Sri Chaitanya

JEE MAIN 2021

PHASE - IV

Key & Solutions

26-Aug-2021 | Shift - 2

Jee-Main_Final_26-August-2021_Shift-02

PHYSICS **Max Marks: 100**

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

01. Match the list

	List-I		List-II
A	Magnetic Induction	I	$ML^2T^{-2}A^{-1}$
B	Magnetic Flux	II	$M^0L^{-1}A$
C	Magnetic Permeability	III	$MT^{-2}A^{-1}$
D	Magnetization	IV	$MLT^{-2}A^{-2}$

Choose the most appropriate answer from the options given below

1) $A \to II, B \to IV, C \to I, D \to III$
2) $A \to III, B \to I, C \to IV, D \to II$
3) $A \to II, B \to I, C \to IV, D \to III$
4) $A \to III, B \to II, C \to IV, D \to I$

Key: 2

Sol: $(i)\ F = mB \Rightarrow MLT^{-2} = ALB \Rightarrow B = MT^{-2}A^{-1}$

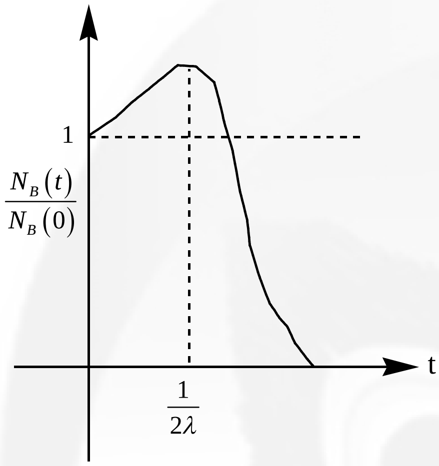
$(ii)\ \phi = BA = MT^{-2}A^{-1} \times L^2 \Rightarrow ML^2T^{-2}A^{-1}$

$(iii)\ \frac{F}{L} = \frac{\mu_0 i_1 i_2}{2\pi r} \Rightarrow \frac{MLT^{-2}}{L} = \frac{\mu_0 A^2}{L} \Rightarrow \mu_0 = MLT^{-2}A^{-2}$

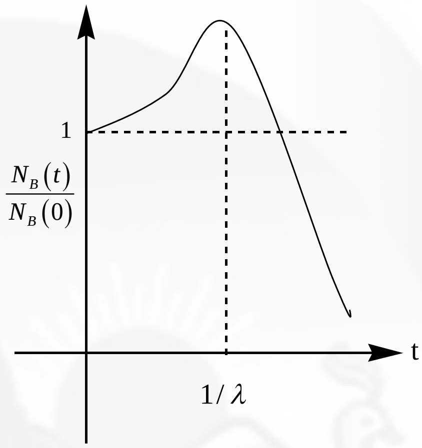
$(iv)\ I = \frac{M}{V} = \frac{AL^2}{L^3} = L^{-1}A$

02. At time t=0,a material is composed of two radioactive atoms A and B, where $N_A(0)=2N_B(0)$.The decay constant of both kind of radioactive atoms is λ.However, A disintegrates to B and B disintegrates to C. Which of the following figures represents the evolution of $N_B(t)/N_B(0)$ with respect to time t?

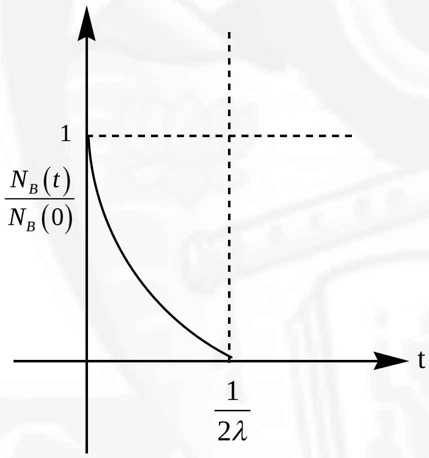
$$\begin{bmatrix} N_A(0)=No.of\ A\,atoms\ at\ t=0 \\ N_{\mathbf{B}}(0)=No.of\ B\,atoms\ at\ t=0 \end{bmatrix}$$



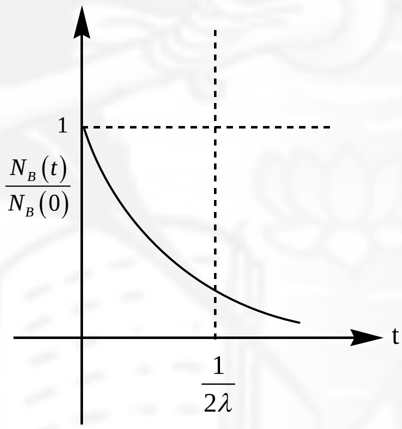
1)



2)



3)



4)

Key: 1

Sol: $\lambda N_A - \lambda N_B = \frac{dN_B}{dt}$

so as time increases $\frac{N_B(t)}{N_B(0)}$ increase and then decreases

03. The temperature of equals of three different liquids x,y and z are $10^0C, 20^0C \& 30^0C$ respectively. The temperature of mixture when x is mixed with y is 16^0C and the at when y is mixed with z is 26^0C.The temperature of mixture when x and z are mixed will be

1) 20.28^0C 2) 23.84^0C 3) 28.32^0C 4) 25.62^0C

Key: 2

Sol: $ms_x 6 = m \times s_y \times 4 \Rightarrow \frac{s_x}{s_y} = \frac{2}{3} \longrightarrow (1)$

$ms_y \times 4 = m \times s_z \times 6 \Rightarrow \frac{s_z}{s_y} = \frac{2}{3} \longrightarrow (2)$

$ms_x \times (t-10) = ms_2 (30-t) \Rightarrow \frac{s_x}{s_z} = \frac{30-t}{t-10} \rightarrow (3)$

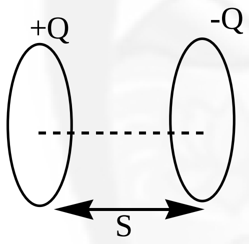
From 1,2,&3 is $t = 23.84^0 C$

04. The two thin coaxial rings, each of radius 'a' and having charges +Q and –Q respectively are separated by a distance of 's'. The potential difference between the centres of the two rings

1) $\frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{a} + \frac{1}{\sqrt{s^2+a^2}}\right]$
2) $\frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{\sqrt{s^2+a^2}}\right]$
3) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} + \frac{1}{\sqrt{s^2+a^2}}\right]$
4) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{\sqrt{s^2+a^2}}\right]$

Key: 2

Sol:



$$\Delta V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r} - \frac{Q}{\sqrt{r^2+s^2}}\right) - \left(\frac{1}{4\pi\varepsilon_0}\frac{-Q}{r} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{r^2+s^2}}\right)$$

$$= \frac{Q}{2\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{\sqrt{s^2+a^2}}\right)$$

05. A light beam is described by $E = 800 \sin\omega\left(t - \frac{x}{c}\right)$. An electron is allowed to move normal to the propagation of light beam with a speed of $3\times10^7 ms^{-1}$. What is the maximum magnetic force exerted on the electron?

1) $12.8\times10^{-18} N$ 2) $12.8\times10^{-17} N$ 3) $1.28\times10^{-21} N$ 4) $1.28\times10^{-18} N$

Key: 1

Sol: $C = \frac{F}{B} \Rightarrow B = \frac{800}{3\times10^8}$

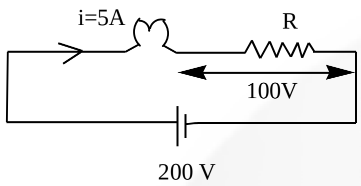
$F = BqV = \frac{800}{3\times10^8} \times 1.6\times10^{-19} \times 3\times10^7 \Rightarrow 12.8\times10^{-18} N$

06. An electric bulb if 500 watt at 100 volt is used in a circuit having a 200V. Calculate the resistance R to be connected in series with the bulb so that the power delivered by the bulb is 500W

1) 20Ω 2) 30Ω 3) 10Ω 4) 5Ω

Key: 1

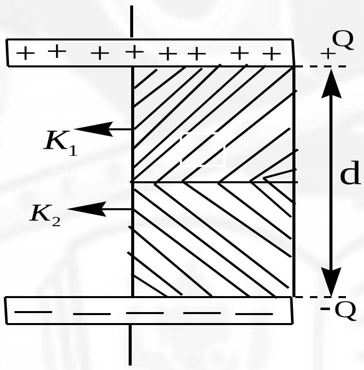
Sol:



$$i=\frac{p}{V}=\frac{500}{100}=5A$$

$$R=\frac{100}{i}=\frac{00}{5}\Rightarrow R=20\Omega$$

07. A parallel plate capacitor with plate area A has separation d between the plates. Two dielectric slabs of dielectric constant $K_1 \& K_2$ of same area A/2 and thickness d/2 are inserted in the space between the plates. The capacitance of the capacitor will be given by

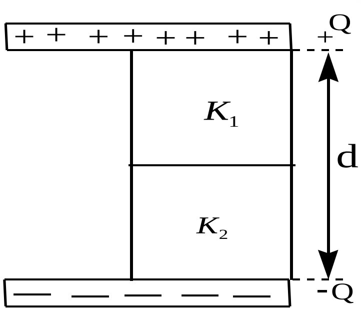


1) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2}+\frac{K_1+K_2}{K_1K_2}\right)$ 2) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2}+\frac{2(K_1+K_2)}{K_1K_2}\right)$

3) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2}+\frac{K_1K_2}{2(K_1+K_2)}\right)$ 4) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2}+\frac{K_1K_2}{K_1+K_2}\right)$

Key: 4

Sol:



$C_1 = \frac{\varepsilon_0 A/2}{d}, C_2 = \frac{\varepsilon_0 A/2}{d/2} k_2, C_3 = \frac{\varepsilon_0 A/2}{d/2} k_1$

$C_{eff} \;\; C_r + \frac{C_2 C_3}{C_2 + C_3} \Rightarrow C = \frac{\varepsilon_0 A}{d}\left(\frac{1}{2} + \frac{k_1 k_2}{k_1 + k_2}\right)$

08. If you are provided a set of resistances $2\Omega, 4\Omega, 6\Omega \& 8\Omega$. Connect these resistances so as to obtain an equivalent resistance of $\frac{46}{3}\Omega$

1) $2\Omega \& 6\Omega$ are in parallel with $4\Omega \& 8\Omega$ in series
2) $4\Omega \& 6\Omega$ are in parallel with $2\Omega \& 8\Omega$ in series
3) $2\Omega \& 4\Omega$ are in parallel with $6\Omega \& 8\Omega$ in series
4) $6\Omega \& 8\Omega$ are in parallel with $2\Omega \& 4\Omega$ in series

Key: 3

Sol: After verifying options we will get 3 option correct

$\frac{2\times 4}{2+6} + 6 + 8 = \frac{4}{3} + 14 = \frac{46}{3}\Omega$

09. A refrigerator consumes an average 35W power to operate between temperature $-10^0 C\, to\, 25^0 C$. If there is no loss of energy then how much average heat per second does it transfer?

1) 298 J/s 2) 350 J/s 3) 35 J/s 4) 263 J/s

Key: 4

Sol: $\frac{Heat\, rejected}{work\, done} = \frac{T_2}{T_1 - T_2}$

$\frac{Heat\, rejected}{35} = \frac{263}{298 - 263}$

$Heat\, rejected = \frac{263}{35} \times 35 \Rightarrow 263J$

10. A transmitting antenna at top of a tower has a height of 50m and the height of receiving antenna is 80m. What is the range of communication for line of sight (Los) mode?

