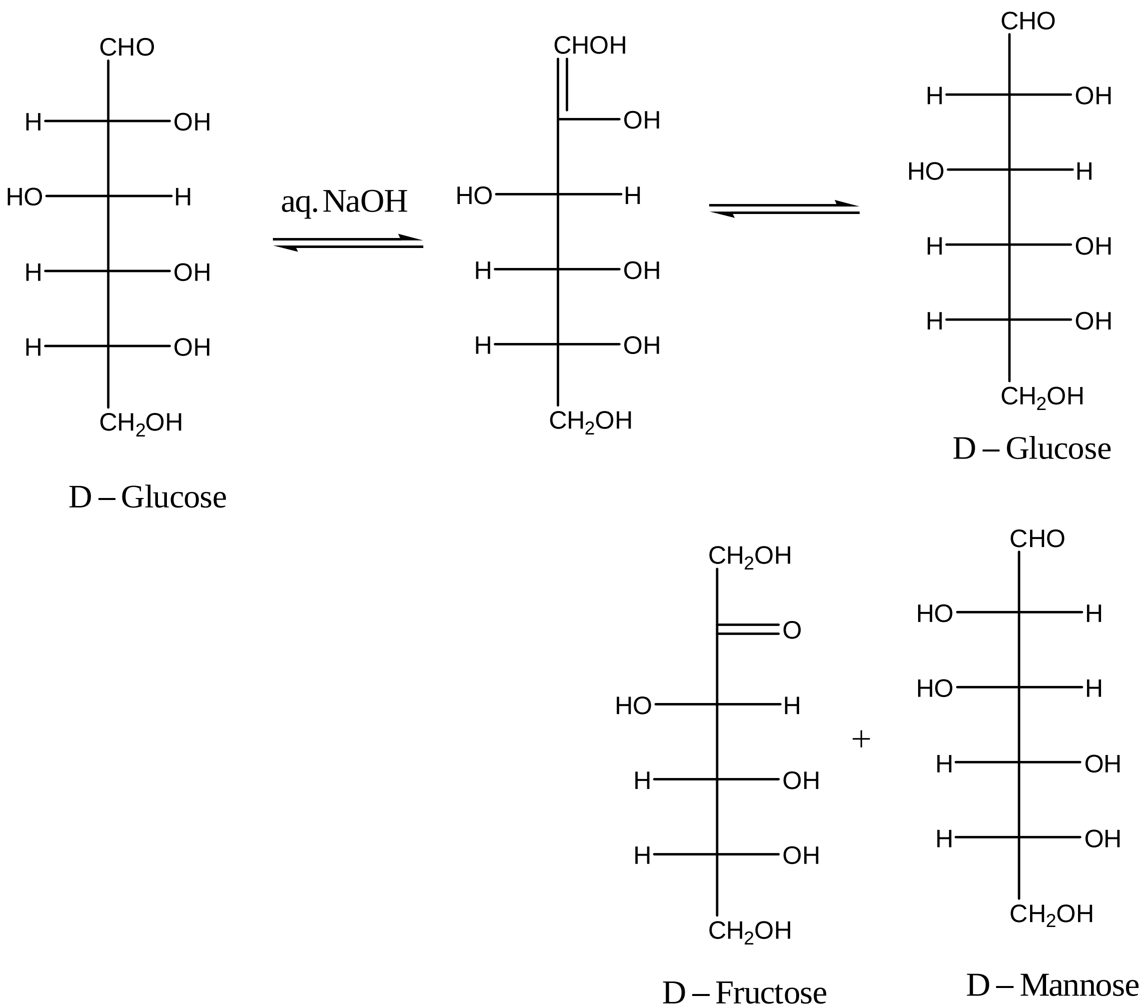


(D)



Ans. C

Sol. Treatment of D-Glucose with aq. NaOH will cause isomerisation of the molecule through tautomerisation.



Thus answer is "C"

MATHEMATICS

SECTION-1 (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER ranging from 0 TO 9, BOTH INCLUSIVE.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct integer is entered;
Zero Marks : 0 If the question is unanswered;
Negative Marks : −1 In all other cases.

1. Let α and β be real numbers such that $-\frac{\pi}{4} < \beta < 0 < \alpha < \frac{\pi}{4}$. If $\sin(\alpha+\beta) = \frac{1}{3}$ and $\cos(\alpha-\beta) = \frac{2}{3}$, then the greatest integer less than or equal to

$$\left(\frac{\sin\alpha}{\cos\beta} + \frac{\cos\beta}{\sin\alpha} + \frac{\cos\alpha}{\sin\beta} + \frac{\sin\beta}{\cos\alpha}\right)^2$$ is ________________.

