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JEE MAIN (JAN) 2023 (24-01-2023-Session-2)

MATHEMATICS, PHYSICS & CHEMISTRY

Sri Chaitanya IIT Academy.,India.

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ICON Central Office - Madhapur - Hyderabad

Jee-Main_24-JAN-2023_Shift-02

IMPORTANT INSTRUCTION:

1. Immediately fill in the Admission number on this page of the Test Booklet with **Blue/Black** Ball Point **Pen** only.

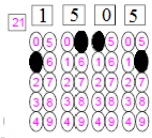
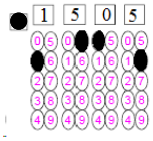
2. The candidates should not write their Admission Number anywhere (except in the specified space) on the Test Booklet/ Answer Sheet.

3. The test is of **3 hours** duration.

4. The Test Booklet consists of 90 questions. The maximum marks are **300.**

5. There are **three** parts in the question paper 1, 2, 3 consisting of **Physics, Chemistry** and **Mathematics** having **30 questions** in each subject and subject having **two sections.**
(I) Section –I contains 20 **multiple choice** questions with only one correct option.
Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.
(II) Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only, if more than 5 questions attempted, First 5 attempted questions will be considered.

■ The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i, e. If answer is above **10** and less than **10.5** round off is **10** and If answer is from **10.5** and less than **11** round off is **11).**
To cancel any attempted question bubble on the question number box.
For example: To cancel attempted question 21. Bubble on 21 as shown below

Question Answered for Marking **Question Cancelled for Marking**
Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases.

6. Use **Blue / Black Point Pen only** for writing particulars / marking responses on the Answer Sheet. **Use of pencil is strictly prohibited.**

7. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electron device etc, except the Identity Card inside the examination hall.

8. Rough work is to be done on the space provided for this purpose in the Test Booklet only.

9. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Hall. **However, the candidate are allowed to take away this Test Booklet with them.**

10. **Do not fold of make any stray marks on the Answer Sheet**

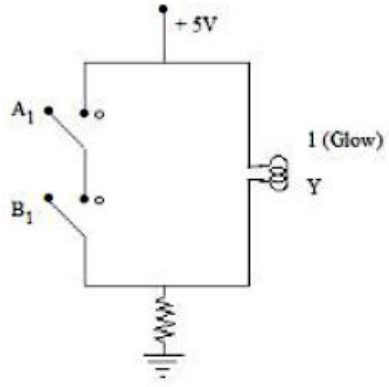
PHYSICS

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1.



The logic gate equivalent to the given circuit diagram is:

1)OR **2)** NAND **3)** AND **4)** NOR

Ans : 2

Sol:

A	B	Y
0	0	1
1	0	1
0	1	0
1	1	0

2. Given below are two statements :

Statement I : Acceleration due to earth's gravity decreases as you go 'up' or 'down' from earth's surface .

Statement II : Acceleration due to earth's gravity 'h' and depth 'd' from earth's surface ; if h = d

In the light of above statements, choose the mast appropriate answer form the options given below

1) Statement I is incorrect but statement II is correct

2) Both Statement I and Statement II are incorrect

3) Statement I is correct but statement II is incorrect

4) Both Statement I and II are correct

Ans : 3

Sol : $g^{1}=g\left(1-\frac{2h}{R}\right)$ $g^{11}=g\left(1-\frac{d}{R}\right)$

3. The electric field and magnetic field components of an electromagnetic wave going through vacuum is described by

$E_x=E_0\sin\left(kz-\omega t\right)$

$B_y=B_0\sin\left(kz-\omega t\right)$

Then the correct relation between E_0 and B_0 is given by

1) $kE_0=\omega B_0$ **2)** $\omega E_0=kB_0$ **3)** $E_0=kB_0$ **4)** $E_0\times B_0=\omega k$

Ans : 1

Sol : $C=\frac{E_0}{B_0}$ $\frac{W}{K}=\frac{E_0}{B_0}$ $E_0k=\omega B_0$

4. Let γ_1 be the ratio of molar specific heat at constant pressure and molar specific heat at constant volume of a monoatomic gas and γ_2 be the similar ratio of diatomic gas .