1)45.5 km 2) 57.28 km 3) 80.2 km 4) 144.1 km

Key: 2

Sol: $d = \sqrt{2Rh_T} + \sqrt{2Rh_R} \Rightarrow 57.28\, km$

11. If the length of the pendulum in pendulum clock increase by 0.1%, then the error in time

1) 43.2 s 2) 4.32 s 3) 8.64 s 4) 86.4 s

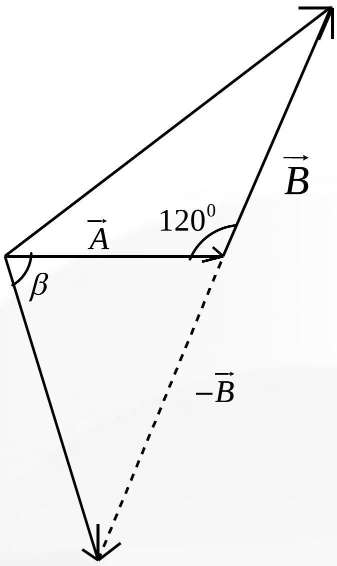
Key: 1

Sol: $\Delta t = \frac{1}{2} \propto \Delta\theta \times 86400$

$= \frac{1}{2}\frac{\Delta l}{l} \times 86400$

$= \frac{1}{2} \times \frac{1}{1000} \times 86400 \Rightarrow 43.2 \text{ sec}$

12. The angle between vector $\left(\vec{A}\right)\&\left(\vec{A}-\vec{B}\right)$ is



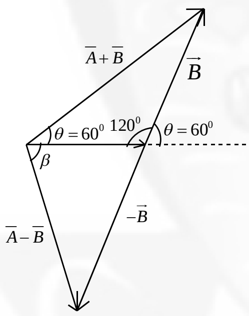
1) $\tan^{-1}\left(\frac{B\cos\theta}{A-B\sin\theta}\right)$

2) $\tan^{-1}\left(\frac{\sqrt{3}B}{2A-B}\right)$

3) $\tan^{-1}\left(\frac{-\frac{B}{2}}{A-B\frac{\sqrt{3}}{2}}\right)$

4) $\tan^{-1}\left(\frac{A}{0.7B}\right)$

Key: 2



Sol:

$$\tan\beta=\frac{B\sin\theta}{A-B\cos\theta}=\frac{B\sin 60^0}{A-B\cos 60^0}$$

$$\frac{\frac{B\sqrt{3}}{2}}{A-\frac{B}{2}}=\frac{B\sqrt{3}}{2A-B}$$

$$\frac{\frac{B\sqrt{3}}{2}}{\frac{A-B}{2}}=\frac{B\sqrt{3}}{2A-B}\Rightarrow\beta=Tan^{-1}\left[\frac{B\sqrt{3}}{2A-B}\right]$$

13. A body is dropped by a fighter plane flying horizontally. To an observer sitting in the plane, the trajectory of the bomb is a

1) Straight line vertically down the plane

2) Parabola in a direction opposite to the motion of plane

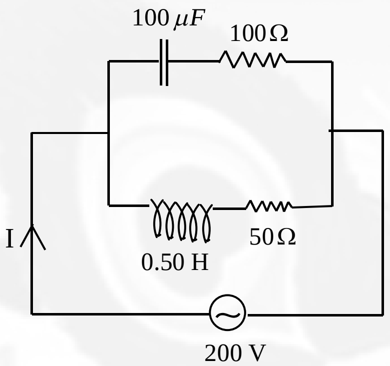
3) Parabola in the direction of motion of plane

4) Hyperbola

Key: 1

Sol: Conceptual

14. In the given circuit the AC source has $\omega = 100 rad\, s^{-1}$. Considering the inductor and capacitor to be ideal, what will be the current I flowing through the circuit?



1) 5.9 A 2) 6A 3) 0.94 A 4) 4.24 A

Key: 4

Sol: $x_c = \frac{1}{\omega c}; x_L = \omega L$

$Z_1 = \sqrt{X_C^2 + R^2},\ Z_1 = 100\sqrt{2}$

$Z_2 = \sqrt{X_L^2 + R^2},\ Z_2 = 50\sqrt{2}$

$R = 100, X_L = wL \Rightarrow 100 \times 0.5 = 50$

In parallel combination

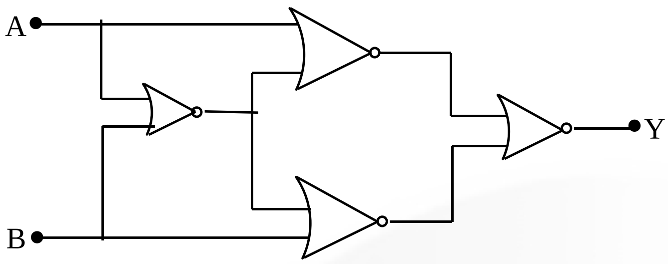
$\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2}$

$= \frac{1}{100\sqrt{2}} + \frac{1}{50\sqrt{2}}$

$\frac{1+2}{100\sqrt{2}} = \frac{3}{100\sqrt{2}} \Rightarrow z = \frac{100\sqrt{2}}{3}$

$I = \frac{V}{Z} = \frac{200 \times 3}{100\sqrt{2}} = \sqrt{2} \times 3 = 1.414 \times 3 \Rightarrow 4.242 \Rightarrow 4.24A$

15. Four NOR gates are connected as shown in figure. The truth table for the given figure is



1)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

2)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

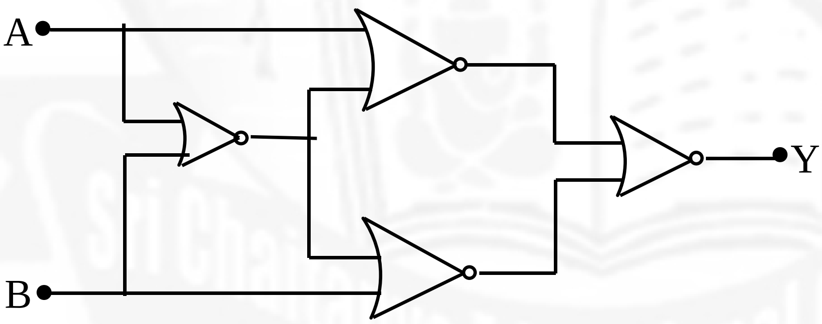
3)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

4)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

Key: 2

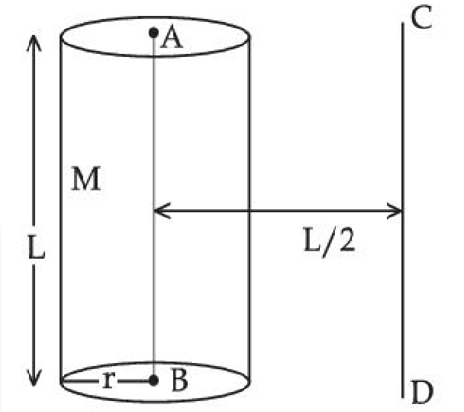


Sol:

By cross verifying by giving values is to input the final result is

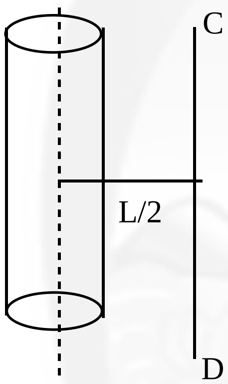
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

16. The solid cylinder of length 80 cm and mass M has a radius of 20 cm. Calculate density of the material used if the moment of inertia of the cylinder about an axis CD parallel to AB as shown in figure is $2.7\, kg\, m^2$



1) $1.49\times10^2 Kg/m^3$ 2) $7.5\times10^1 Kg/m^3$ 3) $7.5\times10^2 Kg/m^3$ 4) $14.9 Kg/m^3$

Key: 1



Sol:

Applying parallel axes theorem, $I_{CD} = I_{AB} + Mh^2$

$$I_{CD} = \frac{MR^2}{2} + \frac{ML^2}{4}$$

$$2.7 = m\left[\left(\frac{20\times10^{-2}}{2}\right) + \frac{\left(80\times10^{-2}\right)^2}{4}\right]$$

$$= M\times10^{-4}\left[\frac{400}{2} + \frac{80\times80}{4}\right]$$

$$= M\times10^{-4}\left[200 + 1600\right]$$

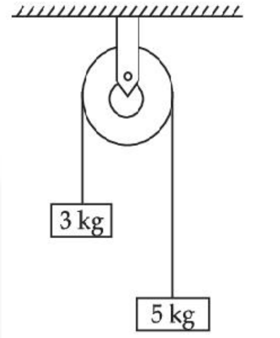
$$M = \frac{2.7}{1800\times10^{-4}} = \frac{2.7\times10^2}{18} = \frac{270}{18} = 15 \text{ kg}$$

$$\rho = \frac{M}{V} = \frac{15}{\pi r^2 l} = \frac{15\times7}{22\times400\times10^{-4}\times80\times10^{-2}}$$

$$\frac{15\times7\times10^3}{22\times4\times8} \Rightarrow \frac{15\times7}{88\times8}\times10^3$$

$$\frac{105}{704} \Rightarrow 0.149\times10^3 \Rightarrow 1.49\times10^2\, kg/m^3$$

17. Two blocks of masses 3kg and 5kg connected by a metal wire going over a smooth pulley. The breaking stress of the metal is $\frac{24}{\pi}\times10^{2}\,Nm^{-2}$. What is the minimum radius of the wire ? $\left(take\ \ g=10\,ms^{-2}\right)$



1) 12.5 cm 2) 1250 cm 3) 1.25 cm 4) 125 cm

Key: 1

Sol: From Atwood machine, the tension in the string $T=\frac{2m_1m_2g}{m_1+m_2}$

$$=\frac{2\times3\times5\times10}{3+5}\Rightarrow\frac{2\times3\times50}{8}=\frac{75}{2}$$

Breaking Stress$=\frac{T}{A}=\frac{T}{\pi r^2}$

$$r^2=\frac{T}{\pi\times stress}=\frac{75}{2\times\pi\times\frac{4}{\pi}\times10^2}\Rightarrow\frac{75}{2\times24\times100}=\frac{1}{8^2}$$

$$r=\frac{1}{8}m=\frac{100}{8}cm=12.5cm$$

18. A cylindrical container of volume $4.0\times10^{-3}m^2$ contains one mole of hydrogen and two moles of carbon dioxide. Assume the temperature of the mixture is 400K. The pressure of the mixture of gases is

$\left[Take\ gas\ \text{constant}\ as\ 8.3\,J\,mol^{-1}K^{-1}\right]$

1) $249\times10^{1}Pa$ 2) $24.9\times10^{3}Pa$ 3) $24.9\times10^{5}Pa$ 4) $24.9Pa$

Key: 3

Sol: From ideal gas equation $PV=nRT \quad P=\frac{nRT}{V}$

Pressure due to 1 mole of hydrogen

$$P_1=\frac{nRT}{V}=\frac{1\times8.314\times400}{4\times10^{-3}}$$

Pressure due to 2 moles of carbon dioxide

$$P_2=\frac{2\times8.314\times400}{4\times10^{-3}}$$

From Daltons'law pressure

$$p^{r}=p_1+p_2\Rightarrow8.314\times10^{5}\left[1+2\right]$$

$$8.314\times3\times10^{5}=24.942\times10^{5}Pa$$

19. The de-Broglie wavelength of a particle having kinetic energy E is λ. How much extra energy must be given to this particle so that the de-Broglie wavelength reduces to 75% of the initial value?