Ans. **1**

Sol. $\sin(\alpha+\beta) = \frac{1}{3}$

$$\cos(\alpha+\beta) = \frac{2\sqrt{2}}{3}$$

$$\cos(\alpha+\beta) = \frac{2}{3}, \sin(\alpha-\beta) = \frac{\sqrt{5}}{3}$$

$$\frac{\sin\alpha\sin\beta + \cos\alpha\cos\beta}{\sin\beta\cos\beta} + \frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\sin\alpha\cos\alpha}$$

$$= \frac{2\cos(\alpha-\beta)}{\sin 2\beta} + \frac{2\cos(\alpha-\beta)}{\sin 2\alpha}$$

$$= 2\cos(\alpha-\beta)\left\{\frac{\sin 2\alpha + \sin 2\beta}{\sin 2\alpha \sin 2\beta}\right\}$$

$$= 4\cos(\alpha-\beta)\left\{\frac{2\sin(\alpha+\beta)\cos(ga-\beta)}{\cos(2\alpha-2\beta)-\cos(2\alpha+2\beta)}\right\}$$

$$= \left\{\frac{8.\cos^2(\alpha-\beta)\cdot\sin(\alpha+\beta)}{\cos 2(\alpha-\beta)-\cos 2(\alpha+\beta)}\right\}$$

$$= \frac{8\cos^2(\alpha-\beta)\sin(\alpha+\beta)}{2\left\{\cos^2(\alpha-\beta)-\cos^2(\alpha+\beta)\right\}}$$

$$= \frac{8\times\frac{4}{9}\times\frac{1}{7}}{2\times\left\{\frac{4}{9}-\frac{8}{9}\right\}} = \frac{-4}{7}$$

$$\left(\frac{\sin\alpha}{\cos\beta}+\frac{\cos\beta}{\sin\alpha}+\frac{\cos\alpha}{\sin\beta}+\frac{\sin\beta}{\cos\alpha}\right)^2$$

$$= \frac{16}{9}.$$

Ans is 1

2. If y(x) is the solution of the differential equation

$$xdy-\left(y^2-4y\right)dx = 0 \;\; fot \;\; x>0,\; y(1)=2,$$

And the slope of the curve y=y(x) is never zero, then the value of $10y\left(\sqrt{2}\right)$ is_________.

Ans. 8

Sol. $xdy = y\left(y-4\right)dx \Rightarrow \int\frac{dx}{x} = \int\frac{dy}{y\left(y-4\right)}$

$$\ln x = \frac{1}{4}\ln\left|\frac{y-4}{y}\right| \Rightarrow \left|\frac{y-4}{y}\right| = x^4$$

$$x^4 = \frac{4-y}{y} \Rightarrow y = \frac{4}{x^4+1} : y\sqrt{2} = \frac{4}{5}$$

$ans : 8$

3. The greatest integer less than or equal to

$$\int_1^2 \log_2\left(x^3+1\right)dx+\int_1^{\log_2 9}\left(2^x-1\right)^{\frac{1}{3}}dx$$ is________________.

Ans. 5

Sol. $y=\log_2\left(x^3+1\right)\Rightarrow x^3+1\Rightarrow x^y\Rightarrow x=\left(z^4-1\right)^{\frac{1}{3}}$

$$\int_a^b f(x)dx+\int_{f(a)}^{f(b)} f^{-1}(x)dx$$

$$=\int_b^a f(x)\,dx+\int_{f(a)}^{f(b)} x.f^1(x)\,dx$$

$$=\left[x\,f(x)\right]_a^b=\left[2\log_2^9-\log 1\right]$$

$$2^3\sqrt{2}=8(1.43)=11.44$$

$$2\times 3-1=5$$

4. The product of all positive real values of x satisfying the equation

$$x^{\left(16(\log_5 x)^3-68\log_5 x\right)}=5^{-16}$$

is________________.

Ans. 1

Sol. $\left(16\left(\log_5^x\right)^3-68\log_5^x\right)\log_5^x=16$

$$16t^3-68t^2+16=0\Rightarrow 4t^4-17t^2+4=0$$

$$1t^2=4\quad :4t^2=1$$

$$t=\pm 2\quad t^2=\pm\frac{1}{2}$$

$\log_5 x=\pm 2;\log_5 x=\pm\frac{1}{2}$ Product=1

5. If $\beta = \lim_{x \to 0} \frac{e^{x^3} - (1-x^3)^{\frac{1}{3}} + \left((1-x^2)^{\frac{1}{2}} - 1\right)\sin x}{x\sin^2 x}$,then the value of 6β is______________.

