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JEE MAIN (JAN) 2023 (24-01-2023-Session-1)

MATHEMATICS, PHYSICS & CHEMISTRY



Sri Chaitanya IIT Academy.,India.

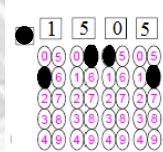
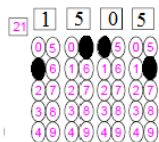
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IMPORTANT INSTRUCTION:

1. Immediately fill in the Admission number on this page of the Test Booklet with **Blue/Black Ball Point Pen** only.
 2. The candidates should not write their Admission Number anywhere (except in the specified space) on the Test Booklet/ Answer Sheet.
 3. The test is of **3 hours** duration.
 4. The Test Booklet consists of 90 questions. The maximum marks are **300**.
 5. There are **three** parts in the question paper 1, 2, 3 consisting of **Physics, Chemistry and Mathematics** having **30 questions** in each subject and subject having **two sections**.
(I) Section –I contains 20 **multiple choice** questions with only one correct option.
Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.
(II) Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only, if more than 5 questions attempted, First 5 attempted questions will be considered.
- The Answer should be within **0 to 9999**. If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i, e. If answer is above **10** and less than **10.5** round off is **10** and If answer is from **10.5** and less than **11** round off is **11**).
- To cancel any attempted question bubble on the question number box.
For example: To cancel attempted question 21. Bubble on 21 as shown below



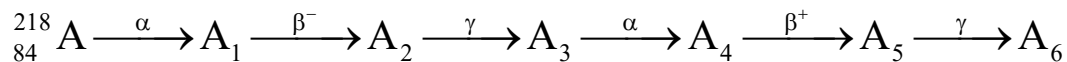
Question Answered for Marking Question Cancelled for Marking

Marking scheme: +4 for correct answer, 0 if **not attempt** and -1 in all other cases.

6. Use **Blue / Black Point Pen only** for writing particulars / marking responses on the Answer Sheet. **Use of pencil is strictly prohibited.**
7. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electron device etc, except the Identity Card inside the examination hall.
8. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
9. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Hall. **However, the candidate are allowed to take away this Test Booklet with them.**
10. **Do not fold of make any stray marks on the Answer Sheet**



04. Consider the following radioactive decay process



The mass number and the atomic number of A_6 are given by:

- 1) 211 and 80 2) 210 and 80 3) 210 and 82 4) 210 and 84

Key: 2

Sol: Effectively change in A and Z is due to α – decays only for the given Reaction.

$$\therefore A^1 = 218 - 2 \times 4 = 210$$

$$Z^1 = 84 - 2 \times 2 = 80$$

05. From the photoelectric effect experiment, following observations are made. Identify which of these are correct.

- A) The stopping potential depends only on the work function of the metal.
 B) The saturation current increases as the intensity of incident light increases.
 C) The maximum kinetic energy of a photo electron depends on the intensity of the incident light.
 D) Photoelectric effect can be explained using wave theory of light.

Choose the correct answer from the options given below:

- 1) A, B, D only 2) B only 3) A, C, D only 4) B,C only

Key: 2

Sol: Only B is correct

06. 1g of a liquid is converted to vapour at 3×10^5 Pa pressure. If 10% of the heat supplied is used for increasing the volume by 1600 cm^3 during this phase change, then the increase in internal energy in the process will be :

- 1) 4800J 2) 4320J 3) 4.32×10^8 J 4) 432000J

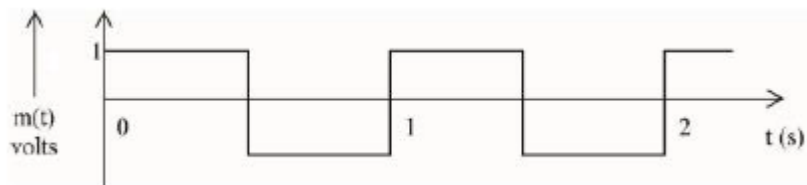
Key: 2

Sol: $\Delta w = P\Delta V = 3 \times 10^5 \times 1600 \times 10^{-6}$
 $= 480 \text{ J}$

$$\therefore \left(\frac{10}{100}\right)(\Delta Q) = 480 \text{ J} \quad \Rightarrow \Delta U = \left(\frac{90}{100}\right)(480) = 4320 \text{ J}$$



07. A modulating signal is a square wave, as shown in the figure.



If the carrier wave is given as $c(t) = 2 \sin(8\pi t)$ volts, the modulation index is

- 1) $\frac{1}{3}$ 2) 1 3) $\frac{1}{2}$ 4) $\frac{1}{4}$

Key: 3

Sol:
$$\mu = \frac{A_m}{A_c} = \frac{1}{2}$$

08. Given below are two statements :

Statement I: The temperature of a gas is -73°C . When the gas is heated to 527°C , the root mean square speed of the molecules is doubled.

Statement-II : The product of pressure and volume of an ideal gas will be equal to translational kinetic energy of the molecules.

In the light of the above statements, choose the correct answer from the options given below:

- 1) Statement I is false but Statement II is true
 2) Both Statement I and Statement II are true
 3) Both Statement I and Statement II are false
 4) Statement I is true but Statement II is false.

Key: 4

Sol: $T_1 = 200\text{K} \quad T_2 = 800\text{K} \therefore \frac{V_2}{V_1} = \sqrt{\frac{T_2}{T_1}} = 2$

$PV = \eta RT$ but $K\varepsilon_{\text{Trans}} = \frac{3}{2}\eta RT.$



09. In \vec{E} and \vec{K} represent electric field and propagation vectors of the EM waves in vacuum, then magnetic field vector is given by : (ω -angular frequency):

- 1) $\frac{1}{\omega}(\vec{K} \times \vec{E})$ 2) $\omega(\vec{K} \times \vec{E})$ 3) $\vec{K} \times \vec{E}$ 4) $\omega(\vec{E} \times \vec{K})$

Key: 1

Sol: $\vec{V} \parallel \vec{E} \times \vec{B}$

$$\text{Also } V = \frac{\omega}{k}$$

10. A circular loop of radius r is carrying current IA . The ratio of magnetic field at the center of circular loop and at a distance r from the center of the loop on its axis is:

- 1) $1:3\sqrt{2}$ 2) $2\sqrt{2}:1$ 3) $3\sqrt{2}:2$ 4) $1:\sqrt{2}$

Key: 2

Sol: At centre, $B_1 = \frac{\mu_0 i}{2r}$

$$\text{On the axis, } B_2 = \frac{\mu_0 i r^2}{2(r^2 + r^2)^{3/2}} = \frac{\mu_0 i r^2}{2 \times 2\sqrt{2} \cdot r^3}$$

$$= \frac{\mu_0 i}{2 \times 2\sqrt{2}r}$$

$$\therefore B_1 : B_2 = 2\sqrt{2} : 1$$

11. A 100 m long wire having cross-sectional area $6.25 \times 10^{-4} \text{ m}^2$ and Young's modulus is 10^{10} Nm^{-2} is subjected to a load of 250N, then the elongation in the wire will be :

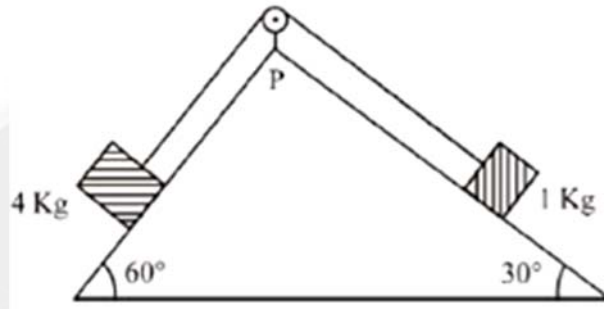
- 1) $6.25 \times 10^{-3} \text{ m}$ 2) $4 \times 10^{-3} \text{ m}$ 3) $4 \times 10^{-4} \text{ m}$ 4) $6.25 \times 10^{-6} \text{ m}$

Key: 2

Sol: 11. $e = \frac{Fl}{YA} = \frac{250 \times 100}{10^{10} \times 6.25 \times 10^{-4}}$
 $= 4 \times 10^{-3} \text{ m}$



12. As per given figure, a weightless pulley P is attached on a double inclined frictionless surface. The tension in the string (massless) will be (if $g = 10\text{m/s}^2$)

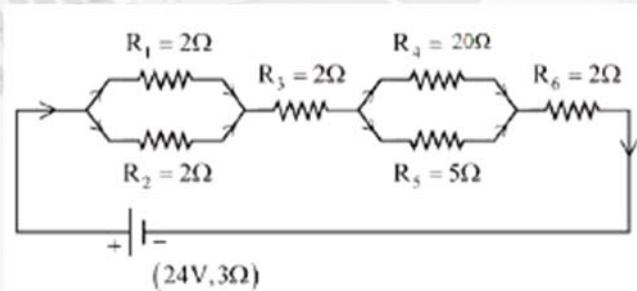


- 1) $(4\sqrt{3} - 1)\text{N}$ 2) $4(\sqrt{3} - 1)\text{N}$ 3) $(4\sqrt{3} + 1)\text{N}$ 4) $4(\sqrt{3} + 1)\text{N}$

Key: 4

Sol: $40 \frac{\sqrt{3}}{2} - T = 4a \quad T - \frac{10}{2} = a \Rightarrow a = 4\sqrt{3} - 1 \quad T = 20\sqrt{3} - 4(4\sqrt{3} - 1)$
 $= 4\sqrt{3} + 4 \quad T = 4(\sqrt{3} + 1)$

13. As shown in the figure a network of resistors is connected to a battery of 24V with an internal resistance of 3Ω . The current through the resistors R_4 and R_5 are I_4 and I_5 respectively. The values of I_4 and I_5 are :



- 1) $I_4 = \frac{8}{5}\text{A}$ and $I_5 = \frac{2}{5}\text{A}$ 2) $I_4 = \frac{6}{5}\text{A}$ and $I_5 = \frac{24}{5}\text{A}$
 3) $I_4 = \frac{2}{5}\text{A}$ and $I_5 = \frac{8}{5}\text{A}$ 4) $I_4 = \frac{24}{5}\text{A}$ and $I_5 = \frac{6}{5}\text{A}$

Key: 3

Sol: $I = \frac{24}{1+2+4+2+3} = \frac{24}{12} \quad I = 2\text{A} \quad I_4 = \left(\frac{5}{25}\right)(2) = \frac{2}{5}\text{A}$
 $I_5 = \left(\frac{20}{25}\right)(2) = \frac{8}{5}\text{A}$



14. Match List I with List II

List I			List II
A.	Planck's constant (h)	I.	$[M^1L^2T^{-2}]$
B.	Stopping potential (Vs)	II.	$[M^1L^1T^{-1}]$
C.	Work function (ϕ)	III.	$[M^1L^2T^{-1}]$
D.	Momentum (p)	IV.	$[M^1L^2T^{-3}A^{-1}]$

Choose the correct answer from the options given below :

- 1) A – II, B – IV, C – III, D – I 2) A – III, B – IV, C – I, D – II
 3) A – I, B – III, C – IV, D – II 4) A – III, B – I, C – II, D – IV

Key: 2

Sol: Planck's constant dimensions \rightarrow Angular Momentum

Stopping potential $\rightarrow \frac{\text{Energy}}{\text{Change}}$

Work function \rightarrow Energy

15. Given below are two statements :

Statement- I: An elevator can go up or down with uniform speed when its weight is balanced with the tension of its cable.