1) $\frac{16}{9}E$ 2) $\frac{7}{9}E$ 3) $\frac{1}{9}E$ 4)E

Key: 2

Sol: The De-Broglie wave length of a charged particle with kinetic energy E is

$$\lambda = \frac{h}{\sqrt{2mE}} \quad \lambda \propto \frac{1}{\sqrt{E}}$$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{E_2}{E_1}} \Rightarrow \frac{\lambda}{\frac{3}{4}\lambda} = \sqrt{\frac{E_2}{E_1}}$$

$$\frac{4}{3} = \sqrt{\frac{E_2}{E_1}} \Rightarrow \frac{E_2}{E_1} = \frac{16}{9}, E_2 = \frac{16E_1}{9}$$

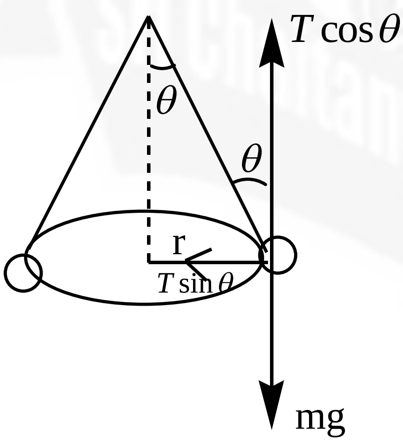
Extra energy= $E_2 - E_1 \Rightarrow \frac{16}{9}E - E = \frac{7E}{9}$

20. A particle of mass m is suspended from a ceiling the rough a string of length L. The particle moves in a horizontal circle of radius r such that $r = \frac{L}{\sqrt{2}}$.The speed of particle will be

1) $\sqrt{rg}$ 2) $\sqrt{\frac{rg}{2}}$ 3) $\sqrt{2rg}$ 4) $2\sqrt{rg}$

Key: 1

Sol:



From diagram

$\sin\theta = \frac{r}{L}$

$r = L\sin\theta$ given $r = \frac{L}{\sqrt{2}}$

$\frac{L}{\sqrt{2}} = L\sin\theta$

$\therefore \theta = 45^0$

$T\sin\theta = \frac{mv^2}{r}$ & $T\cos\theta = mg$ *on dividing we get* $\tan\theta = \frac{v^2}{rg}$

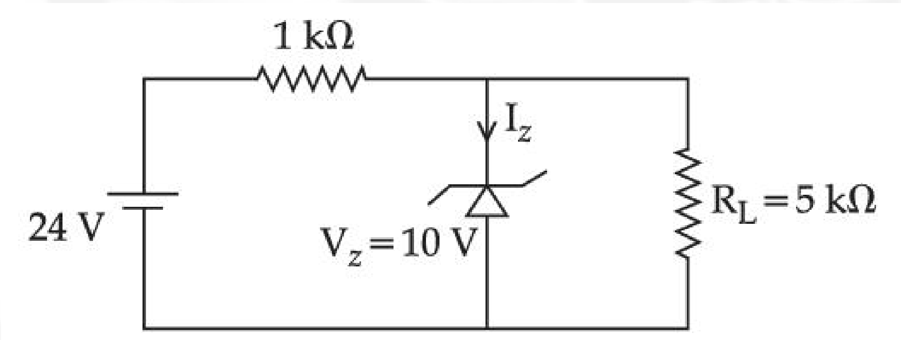
$\therefore \tan 45 = \frac{v^2}{rg}$

$v^2 = rg \Rightarrow v = \sqrt{rg}$

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. For the given circuit, the power across zener diode is _____mW.



Key: 120

Sol: $10 = I_L \times R_L$

$10 = I_L \times 500$

$I_L = 2\times10^{-3} A$ krichoff's law

$1000i + 10 = 24 \Rightarrow i = 14\times10^{-3} A$

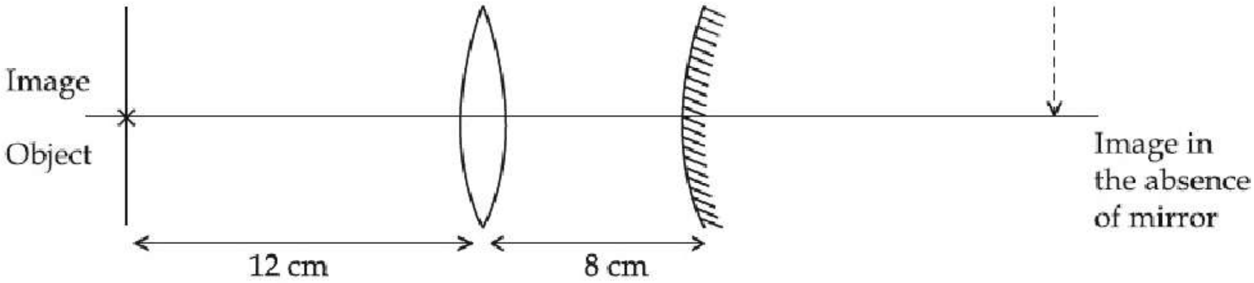
Kirchoffs law

$i_z = (14-2)\times10^{-3} = 12\times10^{-3}$

$p = V_z i_z = 10\times12\times10^{-3}$

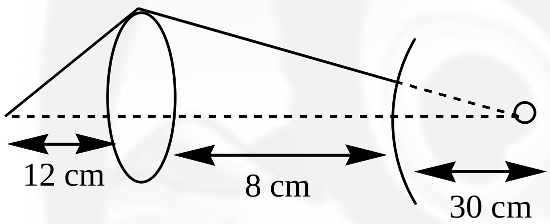
$= 12\times10^{-2} W = 120 mW$

22. An object is placed a distance of 12 cm from a convex lens. A convex mirror of focal length 15 cm is placed on other side of lens at 8cm as shown in the figure. Image of object coincides with the object.



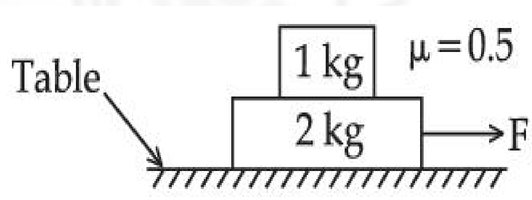
When the convex mirror is removed ,a real and inverted image is formed at a position. The distance of the mage from the object will be____ (cm)

Key: 50



Sol: $d = 12 + 8 + 30 = 50\,cm$

23. The coefficient of static friction between two blocks is 0.5 and the table is smooth. The maximum horizontal force that can be applied to move the blocks together is _____N. $\left(take\ \ g = 10ms^{-2}\right)$



Key: 15

Sol: $f_k = \mu mg$

$= 0.5 \times 1 \times 10 \Rightarrow 5N$

$a = \dfrac{5}{1} = 5ms^{-2}$

$\therefore F = 3 \times 5 = 15N$

24. Two waves are simultaneously passing through a string and their equations are $y_1 = A_1 \sin k(x - vt), y_2 = A_2 \sin k(x - vt + x_0)$. Given amplitudes $A_1 = 12mm \,\&\, A_2 = 5mm, x_0 = 3.5cm$ and wave number $k = 6.28cm^{-1}$.The amplitude of resulting wave will be ____mm.

Key: 7

Sol: $y_1 = 12\times10^{-3}\sin(kx - \omega t)$

$y_2 = 5\times10^{-3}\sin(kx - wt + 7\pi)$

$A_R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\beta}$

$= \sqrt{144 + 25 + 2\times12\times5(-1)}$

$= \sqrt{49\times10^{-6}} = 7\times10^{-3}m \Rightarrow 7\,mm$

25. A source of light is placed in front of a screen. Intensity of light on screen is I/2. P_2 should be rotated by an angle of ____ (degrees) so that the intensity of light on the screen becomes $\frac{3I}{8}$

Key: 30

Sol: $\frac{I}{2}\cos^2\theta = \frac{I}{2} \Rightarrow \cos^2\theta = 1, \theta = 0^0$

The two are parallel now

$\frac{I}{2}\cos^2\theta = \frac{3I}{84}$

$\cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30^0$

26. Two simple harmonic motions are represented by the equations $x_1 = 5\sin\left(2\pi t + \frac{\pi}{4}\right) \,\&\, x_2 = 5\sqrt{2}(\sin 2\pi t + \cos 2\pi t)$.The amplitude of second motion is ____ times the amplitude in first motion.

Key: 2

Sol: $x_1 = 5\sin\left(2\pi t + \frac{\pi}{4}\right), A_1 = 5$

$x_2 = 5\sqrt{2}\sin\pi t + 5\sqrt{2}\cos 2\pi t$

$A_2 = \sqrt{25\times2 + 25 + 2} = 10$

$\frac{A_2}{A_1} = \frac{10}{5} = 2$

27. The acceleration due to gravity is found upto an accuracy of 4% on a planet. The energy supplied to a simple pendulum of known mass 'm' to undertake oscillations of time period T is being estimated. If time period is measured to an accuracy of 3%, the accuracy to which E is known as ____%

Key: 14

Sol: $a = mgl(1-\cos\theta)$

$$T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow T^2 = 4\pi^2\frac{L}{g} \Rightarrow L = \frac{gT^2}{4\pi^2}$$

$$U = \frac{mg.g.T^2}{4\pi^2}(1-\cos\theta)$$

$$\frac{du}{du}\times100 = 2\times\frac{dg}{g}\times100\times\frac{2}{T}dt\times100$$

$$= 2\times4\% + 2\times35 \Rightarrow 14\%$$

28. If the maximum value of accelerating potential provided by a radio frequency oscillator is 12kV. The number of revolution made by a cyclotron to achieve one sixth of the speed of light is ____.

$\left[m_p = 1.67\times10^{-27}kg, e = 1.6\times10^{-19}C, speed\ of\ light = 3\times10^8\ m/s\right]$

Key: 543

Sol: $2nev = \frac{1}{2}m\times v^2$

$$2n\times12\times10^3\times1.6\times10^{-1} = \frac{1}{2}\times1.67\times10^{-27}\times\left(5\times10^7\right)^2$$

N=543

29. A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of $50\,rads^{-1}$ in a uniform horizontal magnetic field of $3.0\times10^{-2}T$ .The maximum emf induced the coil will be ____$\times10^{-2}$ volt (rounded off to the nearest integer)

Key: 60

Sol: $e = BANw$

$$= 3\times10^{-2}\times\pi\times64\times10^{-4}\times20\times50$$

$$= 3\pi\times6.4\times10^{-2}\times9.42\times6.4\times10^{-2}$$

$$= 60.288\times10^{-2} \approx 60\times10^{-2}$$

30. A coil in the shape of an equilateral triangle of side 10 cm lies in a vertical plane between the pole pieces of permanent magnet producing a horizontal magnetic field 20mT. The torque acting on the coil when a current of 0.2A is passed through it and its plane becomes parallel to the magnetic field will be $\sqrt{x}\times10^{-5}\,Nm$.The value of x is____

Key: 3

Sol: $\tau = BinA\sin\theta$

$$= 20\times10^{-3}\times0.2\times1\times\frac{\sqrt{3}}{4}\times100\times10^{-4}\times1$$

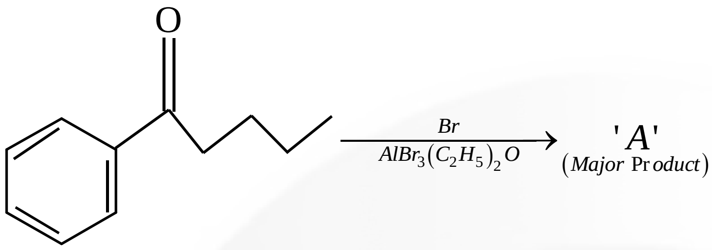
$$= \sqrt{3}\times10^{-5} = \sqrt{x}\times10^{-5} \Rightarrow x = 3$$

(SINGLE CORRECT ANSWER TYPE)

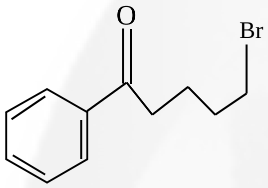
This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

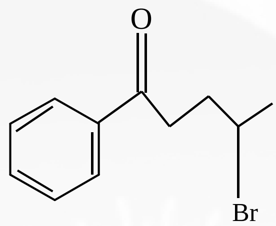
Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

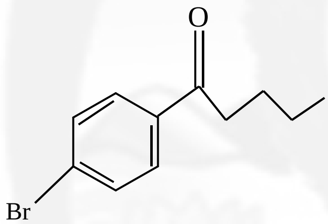
31.

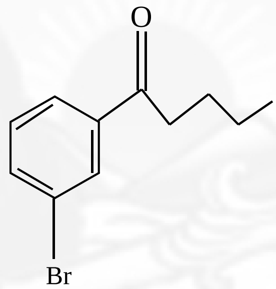


Consider the given reaction, the Product A is

1) 

2) 

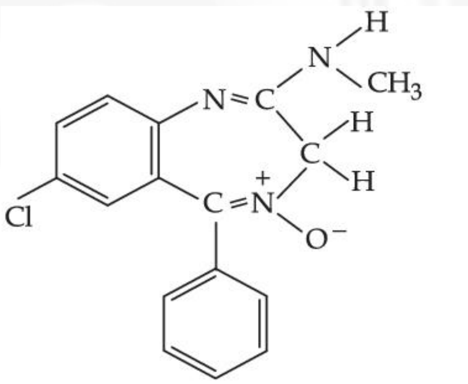
3) 

4) 

Key: 4

Sol: Electrophilic substitution takes place at Meta position with respect to $\rangle C=O$ group

32.