Ans. 5

Sol. $\beta = \lim_{x \to 0} \frac{1 + x^3 - 1 + \frac{1}{3}x^3 + \left(1 - \frac{1}{2}x^2 - 1\right)\sin x}{x\sin^2 x}$

$$\beta = \frac{\frac{4}{3}x^3 - \frac{1}{2}x^2(x)}{x^3} = \frac{4}{3} - \frac{1}{2} = \frac{8-3}{6}$$

Ans: 5

6. Let β be a real number. Consider the matrix

$$A = \begin{pmatrix} \beta & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 1 & -2 \end{pmatrix}$$

If $A^7 - (\beta - 1)A^6 - \beta A^5$ is a singular matrix, then the value of 9β is ________

Ans. 3

Sol. $\left|A^2 - (\beta - 1)A - \beta I\right| = 0 \left(\because |A| \neq 0\right)$

$\left|A^2 + A - \beta(A + I)\right| = 0$

$|A| = -2|3 + 2 - 3 + 2|$

$\left|(A + I)(A - \beta I)\right| = 0 \Rightarrow |A - \beta I| = 0$

$\left|(A + I)(A - \beta I)\right| = 0 \Rightarrow |A - \beta I| = 0$

$$\begin{vmatrix} 0 & 0 & 1 \\ 2 & 1-\beta & -2 \\ 3 & 1 & -2-\beta \end{vmatrix} = 0 \Rightarrow 2 - 3(1 - \beta) = 0$$

$2 - 3 + 3\beta = 0 \Rightarrow 3\beta = 1$

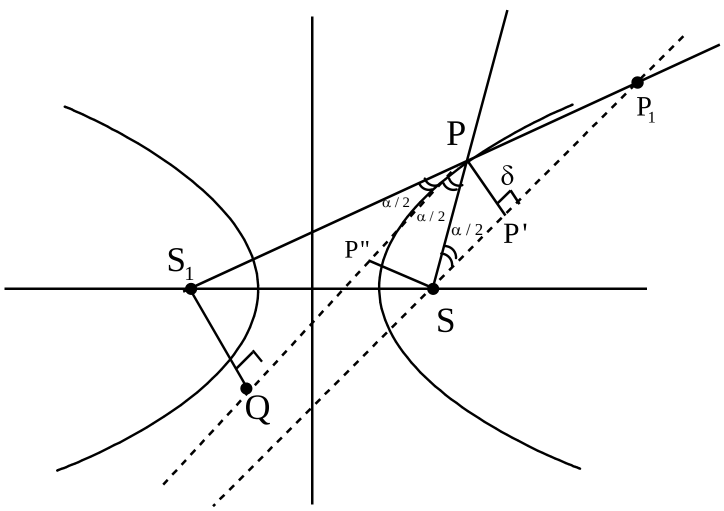
$9\beta = 3$

7. Consider the hyperbola

$$\frac{x^2}{100}-\frac{y^2}{64}=1$$

with foci at S and S_1, where S lies on the positive x-axis. Let P be a point on the hyperbola, in the first quadrant. Let $\angle SPS_1=\alpha$, with $\alpha<\frac{\pi}{2}$. The straight line passing through the point S and having the same slope as that of the tangent at P to the hyperbola, intersects the straight line S_1P at P_1. Let δ be the distance of P from the straight line SP_1, and $\beta=S_1P$. Then the greatest integer less than or equal to $\frac{\beta\delta}{9}\sin\frac{\alpha}{2}$ is _______

Ans. 7



Sol.

$SP''=\delta$ $\qquad\qquad S_1Q\cdot SP''=b^2=64$

$S_1Q=\frac{64}{8}$ ……. (1)

ΔS_1QP & $\Delta SPP''$

$$\sin\frac{\alpha}{2}=\frac{S_1Q}{S_1P}=\frac{S_1Q}{\beta}=\frac{SP''}{SP}=\frac{\delta}{SP}$$

$\Rightarrow \beta.\delta=S_1Q\cdot SP=\frac{64}{\delta}\cdot SP$ ……(2)

from $\Delta SP'P$ $\sin\frac{\alpha}{2}=\frac{\delta}{SP}$ put in (2)

$$\beta.\delta=\frac{64}{8}\cdot SP=\frac{64}{\sin\frac{\alpha}{2}}$$

$$\beta.\delta=\sin\frac{\alpha}{2}=64\Rightarrow\frac{1}{9}\beta\delta\sin\frac{\alpha}{2}=\frac{64}{9}$$

so $\left[\frac{1}{9}\beta\delta\sin\frac{\alpha}{2}\right]=\left[\frac{64}{9}\right]=7$ Ans