Considering the diatomic gas molecule as a rigid , the ratio , $\frac{\gamma_1}{\gamma_2}$ is :

1) $\frac{27}{35}$ **2)** $\frac{25}{21}$ **3)** $\frac{21}{25}$ **4)** $\frac{35}{27}$

Ans : 2

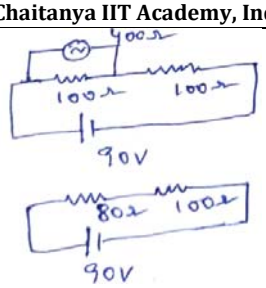
Sol : $\frac{\gamma_1}{\gamma_2}=\frac{5/3}{7/5}=\frac{25}{21}$

5. A cell of emf 90V is connected across series combination of two resistors each of 100Ω resistance. A voltmeter of resistance 400Ω is used to measure the potential difference across each resistor .The reading of the voltmeter will be:

1) 80V **2)** 45V **3)** 40 V **4)** 90V

Ans : 3

Sol :



$R_{eff} = \frac{100\times 400}{500} = 80\ \Omega$ $v_2 = \frac{80}{180}\times 90$ $= 40$ V

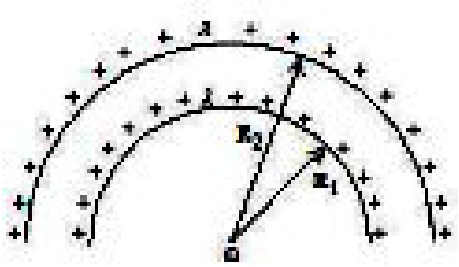
6. A long solenoid is formed by winding 70 turns cm^{-1}.If 2.0A current flows, then the magnetic field produced inside the solenoid is _____$\left(\mu_0 = 4\pi\times 10^{-7} TmA^{-1}\right)$.

1) $1232\times 10^{-4}T$ **2)** $352\times 10^{-4}T$ **3)** $176\times 10^{-4}T$ **4)** $88\times 10^{-4}T$

Ans : 3

Sol: $B = \mu_0 ni$

7. The electric potential at the centre of two concentric half rings of radii R_1 and R_2 having same linear charge density λ is :



1) $\frac{\lambda}{\epsilon_0}$ **2)** $\frac{\lambda}{4\epsilon_0}$ **3)** $\frac{2\lambda}{\epsilon_0}$ **4)** $\frac{\lambda}{2\epsilon_0}$

Ans : 4

Sol : $V = v_1 + v_2$ $\frac{k\lambda(\pi R_1)}{R_1} + \frac{k\lambda(\pi R_2)}{R_2} = \frac{\lambda}{2\epsilon_0}$

8. Match List I with List II

LIST -1	**LIST -11**
1) AM Broadcast	I) 88-108 MHz
2) FM Broadcast	II) 540-1600 kHz
3) Television	III) 3.7-4.2 GHz
4) Satellite Communication	IV) 54MH2-890MH2

Choose the correct answer from the options given below

1)A-IV, B- III,C-I , D – II **2)** A-II,B-III,C-I,D-IV

3)A-II,B-I,C-IV,D-III **4)** A-I,B-III,C-II,D-IV

Ans :3

Sol : Conceptual

9. If two vectors $\vec{P}=\hat{i}+2m\hat{j}+m\hat{k}$ and $\vec{Q}=4\hat{i}-2\hat{j}+m\hat{k}$ are perpendicular to each other. Then , the value of m will be :

1) 3 **2)** 2 **3)** -1 **4)** 1

Ans : 2

Sol : $\bar{P}.\bar{Q}=0$

10. The frequency (v) of an oscillating liquid drop may depend upon radius (r) of the drop, density (ρ) of liquid and the surface tension (s) of the liquid as : $v=r^a\rho^b s^c$. The values of a,b, and c respectively are

1) $\left(-\frac{3}{2},-\frac{1}{2},\frac{1}{2}\right)$ **2)** $\left(\frac{3}{2},-\frac{1}{2},\frac{1}{2}\right)$ **3)** $\left(\frac{3}{2},\frac{1}{2},-\frac{1}{2}\right)$ **4)** $\left(-\frac{3}{2},\frac{1}{2},\frac{1}{2}\right)$