Chlordiazepoxide

The class of during to which chlordiazepoxide with above structure belongs is

1) Analgesic 2) Antibiotic 3) Tranquilizer 4) Antacid

Key: 3

Sol: The drug chlorodiazepoxide is a tranquiliser

33. Given below are two statements: one is labeled as **Assertion (A):** and the other is labeled as **Reason (R).**

Assertion (A): Heavy water is used for the study of reaction mechanism

Reason (R): The rate of reaction for the cleavage of O-H bond is slower than that of O-D bond

Choose the most appropriate answer from the options given below:

1) Both **(A)** and (R) are true but (R) is not true explanation of (A)

2) (A) is true bur (R) is false

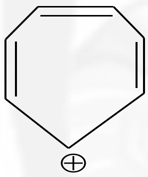
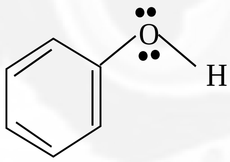
3) (A) is false but (R) is true

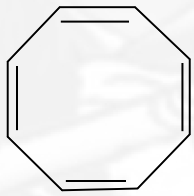
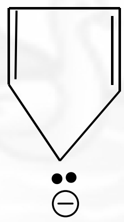
4) Both (A) and (R) are true and (R) is the true explanation of (A) .

Key: 2

Sol: A is true. But $O-D$ dissociation is slower than $O-H$ bond

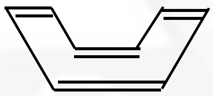
34. Which one of the following compounds is not aromatic?

1)  2) 

3)  4) 

Key: 3

Sol: Cyclocta tetra ene is not planar. Hence it is not asomatic



35. The sol given below with negativity charged colloidal particles is

1) $KI\ added\ to\ AgNO_3\ solution$ 2) $Al_2O_3.xH_2O$ in water

3) $AgNO_3\ added\ to\ KI\ solution$ 4) $FeCl_3$ added to hot water

Key: 3

Sol: When $AgNO_3$ is added to KI solution Negatively charged colloid is formed due to preferential adsoroption by I- ions

$KI \xrightarrow{AgNO_3} AgI\downarrow \xrightarrow{KI} AgI/I^-$

36. Given below are two statements: one is labeled as **Assertion (A)** and other is labeled as **Reason (R).**

Assertion (A): Barium carbonate is insoluble in water and is highly stable

Reason (R): The thermal stability of the carbonates increases with increasing cationic size

Choose the **most appropriate** answer the options given below:

1) Both (A) and (R) are true but (R) is not the true explanation of (A)

2) (A) is true but (R) is false

3) Both (A) and (R) are true and (R) is the true explanation of (A)

4) (A) is false but (R) is true

Key: 3

Sol: The stability of the carbonates of II A group increases with increase of electropositive character of element.The solubility of carbonates decreases from berillium carbonate to barium carbonate.

$BeCO_3 < MgCO_3 < CaCO_3 < SrCO_3 < CaCO_3$ stability

$BeCO_3 > MgCO_3 > CaCO_3 > SrCO_3 > CaCO_3$ solubility

37. The number of stereoisomers possible for 1, 2-dimethyl cyclopropane is

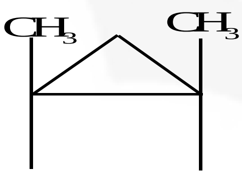
1) Two 2) Four 3) One 4) Three

Key: 4

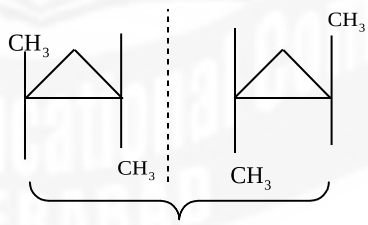
Sol: Three isomers are possible

Cis isomer - (1) isomer

Trans isomer - (2) isomers



Cis



Trans isomers

38. Chalcogen group elements are

1) O, Ti and Po 2) Se, Te and Po 3) Se, Tb and Pu 4) S, Te and Pm

Key: 2

Sol: O, S, Se, Te, Po belong to chalcogen family

39. Given below are two statements: one is labeled as **Assertion (A)** and other is labeled as **Reason (R).**

Assertion (A): Photochemical smog causes cracking of rubber

Reason (R): Presence of ozone, nitric oxide, acrolein, formaldehyde and peroxyacetyl nitrate ohotochemical smog makes its oxidizing

Choose the **most appropriate** answer the options given below:

1) Both (A) and (R) are true but (R) is not the true explanation of (A)

2) Both (A) and (R) are true and (R) is the true explanation of (A)

3) (A) is true but (R) is false

4) (A) is false but (R) is true

Key: 0

Sol: The photochemical smog is causes crocking of rubber products.This is to ozone and other oxidants in atmospher.

40. Indicate the complex/complex ion which did not show any geometrical isomerism

1) $\left[CoCl_2(en)_2\right]$ 2) $\left[Co(NH_3)_3(NO_3)_3\right]$ 3) $\left[Co(NH_3)_4Cl_2\right]^+$ 4) $\left[Co(CN)_5(NC)\right]^{3-}$

Key: 4

Sol: $\left[\mathrm{Co(CN)_5(NC)}\right]^{-3}$ doesnot exhibit geometrical isomerism as 5 ligands are same.

41. Given below are two statements:

Statement I : Sphalerite is a sulphide ore of zinc copper glance is a sulphide ore of copper

Statement II : It is possible to separate two sulphide ores by adjusting proportion of oil to water or by using 'depressants' in a forth flotation method.

Choose the most approximate answer from the options given below

1) Both Statement I and Statement II are false

2) Statement I is true but Statement II is false

3) Both Statement I and Statement II are true

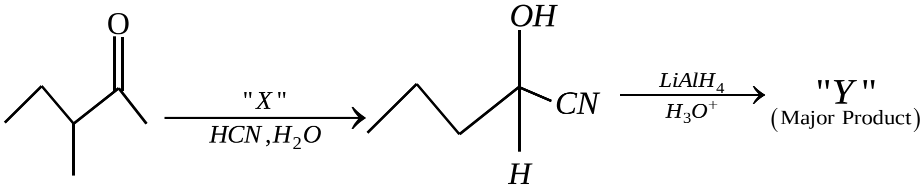
4) Statement I is false but Statement II is true

Key: 3

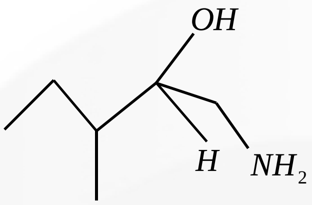
Sol: Sphalerite (ZnS) & Copper glans (Cu_2S) are concentrated by froth flotation.

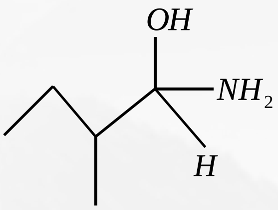
If is possible to separate them by adding depressents like NaCN

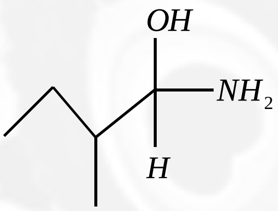
42.

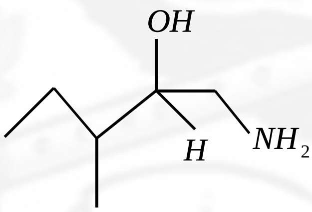


Consider the given reaction, Identify “X” & “Y”

1) $X - NaOH$ Y- 

2) $X - NaOH$ Y- 

3) $X - HNO_3$ Y- 

4) $X - HNO_3$ Y- 

Key: 1

Sol: Aldehydes react with HCN in the presence of base. $LiAlH_4$ reduces cyanides to amines.

43. The number of non-ionisable product obtained from the hydrolysis of PCl_5 is

1) 2 2) 0 3) 3 4) 1

Key: 2

Sol: $PCl_5 + H_2O \rightarrow H_3PO_4$

$H_3PO_4 \rightarrow 3H^+ + PO_4^{-3}$

No. of non Isonisable Hydrogens = Zero

44. The bond order and magnetic behavior of O_2^- ion are, respectively :

1) 2 and diamagnetic 2) 1.5 and diamagnetic

3) 1 and paramagnetic 4) 1.5 and paramagnetic

Key: 4

Sol: $\overset{\ominus}{O}_2 : \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \sigma 2P_z^2 \begin{matrix} \pi 2p_x^2\ \pi^* 2p_x^2 \\ \pi 2p_y^2\ \pi^* 2p_y^1 \end{matrix} \sigma * 2P_z^0$

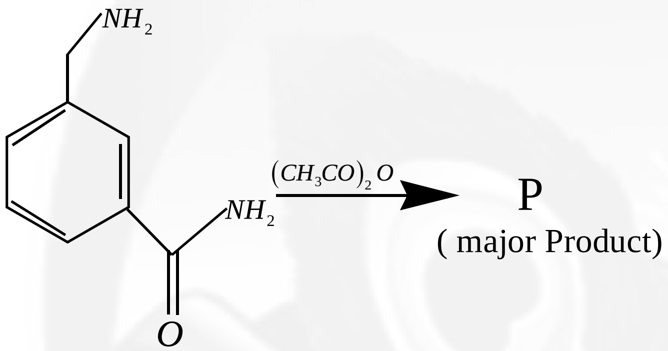
$\sigma^* 2p_z^0$

$\text{Bond order} = \dfrac{10-7}{2} = 1.5$

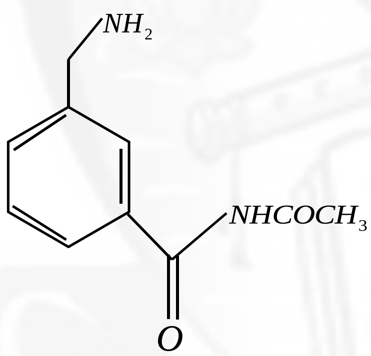
$\left[\text{B.O.} = \dfrac{N_b - N_a}{2} \right]$

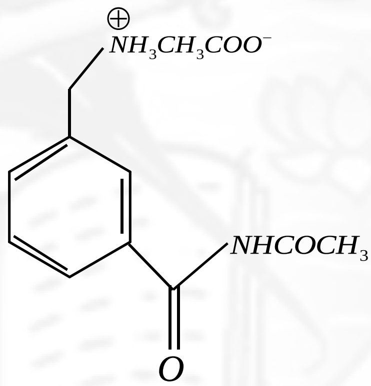
$\overset{\ominus}{O}_2$ is paramagnetic as there is 1 cenpaired electron

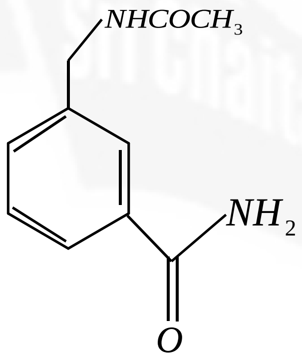
45.

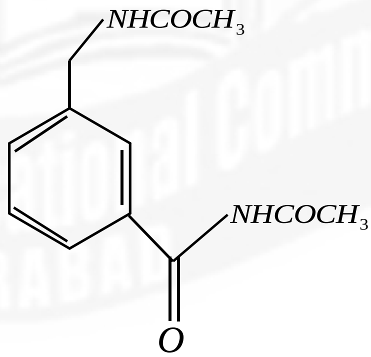


The major product in the above reaction is

1) 

2) 

3) 

4) 

Key: 3

Sol: Aeetylation occurs at $-CH_2-NH_2$ rather than at $-\overset{\overset{O}{\|}}{C}-NH_2$ as $-\overset{\overset{O}{\|}}{C}-NH_2$ is involved in resonance.