8. Consider the functions f, $g:R\rightarrow R$ defined by

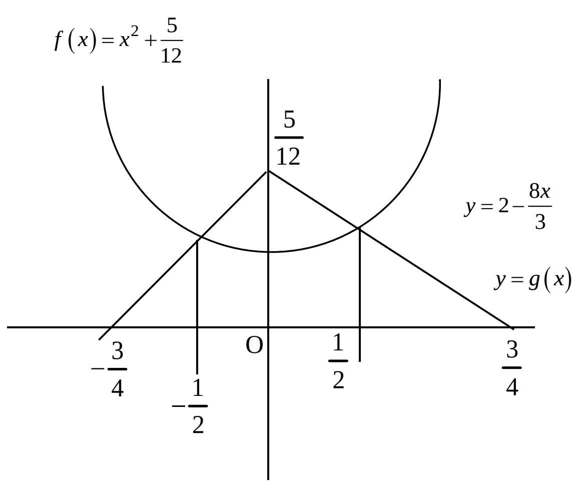
$$f(x)=x^2+\frac{5}{12} \text{ and } g(x)=\begin{cases}2\left(1-\frac{4|x|}{3}\right), & |x|\le\frac{3}{4},\\ 0, & |x|>\frac{3}{4}\end{cases}$$

If α is the area of the region

$$\left\{(x,y)\in R\times R:|x|\le\frac{3}{4},\ 0\le y\le\min\{f(x),g(x)\}\right\},$$

then the value of 9α is ___________

Ans. 6



Sol.

$$A = 2\left[\int_0^{\frac{1}{2}}\left(x^2 + \frac{5}{12}\right)dx + \int_{\frac{1}{2}}^{\frac{3}{4}}\left(2 - \frac{8x}{3}\right)dx\right]$$

$$= 2\left[\left(\frac{x^3}{3} + \frac{5}{12}\right)_0^{\frac{1}{2}} + \left(2x - \frac{4x^2}{3}\right)_{\frac{1}{2}}^{\frac{3}{4}}\right] = \frac{4}{6}$$

$$\frac{2}{3} \times 9 = 6$$

SECTION-2 (Maximum Marks : 24)

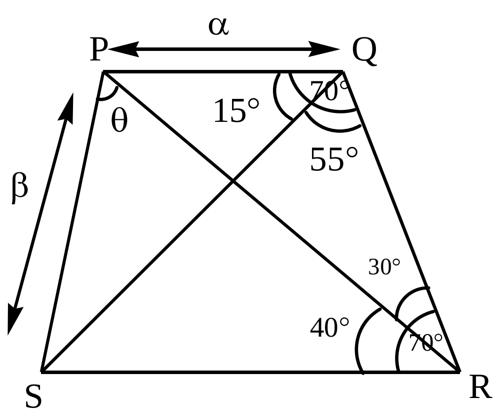
- This section contains **NINE (09)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct ;
Partial Marks : +1 If two or more options are correct but **ONLY** two options are chosen, and it is a correct option ;
Zero Marks : 0 If unanswered;
Negative Marks : –2 In all other cases.

9. Let PQRS be a quadrilateral in a plane, where $QR = 1$, $\angle PQR = \angle QRS = 70^\circ$, $\angle PQS = 15^\circ$ and $\angle PRS = 40^\circ$. If $\angle RPS = \theta^\circ$, $PQ = \alpha$ and $PS = \beta$, then the interval(s) that contain(s) the value of $4\alpha\beta\sin\theta^\circ$ is/are

(A) $(0, \sqrt{2})$ (B) $(1, 2)$ (C) $(\sqrt{2}, 3)$ (D) $\left(2\sqrt{2}, 3\sqrt{2}\right)$

Ans. AB



Sol.

$$\frac{PR}{\sin 70^\circ} = \frac{\alpha}{\sin 30^\circ} = \frac{1}{\sin 80^\circ} \Rightarrow \alpha = \frac{1}{2\sin 80^\circ}$$

$$\frac{SR}{\sin\theta} = \frac{\beta}{\sin 40^\circ} = \frac{PR}{\sin(40+\theta)} \Rightarrow \beta = \frac{PR \cdot \sin 40^\circ}{\sin(40+\theta)}$$