Statement-II: Force exerted by the floor of an elevator on the foot of a person standing on it is more than his/her weight when the elevator goes down with increasing speed.

In the light of the above statements, choose the correct answer from the options given below:

1. Statement I is false but Statement II is true.
2. Statement I is true but Statement II is false.
3. Both Statement I and Statement II are false.
4. Both Statement I and Statement II are true.

Key: 2

Sol: $T = m(g \pm a)$ for uniform speed, $a=0$

$$\therefore T = mg$$

When speeding down,

$$N = m(g - a) \therefore N < mg$$



16. The weight of a body at the surface of earth is 18 N. The weight of the body at an altitude of 3200 km above the earth's surface is (given, radius of earth $R_e = 6400$ km):

1. 19.6 N 2. 9.8 N 3. 4.9 N 4. 8 N

Key: 4

Sol:
$$W_G = mg \left[\frac{R}{R+G} \right]^2$$
$$= 18 \left[\frac{6400}{6400 + 3200} \right]^2$$
$$= 18 \left[\frac{2}{3} \right]^2 = 8N$$

17. A conducting circular loop of radius $\frac{10}{\sqrt{\pi}}$ cm is placed perpendicular to a uniform magnetic field of 0.5 T. The magnetic field is decreased to zero in 0.5 s at a steady rate. The induced emf in the circular loop at 0.25 s is :

- 1) emf = 1 mV 2) emf = 100 mV 3) emf = 5 mV 4) emf = 10 mV

Key: 4

Sol:
$$e = - \frac{\Delta \Phi}{\Delta t} = \frac{(B_i - B_f) A}{\Delta t}$$
$$= \frac{(0.5 - 0)}{0.5} \pi \left[\frac{10 \times 10^{-2}}{\sqrt{\pi}} \right]^2$$
$$= 10^{-2} V = 10 \text{ mV}$$



18. Given below are two statements :

Statement I : If the Brewster's angle for the light propagating from air to glass is θ_B . Then the Brewster's angle for the light propagating from glass to air is $\frac{\pi}{2} - \theta_B$

Statement II : The Brewster's angle for the light propagating from glass to air is $\tan^{-1}(\mu_g)$ where μ_g is the refractive index of glass.

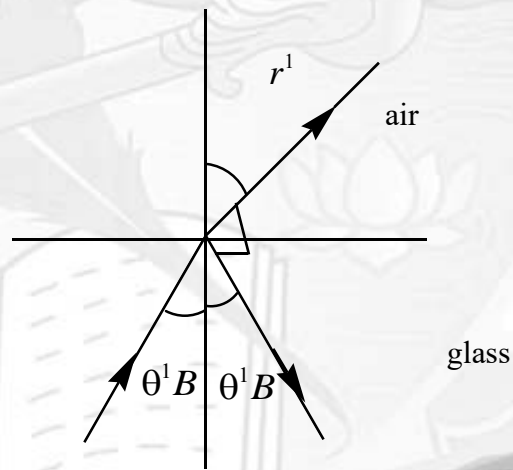
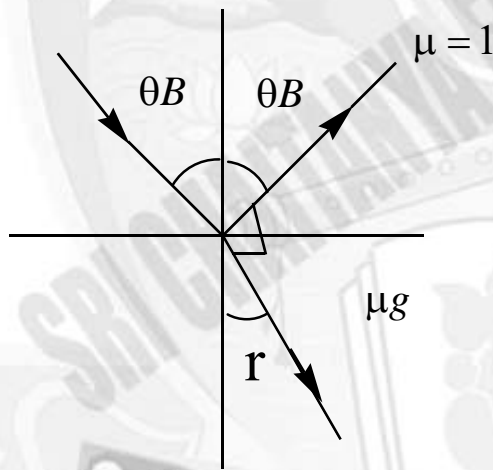
In the light of the above statements, choose the correct answer from the options given below :

- 1) Both Statement I and Statement II are true.
- 2) Both Statement I and Statement II are false
- 3) Statement I is true but Statement II is false.
- 4) Statement I is false but Statement II is true.

Key: 3

Sol: Air \rightarrow glass

glass \rightarrow air



$$(1) \sin \theta_B = \mu_g \sin r$$

$$= \mu_g \sin(90 - \theta_B)$$

$$\mu_g \sin \theta_B^1 = (1) \sin r^1$$

$$= \sin(90 - \theta_B^1)$$

$$\tan \theta_B = \mu_g$$

$$\cot \theta_B^1 = \mu_g = \tan \theta_B \therefore \theta_B^1 = \frac{\pi}{2} - \theta_B$$



19. The maximum vertical height to which a man can throw a ball is 136 m. The maximum horizontal distance upto which he can throw the same ball is :

- 1) 68 m 2) 136m 3) 192 m 4) 272 m

Key: 4

$$\text{Sol: } H_{\max} = \frac{u^2}{2g} \Rightarrow \frac{u^2}{g} = 2(H_{\max}) = 2(136) \quad R_{\max} = 272m$$

20. If two charges q_1 and q_2 are separated with distance 'd' and placed in a medium of dielectric constant K. What will be the equivalent distance between charges in air for the same electrostatic force ?

- 1) $k\sqrt{d}$ 2) $1.5d\sqrt{k}$ 3) $2d\sqrt{k}$ 4) $d\sqrt{k}$

Key: 4

$$\text{Sol: } F_{\text{air}} = F_{\text{dielectric}}$$

$$\frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{d_a^2} = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{kd^2} \Rightarrow d_a = \sqrt{k}d$$

(NUMERICAL VALUE TYPE)

Section-II contains 10 Numerical Value Type questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within 0 to 9999. If the Answer is in Decimal then round off to the nearest Integer value (Example i, e. If answer is above 10 and less than 10.5 round off is 10 and if answer is from 10.5 and less than 11 round off is 11).

Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.

21. A hole is drilled in a metal sheet. At 27°C the diameter of hole is 5 cm. When the sheet is heated to 177°C , the change in the diameter of hole is $d \times 10^{-3}$ cm. The value of d will be _____ if coefficient of linear expansion of the metal is $1.6 \times 10^{-5} / ^\circ\text{C}$

Key: 12

$$\text{Sol: } \Delta D = D \alpha \Delta t$$

$$d \times 10^{-3} = (5)(1.6 \times 10^{-5})(177 - 27)$$

$$= 8 \times 10^{-5} \times 150$$

$$d = 12 \times 10^{-3}$$



22. A spherical body of mass 2 kg starting from rest acquires a kinetic energy of 10000 J at the end of 5th second. The force acted on the body is _____ N.

Key: 40

Sol: $10^4 = \frac{1}{2}mv^2$

$$= \frac{1}{2}(2)v^2 \Rightarrow v = 100 \text{ m/s}$$

$$F = ma = m \left[\frac{V - u}{t} \right] = 2 \left[\frac{100 - 0}{5} \right]$$

$$= 40 \text{ N}$$

23. A hollow cylindrical conductor has length of 3.14 m. while its inner and outer diameters are 4 mm and 8mm respectively. The resistance of the conductor is $n \times 10^{-3} \Omega$. If the resistivity of the material is $2.4 \times 10^{-8} \Omega \text{m}$. The value of n is _____

Key: 2

Sol: $R = \frac{gl}{A} = \frac{gl}{\frac{\pi}{4}[D_2^2 - D_1^2]} = \frac{2.4 \times 10^{-8} \times 3.14}{\frac{3.14}{4}[64 - 16] \times 10^{-6}}$

$$= \frac{9.6}{48} \times 10^{-2} = 2 \times 10^{-3} \Omega$$

24. Assume that protons and neutrons have equal masses. Mass of a nucleon is $1.6 \times 10^{-27} \text{ kg}$ and radius of nucleus is $1.5 \times 10^{-15} \text{ A}^{1/3} \text{ m}$. The approximate ratio of the nuclear density and water density is $n \times 10^{13}$. The value of n is _____.

Key: 11

Sol: $S_{nucleus} = \frac{(m)A}{\frac{4}{3}\pi R^3} = \frac{1.6 \times 10^{-27} \times A}{\frac{4}{3} \times \pi \times 1.5^3 \times 10^{-45} \times A} = \frac{1.2 \times 10^{-27}}{\pi \times \frac{27}{8} \times 10^{-45}} = \frac{9.6}{\pi \times 27} \times 10^{18}$

$$\text{Ratio } R = \frac{S_N}{\ell_w} = \frac{960}{3.142 \times 27} \times \frac{10^{16}}{10^3} = 11.32 \times 10^{13}$$

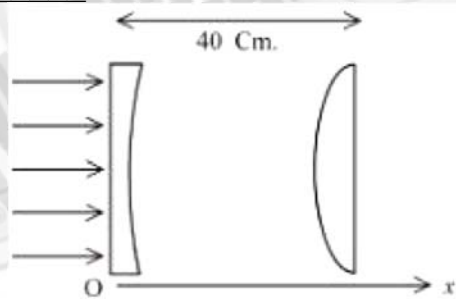


25. Vectors $a\hat{i} + b\hat{j} + \hat{k}$ and $2\hat{i} - 3\hat{j} + 4\hat{k}$ are perpendicular to each other when $3a + 2b = 7$, the ratio of a to b is $\frac{x}{2}$. The value of x is _____

Key: 1

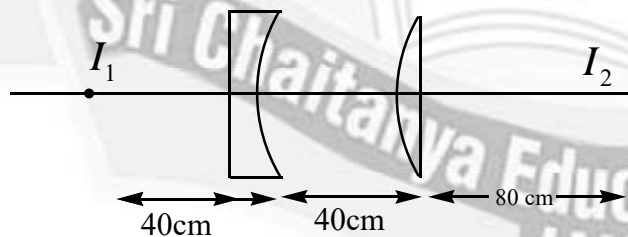
Sol: $(a\hat{i} + b\hat{j} + \hat{k}) \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 0, 2a - 3b + 4 = 0 \dots\dots(1)$
 $3a + 2b - 7 \geq 0 \dots\dots(2)$ (given), $(1) \times 3(-2) \times 2 \Rightarrow -9b - 4b + 12 + 14 = 0$
 $\Rightarrow 13b = 26 \Rightarrow b = 2$, From (1), $2a - 3(2) + 4 = 0 \Rightarrow a = 1$
 $\therefore \frac{a}{b} = \frac{1}{2} \Rightarrow x = 1$

26. As shown in the figure, a combination of a thin plano concave lens and a thin plano convex lens is used to image an object placed at infinity. The radius of curvature of both the lenses is 30 cm and refraction index of the material for both the lenses is 1.75. Both the lenses are placed at distance of 40 cm from each other. Due to the combination, the image of the object is formed at distance $x =$ _____ cm. from concave lens.