46. Given below are two statements: one is labeled as **Assertion (A)** and other is labeled as **Reason (R).**

Assertion (A) : Sucrose is a disaccharide and a non-reducing sugar

Reason (R) : Sucrose involves glycosidic linkage between

C_1 *of* $\beta-$glucose *and* C_2 *of* $\alpha-$ *fructose*

Choose the **most appropriate** answer the options given below :

1) (A) is true but (R) is false

2) Both (A) and (R) are true but (R) is not the true explanation of (A)

3) (A) is false but (R) is true

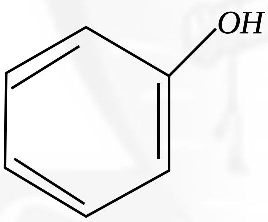
4) Both (A) and (R) are true and (R) is the true explanation of (A)

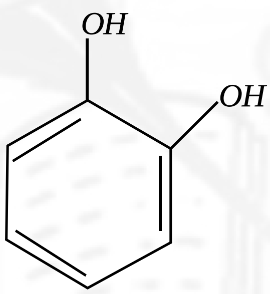
Key: 1

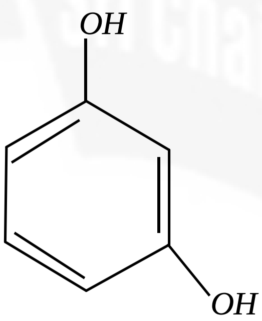
Sol: Sucrose is a non reducing sugar as the both reducing carbons are involved in glycosidc linkage.

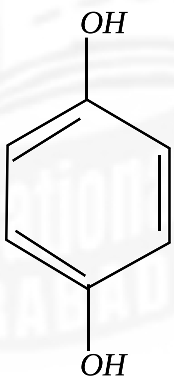
In Sucrose C-1 of $\alpha-$glucose is linked to C-2 of $\beta-$*Fructose*

47. Which one of the following phenols does not give colour when condensed with phtalic anhydride in presence of $conc.H_2SO_4$?

1) 

2) 

3) 

4) 

Key: 4

Sol: In para cresol, para position is blocked.

48. The interaction energy of London forces between two particles us proportional to r^x, where r is the distance between the particles. The value of x is

1) -6 2) -3 3) 6 4) 3

Key: 1

Sol: In London forces the interaction energy is directly proportional to $\frac{1}{r^6}$

$\therefore x = -6$

49. Arrange the following Cobalt complex in the order of increasing Crystal Field Stabilization Energy (CFSE)

Complexes $A:[CoF_6]^{3-}, B:\left[Co(H_2O)_6\right]^{2+}, C:\left[Co(NH_3)_6\right]^{3+} \& D:\left[Co(en)_3\right]^{3+}$

1) $B < C < D < A$ 2) $B < A < C < D$ 3) $A < B < C < D$ 4) $C < D < B < A$

Key: 2

Sol: The CSFE value depends upon the ligand field strength and oxidation states of metal ion. Hence the order is $B < A < C < D$

50. Match List –I with List-II

	List-I (Chemical Reaction)		List-II (Reagent used)
A	$CH_3COOCH_2CH_3 \to CH_3CH_2OH$	I	CH_3MgBr / H_3O^+ $(1.equivalent)$
B	$CH_3COOCH_3 \to CH_3CHO$	II	H_2SO_4 / H_2O
C	$CH_3 \equiv N \to CH_3CHO$	III	$DIBAL-H / H_2O$
D	$CH_3C \equiv NN \to CH_3-\overset{\overset{O}{\|}}{C}-CH_3$	IV	$SnCl_2, HCl / H_2O$

1) $A \to III; B \to II; C \to I; D \to IV$ 2) $A \to II; B \to III; C \to IV; D \to I$
3) $A \to IV; B \to II; C \to III; D \to I$ 4) $A \to II; B \to IV; C \to III; D \to I$

Key: 2

Sol: $(A) CH_3-\overset{\overset{o}{\|}}{C}-O-CH_2-CH_3 \xrightarrow[H_2O]{H_2SO_4/} CH_3-CH_2OH + CH_3-\overset{\overset{o}{\|}}{C}-OH$

$(B) CH_3-\overset{\overset{o}{\|}}{C}-O-CH_3 \xrightarrow[H_2O]{DIBAL-H} CH_3-\overset{\overset{o}{\|}}{C}-H + CH_3-OH$

$(C) CH_3-C \equiv N \xrightarrow[H_2O]{SnCl_2\ HCl} CH_3-\overset{\overset{o}{\|}}{C}-H$

$(D) CH_3-C \equiv N \xrightarrow{CH_3mgBr} CH_3-\underset{\underset{CH_3}{|}}{C} = NMgBr \xrightarrow{H_2O} CH_3-\overset{\overset{o}{\|}}{C}-CH_3$

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

51. 100 mL of Na_3PO_4 solution contains 3.45g of sodium. The molarity of the solution is _______ $\times 10^{-2}\, mol\, L^{-1}$.$(Nearest\ integer)$ $[Atomic\ mass - Na = 23.0u, O:16.0u, P:31.0u]$

Key: 50

Sol: molecular weight of Na_3PO_4 is = 164

69 gm of Na is present in 164 gm of Na_3PO_4

3.45 gm of Na is present in $\frac{164\times 3.45}{69} \Rightarrow 8.2\, gm\ Na_3PO_4$

Molarity of the solution $= \frac{wt}{MW}\times\frac{1000}{V\ ml} \Rightarrow \frac{8.2}{164}\times\frac{1000}{100} = 0.5$

$= 50\times 10^{-2}$

52. A chloro compound "A"

(i) Forms aldehydes on ozonolysis followed by the hydrolysis

(ii) When vaporized completely 1.53 g of A, gives 448mL of vapour at STP.

The number of carbon atoms in a molecule of compounds A is_____

Key: 3

Sol: Molecular weight $= \frac{22400\times 1.53}{448} = 76.5$

Weight of carbon & Hydrogen residue=76.5-35.5=41 so the molecular formula may be C_3H_5Cl

So the No. of carbons is 3

53. 83g of ethylene glycol dissolved in 625g of water. The freezing point of the solution is _____K.(Nearest Integer)

[use : Molal freezing point depression constant of water=$1.86K\, kg\, mol^{-1}$, freezing point of water=273 K, Atomic masses : C: 12.0 u, O: 16.0 u, H:1.0 u]

Key: 269

Sol: $\Delta T_f = k_f.\frac{wt}{mw}\times\frac{1000}{W} \Rightarrow 1.86\times\frac{83}{62}\times\frac{1000}{625} = 3.984$

Freezing point $= 0 - 3.984 = -3.984^0 C$

$= -3.984 + 273 = 269.01K$

54. A metal surface is exposed to 500nm radiation. The threshold frequency of the metal for photoelectric current is $4.3\times10^{14}\,Hz$. The velocity of ejected electron is _____ $\times10^{5}\,ms^{-1}$.

(Nearest integer)

$\left[Use: h = 6.63\times10^{-34}\,Js, m_e = 9.0\times10^{-31}\,kg\right]$

Key: 5

Sol: velocity is given by $V = \sqrt{\dfrac{2h(\nu-\nu_0)}{m}} = \sqrt{\dfrac{2h\left(\dfrac{c}{\lambda}-\nu_0\right)}{m}}$

$$\sqrt{\frac{2\times6.63\times10^{-3}\left(\dfrac{3\times10^{8}}{500\times10^{-9}}-4.3\times10^{14}\right)}{9\times10^{-31}}} = 5\times10^{5}$$

55. In the sulphur estimation 0.471 g of an organic compound gave 1.44g of barium sulfate. The percentage of sulphur in the compound is_______% (nearest Integer) (Atomic mass of Ba=137 u)

Key: 42

Sol: % of sulphur = $= \dfrac{32}{233}\times\dfrac{wt\ of\ BaSO_4}{wt\ of\ organic\ compound}\times100$

$$= \frac{32}{233}\times\frac{1.44}{0.471}\times100 = 41.98\% \approx 42$$

56. For water $\Delta_{vap}H = 41\,kJ\,mol^{-1}$ at 373K and 1 bar pressure. Assuming that water vapour is an ideal gas that occupies a much larger volume than liquid water, the internal energy change during evaporation of water is _____ $kJ\,mol^{-1}$ $\left[use: R = 8.3\,J\,mol^{-1}K^{-1}\right]$

Key: 38

Sol: $H_2O(l) \rightarrow H_2O(Vap)$

$\Delta H = \Delta U + \Delta nRT$

$\Delta U = \Delta H - \Delta nRT$

$\Delta H = 41KJ$ $\quad\Delta n = 1-0 = 1$; $\quad R = 8.3J = 8.3\times10^{-3}kJ$; $T = 373K$

$\Delta U = 4.1 - \left(1\times8.3\times10^{-3}\times373\right)$

$= 4.1 - 3.0959 = 37.9 \approx 38$

57. The equilibrium constant K_c at 298K for the reaction $A+B \rightleftharpoons C+D$ is 100. Starting with an equimolar solution with concentration of A, B, C and D all equal 1m, the equilibrium concentration of D is ____$\times 10^{-2} M$.(Nearest integer)

Key: 182

Sol: $A \; + \; B \rightleftharpoons C \; + \; D$

$t=0 \quad 1 \quad 1 \quad 1 \quad 1$

$t_{eq} \quad 1-x \quad 1-x \quad 1+x \quad 1+x$

$K_a = \dfrac{(1+x)^2}{(1-X)^2} \Rightarrow \quad 100 = \dfrac{(1+X)^2}{(1-X)^2}$

$\dfrac{1+x}{1-x} = 10 \quad \Rightarrow x = \dfrac{9}{11}$

$\therefore \text{conc of } D = 1 + \dfrac{9}{11} = \dfrac{1.818}{181.8\times10^{-2}} = 182\times10^{-2}$

58. The overall stability constant of the complex ion $\left[Cu\left(NH_3\right)_4\right]^{2+}$ $is\, 2.1\times10^{13}$.The over all dissociation constant is $y\times10^{-14}$.Then y is _______. (Nearest integer)

Key: 5

Sol: $\text{Dissociation constant} = \dfrac{1}{\text{stability const}} = \dfrac{1}{2.1\times10^{13}} = 0.476\times10^{-13}$

$= 4.76\times10^{-14}$

Nearest in fig: 5 x 10^{-14}

59. The reaction rate for the reaction $\left[PtCl_4\right]^{2-} + H_2O \rightleftharpoons \left[Pt\left(H_2O\right)Cl_3\right]^{-} + Cl^{-}$ was measured as a function of concentrations of different species. It was observed that $\dfrac{-d\left[\left[PtCl_4\right]^{2-}\right]}{dt} = 4.8\times10^{-5}\left[\left[PtCl_4\right]^{2-}\right] - 2.4\times10^{-3}\left[\left[Pt\left(H_2O\right)Cl_3\right]^{-}\right]\left[Cl^{-}\right]$ where square brackets are used to denote molar concentrations. The equilibrium constant $K_C =$ ______ (Nearest integer)

Key: 50

Sol: equilibrium constant $K_c = \dfrac{k_f}{k_b} = \dfrac{2.4\times10^{-3}}{4.8\times10^{-5}} = 50$

60. For the galvanic cell, $Zn(s)+Cu^{2+}(0.02M)\rightarrow Zn^{2+}(0.04M)+Cu(s)$,

$E_{cell} =$ _____ $\times10^{-2}V$.(Nearest integer) $\left[Use: E^{0}_{Cu/Cu^{2+}}=-0.34V, E^{0}_{Zn/Zn^{2+}}=+0.76V, \frac{2.303RT}{F}=0.059V\right]$

Key: 109

Sol: $Zn(s)+Cu^{+2}(0.02)\longrightarrow zn^{+2}(0.04)+Cu$

$$E_{cell}=E^{0}_{cell}-\frac{0.059}{x}\log\frac{\left[zn^{+2}\right]}{\left[Cu^{+1}\right]}$$

$$E_{cell}=+0.64-(-0.76)=1.1V$$

$$E_{cell}=1.1-\frac{0.059}{2}\log\frac{0.04}{0.02}=1.09V\Rightarrow109\times10^{-2}$$

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

61. $\lim\limits_{x\to 2}\left(\sum\limits_{n=1}^{9}\dfrac{x}{n(n+1)x^2+2(2n+1)x+4}\right)$ is equal to

1) $\dfrac{5}{24}$ 2) $\dfrac{7}{36}$ 3) $\dfrac{1}{5}$ 4) $\dfrac{9}{44}$

Key: 4

Sol: $\lim\limits_{x\to 2}\sum\limits_{n=1}^{9}\dfrac{x}{n(n+1)x^2+2(2n+1)x+4}$

$$\sum_{n=1}^{9}\frac{2}{n(n+1)4+2(2n+1)2+4}$$

$$\sum_{n=1}^{9}\frac{2}{4(n^2+n+2n+1)}$$

$$\frac{1}{2}\sum_{n=1}^{9}\left(\frac{1}{n^2+3n+2}\right)$$

$$\frac{1}{2}\sum_{n=1}^{9}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$$

Tabular sum

$$=\frac{1}{2}\left(\frac{1}{2}-\cancel{\frac{1}{3}}\right)$$

$$+\frac{1}{2}\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{4}}\right)$$

$$+\frac{1}{2}\left(\cancel{\frac{1}{4}}-\cancel{\frac{1}{5}}\right)$$

$$+\frac{1}{2}\left(\cancel{\frac{1}{10}}-\frac{1}{11}\right)$$

$$\frac{1}{2}\left(\frac{1}{2}-\frac{1}{11}\right)=\frac{1}{2}\left(\frac{9}{22}\right)=\frac{9}{44}$$

62. Let P be the plane passing through the point (1,2,3) and the line of intersection of the planes . $\vec{r}.\left(\hat{i}+\hat{j}+4\hat{k}\right)=16\ \&\ \vec{r}.\left(-\hat{i}+\hat{j}+\hat{k}\right)=6$. Then which of the following points does NOT lie on P?