Also $\frac{Sr}{\sin 55°} = \frac{1}{\sin 55°}$ $\beta = \frac{\sin 70°}{\sin 80°} \cdot \frac{\sin 40°}{\sin(40+\theta)}$

$SR = 1$ $\beta = \frac{\sin 70°}{2\cos 40° \sin(40+\theta)}$

$\frac{1}{\sin\theta} = \frac{\beta}{\sin 40°} \Rightarrow \beta = \frac{\sin 40°}{\sin\theta}$

$24 \times \frac{1}{2\sin 80°} \cdot \sin\theta = \frac{2\sin 40°}{2\sin 40° \cos 40°} = \sec 40°$

$\sec 45° > \sec 40° > \sec 60°$

10. Let $\alpha = \sum_{k=1}^{\infty} \sin^{2k}\left(\frac{\pi}{6}\right)$.

Let $g:[0,1] \to R$ be the function defined by

$g(x) = 2^{\alpha x} + 2^{\alpha(1-x)}$.

Then, which of the following statements is/are TRUE?

(A) The minimum value of $g(x)$ is $2^{\frac{7}{6}}$

(B) The maximum value of $g(x)$ is $1+2^{\frac{1}{3}}$

(C) The function $g(x)$ attains its maximum at more than one point

(D) The function $g(x)$ attains its minimum at more than one point

Ans. ABC

Sol. $\alpha = \frac{1}{2^2} + \frac{1}{2^4} + \ldots\ldots = \frac{\frac{1}{22}}{1-\frac{1}{4}} = \frac{1}{3}$

$g(x) = 2^{\frac{1}{3}x} + 2^{\frac{1}{3}(1-x)} = 2^{\frac{1}{3}x} + \frac{2^{\frac{1}{3}}}{2^{x/3}} \geq 2.2^2$

$2^{\frac{2}{3}x} = 2^{\frac{1}{3}}$

$$\frac{2}{3}x = \frac{1}{3}$$

$$x = \frac{1}{2}$$

Max $1 + \sqrt[3]{2}$.

11. Let $\overline{z}$ denote the complex conjugate of a complex number z. If z is a non-zero complex number for which both real and imaginary parts of

$$(\overline{z})^2 + \frac{1}{z^2}$$

are integers, then which of the following is/are possible value(s) of |z| ?

(A) $\left(\frac{43+3\sqrt{205}}{2}\right)^{\frac{1}{4}}$

(B) $\left(\frac{7+\sqrt{33}}{4}\right)^{\frac{1}{4}}$

(C) $\left(\frac{9+\sqrt{65}}{4}\right)^{\frac{1}{4}}$

(D) $\left(\frac{7+\sqrt{13}}{6}\right)^{\frac{1}{4}}$

Ans. **A**

Sol. $\overline{Z}^2 + \frac{1}{Z^2} + Z^2 + \frac{1}{\overline{Z}^2} = 2\ (\text{Integer})\ldots\ldots\ldots(1)$

$\overline{Z}^2 + \frac{1}{Z^2} - Z^2 - \frac{1}{\overline{Z}^2} = 2\ (\text{Integer})\ldots\ldots\ldots(2)$

from (1) + (2)

$$\overline{Z}^2 + \frac{1}{Z^2} = 2(\text{Integer})$$

$$\overline{Z}^2 + \frac{\overline{Z}^2}{r^4} = \text{Integer}$$

$$\overline{Z}^2\left(1 + \frac{1}{r^4}\right) = \text{Integer}$$

From (1) – (2)

$$Z^2\left(1+\frac{1}{r^4}\right)=\text{integer}$$

$$r^4\left(1+\frac{1}{r^4}\right)^2=\text{integer}=k.$$

$$r^4=T$$

$$T\left(1+\frac{1}{T}\right)^2=\text{integer}$$

$$T\left(1+\frac{1}{T^2}+\frac{2}{T}\right)=\text{integer}$$

$$T+\frac{1}{T}+2=\text{integer}$$

$$T+\frac{1}{T}=\text{integer}-2=\text{integer}=k$$

$$T^2-KT+1=0$$

$$T=\frac{K+\sqrt{K^2-4}}{2}$$

Now check options only (A) is answer

12. Let G be a circle of radius R > 0. Let $G_1, G_2, \ldots\ldots, G_n$ be n circles of equal radius $r>0$. Suppose each of the n circles $G_1, G_2, \ldots\ldots, G_n$ touches the circle G externally. Also, for $i=1,2,\ldots.,n-1$, the circle G_i touches G_{i+1} externally, and G_n touches G_1 externally. Then, which of the following statements is/are TRUE?