Key: 120

Sol: $f = \frac{R}{\mu - 1} = \frac{30}{1.75 - 1} = \frac{30}{(3/4)} = 40 \text{ cm}$



$I_1 = V.I.$ formed by concave lens = R.O to the convex lens

Convex lens: $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \frac{1}{v} - \frac{1}{-80} = \frac{1}{40} \Rightarrow \frac{1}{v} = \frac{1}{40} - \frac{1}{80} = \frac{2-1}{80} = \frac{1}{80}$

$\therefore v = +80 \text{ cm}, I_2 = R.I$ of I_1 formed by convex lens, $d = 40 + 80 = 120 \text{ cm}$



27. A stream of a positively charged particles having $\frac{q}{m} = 2 \times 10^{11} \frac{C}{kg}$ and velocity

$\vec{v}_0 = 3 \times 10^7 \hat{i} \text{ m/s}$ is deflected by an electric field $1.8 \hat{j} \text{ kV/m}$. The electric field exists in a region of 10cm along x direction. Due to the electric field, the deflection of the charge particles in the y direction is _____ mm.

Key: 2

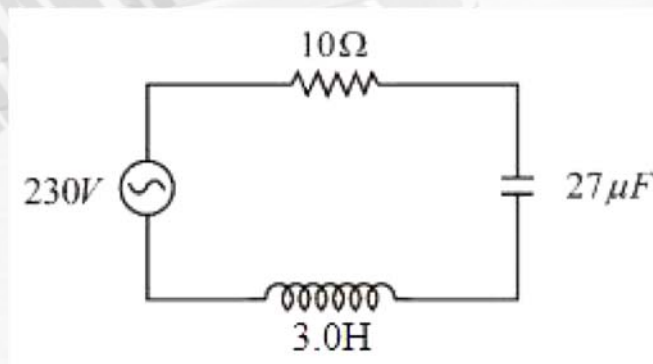
Sol:

$$y = \frac{1}{2} \left(\frac{eq}{m} \right) \left(\frac{x}{v_0} \right)^2$$

$$= \frac{1}{2} \times (1.8 \times 10^3 \times 2 \times 10^{11}) \left[\frac{10 \times 10^{-2}}{3 \times 10^7} \right]^2$$

$$= \frac{1.8}{9} \times \frac{10^{14}}{10^{16}} = 2 \text{ mm}$$

28. In the circuit shown in the figure, the ratio of the quality factor and the band width is _____ s.



Key: 10

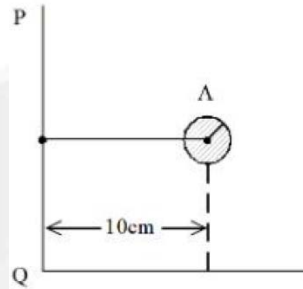
Sol:

$$\frac{Q}{\Delta\omega} = \frac{\frac{1}{R} \sqrt{\frac{L}{C}}}{(R/L)} = \frac{L}{R^2} \sqrt{\frac{L}{C}}$$

$$= \frac{3}{10^2} \sqrt{\frac{3}{27 \times 10^{-6}}} = 10$$



29. Solid sphere A is rotating about an axis PQ. If the radius of the sphere is 5 cm then its radius of gyration about PQ will be \sqrt{x} cm. The value of x is _____.

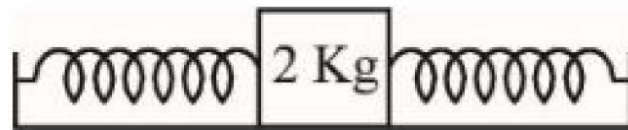


Key: 110

Sol:
$$K^2 = \frac{2}{5}R^2 + x^2 = \frac{2}{5}(25) + 10^2 = 110$$

$$k = \sqrt{110}$$

30.



A block of mass 2 kg is attached with two identical springs of spring constant 20 N/m each. The block is placed on a frictionless surface and the ends of the springs are attached to rigid supports (see figure). When the mass is displaced from its equilibrium position. It executes a simple harmonic motion. The time period of oscillations is $\frac{\pi}{\sqrt{x}}$ in SI units. The value of x is

Key: 5

Sol: $k_{eff} = 2k$

$$T = 2\pi \sqrt{\frac{m}{K_e}} = 2\pi \sqrt{\frac{2}{2 \times 20}} = \frac{\pi}{\sqrt{5}} \text{ s}$$

**CHEMISTRY****(SINGLE CORRECT ANSWER TYPE)**

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 in all other cases.

31. In the depression of freezing point experiment

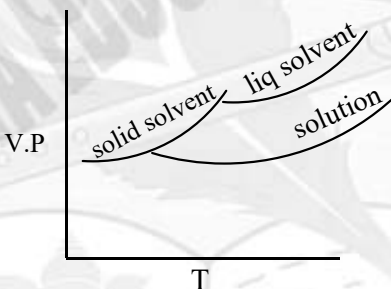
- A) Vapour pressure of the solution is less than that of pure solvent
- B) Vapour pressure of the solution is more than that of pure solvent
- C) Only solute molecules solidify at the freezing point
- D) Only solvent molecules solidify at the freezing point

Choose the most appropriate answer from the options given below:

- 1) A only 2) B and C only 3) A and C only 4) A and D only

Key. 4

Sol. $P_{\text{soln}} < P^0$ and during freezing only solvent particles freeze not solute



A and D are correct

32. Order of Covalent bond :

- A) $KF > KI$; $LiF > KF$
- B) $KF < KI$; $LiF > KF$
- C) $SnCl_4 > SnCl_2$; $CuCl > NaCl$
- D) $LiF > KF$; $CuCl < NaCl$
- E) $KF < KI$; $CuCl > NaCl$

Choose the correct answer from the options given below :

- 1) A, B only 2) B, C only 3) B, C, E only 4) C, E only

**Key. 3**

Sol. Covalent character \propto size of anion, Covalent character \propto $1/\text{size of cation}$

Covalent character \propto charge on ions

A) Covalent character of KI > KF, B) Covalent character KI > KF, LiF > KF

C) Cations with pseudo octet are more covalent than cations having octet

\therefore CuCl > NaCl (covalent character)

D) CuCl > NaCl, E) KI > KF, CuCl > NaCl, \therefore B, C & E are correct

33. Match List-I with List-II

List-I

List-II

A) Reverberatory furnace

I) Pig Iron

B) Electrolytic cell

II) Aluminum

C) Blast furnace

III) Silicon

D) Zone Refining furnace

IV) Copper

Choose the correct answer from the options given below :

1) A-I, B-III, C-II, D-IV

2) A-III, B-IV, C-I, D-II

3) A-I, B-IV, C-II, D-III

4) A-IV, B-II, C-I, D-III

Key. 4

Sol. Reverberatory furnace \rightarrow Roasting of Cu

Electrolytic cell \rightarrow Al_2O_3 electrolytic reduction

Blast furnace \rightarrow Smelting of Iron

Zone refining \rightarrow metals used in semiconductors.

34. Which of the following is true about freons?

1) These are radicals of chlorine and chlorine monoxide

2) These are chlorofluorocarbon compounds

3) All radicals are called freons

4) These are chemicals causing skin cancer

**Key. 2****Sol.** Freons are chlorofluorocarbonsEg :- CCl_2F_2 , CFCl_3 **35.** Match List-I with List-II

List-I

A) Chlorophyll

B) Soda ash

C) Dentistry, Ornamental work

D) Used in white washing

List-II

I) Na_2CO_3 II) CaSO_4 III) Mg^{2+} IV) $\text{Ca}(\text{OH})_2$

Choose the correct answer from the options given below :

1) A-III, B-I, C-II, D-IV

2) A-II, B-III, C-IV, D-I

3) A-III, B-IV, C-I, D-II

4) A-II, B-I, C-III, D-IV

Key. 1**Sol.** Chlorophyll \rightarrow Contain Mg^{+2} ionsSoda ash \rightarrow Anhydrous Na_2CO_3 Dentistry \rightarrow CaSO_4 White wash \rightarrow $\text{Ca}(\text{OH})_2$ absorb CO_2 to form hard coating of CaCO_3 **36.** The magnetic moment of a transition metal compound has been calculated to be 3.87 B.M.

The metal ion is

1) Ti^{2+} 2) Mn^{2+} 3) V^{2+} 4) Cr^{2+} **Key. 3****Sol.** $M.M = \sqrt{n(n+2)}$ ($n =$ no. of unpaired e^- s) $\text{Ti}^{+2} \rightarrow [\text{Ar}]3d^2$ $n = 2$ $M.M = 2.44$ $\text{Mn}^{+2} \rightarrow [\text{Ar}]3d^5$ $n = 5$ $M.M = 5.91$ $\text{V}^{+2} \rightarrow [\text{Ar}]3d^3$ $n = 3$ $M.M = 3.87$ $\text{Cu}^{+2} \rightarrow [\text{Ar}]3d^9$ $n = 1$ $M.M = 1.73$