1) $(-8,8,6)$ 2) $(6,-6,2)$ 3) $(4,2,2)$ 4) $(3,3,2)$

Key: 3

Sol: Equation of plane

$(x+y+4z-16)+\lambda(-x+y+z-6)=0$

It passes through point (1,2,3)

$(1+2+12-16)+\lambda(-1+2+3-6)=0$

$-1+\lambda(-2)=0 \Rightarrow 2\lambda=1 \Rightarrow \lambda=\frac{-1}{2}$

So plane

$(x+y+4z-16)-\frac{1}{2}(-x+y+z-6)=0$

$2x+2y+8z-32+x-y-z+6=0$

$3x+y+7z-26=0$

So (4,2,2) not passing through the plane

63. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{1+\sin^2 x}{1+\pi^{\sin x}}\right)dx\ is$

1) $\frac{5\pi}{4}$ 2) $\frac{\pi}{2}$ 3) $\frac{3\pi}{4}$ 4) $\frac{3\pi}{2}$

Key: 3

Sol: $I=\int_a^b f(x)dx$

$f(a+b-x)=f(x)$

$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1+\sin^2 x}{1+\pi^{\sin x}}dx \longrightarrow (1)$

$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1+\sin^2 x}{1+\pi^{\sin x}}dx \longrightarrow (2)$

$(1)+(2)$

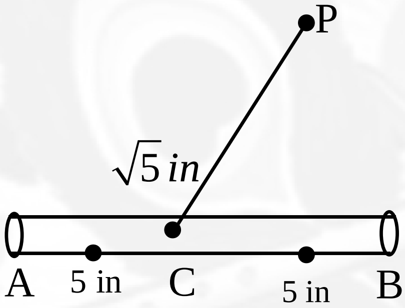
$$2I=\int_{-\pi/2}^{\pi}\left(1+\sin^2 x\right)dx$$

$$2I=\int_{-\pi/2}^{\pi}\left(1+\frac{1-\cos 2x}{2}\right)dx$$

$$2I=\int_{-\pi/2}^{\pi}\left(\frac{3-\cos 2x}{2}\right)dx$$

$$I=\frac{1}{4}\left[3x-\frac{\sin 2x}{2}\right]_{-\pi/2}^{\pi/2}\Rightarrow I=\frac{3\pi}{4}$$

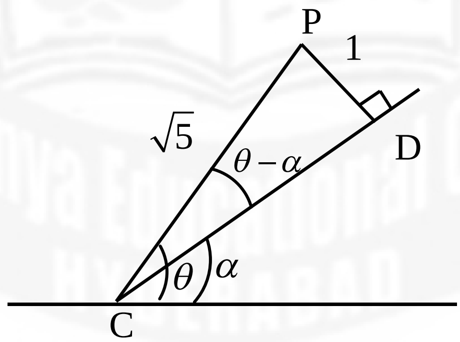
64. A 10 inches long pencil AB with mid point C and a small eraser p are placed on the horizontal top of a table such that $PC=\sqrt{5}\ inches\ \&\ \angle PCB=\tan^{-1}(2)$.The acute angle through which the pencil must be rotated about C so that the perpendicular distance between eraser and pencil becomes exactly 1 inch is



1) $\tan^{-1}(1)$ 2) $\tan^{-1}\left(\frac{4}{3}\right)$ 3) $\tan^{-1}\left(\frac{3}{4}\right)$ 4) $\tan^{-1}\left(\frac{1}{2}\right)$

Key: 3

Sol:



Let pencil rotated with angle α

So, ΔPCD

$PD=1\left(given\ ,\tan\theta=2\right)$

$DC=\sqrt{5}$

CD=2

ΔPCD

$\tan\left(\theta-\alpha\right)=\frac{1}{2}$

$\frac{\tan\theta-\tan\alpha}{1+\tan\theta\tan\alpha}=\frac{1}{2}$

$\frac{2-\tan\alpha}{1+2\tan\alpha}=\frac{1}{2}$

$4-2\tan\alpha=1+2\tan\alpha$

$4\tan\alpha=3\Rightarrow\tan\alpha=\frac{3}{4}\Rightarrow\alpha=\tan^{-1}\left(\frac{3}{4}\right)$

65. Two fair dice are thrown. The numbers on them are taken as $\lambda\,\&\,\mu$ and a system of linear equations. $x+y+z=5, x+2y+3z=\mu, x+3y+\lambda z=1$ is constructed. If P is the probability that the system has a unique solution and q is the probability that the system has no solution, then

1) $p=\frac{1}{6}\,\&\,q=\frac{5}{36}$ 2) $p=\frac{5}{6}\,\&\,q=\frac{1}{36}$ 3) $p=\frac{1}{6}\,\&\,q=\frac{1}{36}$ 4) $p=\frac{5}{6}\,\&\,q=\frac{5}{36}$

Key: 4

Sol: $x+1+z=5$

$x+2y+3z=\mu$

$x+3y+\lambda z=1$

$\Delta=\begin{vmatrix}1&1&1\\1&2&3\\1&3&\lambda\end{vmatrix}$

$\Delta=1\left(2\lambda-9\right)-1\left(\lambda-3\right)+1\left(3-2\right)$

$\Delta=2\lambda-9-\lambda+3+1$

$\Delta=\lambda-5$

For unique solution $\Delta\neq0, so\,\lambda\neq5$

So, probability for unique solution $=\frac{favourable\,case}{total\,case}=\left(\frac{5}{6}\right)$

For no solution

$\Delta=0\,\&\,\Delta_1,\Delta_2,\Delta_3$ any of them non-zero $\Delta=0$ *for* $\lambda=5$

$$\Delta_1 = \begin{vmatrix} 5 & 1 & 1 \\ \mu & 2 & 3 \\ 1 & 3 & 5 \end{vmatrix}$$

$$\Delta_1 = 5(10-9) - 1(5\mu - 3) + 1(3\mu - 2)$$

$$= 5 - 5\mu + 3 + 3\mu - 2$$

$$\Delta_1 = 6 - 2\mu$$

If $\Delta_1 = 0 \Rightarrow \mu = 3$

$$\Delta_2 = \begin{vmatrix} 1 & 5 & 1 \\ 1 & \mu & 3 \\ 1 & 1 & 5 \end{vmatrix}$$

$$\Delta_2 = 1(5\mu - 3) - 5(2) + 1(1-\mu)$$

$$= 5\mu - 3 - 10 + 1 - \mu$$

$$\Delta_2 = 4\mu - 12$$

If $\Delta_2 = 0, \mu = 3$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & \mu \\ 1 & 3 & 1 \end{vmatrix}$$

$$\Delta_3 = 1(2 - 3\mu) - 1(1-\mu) + 5(3-2)$$

$$= 2 - 3\mu - 1 + \mu + 5$$

$$\Delta_3 = -2\mu + 6$$

If $\Delta_3 = 0 \quad \mu = 3$

Favorable case

$(5,1)(5,2)(5,4)(5,5)(5,6)$ Total =5

Sample space =36

Probability for No solution $= \frac{5}{36}$

66. Let $[t]$ denote the greatest integer less than or equal to t. let

$f(x) = x - [x], g(x) 1 - x + [x], \& h(x) = \min\{f(x), g(x)\}, x \in [-2,2]$, then h is:

1) Continuous in [-2,2] but not differentiable at more than four points in (-2,2)

2) Not continuous at exactly three points in [-2,2]

3) Continuous in [-2,2] but not differentiable at exactly three points in (-2,2)

4) Not continuous at exactly four points in [-2,2]

Key: 1

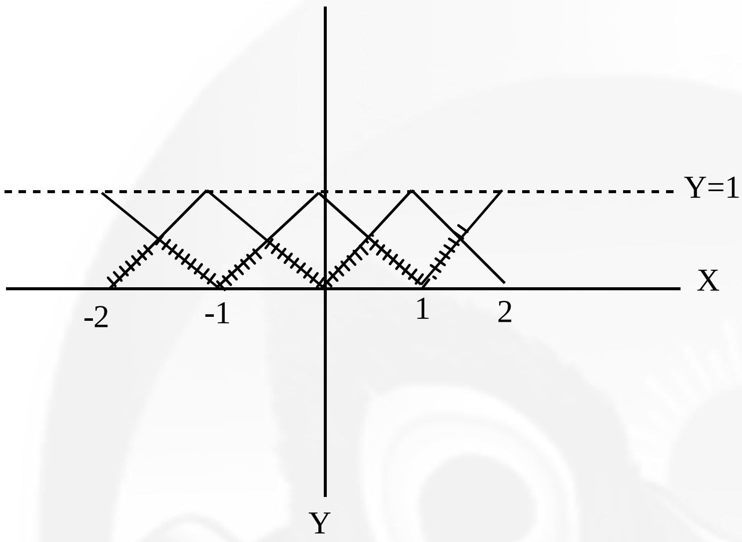
Sol: Given $f(x)=x-[x]=\{x\}$

$g(x)=1-x+[x]\Rightarrow 1-(x-[x])$

$g(x)=1-\{x\}$

$h(x)=\min.\{f(x),g(x)\}\ x\in[-2,2]$

Graph



$h(x)$ is dotted line, which is continuous but not differentiable at more than 4 points.

67. The locus of the mid points of the chords of the hyperbola $x^2-y^2=4$. Which touch the parabola y^2-8x, is

1) $y^2(x-2)=x^3$ 2) $y^3(x-2)=x^2$ 3) $x^3(x-2)=y^2$ 4) $x^2(x-2)=y^3$

Key: 1

Sol: Let the mid point of chord is P(h,k) locus of chord whose mid point is known given as

$T=S_1$

$hx-ky=h^2-k^2$

$ky=hx-(h^2-k^2)$

$y=\frac{h}{k}x-\frac{(h^2-k^2)}{k}$ we know $y=mx+c$ is tangent to $y^2=4ax\ If\ C=\frac{a}{m}$ so

$\frac{-(h^2-k^2)}{k}=\frac{2}{\frac{h}{k}}$

$\frac{k^2-h^2}{k}=\frac{2k}{h}$

Locus $\frac{y^2-x^2}{y}=\frac{2y}{x}$

$xy^2-x^3=2y^2$

$y^2(x-2)=x^3$

68. The value of $2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$ is:

1) $\frac{1}{8}$ 2) $\frac{1}{8\sqrt{2}}$ 3) $\frac{1}{4}$ 4) $\frac{1}{4\sqrt{2}}$

Key: 1

Sol: $2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$

$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{\pi}{4}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\pi-\frac{3\pi}{8}\right)\sin\left(\frac{3\pi}{4}\right)\sin\left(\pi-\frac{\pi}{8}\right)$

$2\sin\left(\frac{\pi}{8}\right)\frac{1}{\sqrt{2}}\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\left(\frac{1}{\sqrt{2}}\right)\sin\left(\frac{\pi}{8}\right)\sin^2\left(\frac{\pi}{8}\right)\sin^2\left(\frac{3\pi}{8}\right)$