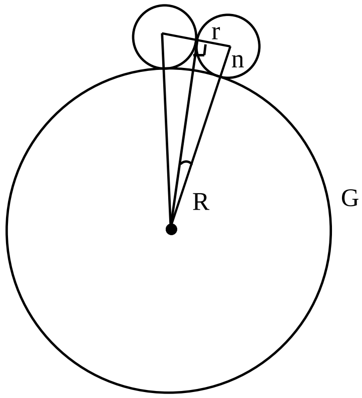
(A) If n = 4, then $(\sqrt{2}-1)r<R$

(B) If n = 5, then $r<R$

(C) If n = 8, then $(\sqrt{2}-1)r<R$

(D) If n = 12, then $\sqrt{2}(\sqrt{3}+1)r>R$.

Ans. CD



Sol.

a) n=4

$$r=\frac{R\sin\frac{\pi}{m}}{1-\sin\frac{\pi}{m}}=\frac{R}{\sqrt{2}-1}$$

$$\left(\sqrt{2}-1\right)R$$

b) n = 5

$$r=\frac{R\sin\frac{\pi}{5}}{1-\sin\frac{\pi}{5}}=\frac{R\sin 36^\circ}{1-\sin 36^\circ}>R$$

$$\because \sin 36>\frac{1}{2}\Rightarrow 1-\sin 36^\circ<\frac{1}{2}$$

$$\Rightarrow\frac{\sin 36}{1-\sin 36}>1$$

C) n = 8

$$r=\frac{R\sin\frac{\pi}{8}}{1-\sin\frac{\pi}{8}}$$

$$(\sqrt{2}-1)r = \frac{R\sin\frac{\pi}{8}}{1-\sin\frac{\pi}{8}}(\sqrt{2}-1)$$

$$\sin\frac{\pi}{8} < \frac{1}{\sqrt{2}} \Rightarrow 1-\sin\frac{\pi}{8} > \frac{\sqrt{2}-1}{\sqrt{2}}$$

$$\frac{\sqrt{2}-1}{1-\sin\frac{\pi}{8}} < \sqrt{2} \text{ and } R\sin\frac{\pi}{8} < \frac{R}{\sqrt{12}}$$

$$r = (R+r)\sin\theta$$

$$r = \frac{R\sin\theta}{1-\sin\theta}$$

$$r = \frac{R\sin\frac{\pi}{n}}{1-\sin\frac{\pi}{n}}.$$

13. Let $\hat{i}, \hat{j}$ and $\hat{k}$ be the unit vectors along the three positive coordinate axes. Let

$\vec{a} = 3\hat{i} + \hat{j} - \hat{k}$,

$\vec{b} = \hat{i} + b_2\hat{j} + b_3\hat{k}$, $\quad b_2, b_3 \in R$

$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$, $\quad c_1, c_2, c_3 \in R$

be three vectors such that $b_2 b_3 > 0$, $\vec{a}\cdot\vec{b} = 0$ and

$$\begin{pmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 3-c_1 \\ 1-c_2 \\ -1-c_3 \end{pmatrix}.$$

Then, which of the following is/are TRUE?

(A) $\vec{a}\cdot\vec{c} = 0$ (B) $\vec{b}\cdot\vec{c} = 0$

(C) $|\vec{b}| > \sqrt{10}$ (D) $|\vec{c}| \le \sqrt{11}$.

Ans. BCD

Sol. $\bar{c}\times\bar{b}=\bar{a}-\bar{c}$ ....(1) [Observe it]

$\bar{a}\cdot\bar{b}=0$ ....(2)

(1) $\bar{b}$ $\Rightarrow$ $0=\bar{a}\cdot\bar{b}-\bar{c}\cdot\bar{b}\Rightarrow\bar{b}\cdot\bar{c}=0$

$\bar{a}\cdot\bar{b}=0\Rightarrow b_3=3+b_2,\ b_2>0, b_3>3$

Check (c) i.e. $|\bar{b}|$

From (1) $\bar{a}=\bar{c}\times\bar{b}+\bar{c}$; $|\bar{a}|^2=|\bar{c}\times\bar{b}|^2+|\bar{c}|^2$

$11=|\bar{c}\times\bar{b}|^2+|c|^2\Rightarrow|\bar{c}|=11-|c\times b|^2$

14. For $x\in R$, let the function $y(x)$ be the solution of the differential equation

$$\frac{dy}{dx}+12y=\cos\left(\frac{\pi}{12}x\right),\ y(0)=0.$$

Then, which of the following statements is/are TRUE?