Ans : 1

Sol : $m^0l^0t^{-1}=l^a\left(ml^{-3}\right)^b\left(mt^{-2}\right)^c$

Compare power on both sides

$a=-\frac{3}{2}$ $b=-\frac{1}{2}$ $c=\frac{1}{2}$

11. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : Steel is used in the construction of buildings and bridges

Reason R : Steel is more elastic and its elastic limit is high

In the light of above statements , choose the most appropriate answer from the options given below

1) A is not correct but R is correct

2) A is correct but R is not correct

3)Both A and R are correct and R is is the correct explanation of A

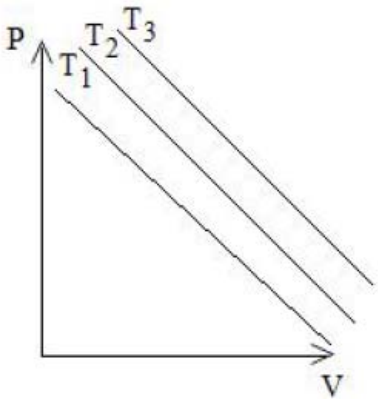
4)Both A and R are correct and R is NOT the correct explanation of A

Ans : 3

Sol : Conceptual

12. In an Isothermal change, the change in pressure and volume of a gas can be represented for three different temperature ; $T_3 > T_2 > T_1$ as :

1)



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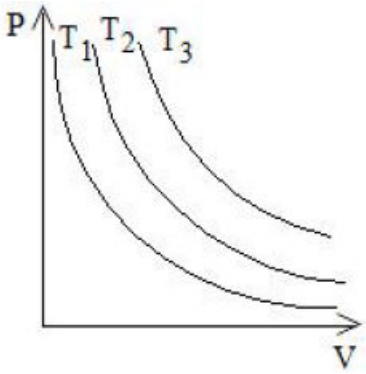

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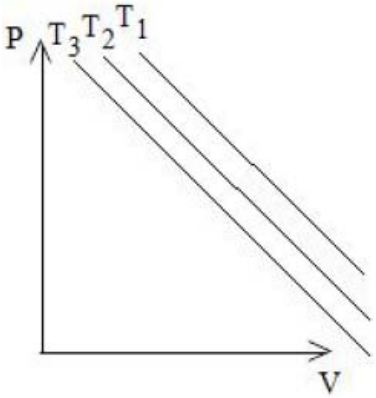
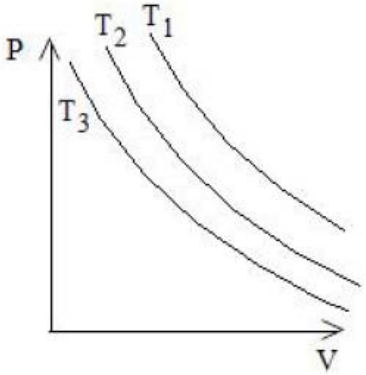


4)



Ans : 2

Sol : $Pv = nRT$

$\Rightarrow PV \alpha T$ As Area of P-V graph is more then T is more . Nature of group is hyperbola

13. A body of mass 200g is tied to a spring constant 12.5 Nm, while the other end of spring is fixed at point O. If the body movies about O in a circular path on a smooth horizontal surface with constant angular speed 5 rad/s . Then the ratio of extension in the spring to its natural length will be :

1) 1 : 2 **2)** 2 : 5 **3)** 1 : 1 **4)** 2 : 3

Ans : 4

Sol : $m(L+e)\omega^2 = Ke$

$$200\times10^{-3}(l+e)25 = \frac{25}{2}e$$

$$\frac{l+e}{e}=\frac{10}{4}=\frac{5}{2}\Rightarrow\frac{l}{e}=\frac{3}{2}$$

$$\frac{e}{l}=\frac{2}{3}$$

14. An α-particle, a proton and an electron have the same kinetic energy. Which one of the following is correct in case of their de-Broglie wavelength :