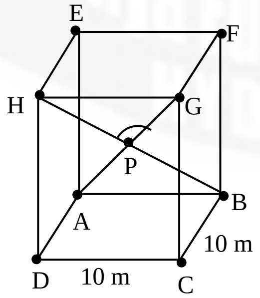
$=\frac{1}{4}\left(2\sin^2\frac{\pi}{8}\right)\left(2\sin^2\frac{3\pi}{8}\right)$

$=\frac{1}{4}\left(1-\cos\frac{\pi}{4}\right)\left(1-\cos\left(\frac{3\pi}{4}\right)\right)$

$=\frac{1}{4}\left(1-\frac{1}{\sqrt{2}}\right)\left(1+\frac{1}{\sqrt{2}}\right)$

$=\frac{1}{4}\left(1-\frac{1}{2}\right)=\frac{1}{8}$

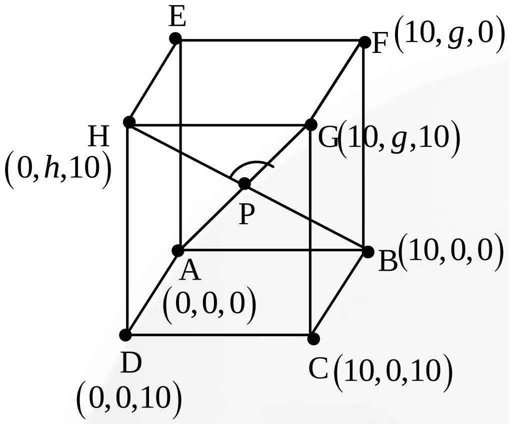
69. A hall has a square floor of dimension $10m\times10m$ (see the figure) and vertical walls. If the angle GPH between the diagonals AG and BH is $\cos^{-1}\frac{1}{5}$, then the height of the hall (in meters) is:



1) $5\sqrt{2}$ 2) $5\sqrt{3}$ 3) 5 4) $2\sqrt{10}$

Key: 1

Sol: Given $\angle GPH = \cos^{-1}\left(\frac{1}{5}\right)$



$\overline{AG} = (10, g, 10), \overline{BH} = (-10), g, 10$

$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{\sum a_1^2}\sqrt{\sum a_2^2}}\right|$

$\frac{1}{5} = \frac{h^2}{200 + h^2}$

$200 + h^2 = 5h^2$

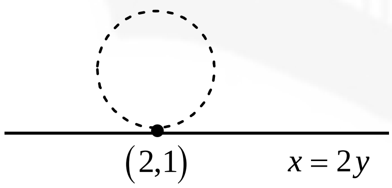
$4h^2 = 200 \Rightarrow h^2 = 50 \Rightarrow h = 5\sqrt{2}$

70. A circle C touches the line x=2y at the point (2,1) and intersects the circle $C_1 : x^2 + y^2 + 2y - 5 = 0$ at two points P and Q such that PQ is a diameter of C_1. Then the diameter of C is

1) 15 2) $\sqrt{285}$ 3) $7\sqrt{5}$ 4) $4\sqrt{15}$

Key: 3

Sol: family of circle touching line x=2y at (2,1) given as



$(x-2)^2 + (y-1)^2 + \lambda(x - 2y) = 0$

Common chord of two circle $(s_1 - s_2 = 0)$

$(x-2)^2 + (y-1)^2 + \lambda(x-2y) - (x^2 + y^2 + 2y - 5) = 0$

This chord is diameter of $x^2+y^2+2y-5=0$ so,it will pass through centre (0,-1)

$$(0-2)^2+(-1-1)^2+\lambda(0+2)-(0+1-2-5)=0$$

$$4+4+2\lambda+6=0$$

$$2\lambda=-14 \Rightarrow \lambda=-7$$

Equation of required circle

$$(x-2)^2+(y-1)^2-7(x-2y)=0$$

$$x^2+y^2-11x+12y+5=0$$

Radius= $\sqrt{g^2+f^2-c}$

$$=\sqrt{\left(\frac{11}{2}\right)^2+\left(\frac{12^2}{2}\right)-5}$$

$$=\sqrt{\frac{245}{2\times 2}}$$

Diameter $=\sqrt{245}=7\sqrt{5}$

71. Let $A=\begin{pmatrix}1&0&0\\0&1&1\\1&0&0\end{pmatrix}$.*Then* $A^{2025}-A^{2020}$ is equal to

1) A^6-A 2) A^5 3) A^6 4) A^5-A

Key: 1

Sol: $A=\begin{bmatrix}1&0&0\\0&1&1\\1&0&0\end{bmatrix} \Rightarrow A^2=\begin{bmatrix}1&0&0\\1&1&1\\1&0&0\end{bmatrix} \Rightarrow A^3=\begin{bmatrix}1&0&0\\2&1&1\\1&0&0\end{bmatrix} \Rightarrow A^4=\begin{bmatrix}1&0&0\\3&1&1\\1&0&0\end{bmatrix} \Rightarrow A^n=\begin{bmatrix}1&0&0\\n-1&1&1\\1&0&0\end{bmatrix}$

$$A^{2025}-A^{2020}=\begin{bmatrix}0&0&0\\5&0&0\\0&0&0\end{bmatrix}$$

$$A^6-A=\begin{bmatrix}0&0&0\\5&0&0\\0&0&0\end{bmatrix}$$

72. If the value of the integer $\int_0^5 \frac{x+[x]}{e^{x-[x]}}dx=\alpha e^{-1}+\beta$, where $\alpha,\beta\in R, 5\theta+6\beta=0, \&[x]$ denotes the greatest integer less than or equal to x; then the value of $(\alpha+\beta)^2$ is equal to

1) 25 2) 36 3) 16 4) 100

Key: 1

Sol: $I=\int_0^5 \frac{x+[x]}{e^{x-[x]}}dx$

$$\int_0^1 \frac{x}{e^x}dx+\int_1^2 \frac{x+1}{e^{x-1}}dx+\int_2^3 \frac{x+2}{e^{x-2}}dx+.....\int_4^5 \frac{x+4}{e^{x-4}}dx$$

$$\Downarrow_{x=t+1} \qquad \Downarrow_{x=z+2} \qquad \Downarrow_{x=y+4}$$

$$\int_0^1 \frac{t+2}{e^t}dt+\int_0^1 \frac{z+4}{e^z}dz+......\int_0^1 \frac{y+8}{e^y}dy$$

$$\Rightarrow \int_0^5 \frac{5x+20}{e^x}dx=5\int_0^1 \frac{x+4}{e^x}dx$$

$$\Rightarrow 5\int_0^1 (x+4)e^{-x}dx \Rightarrow 5e^{-x}(-x-5)\Big]_0^1 \Rightarrow \frac{30}{e}+25$$

$$\alpha=-30, \beta=25 \Rightarrow 5\alpha+6\beta=0 \Rightarrow (\alpha+\beta)^2=5^2=25$$

73. The domain of the function $\operatorname{cosec}^{-1}\left(\frac{1+x}{x}\right)$ is:

1) $\left(-\frac{1}{2},\infty\right)-\{0\}$ 2) $\left[-\frac{1}{2},0\right)\cup[1,\infty)$ 3) $\left(-1,-\frac{1}{2}\right]\cup(0,\infty)$ 4) $\left[-\frac{1}{2},\infty\right)-\{0\}$

Key: 4

Sol: $\frac{1+x}{x}\in(-\infty,-1]\cup[1,\infty)$

$\frac{1+x}{x}\geq 1,\quad \frac{1+x}{x}-1\geq 0 \qquad \frac{1+x}{x}\leq -1$

$\frac{1}{x}\geq 0 \qquad \frac{1+x}{x}+1\leq 0$

$x<\infty \qquad \frac{1+2x}{x}\leq 0 \quad x\in\left[-\frac{1}{2},0\right)$

$x\in\left[\frac{-1}{2},\infty\right)-\{0\}$

74. The local maximum value of the function $f(x)=\left(\frac{2}{x}\right)^{x^2}, x>0$ is

1) $\left(\frac{4}{\sqrt{e}}\right)^{\frac{e}{4}}$ 2) $(e)^{\frac{2}{e}}$ 3) 1 4) $\left(2\sqrt{e}\right)^{\frac{1}{e}}$

Key: 2

Sol: $f(x)=\left(\frac{2}{x}\right)^{x^2}$

$f^1(x)=0$

$$y=\left(\frac{2}{x}\right)^{x^2}$$

$$\log y=x^2\ln\left(\frac{2}{x}\right)$$

$$\frac{1}{y}y^1=2x\ln\left(\frac{2}{x}\right)+x^2\times\frac{1}{\frac{2}{x}}\times\frac{-2}{x^2}$$

$$y^1=y\left[2\ln\frac{2}{x}-1\right]x$$

$$y^1=\left(\frac{2}{x}\right)^{x^2}x\left(2\ln\frac{2}{x}-1\right)=0$$

$$2\ln\frac{2}{x}=1$$

$$\ln\frac{2}{x}=\frac{1}{2}$$

$$\frac{2}{x}=e^{\frac{1}{2}}$$

$$\begin{array}{cccc} & + & & - \\ \hline O & \text{inc} & \frac{2}{\sqrt{e}} & \text{dec} \end{array}$$

$$x=2e^{-1/2}$$

$$f\left(2e^{-1/2}\right)=\left(\frac{2}{2e^{-1/2}}\right)=e^{2/e}$$

75. Let $y(x)$ be the solution of the differential equation $2x^2dy+\left(e^y-2x\right)dx=0, x>0. If\ y(e)=1$ then $y(1)$ is equal to

1) 0 2) $\log_e(2e)$ 3) 2 4) $\log_e 2$

Key: 4

Sol: $2x^2dy+\left(e^y-2x\right)dx=0 \quad y(e)=1$

$$\frac{dy}{dx}=\frac{-e^y}{2x^2}+\frac{1}{x}$$

$$e^{-y}\frac{dy}{dx}=\frac{e^y}{x}-\frac{1}{2x^2}$$

$$e^{-y}=t$$

$$e^{-y}(-1)\frac{dy}{dx}=\frac{dt}{dx}$$

$$\frac{dt}{dx}+\frac{t}{x}=\frac{1}{2x^2}$$

$$IF=e^{\int\frac{1}{x}dx}=x$$

$$tx=\int\frac{1}{2x^2}x\,dx\Rightarrow e^{-y}x=\frac{1}{2}\ln x+c$$

$$e^{-1}e=\frac{1}{2}+x\Rightarrow C=\frac{1}{2}$$

$$e^{-y}x=\frac{1}{2}(1+\ln x)$$

$$e^{-y}=\frac{1}{2}(1+0)$$

$$e^{y}=2$$

$$y=\log_e^2$$

76. The point $P\left(-2\sqrt{6},\sqrt{3}\right)$ lies on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ having eccentricity $\frac{\sqrt{5}}{2}$.If the tangent and normal at P to the hyperbola intersect its conjugate axis at the points Q and R respectively, then QR is equal to

1) $3\sqrt{6}$ 2) $6\sqrt{3}$ 3) 6 4) $4\sqrt{3}$

Key: 2

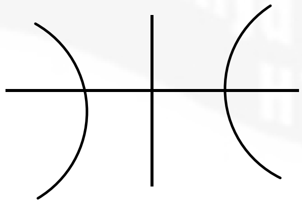
Sol: $P\left(-2\sqrt{6},\sqrt{3}\right)$ lies on hyperbola

$$\Rightarrow\frac{24}{a^2}-\frac{3}{b^2}=1\longrightarrow(i)$$

$$e=\frac{\sqrt{5}}{2}\Rightarrow b^2=a^2\left(\frac{5}{4}-1\right)\Rightarrow 4b^2=a^2$$

Put on (i) $\frac{6}{b^2}-\frac{3}{b^2}=1\Rightarrow b=\sqrt{3}\Rightarrow a=\sqrt{12}$

$$\frac{x^2}{12}-\frac{y^2}{3}=1$$



Tangent at P :

$$\frac{-x}{\sqrt{6}}-\frac{y}{\sqrt{3}}=1\Rightarrow Q=\left(0,\sqrt{3}\right)$$