(A) $y(x)$ is an increasing function

(B) y(x) is a decreasing function

(C) There exists a real number β such that the line $y=\beta$ intersects the curve $y=y(x)$ at infinitely many points

(D) $y(x)$ is a periodic function

Ans. C

Sol. $x\in R$

$$\frac{dy}{dx}+12y=\cos\frac{\pi x}{12} \qquad y(0)=0$$

its L.d.c with I.F = $e^{\int 12\,dx}=e^{12x}$

solving gives

$$y\cdot e^{12x}=\int e^{12x}\cos\frac{\pi x}{12}dx$$

$$y \cdot e^{12x} = \frac{e^{12x}}{\left(12^2 + \left(\frac{\pi}{12}\right)^2\right)}\left(12\cos\frac{\pi x}{12} + \frac{\pi}{12}\sin\frac{\pi x}{12}\right) + C$$

$$y(0) = 0 \Rightarrow C = \frac{-12}{12^2 + \left(\frac{\pi}{12}\right)^2}$$

$$y = \frac{12\cos\frac{\pi x}{12} + \frac{\pi}{12}\sin\frac{\pi x}{12} - 12e^{-12x}}{12^2 + \left(\frac{\pi}{12}\right)^2}$$

y is of the form

$$y = \frac{p\cos qx + q\sin qx}{(p^2 + q^2)} - \frac{pe^{-px}}{p^2 + q^2}$$

$p = 12 \qquad q = \frac{\pi}{12}$

here y is neither Increasing nor decreasing and not periodic as well further I real no β straight line $y = \beta$ intersect the curve $y = y(x)$ at ∞ many points

so Ans C

SECTION-3 (Maximum Marks : 12)

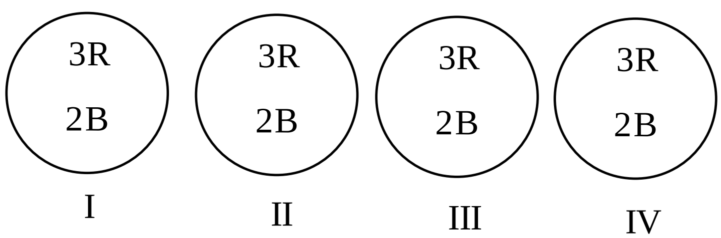
- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

15. Consider 4boxes, where each box contains 3 red balls and 2 blue balls. Assume that all 20 balls are distinct. In how many different ways can 10 balls be chosen from these 4 boxes so that from each box at least one red ball and one blue ball are chosen ?

(A) 21816 (B) 85536 (C) 12096 (D) 156816

Ans. A



Sol.

Alls are to be chosen in the group of

2 2 2 4

or

2 2 3 3

Now for 2 2 2 4

Number of ways of selection = ${}^5C_4\left({}^3C_1{}^2C_1\right)^3 \times 4 = 4320$ ….. (1)

for 2 2 3 3

Number of ways of selection = $\dfrac{\left({}^5C_3 - {}^3C_3\right)^2\left({}^3C_1\,{}^2C_1\right)^2_{41}}{2!2!}$

$$=\frac{81\times 36\times 24}{4}=17496 \quad \text{........ (2)}$$

Ans = (1) + (2) = 21816

Ans = A

16. If $M=\begin{pmatrix}\frac{5}{2} & \frac{3}{2}\\ -\frac{3}{2} & -\frac{1}{2}\end{pmatrix}$, then which of the following matrices is equal to M^{2022} ?

(A) $\begin{pmatrix}3034 & 3033\\ -3033 & -3032\end{pmatrix}$ (B) $\begin{pmatrix}3034 & -3033\\ 3033 & -3032\end{pmatrix}$

(C) $\begin{pmatrix}3033 & 3032\\ -3032 & -3031\end{pmatrix}$ (D) $\begin{pmatrix}3032 & 3031\\ -3031 & -3030\end{pmatrix}$.