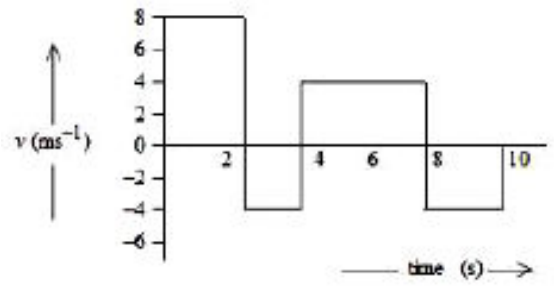
1) $\lambda_\alpha > \lambda_p > \lambda_e$ **2)** $\lambda_\alpha < \lambda_p < \lambda_e$ **3)** $\lambda_\alpha = \lambda_p = \lambda_e$ **4)** $\lambda_\alpha > \lambda_p < \lambda_e$

Ans : 2

Sol : $\lambda=\frac{h}{P}=\frac{h}{\sqrt{2mKE}}$ $m_e < m_p < m_\alpha$

$\Rightarrow \lambda\alpha\frac{1}{\sqrt{m}}\Rightarrow$ $\lambda_\alpha < \lambda_p < \lambda_e$

15. The velocity time graph of a body moving in a straight line is shown in figure :



The ratio of displacement to distance travelled by the body in time 0 to 10s is :

1) 1 : 2 **2)** 1 : 3 **3)** 1 : 1 **4)** 1 : 4

Ans : 2

Sol : Total area = distance $=|S_1|+|S_2|+|S_3|+|S_4|$

$=|2\times8|+|2\times4|+|4\times4|+|2\times4|$ $=16+8+16+8=48$

Displacement $=16-8+16-8=16$

16. If the distance of the earth from Sun is $1.5\times10^6\,km$ .Then the distance of an imaginary planet from Sun, if its period of revolution is 2.83 years is :

1) $3\times10^6\,km$ **2)** $3\times10^7\,km$ **3)** $6\times10^6\,km$ **4)** $6\times10^7\,km$

Ans : 1

Sol: $T^2\alpha R^3$

$$\left(\frac{T_2}{T_1}\right)^2=\left(\frac{R_2}{R_1}\right)^3 \qquad R_2=\left(\frac{T_2}{T_1}\right)^{2/3}\times R_1$$

$$R_2=(2.83)^{2/3}\times 1.5\times 10^6=3\times 10^6\,Km.$$

17. When a beam of white light is allowed to pass through convex lens parallel to principal axis , the different colours of light converge at different point on the principle axis after refraction . This is called .

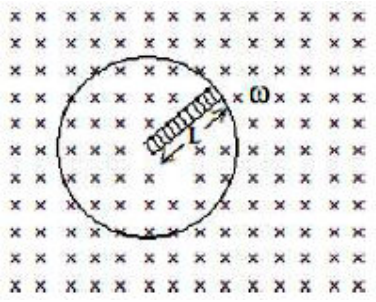
1) Scattering **2)** Chromatic aberration

3) Spherical aberration **4)** Polarisation

Ans : 2

Sol: A lens will not focus different colours in exactly the same place. Because the focal length depends on refraction . This is called chromatic aberration.

18. A metallic rod of length 'L' is rotated with an angular speed of 'ω' normal to a uniform magnetic field 'B' about an axis passing through one end of rod as shown in figure . The induced emf will be :



1) $\frac{1}{2}BL^2\omega$ **2)** $\frac{1}{4}B^2L\omega$ **3)** $\frac{1}{4}BL^2\omega$ **4)** $\frac{1}{2}B^2L^2\omega$

Ans : 1

Sol :

$$de=B.l.\upsilon$$

$$de=B.dx.xw$$

$$\int de=\int_0^1 BWX.dX$$

$$e = BW\frac{l^2}{2}$$

$$e = \frac{1}{2}Bl^2w$$

19. A photon is emitted in transition from n = 4 to n = 1 level in hydrogen atom .The corresponding wavelength for this transition is (given , $h = 4\times10^{-15}eVs$) :

1) 94.1 nm **2)** 974 nm **3)** 99.3 nm **4)** 941 nm

Ans : 1

Sol : $E_2 - E_1 = \frac{h_c}{\lambda}$

$$13.6z^2\left(\frac{1}{1}-\frac{1}{4^2}\right) = \frac{h\times c}{\lambda}$$

$$\lambda = \frac{4\times10^{-15}\times3\times10^8}{13.6}\times\frac{16}{15} = 94.1nm$$

20. Given below are two statement : one is labelled as Assertion A and the labelled as Reason R.

Assertion A : A pendulum clock when taken to Mount Everest becomes fast

Reason R : The value of g (acceleration due to gravity) is less at Mount Everest than its value on the surface of earth.