37. Decreasing order of the hydrogen bonding in following forms of water is correctly represented by

- A) Liquid water B) Ice C) Impure water

Choose the correct answer from the options given below

- 1) C > B > A 2) B > A > C 3) A = B > C 4) A > B > C

Key. 2

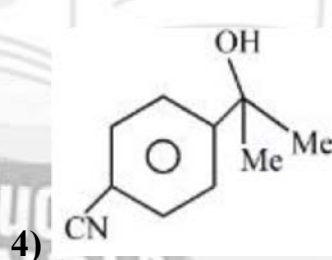
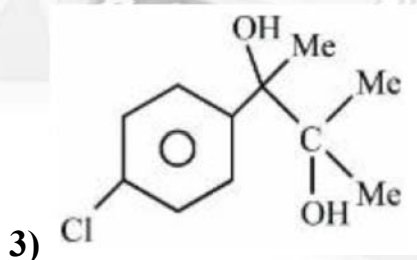
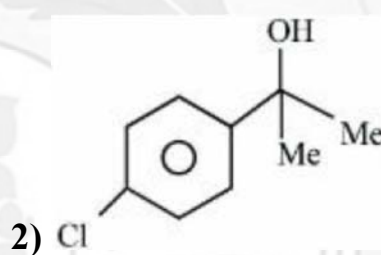
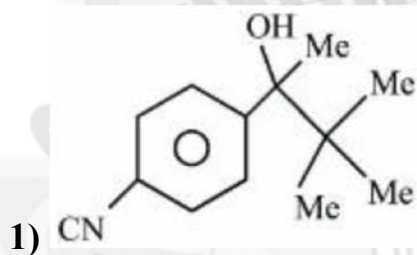
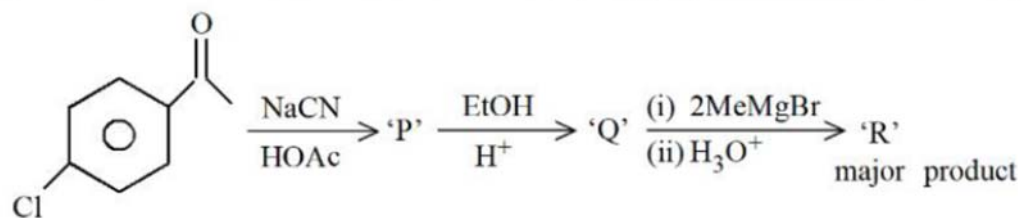
Sol. Strength of hydrogen bonding of depends on various factors like (for same compound)

1) Physical state (solid > liquid > gas)

2) Purity

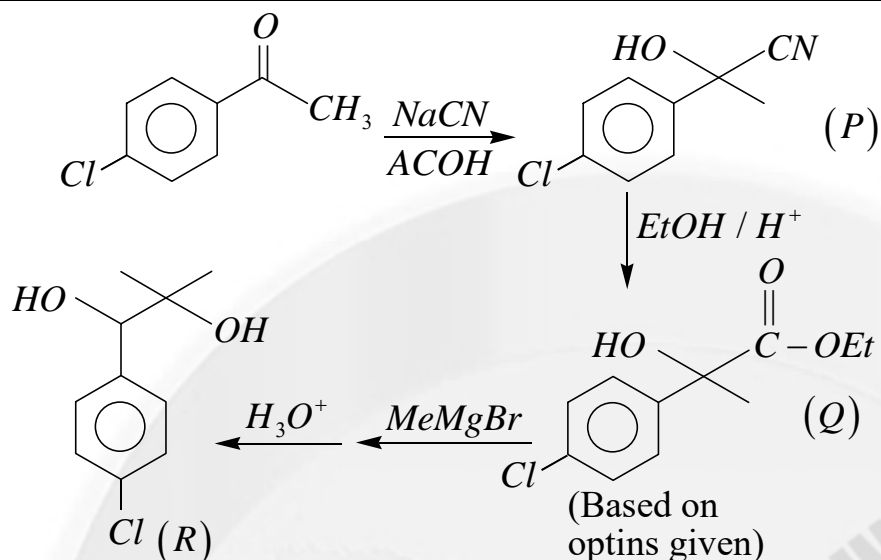
∴ Ice > Water > Impure water

38. 'R' formed in the following sequence of reactions is



Key. 3

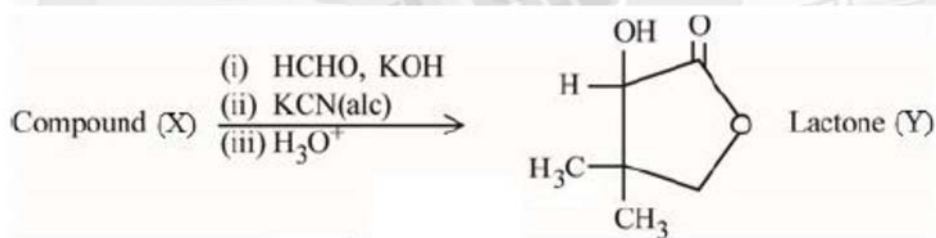
Sol.



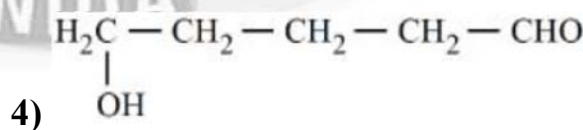
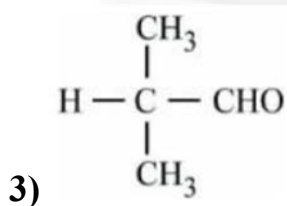
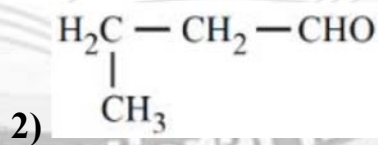
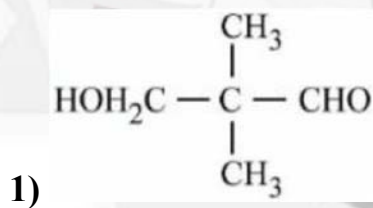
Note: Above solution is based on options given the question. In general cyanides do not convert into esters directly with EtOH

∴ It is wrong question

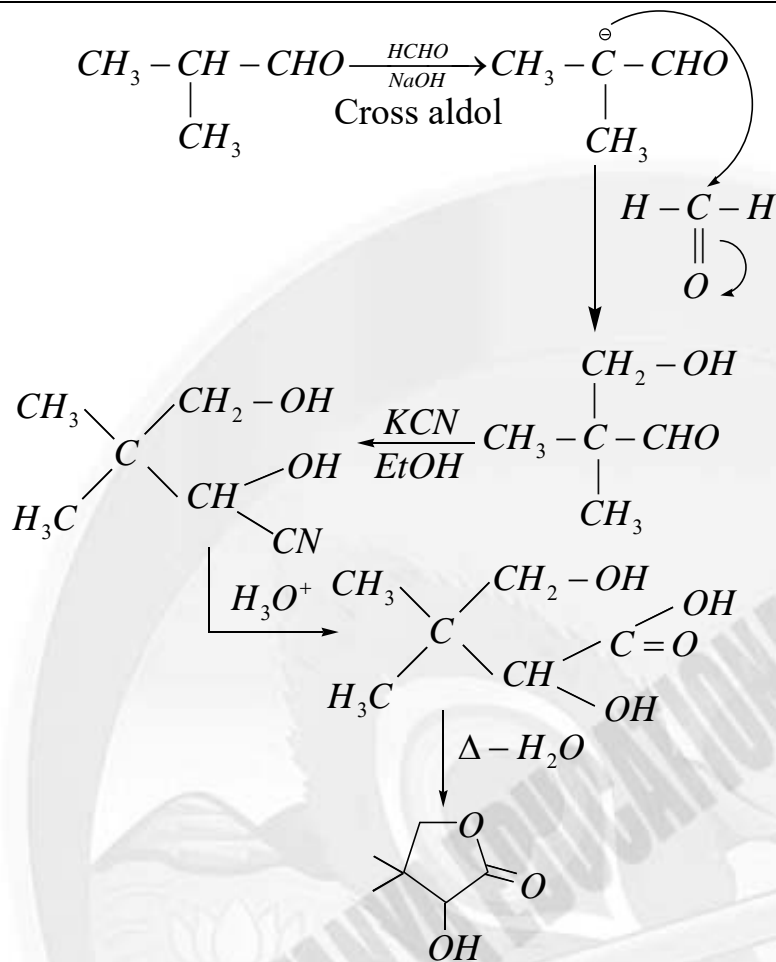
39. Compound (X) undergoes following sequence of reactions of given the Lactone (Y).



Compound X is



Key. 3

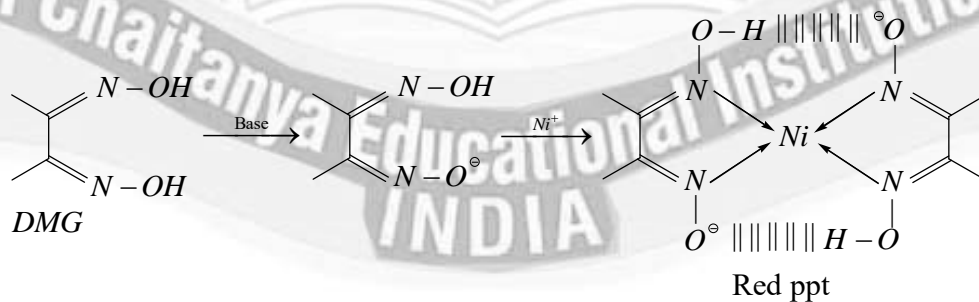


Sol.

40. An ammoniacal metal salt solution gives a brilliant red precipitate on addition of dimethylglyoxime. The metal ion is :

- 1) Co^{2+} 2) Fe^{2+} 3) Ni^{2+} 4) Cu^{2+}

Key. 3

Sol. $\text{Ni}^{2+} \xrightarrow[\text{base}]{\text{DMG}} \text{Red ppt}$ 

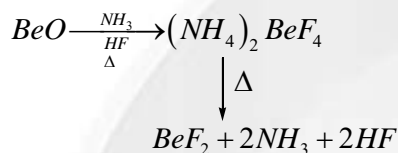


41. Reaction of BeO with ammonia and hydrogen fluoride gives A which on thermal decomposition gives BeF₂ and NH₄F. What is 'A'?

- 1) (NH₄)BeF₃ 2) H₃NBeF₃ 3) (NH₄)Be₂F₅ 4) (NH₄)₂BeF₄

Key. 4

Sol.



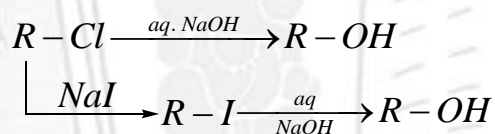
42. Assertion (A) : Hydrolysis of an alkyl chloride is a slow reaction but in the presence of NaI, the rate of the hydrolysis increases.

Reason (R) : I⁻ is a good nucleophile as well as a good leaving group

In the light of the above statements, choose the correct answer from the options given below

- 1) A is false but R is true
 2) A is true but R is false
 3) Both A and R true but R is NOT the correct explanation of A
 4) Both A and R are true and R is the correct explanation of A

Key. 4



Sol.