Ans. A

Sol. $M=\begin{bmatrix}\frac{5}{2} & \frac{3}{2}\\ -\frac{3}{2} & -\frac{1}{2}\end{bmatrix}=\begin{bmatrix}\frac{3}{2}+1 & \frac{3}{2}\\ -\frac{3}{2} & -\frac{3}{2}+1\end{bmatrix}$

$=\begin{bmatrix}\frac{3}{2} & \frac{3}{2}\\ -\frac{3}{2} & -\frac{3}{2}\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$M=A+I$

$M^{2022}=(A+I)^{2022}+(I+A)^{2022}$

$I+2022A+{}^{2022}C_2A^2+......$

But $\because A^2=O=A^3\mp A^4......$

$\therefore M^{2022}=I+2022A$

$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}3033 & 3033\\ -3033 & -3033\end{bmatrix}$

$$\Rightarrow M^{2022} = \begin{bmatrix} 3034 & 3033 \\ -3033 & -3032 \end{bmatrix}$$

$\Rightarrow$ Ans: A

17. Suppose that

Box-I contains 8 red, 3 blue and 5 green balls,

Box-II contains 24 red, 9 blue and 15 green balls,

Box-III contains 1 blue, 12 green and 3 yellow balls,

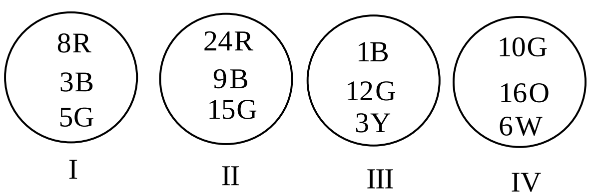
Box-IV contains 10 green, 16 orange and 6 white balls.

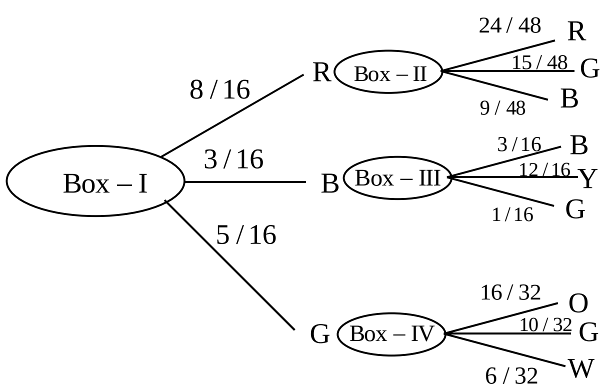
A ball is chosen randomly from Box-I; Call this ball b. If b is red then a ball is chosen randomly from Box-II, if b is blue then a ball is chosen randomly from Box-III, and if b is green then a ball is chosen randomly from Box-IV. The conditional probability of the event 'one or the chosen balls is white' given that the event 'at least one of the chosen balls is green' has happened, is eqla to

(A) $\frac{15}{256}$ (B) $\frac{3}{16}$ (C) $\frac{5}{52}$ (D) $\frac{1}{8}$

Ans. C

Sol.





Let

E: atleast one of the chosen ball is green

E_1: One of the chosen ball is white to calculate $P(E_1 / E)$

$$=\frac{\frac{5}{16}\times\frac{6}{32}}{\frac{3}{16}\times\frac{15}{48}+\frac{3}{16}\times\frac{12}{16}+\frac{5}{16}\times 1}=\frac{5}{52}$$ Ans: C

18. For positive integer n, define

$$f(n)=n+\frac{16+5n-3n^2}{4n+3n^2}+\frac{32+n-3n^2}{8n+3n^2}+\frac{48-3n-3n^2}{12n+3n^2}+.....+\frac{25n-7n^2}{7n^2}.$$

Then, the value of $\lim_{n\to\infty} f(n)$ is equal to

(A) $3+\frac{4}{3}\log_e 7$

(B) $4-\frac{3}{4}\log_e\left(\frac{7}{3}\right)$

(C) $4-\frac{4}{3}\log_e\left(\frac{7}{3}\right)$

(D) $3+\frac{3}{4}\log_e 7$.

Ans. B

Sol. $f(n)=n+\frac{16+5n-3n^2}{4n+3n^2}+\frac{32+n-3n^2}{8n+3n^2}+......+\frac{25n-7n^2}{7n^2}$

$$f(n)=n+\sum_{r=1}^{n}\frac{16r+(9-4r)n-3n^2}{4nr+3n^2}$$

$$=\sum_{r=1}^{n}\left(\frac{16r+(9-4r)n-3n^2}{4nr+3n^2}+1\right)$$

$$=\sum_{r=1}^{n}\frac{16r+9n}{4nr+3n^2}$$

$$\lim_{n\to\infty} f(x)=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\frac{16\frac{r}{n}+9}{4\frac{r}{n}+3}$$

$$=\int_0^1\frac{16x+9}{4x+3}dx \qquad =4-\frac{3}{4}\ln\frac{7}{3}$$

Ans: B