Slope of $T = -\frac{1}{\sqrt{2}}$

Normal at P: $y-\sqrt{3}=\sqrt{2}\left(x+2\sqrt{6}\right)$

$\Rightarrow R=\left(0,5\sqrt{3}\right)$

$QR=6\sqrt{3}$

77. A fair die is tossed until six is obtained on it. Let X be the number of required tosses, then the conditional probability $P\left(X\geq 5 \mid X>2\right)$ is

1) $\frac{5}{6}$ 2) $\frac{11}{36}$ 3) $\frac{25}{36}$ 4) $\frac{125}{216}$

Key: 3

Sol: $p\left(\frac{x\geq 5}{x>2}\right)=\frac{p\left(x\geq 5\cap x>2\right)}{p\left(x>2\right)}$

$$=\frac{p\left(x=5\right)+p\left(x=6\right)+p\left(x=7\right)....}{p\left(x=3\right)+p\left(x=4\right)......}$$

$$=\frac{\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^5\frac{1}{6}.....}{\left(\frac{5}{6}\right)^2\frac{1}{6}+\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)}$$

$$=\frac{\frac{\left(\frac{5}{6}\right)^4\frac{1}{6}}{1-\frac{5}{6}}}{\frac{\left(\frac{5}{6}\right)^2\frac{1}{6}}{1-\frac{5}{6}}}=\left(\frac{5}{6}\right)^2=\frac{25}{36}$$

78. If $\sum_{r=1}^{50}\tan^{-1}\frac{1}{2r^2}=p$,then the value of $\tan p$ is

1) $\frac{101}{102}$ 2) $\frac{51}{50}$ 3) 100 4) $\frac{50}{51}$

Key: 4

Sol: $\sum_{r=1}^{50}\tan^{-1}\frac{1}{2r^2}=p$

$$\tan^{-1}\left(\frac{2}{4r^2}\right)=Tan^{-1}\left(\frac{2}{1+4r^2-1}\right)=\tan^{-1}\left(\frac{2}{1+\left(2r-1\right)\left(2r+1\right)}\right)$$

$$= \tan^{-1}\left(\frac{(2r+1)-(2r-1)}{1+(2r-1)(2r+1)}\right) = \tan^{-1}(2r+1) - \tan^{-1}(2r-1)$$

$$\sum_{r=1}^{50} \tan^{-1}\frac{1}{2r^2} = \sum_{r=1}^{50} \tan^{-1}(2r+1) - \tan^{-1}(2r-1) = \frac{\tan^{-1}(101) - \tan^{-1} 99}{\tan^{-1}(101) - \tan^{-1}}$$

$$= \tan^{-1}(101) - \tan^{-1}(1)$$

$$P = \tan^{-1}\left(\frac{100}{1+101}\right)$$

$$\tan P = \frac{100}{102} = \frac{50}{51}$$

79. Consider the two statements:

$(S1):(p \to q) \vee (\sim q \to p)$ *is a* tautology

$(S2):(p \wedge \sim q) \wedge (\sim p \vee q)$ *is a fallacy*

Then

1) Only $(S2)$ is true
2) Only $(S1)$ is true
3) both $(S1)\&(S2)$ are true
4) both $(S1)\&(S2)$ are false

Key: 3

Sol: $S_1 : (\sim p \vee q) \vee (q \vee p) = (q \vee \sim p) \vee (q \vee p)$

$S_1 : q \vee (\sim p \vee q) = q \vee t = t =$ tautology

$S_2 : (p \wedge \sim q) \wedge (\sim p \vee q) = (p \wedge \sim q) \wedge \sim (p \wedge \sim q) = C = fallacy$

80. If $\left(\sqrt{3}+i\right)^{100} = 2^{99}(p+iq)$, then p and q are roots of the equation

1) $x^2 + \left(\sqrt{3}-1\right)x - \sqrt{3} = 0$
2) $x^2 - \left(\sqrt{3}-1\right)x - \sqrt{3} = 0$
3) $x^2 + \left(\sqrt{3}+1\right)x + \sqrt{3} = 0$
4) $x^2 - \left(\sqrt{3}+1\right)x + \sqrt{3} = 0$

Key: 2

Sol: $\left(2e^{i\pi/6}\right)^{100} = 2^{99}(p+iq)$

$$2^{100}\left(\cos\frac{50\pi}{3} + i\sin\frac{50\pi}{3}\right) = 2^{99}(p+iq)$$

$$p+iq = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$$p = -1, q = \sqrt{3}$$

$$x^2 - \left(\sqrt{3}-1\right)x - \sqrt{3} = 0$$

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

81. Let $a_1, a_2, ...a_{10}$ be an AP with common difference -3 and $b_1, b_2, ...b_{10}$ be a GP with common ratio 2, Let $c_k = a_k + b_k, k = 1, 2, ..., 10. If\ C_2 = 12 \ \& \ C_3 = 13 \ then \sum_{k=1}^{10} c_k$ is equal to ______

Key: 2021

Sol:

$C_2 = 12 \qquad C_3 = 13$

$a_2 + b_2 = 12 \qquad a_3 + b_3 = 13$

$a_1 - 3 + 2b_1 = 12 \qquad a_1 - 6 + 4b_1 = 13$

$a_1 + 2b_1 = 15 \qquad a_1 + 4b_1 = 19$

$a_1 = 11, b_1 = 2$

$$\sum_{k=1}^{10} C_k = \sum_{k=1}^{10} a_k + b_k = \sum_{k=1}^{10} a_k + \sum_{k=1}^{10} b_k = -25 + 2046 = 2021$$

82. Let the mean and variance of four numbers 3,7,x and y (x>y) be 5 and 10 respectively. Then the mean of four numbers $3 + 2x, 7 + 2y, x + y \ \& \ x - y$ ___ is

Key: 12

Sol: $\bar{x} = 5, \sigma^2 = 10$

$\frac{3 + 7 + x + y}{4} = 5 \qquad \frac{\sum x_i^2}{n} - \left(\bar{x}\right)^2 = 10$

$\Rightarrow x + y = 10 \qquad \frac{9 + 49 + x^2 + y^2}{4} = 35$

$x^2 + y^2 = 82$

$\therefore x = 9, y = 1$

Mean of $\frac{21 + 9 + 10 + 8}{4} = 12$

83. Let A be a 3×3 real matrix. If $\det\left(2Adj\left(2Adj\left(Adj\left(2A\right)\right)\right)\right) = 2^{41}$ then the value of $\det\left(A^2\right)$

Key: 4

Sol: $\left|2\,adj\left(2adj\left(adj2A\right)\right)\right|$

$= 2^3\left|adj2adj\left(adj2A\right)\right| \Rightarrow 2^3\left|2^2\,adjadjadj\left(2A\right)\right|$

$= 2^3.\left(2^2\right)^3\left|2A\right|^8 \Rightarrow 2^9\left(2^3\right)^8\left|A\right|^8 \Rightarrow 2^{33}\left|A\right|^8$

G.T $2^{33}\left|A\right|^8 = 2^{41}$

$\left|A\right|^8 = 2^8 \ \left|A\right|^8 = 2^8 \Rightarrow \left|A\right| = 2 \Rightarrow \left|A\right|^2 = 4$

84. Let Q be the foot of the perpendicular from the point $P(7,-2,13)$ on the plane containing the lines $\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8}$ & $\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-3}{7}$ Then $(PQ)^2$ is equal to

Key: 96

Sol: Equation of plane containg

$$\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8} \;\&\; \frac{x-1}{3}=\frac{y-2}{5}=\frac{z-3}{7}$$

$$\begin{vmatrix} x+1 & y-1 & z-3 \\ 6 & 7 & 8 \\ 3 & 5 & 7 \end{vmatrix}=0 \Rightarrow x-2y+z=0$$

Perpendicular distance from $P(7,-2,13)$ to $x-2y+z=0$

$$PQ=\frac{|7+4+13|}{\sqrt{1+4+1}}=\frac{24}{\sqrt{6}}$$

$$\therefore PQ^2=96$$

85. Let $\lambda \neq 0$ be in R. If $\alpha \,\&\, \beta$ are the roots of the equation $x^2-x+2\lambda=0$ and $\alpha \,\&\, \gamma$ are the roots of the equation $3x^2-10x+27\lambda=0$ then $\frac{\beta\gamma}{\lambda}$ is equal to__________

Key: 18

Sol: Let α be the common root of $x^2-x+2\lambda=0, 3x^2-10x+27\lambda=0$

$\alpha^2 \quad \alpha \quad 1$

$-1 \quad 2\lambda \quad 1 \quad -1$

$-10 \quad 27\lambda \quad 3 \quad -10$

$$\frac{\alpha^2}{-7\lambda}=\frac{\alpha}{-21\lambda}=\frac{-1}{7}$$

$$\alpha=3\lambda, \alpha^2=\lambda$$

$$9\lambda^2=\lambda$$

$$\lambda=\frac{1}{9} \Rightarrow \alpha=\frac{1}{3}, \therefore \beta=\frac{2}{3}, \gamma=3 \Rightarrow \frac{\beta\times\gamma}{\lambda}=18$$

86. Let a and b respectively be the points of local maximum and local minimum of the function $f(x)=2x^3-3x^2-12x$. If A is the total area of the region bounded by $y=f(x)$, the x-axis and the lines $x=a$ & $x=b$, then 4A is equal to_____

Key: 114

Sol: $f(x)=2x^3-3x^2-12x$

$f'(x)=6x^2-6x-12$

$=6\left[x^2-x-2\right]$

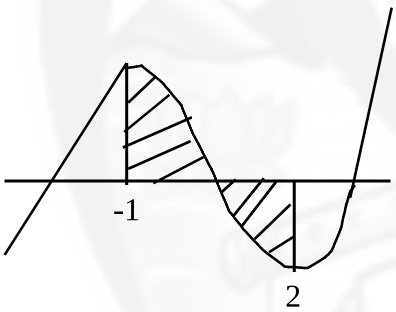
$=6\left[(x-2)(x+1)\right]$

At $x=-1\max x=2\min$

$a=-1, b=2$

$$A=\int_{-1}^{0}2x^3-3x^2-12xdx-\int_{0}^{2}2x^3-3x^2-12xdx \Rightarrow \frac{57}{2}$$

$4A=114$



87. If the projection of the vector $\hat{i}+2\hat{j}+\hat{k}$ on the sum of the two vectors $2\hat{i}+4\hat{j}-5\hat{k}$ & $-\lambda\hat{i}+2\hat{j}+3\hat{k}$ is 1, then λ is equal to ____

Key: 5

Sol: Projection of $i+2j+k$ on $i(2-\lambda)+6j-2k$ is 1

$$\frac{|2-\lambda+12-2|}{\sqrt{(2-\lambda)^2+36+4}}=1\Rightarrow\lambda=5$$

88. The sum of all 3-digit numbers less than or equal to 500, that are formed without using the digit "1" and they all are multiple of 11, is____

Key: 7744

Sol: Sum 3 digited numbers divisible by 11 not contains 1

$[209+220+231+.......495]-[231+319+341+418+451]$

$\frac{27}{2}[2\times 209+26\times 11]-1760$

$\Rightarrow 9504-1760=7744$

89. The least positive integer n such that $\frac{(2i)^n}{(1-i)^{n-2}}, i=\sqrt{-1}$, is a positive integer is____

Key: 6

Sol: $\frac{(2i)^n}{(1-i)^{n-2}}=\left[\frac{2i}{1-i}\right]^n(1-i)^2$

$=(-1+i)^n(-1+i)^2$

$(-1+i)^{n+2}=(-2i)^{\frac{n+2}{2}}$

N=6 is least the integer

90. Let $\binom{n}{k}$ denote ${}^nC_k$ & $\begin{bmatrix} n \\ k \end{bmatrix}=\begin{cases}\binom{n}{k}, if\ \ 0\le k\le n \\ 0,\ \ other\ wise\end{cases}$

If $A_k=\sum_{i=0}^{9}\binom{9}{1}\begin{bmatrix}12 \\ 12-k+i\end{bmatrix}+\sum_{i=0}^{8}\binom{8}{i}\begin{bmatrix}13 \\ 13-k+i\end{bmatrix}$ & $A_4-A_3=190p$, then p is equal to

Key: 49

Sol: $A_4=2\times 21C_4$

$A_3=2\times 21C_3$

$A_4-A_3=190P$

$2[21C_4-21C_3]=190p\Rightarrow p=49$

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