In presence of NaI, Cl gets replaced by I, as I is a better L.G. than Cl reaction becomes faster

Nucleophilicity \propto polarizing power

Nucleophilicity \propto size of anion, I⁻ is a good Nu⁻ due to high polarizability

43. Given below are two statements

Statement-I : Noradrenaline is a neurotransmitter

Statement-II : Low level of noradrenaline is not the cause of depression in human

In the light of the above statements, choose the correct answer from the options given below



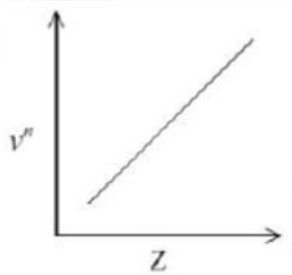
- 1) Statement-I is incorrect but Statement-II is correct
- 2) Both Statement-I and Statement-II are correct
- 3) Statement-I is correct but Statement-II is incorrect
- 4) Both Statement-I and Statement-II are incorrect

Key. 3

Sol. Noradrenaline is a neuro transmitter

Its production of Noradrenaline is low then person will suffer with depression

44. It is observed that characteristic X-ray spectra of elements show regularity. When frequency to the power “n” i.e, ν^n of X-rays emitted is plotted against atomic number “Z”, following graph is obtained.



The value of “n” is

1) 2

2) 3

3) 1

4) $\frac{1}{2}$

Key. 4

Sol. $\sqrt{\nu} = a(z-b)$

$$\nu^n \propto Z \Rightarrow n = \frac{1}{2}$$



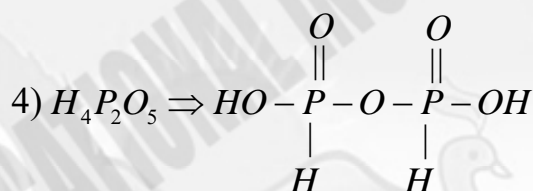
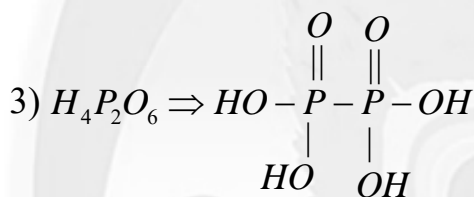
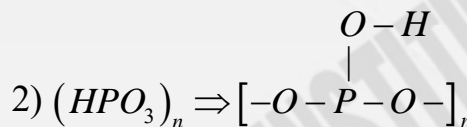
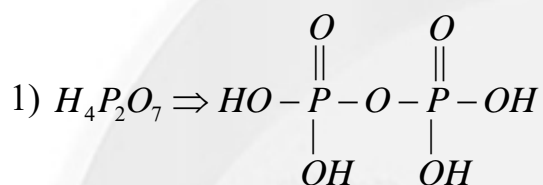
45. Which of the Phosphorus oxoacid can create silver mirror from AgNO_3 solution?

- 1) $\text{H}_4\text{P}_2\text{O}_7$ 2) $(\text{HPO}_3)_n$ 3) $\text{H}_4\text{P}_2\text{O}_6$ 4) $\text{H}_4\text{P}_2\text{O}_5$

Key. 4

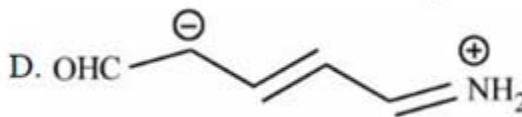
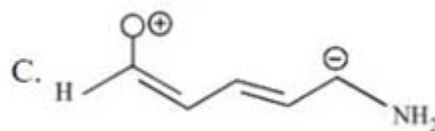
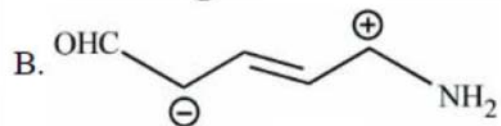
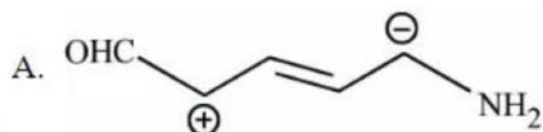
Sol. $\text{AgNO}_3 \longrightarrow \text{Ag}$

Above reaction is possible if oxoacid contain P-H bond.



Only $\text{H}_4\text{P}_2\text{O}_5$ contains P-H bonds and acts as a reducing agent

46. Increasing order of stability of the resonance structures is :



Choose the correct answer from the options given below :

- 1) D, C, A, B 2) C, D, B, A 3) D, C, B, A 4) C, D, A, B

Key. 4



Sol. Stability of resonating structures

$$\alpha \frac{1}{\text{No of charges}}$$

α No. of octet atoms (or) No. of covalent bonds

α -ve / +ve charge stability

α distance b/w charges

D is most stable as all atoms are octet.

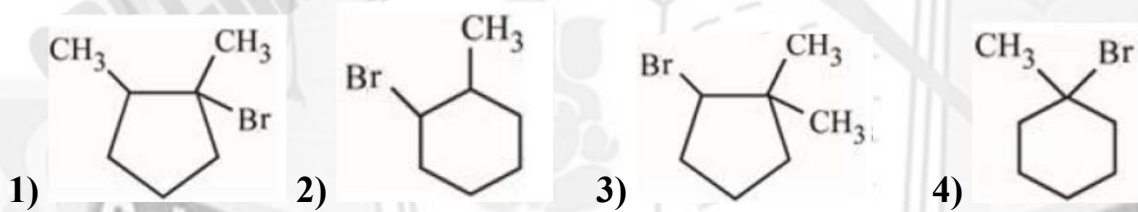
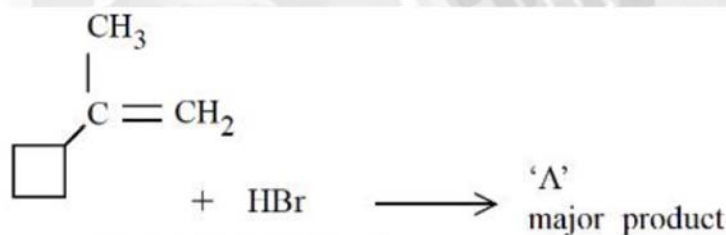
C is least stable as oxygen has only $6e^-$ (Non-octet oxygen canonicals do not form)

Between A and B, A is less stable due to repulsions b/w L.P. of 'N' and C^\oplus

\therefore Correct stability order is $D > B > A > C$

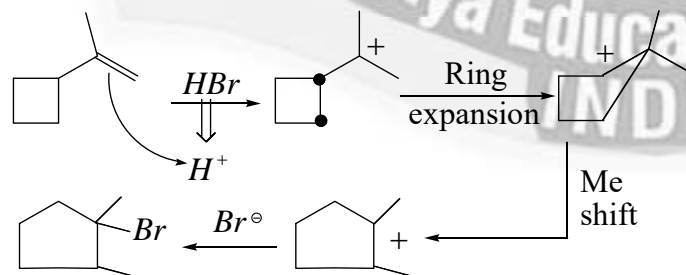
\therefore No answer

47. In the following given reaction, 'A' is



Key. 1

Sol.





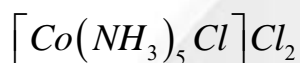
48. The primary and secondary valencies of cobalt respectively in $[\text{Co}(\text{NH}_3)_5\text{Cl}]\text{Cl}_2$ are :

- 1) 3 and 5 2) 2 and 6 3) 2 and 8 4) 3 and 6

Key. 4

Sol. Primary valency = +ve charge on metal

Secondary valency = coordination number



49. Statement-I : For colloidal particles, the values of colligative properties are of small order as compared to values shown by true solutions at same concentration.

Statement-II : For colloidal particles, the potential difference between the fixed layer and the diffused layer of same charges is called the electrokinetic potential or zeta potential.

In the light of the above statements, choose the correct answer from the options given below

- 1) Statement-I is false but Statement-II is true
2) Both Statement-I and Statement-II are true
3) Statement-I is true but Statement-II is false
4) Both Statement-I and Statement-II are false

Key. 3

Sol. Statement-I : Colloidal particles have bigger size and hence smaller values of colligative properties. \therefore true

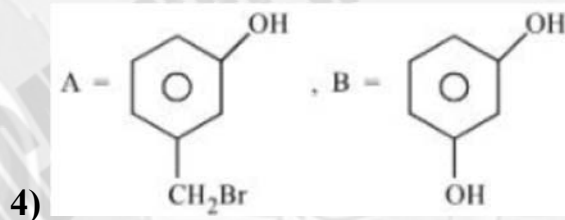
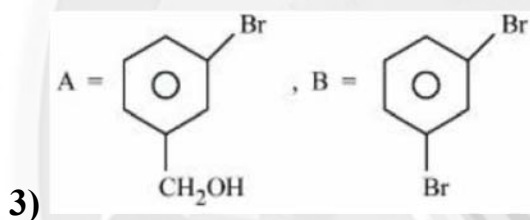
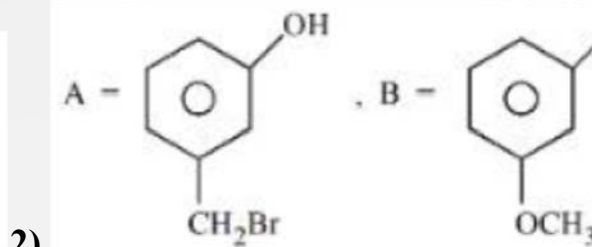
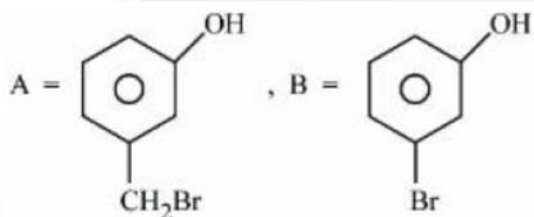
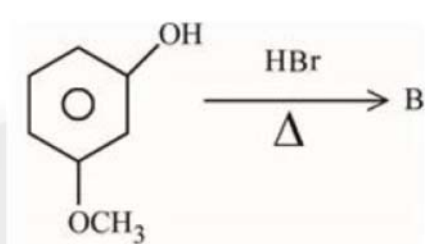
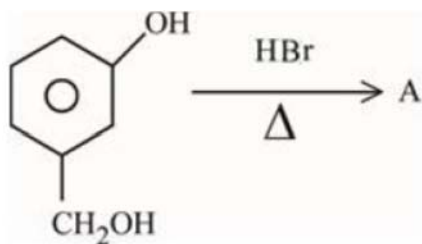
Statement-II : Zeta potential is the potential difference between diffused layer and fixed layer of opposite charges.

\therefore false

Statement-I : true Statement-II : false

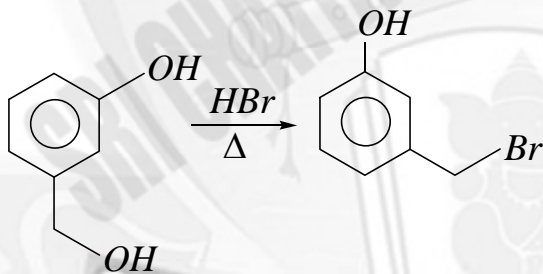


50. 'A' and 'B' formed in the following set of reaction are :

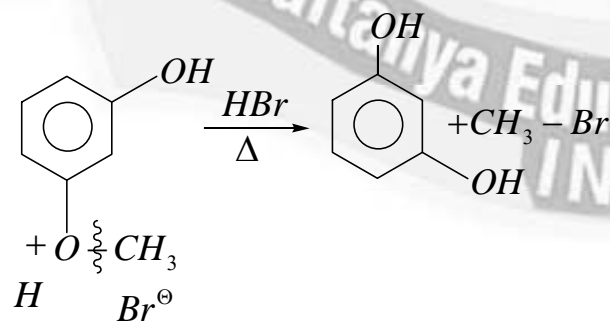


Key. 4

Sol.



Phenols fail to react with HBr

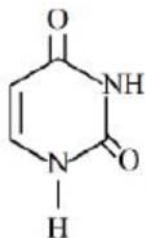


**(NUMERICAL VALUE TYPE)**

Section-II contains 10 Numerical Value Type questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within 0 to 9999. If the Answer is in Decimal then round off to the nearest Integer value (Example i, e. If answer is above 10 and less than 10.5 round off is 10 and if answer is from 10.5 and less than 11 round off is 11).

Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.

51. Uracil is a base present in RNA with the following structure % of N in uracil is _____



Given :

Molar mass N = 14g mol⁻¹
 O = 16g mol⁻¹
 C = 12g mol⁻¹
 H = 1g mol⁻¹

Key. 25

Sol. $\%N = \frac{14 \times \text{no. of N-atoms}}{\text{M.wt}} \times 100$
 $= \frac{14 \times 2}{112} \times 100 = 25\%$

52. If wavelength of the first line of the Paschen series of hydrogen atom is 720 nm, then the wavelength of the second line of this series is _____ nm. (Nearest integer)

Key. 492

Sol. $\frac{1}{\lambda} = R_H \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$

1st line of paschen series, $\frac{1}{\lambda_1} = R_H \left[\frac{1}{9} - \frac{1}{16} \right]$

2nd line of paschen series, $\frac{1}{\lambda_2} = R_H \left[\frac{1}{9} - \frac{1}{25} \right]$

$$\frac{\lambda_2}{\lambda_1} = \frac{\left[\frac{1}{9} - \frac{1}{16} \right]}{\left[\frac{1}{9} - \frac{1}{25} \right]} \Rightarrow \lambda_2 = 720 \times \frac{\left[\frac{7}{9 \times 16} \right]}{\left[\frac{16}{9 \times 25} \right]}$$

= 492.19 nm

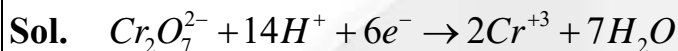


53. At 298 K, a 1 litre solution containing 10m mol of $\text{Cr}_2\text{O}_7^{2-}$ and 100m mol Cr^{3+} shows a pH of 3.0.

Given: $\text{Cr}_2\text{O}_7^{2+} \rightarrow \text{Cr}^{3+}$; $E^0 = 1.330\text{V}$ and $\frac{2.303 RT}{F} = 0.059 \text{ V}$

The potential for the half cell reaction is $x \times 10^{-3}\text{V}$. The value of x is _____

Key. 917



$$E_{H.cell} = E^0 - \frac{0.059}{6} \log \frac{[\text{Cr}^{3+}]^2}{[\text{Cr}_2\text{O}_7^{2-}][\text{H}^+]^{14}}$$

$$= 1.33 - \frac{0.059}{6} \log \frac{(0.1)^2}{(0.01)(10^{-3})^{14}} = 0.917 \text{ V} = 917 \times 10^{-3} \text{ V}$$

54. The dissociation constant fo acetic acid is $x \times 10^{-5}$. When 25 mL of 0.2 M CH_3COONa solution is mixed with 25 mL of 0.02 M CH_3COOH soluiton the pH of the resultant solution is found to be equal to 5. The value of x is _____

Key. 10



Buffer soln :

$$\text{pH} = \text{pKa} + \log \frac{[\text{salt}]}{[\text{acid}]}$$

$$5 = \text{pKa} + \log \frac{0.2}{0.02} \Rightarrow \text{pKa} = 5 - 1 = 4$$

$$\Rightarrow \text{Ka} = 10^{-4} = 10 \times 10^{-5}$$

\therefore Ans = 10

55. For independent processes at 300 K

Process	$\Delta H / \text{kJ mol}^{-1}$	$\Delta S / \text{J K}^{-1}$
A	-25	-80
B	-22	40
C	25	-50
D	22	20

The number of non-spontaneous processes from the following is _____

Key. 2



Sol. $\Delta G = \Delta H - T\Delta S$

$$\text{Process A} = \Delta G = -25 - \frac{300(-80)}{1000}$$

$$= -25 + 24$$

$$= -1 < 0$$

$$\text{Process B} : \Delta G = -22 - \frac{300(+40)}{1000}$$

$$< 0$$

$$\text{Process C} : \Delta G = 25 - \frac{300(-50)}{1000}$$

$$> 0$$

$$\text{Process D} : \Delta G = 25 - \frac{300(20)}{1000}$$

$$= 22 - 6$$

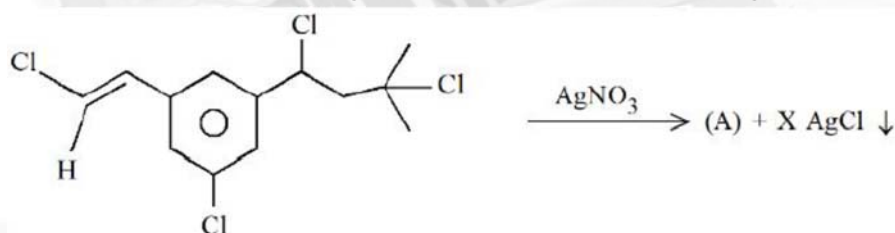
$$= 16$$

$$> 0$$

\therefore Process C & D are non spontaneous

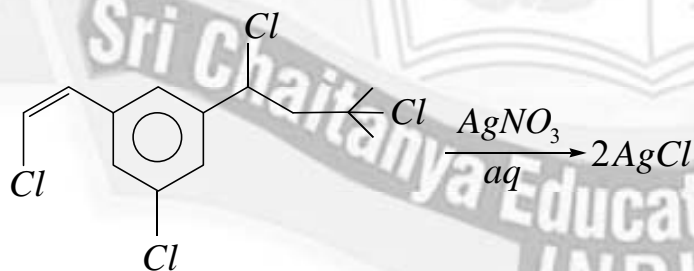
Ans = 2

56. Number of moles of AgCl formed in the following reaction is _____



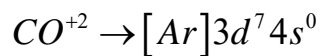
Key. 2

Sol.

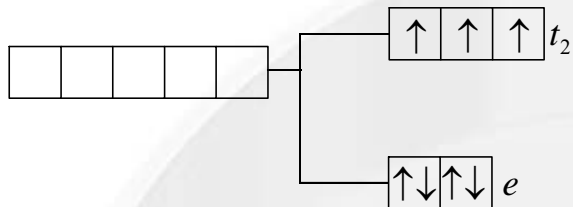


aq. AgNO_3 can't ionize vinyl and Arylhalides

57. The d-electronic configuration of $[\text{CoCl}_4]^{2-}$ in tetrahedral crystal field is $e^m t_2^n$. Sum of "m" and "number of unpaired electrons" is _____

**Key. 7****Sol.** $[CoCl_4]^{-2}$ 

In tetrahedral complex



$$\therefore m = 4 \text{ \& U.P. } e^- = 3$$

$$\therefore \text{Ans} = 7$$

58. 5g of NaOH was dissolved in deionized water to prepare a 450 mL stock solution. What volume (in mL) of this solution would be required to prepare 500 mL of 0.1 M solution? _____Given : Molar Mass of Na, O and H is 23, 16 and 1g mol⁻¹ respectively.**Key. 180**

Sol. Molarity of stock soln = $\frac{n}{V} = \frac{(5/40)}{0.45}$

On dilution : $M_1V_1 = M_2V_2$

$$\frac{5/40}{0.45} \times V = 0.1 \times 500 \quad \Rightarrow V = 180 \text{ ml}$$

59. The number of correct statements from the following is _____

A) Large the activation energy smaller is the value of the rate constant.

B) The higher is the activation energy higher is the value of the temperature coefficient.

C) At lower temperatures increase in temperature causes more change in the value of k than at higher temperature.

D) A plot of $\ln k$ vs $\frac{1}{T}$ is a straight line with slope equal to $-\frac{E_a}{R}$ **Key. 3**

Sol. $K = Ae^{-E_a/RT}$

A) as E_a inc, K dec

B) temp coeff = $\frac{K_{t+10}}{K_t}$

$$\ln \frac{K_2}{K_1} = \frac{E_a}{R} \left(\frac{10}{T_1 T_2} \right)$$



For same T_1, T_2 , as E_a inc, temp coeff inc.

C) At lower temperatures, inc in temp causes less change in value of K than at higher temperature

$$D) \ln K = \ln A - \frac{E_a}{RT}$$

$$\therefore \ln K \text{ vs } \frac{1}{T} \text{ is a straight line with slope } = \frac{-E_a}{R}$$

\therefore true statements are : ABD

Ans = 3

60. When $Fe_{0.93}O$ is heated in presence of oxygen, it converts to Fe_2O_3 . The number of correct statement/s from the following is ___

A) The equivalent weight of $Fe_{0.93}O$ is $\frac{\text{Molecular weight}}{0.79}$

B) The number of moles of Fe^{2+} and Fe^{3+} in 1 mole of $Fe_{0.93}O$ is 0.79 and 0.14 respectively

C) $Fe_{0.93}O$ is metal deficient with lattice comprising of cubic closed packed arrangement of O^{2-} ions.

D) The % composition of Fe^{2+} and Fe^{3+} in $Fe_{0.93}O$ is 85% and 15% respectively

Key. 4

Sol. $Fe_{0.93}O \longrightarrow Fe_2O_3$

For every 100 O atoms, 93 Fe atoms are present. If no. of $Fe^{+2} = x$, $Fe^{+3} = 93 - x$

Charge balanced $\Rightarrow 2x + 3(93 - x) = 200$

$$\Rightarrow x = 79 \text{ } Fe^{+2} \quad \Rightarrow 14 \text{ } Fe^{+3}$$

A) Total change of oxidation state = $0.79 \times 1 = 0.79$ for 1 mole

$$\therefore \text{Eq.wt} = \frac{\text{M.wt}}{0.79}$$

B) In 1 mole of $Fe_{0.93}O$, 0.79 moles of Fe^{+2} , 0.14 moles of Fe^{+3} .

C) $2Fe^{+3}$ ions replace $3Fe^{+2}$ ions. It is a metal deficiency defect

$$D) \%Fe^{+2} = \frac{79}{93} \times 100 \approx 85\%$$

$$Fe^{+3} = 15\%$$

All statements are true

**MATHEMATICS****(SINGLE CORRECT ANSWER TYPE)**

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 in all other cases.

61. Let $y = y(x)$ be the solution of the differential equation $x^3 dy + (xy - 1) dx = 0, x > 0, y\left(\frac{1}{2}\right) = 3 - e$.

Then $y(1)$ is equal to

- 1) $2 - e$ 2) e 3) 1 4) 3

Key: 3

Sol: Given $x^3 dy + (xy - 1) dx = 0$

$$\frac{dy}{dx} + \frac{y}{x^2} = \frac{1}{x^3}$$

$$\text{I.F} = e^{\frac{1}{x}}$$

$$\text{Solution is } y\left(e^{\frac{1}{x}}\right) = \int \frac{1}{x^3} \left(e^{\frac{1}{x}}\right) dx + c$$

$$\text{Put } -\frac{1}{x} = t \Rightarrow \frac{1}{x^2} dx = dt$$

$$y\left(e^{\frac{1}{x}}\right) = \int -te^t + c = e^{-\frac{1}{x}} \left(\frac{1}{x} + 1\right) + c$$

$$\text{Given } y\left(\frac{1}{2}\right) = 3 - e \Rightarrow y(1) = 1$$

62. The area enclosed by the curves $y^2 + 4x = 4$ and $y - 2x = 2$ is

- 1) $\frac{23}{3}$ 2) $\frac{22}{3}$ 3) 9 4) $\frac{25}{3}$

Key: 3

Sol: Given curves are $y^2 + 4x = 4$ and $y - 2x = 2$

Curves are Intersecting

$$x = \frac{4 - y^2}{4} = \frac{y - 2}{2} \Rightarrow \frac{y^2}{4} + \frac{y}{2} - 2 \quad \text{Area} = \frac{\Delta^{3/2}}{6a^2} = 9$$



63. Let Ω be the sample space and $A \subseteq \Omega$ be an event. Given below are two statements

(S1): If $P(A) = 0$, then $A = \phi$

(S2): If $P(A) = 1$, then $A = \Omega$

Then

1) both (S1) and (S2) are true

2) only (S1) is true

3) only (S2) is true

4) both (S1) and (S2) are false

Key: 1

Sol: Probability of Impossible event is 0 & Probability of certain event is 1

64. The value of $\sum_{r=0}^{22} {}^{22}C_r \cdot {}^{23}C_r$ is

1) ${}^{45}C_{24}$

2) ${}^{45}C_{23}$

3) ${}^{44}C_{23}$

4) ${}^{44}C_{22}$

Key: 2

Sol:
$$\sum_{r=0}^{22} {}^{22}C_r \cdot {}^{23}C_r = \sum_{r=0}^{22} {}^{22}C_r \cdot {}^{23}C_{(23-r)}$$

$$= {}^{22}C_0 \cdot {}^{23}C_{23} + {}^{22}C_1 \cdot {}^{23}C_{22} + \dots + {}^{22}C_{22} \cdot {}^{23}C_1$$

$$= {}^{45}C_{23} \text{ (using Vander Mond's theorem)}$$

65. Let α be a root of the equation $(a-c)x^2 + (b-a)x + (c-b) = 0$ where a, b, c are distinct real

numbers such that the matrix $\begin{bmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{bmatrix}$ is singular. Then the value of

$\frac{(a-c)^2}{(b-a)(c-b)} + \frac{(b-a)^2}{(a-c)(c-b)} + \frac{(c-b)^2}{(a-c)(b-a)}$ is

1) 6

2) 3

3) 9

4) 12

Key: 2



Sol: $(a-c)\alpha^2 + (b-a)\alpha + (c-b) = 0$

$$\frac{(a-c)^2}{(b-a)(c-b)} + \frac{(b-a)^2}{(a-c)(c-b)} + \frac{(c-b)^2}{(a-c)(b-a)}$$

$$= \frac{(a-c)^3 + (b-a)^3 + (c-b)^3}{(b-a)(c-b)(a-c)}$$

$$= 3 \quad \left(\begin{array}{l} \because x + y + z = 0 \\ \Rightarrow x^2 + y^2 + z^2 = 3xyz \end{array} \right)$$

66. $\lim_{t \rightarrow 0} \left(1^{\frac{1}{\sin^2 t}} + 2^{\frac{1}{\sin^2 t}} + \dots + n^{\frac{1}{\sin^2 t}} \right)^{\sin^2 t}$ is equal to

- 1) n^2 2) $n^2 + n$ 3) n 4) $\frac{n(n+1)}{2}$

Key: 3

Sol: $\lim_{t \rightarrow 0} \left(n^{\frac{1}{\sin^2 t}} \right)^{\sin^2 t} \left(\left(\frac{1}{n} \right)^{\cos^2 t} + \left(\frac{2}{n} \right)^{\cos^2 t} + \dots + 1^{\cos^2 t} \right)^{\sin^2 t}$

(As $n \rightarrow \infty$, $r^n \rightarrow 0$ when $0 < x < 1$)

$$(n)(0 + 0 + \dots + 1) = n$$

67. The distance of the point $(-1, 9, -16)$ from the plane $2x + 3y - z = 5$ measured parallel to the

line $\frac{x+4}{3} = \frac{2-y}{4} = \frac{z-3}{12}$ is

- 1) $20\sqrt{2}$ 2) 31 3) $13\sqrt{2}$ 4) 26

Key: 4

Sol: Distance of $(-1, 9, -16)$ from plane $2x + 3y - z - 5 = 0$

Measured parallel to line $\frac{x+4}{3} = \frac{y-2}{-4} = \frac{z-3}{12}$

Is $\left| \frac{ax_1 + by_1 + cz_1 + d}{al + bm + cn} \right|$ where l, m, n are d.c's of line

$$= 26$$



68. Let N denote the number that turns up when a fair die is rolled. If the probability that the system of equations

$$x + y + z = 1$$

$$2x + Ny + 2z = 2$$

$$3x + 3y + Nz = 3$$

has unique solution is $\frac{k}{6}$, then the sum of value of k and all possible value of N is

- 1) 18 2) 19 3) 20 4) 21

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & N & 2 \\ 3 & 3 & N \end{vmatrix} \neq 0 \Rightarrow N \neq 4, N = 1, 2, 3, 5, 6$$

But when $N = 2$ or 3 system has Infinite solution

$$\therefore \text{Probability} = \frac{3}{6}$$

$$\therefore \text{Sum} = (1 + 2 + 3 + 5 + 6) + 3 = 20$$

69. Let $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$

Then at $x = 0$

- 1) f is continuous but not differentiable
 2) f is continuous but f' is not continuous
 3) f' is continuous but not differentiable
 4) f and f' both are continuous

Key: 2

Sol: $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right) = 0 = f(0)$

f is continuous at 0

$$f'(x) = -\cos\left(\frac{1}{x}\right) + 2x \sin\left(\frac{1}{x}\right), f'(x) \text{ is discontinuous at } x = 0$$

Because $\lim_{x \rightarrow 0} f'(x)$ does not exist



70. Let PQR be a triangle. The point A, B and C are on the sides QR, RP and PQ respectively

such that $\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}$. Then $\frac{\text{Area}(\Delta PQR)}{\text{Area}(\Delta ABC)}$ is equal to

1) 4

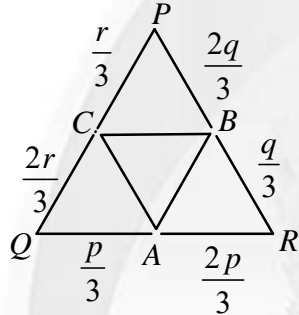
2) 3

3) 2

4) $\frac{5}{2}$

Key: 2

Sol: Area of ΔABC



$$= \Delta PQR - \{\Delta PBC + \Delta AQC + \Delta ABR\}$$

$$= \Delta PQR - \left\{ \frac{1}{2} \left(\frac{r}{3} \right) \left(\frac{2q}{3} \right) \sin P + \frac{1}{2} \left(\frac{p}{3} \right) \left(\frac{2r}{3} \right) \sin Q + \frac{1}{2} \left(\frac{q}{3} \right) \left(\frac{2p}{3} \right) \sin R \right\}$$

$$= \Delta PQR - \frac{2}{3} \Delta PQR = \frac{1}{3} \Delta PQR = \frac{\Delta PQR}{\Delta ABC} = 3$$

71. For three positive integers $p, q, r, x^{pq^2} = y^{qr} = z^{p^2r}$ and $r = pq + 1$ such that

$3, 3 \log_y x, 3 \log_z y, 7 \log_x z$ are in A.P. with common difference $\frac{1}{2}$. Then $r - p - q$ is equal to

1) 6

2) 2

3) 12

4) -6

Key: 2

Sol: $3 \log_y x - 3 = \frac{1}{2} \Rightarrow x = y^{7/6}$

$$\Rightarrow pq = 6$$

Also $3 \log_z y - 3 = 1 \Rightarrow y = z^{4/3}$

$$\Rightarrow 4q = 3p^2 \quad \Rightarrow p = 2, q = 3, r = 7$$

$$r - p - q = 2$$



72. $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$ is equal to

- 1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{3}$

Key: 4

Sol: $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$
 $= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$

73. The equation $x^2 - 4x + [x] + 3 = x[x]$, where $[x]$ denote the greatest integer function, has

- 1) no solution 2) exactly two solutions in $(-\infty, \infty)$
 3) a unique solution in $(-\infty, 1)$ 4) a unique solution in $(-\infty, \infty)$

Key: 4

Sol: $x^2 - 4x + 3 = x[x] - [x]$
 $(x-1)(x-3) = (x-1)[x] \Rightarrow x=1 \quad \text{or} \quad x-3=[x]$
 $\Rightarrow x - [x] = 3$ not possible $\Rightarrow \{x\} = 3$
 $\therefore x=1$ is only solution

74. Let $p, q \in \mathbb{R}$ and $(1 - \sqrt{3}i)^{200} = 2^{199}(p+iq), i = \sqrt{-1}$ Then $p+q+q^2$ and $p-q+q^2$ are roots of the equation.

- 1) $x^2 - 4x + 1 = 0$ 2) $x^2 + 4x + 1 = 0$ 3) $x^2 - 4x - 1 = 0$ 4) $x^2 + 4x - 1 = 0$

Key: 1

Sol: $(1 + \sqrt{3}i)^{200} = 2^{199}(p+iq)$
 $2^{200}\left(\frac{1}{2} + i \cdot \frac{\sqrt{3}}{2}\right)^{200} = 2^{199}(p+iq) \Rightarrow 2(-w^2)^{200} = p+iq$

$\Rightarrow 2w = p+iq \Rightarrow P = -1, q = \sqrt{3}$

Required equation is $x^2 - 4x + 1 = 0$



75. If A and B are two non-zero $n \times n$ matrices such that $A^2 + B = A^2B$, then

- 1) $A^2 = I$ or $B = I$ 2) $A^2B = BA^2$ 3) $AB = I$ 4) $A^2B = I$

Key: 2

Sol: $A^2 + B = A^2B$

$$A^2B - A^2 - B = 0$$

$$B(A^2 - I) - (A^2 - I) = I$$

$$(B - I)(A^2 - I) = I$$

$$\Rightarrow BA^2 - B - A^2 = 0 \Rightarrow BA^2 = B + A^2 = A^2B$$

$$\therefore BA^2 = A^2B$$

76. Let a tangent to the curve $y^2 = 24x$ meet the curve $xy = 2$ at the points A and B. Then the mid points of such line segments AB lie on a parabola with the

- 1) length of latus rectum $\frac{3}{2}$ 2) length of latus rectum 2
3) directrix $4x = 3$ 4) directrix $4x = -3$

Key: 3

Sol: Let $p(x_1, y_1)$ be midpoint of AB of curve $xy = 2$ then equation of AB is $S_1 = S_{11}$

$$\Rightarrow \frac{1}{2}(xy_1 + x_1y) = \frac{1}{2}x_1y_1$$

$$\Rightarrow xy_1 + x_1y - x_1y_1 = 0 \text{ is tangent of } y^2 = 24x$$

$$\Rightarrow am^2 = 1n$$

$$\text{Locus is } 6(x_1^2) = y_1(-x_1y_1)$$

$$\text{Locus is } y^2 = -6x$$

$$\text{Directrix is } 4x = 3$$



77. The distance of the point $(7, -3, -4)$ from the plane passing through the points $(2, -3, 1), (-1, 1, -2)$ and $(3, -4, 2)$ is
- 1) $5\sqrt{2}$ 2) $4\sqrt{2}$ 3) 4 4) 5

Key: 1

Sol: Plane passing through the points $(2, -3, 1), (-1, 1, -2)$ and $(3, -4, 2)$ is

$$\begin{vmatrix} x-2 & y+3 & z-1 \\ 3 & -4 & 3 \\ -1 & 1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow x - z - 1 = 0$$

$$\text{Required distance} = \frac{10}{\sqrt{2}} = 5\sqrt{2}$$

78. The compound statement $(\sim(P \wedge Q)) \vee ((\sim P) \wedge Q) \Rightarrow ((\sim P) \wedge (\sim Q))$ is equivalent to
- 1) $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$ 2) $(\sim Q) \vee P$
 3) $((\sim P) \vee Q) \wedge (\sim Q)$ 4) $(\sim P) \vee Q$

Key: 1

Sol: Use Truth Table

79. The relation $R = \{(a, b) : \gcd(a, b) = 1, 2a \neq b, a, b \in \mathbb{Z}\}$ is

- 1) reflexive but not symmetric 2) transitive but not reflexive
 3) symmetric but not transitive 4) neither symmetric nor transitive

Key: 4

Sol: $a, b \in \mathbb{Z}, \gcd(a, b) = 1, b \neq 2a$

$$\gcd(a, a) \neq 1 \Rightarrow \text{Not Reflexive}$$

$$\gcd(3, 7) = 1 \ \& \ \gcd(7, 15) = 1$$

$$\text{But } \gcd(3, 15) \neq 1 \Rightarrow \text{Not Transitive}$$

$$\gcd(a, b = 1) \Rightarrow \gcd(b, a) = 1 \Rightarrow \text{Symmetric}$$



80. Let $\vec{u} = \hat{i} - \hat{j} - 2\hat{k}$, $\vec{v} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{v} \cdot \vec{w} = 2$ and $\vec{v} \times \vec{w} = \vec{u} + \lambda \vec{v}$. Then $\vec{u} \cdot \vec{w}$ is equal to

1) 1

2) 2

3) $\frac{3}{2}$ 4) $-\frac{2}{3}$

Key: 1

Sol: $(\vec{v} \times \vec{w}) \times \vec{v} = \vec{u} \times \vec{v}$

$$\Rightarrow |\vec{v}|^2 \vec{w} - (\vec{v} \cdot \vec{w}) \vec{v} = \vec{u} \times \vec{v}$$

$$\Rightarrow \vec{w} = \left(\frac{7}{6}, \frac{-1}{6}, \frac{1}{6} \right)$$

$$\vec{u} \cdot \vec{w} = 1$$

(NUMERICAL VALUE TYPE)

Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within **0 to 9999**. If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i, e. If answer is above **10** and less than **10.5** round off is **10** and If answer is from **10.5** and less than **11** round off is **11**).

Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.

81. Let $\lambda \in \mathbb{R}$ and let the equation E be $|x|^2 - 2|x| + |\lambda - 3| = 0$. Then the largest element in the set $S = \{x + \lambda : x \text{ is an integer solution of E}\}$ is _____

Key: 5

Sol: $|x|^2 - 2|x| + |\lambda - 3| = 0$

For real roots $\Delta \geq 0$

$$\Rightarrow |\lambda - 3| \leq 1$$

$$\Rightarrow 2 \leq \lambda \leq 4$$

$$\text{When } l = 4 \Rightarrow |x|^2 - 2|x| + 1 = 0$$

$$\Rightarrow |x| = 1 \Rightarrow x = -1, 1$$

$$\text{Largest element} = x + \lambda = 5$$

82. A boy needs to select five courses from 12 available courses, out of which 5 courses are language courses. If he can choose at most two language courses, then the number of ways he can choose five courses is _____

Key: 546

Sol:

Languages (5)	Other (7)
2	$3 \rightarrow 5C_2 \cdot 7C_3 = 350$
1	$4 \rightarrow 5C_1 \cdot 7C_4 = 175$
0	$5 \rightarrow 5C_0 \cdot 7C_5 = 21$
	$= 546$



83. The value of $\frac{8}{\pi} \int_0^{\frac{\pi}{2}} \frac{(\cos x)^{2023}}{(\sin x)^{2023} + (\cos x)^{2023}} dx$ is _____

Key: 2

Sol:
$$I = \int_0^{\frac{\pi}{2}} \frac{(\cos x)^{2023}}{(\sin x)^{2023} + (\cos x)^{2023}}$$

Using kings property
$$I = \int_0^{\frac{\pi}{2}} \frac{(\sin x)^{2023}}{(\cos x)^{2023} + (\sin x)^{2023}}, 2I = \int_0^{\frac{\pi}{2}} 1 dx \Rightarrow I = \frac{\pi}{4}$$

84. The shortest distance between the lines $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-6}{2}$ and $\frac{x-6}{3} = \frac{y-1}{2} = \frac{z+8}{0}$ is equal to _____

Key: 14

Sol: Lines are $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-6}{2}$

$$\frac{x-6}{3} = \frac{y-1}{-2} = \frac{z+8}{0} \quad S.D = \frac{[\vec{c} - \vec{a} \quad \vec{b} \quad \vec{d}]}{|\vec{b} \times \vec{d}|} = 14$$

85. The 4th term of GP is 500 and its common ratio is $\frac{1}{m}, m \in \mathbb{N}$. Let S_n denote the sum of the first n terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$. then the number of possible values of m is _____

Key: 12

Sol: $S_6 - S_5 > 1 \Rightarrow ar^5 > 1$

$$S_7 - S_6 < \frac{1}{2} \Rightarrow ar_6 < \frac{1}{2}$$

Also given $ar^3 = 500$

$$\Rightarrow 500r^2 > 1 \Rightarrow r^2 > \frac{1}{500} \quad \forall \quad r^3 < \frac{1}{1000}$$

No. of possible values are 12

86. Let C be the largest circle centred at (2,0) and inscribed in the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$.

If $(1, \alpha)$ lies on C, then $10\alpha^2$ is equal to _____

Key: 118

Sol: Let circle is $(x-2)^2 + y^2 = r^2, \frac{x^2}{36} + \frac{y^2}{16} = 1 \Rightarrow \frac{x^2}{36} + \frac{r^2 - (x-2)^2}{16} = 1$

$$\Rightarrow 16x^2 + 36(r^2 - (x-2)^2) = (36)(16)$$



Using $\Delta = 0$ we get $r^2 = \frac{320}{25}$, circle is $(x-2)^2 + y^2 = \frac{320}{25}$

$$(1, \alpha) \text{ lies on this circle} \Rightarrow \alpha^2 = \frac{59}{5} \quad 10\alpha^2 = 118$$

87. Let a tangent to the curve $9x^2 + 16y^2 = 144$ intersect the coordinate axes at the points A and B. Then, the minimum length of the line segment AB is _____

Key: 7

Sol: Ellipse is $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Minimum length of tangent to ellipse between axis = $a + b = 7$

88. Suppose $\sum_{r=0}^{2023} r^2 \cdot {}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$. Then the value of α is _____

Key: 1012

$$\begin{aligned} \text{Sol: } \sum r^2 \cdot C_r &= n(n+1)2^{n-2} &&= (2023)(2024) \cdot 2^{2021} \\ &= (2023) \times 2^{2022} (1012) &&\alpha = 1012 \end{aligned}$$

89. The value of $12 \int_0^3 |x^2 - 3x + 2| dx$ is _____

Key: 22

$$\begin{aligned} \text{Sol: } &\int_0^1 (x^2 - 3x + 2) dx + \int_1^2 (-2 + 3x - x^2) dx + \int_2^3 (x^2 - 3x + 2) dx \\ &= \left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x \right)_0^1 + \left(-2x + \frac{3x^2}{2} - \frac{x^3}{3} \right)_1^2 + \left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x \right)_2^3 \\ &= \left(\frac{1}{3} - \frac{3}{2} + 2 \right) + \left(-4 + \frac{12}{2} - \frac{8}{3} \right) - \left(-2 + \frac{3}{2} - \frac{1}{3} \right) + \left(\frac{3^3}{3} - \frac{3(3)^2}{2} + 2(3) \right) - \left(\frac{2^3}{3} - \frac{3(2)^2}{2} + 2(2) \right) \\ \text{Rv} &= (4 - 18 + 24) + (-48 + 72 - 32) - (-24 + 18 - 4) + (108 - 162 + 72) - (32 - 72 + 48) \end{aligned}$$

90. The number of 9 digit numbers, that can be formed using all the digits of the number 123412341 so that the even digits occupy only even places, is _____

Key: 60

Sol: 1 2 3 4 1 2 3 4 1

_____ ○ _____ ○ _____ ○ _____ ○ _____

$$\text{No. of ways} = \frac{4!}{2!2!} \times \frac{5!}{3!2!} = 60$$