In the light of the above statements, choose the most appropriate answer from the options given below

1)Both A and R are correct and R is is the correct explanation of A

2)A is correct but R is not correct

3)A is not correct but R is correct

4)Both A and R are correct and R is NOT the correct explanation of A

Ans : 3

Sol : $T = 2\pi\sqrt{\frac{l}{g}}$ 'g' value decrease an mount everest. So time period increases .It's becomes slow

(NUMERICAL VALUE TYPE)

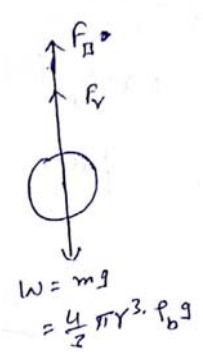
Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i, e. If answer is above **10** and less than **10.5** round off is **10** andIf answer is from **10.5** and less than **11** round off is **11).**

Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases

21. A spherical ball of radius 1 mm and density 10.5g/cc is dropped in glycerine of coefficient of viscosity 9.8 poise and density 1.5 g /cc. Viscous force on the ball when it attains constant velocity is $3696\times10^{-x}N$ .The value of x is (given $g=9.8m/s^2$ and $\pi=\frac{22}{7}$)

Ans : x = 7

Sol : $r=1mm=1\times10^{-3}m$



$\rho_b=10.5\ gm/c.c, \sigma_g=1.5\ gm/cc.$

$\eta=9.8\ poise, F_V=3696\times10^{-X}.$

$x=?$

$F_b+F_V=W$

$F_v-\frac{4}{3}\pi r^3.\rho_b g-\frac{4}{3}or\,r^3.\sigma_g g$

$F_v-\frac{4}{3}\pi r^3.(\rho_b-\rho_g).g$

$F_v-\frac{4}{3}\times\frac{22}{7}\times(10^{-3})^3\times9\times10^3\times9.8$

$=3696\times10^{-7}N$

$\therefore x=7$

22. A body of mass 1 kg begins to move under the action of a time dependent force $\vec{F}=(t\hat{i}+3t^2\hat{j})N$,where $\hat{i}$ and $\hat{j}$ are the unit vectors along x and y axis .The power developed by above force, at the time t = 2s,will be _____w.

Ans : 100

Sol : $\vec{F}=(t\hat{i}+3t^2\hat{j})N \qquad m\vec{a}=t\hat{i}+3t^2\hat{j}$

$\vec{a}=\frac{t\hat{i}+2t^2\hat{j}}{m}=\frac{t\hat{i}+3t^2\hat{j}}{1}=t\hat{i}+3t^2\hat{j} \qquad \vec{a}=t\hat{i}+3t^2\hat{j}$

$\frac{dv}{dt}=t\hat{i}+3t^2\hat{j} \qquad \alpha v=(t\hat{i}+3t^2\hat{j})dt$

$\int dv=\int(t\hat{i}+3t^2\hat{j})dt \quad \vec{V}=\frac{t^2}{2}\hat{i}\to3.\frac{t^3}{3}\hat{j}=\frac{t^2}{2}\hat{i}+t^3\hat{j}.$

At $t=2$ sec $\to\vec{F}=2\hat{i}+12\hat{j} \qquad \vec{v}=2\hat{i}+8\hat{j}$

$P = \vec{F}.\vec{V} = \left(2\hat{i} + 12\hat{j}\right).\left(2\hat{i} + 8\hat{j}\right) = 4 + 96 = 100W$

23. The energy released per fission of nucleus of ^{240}X is 200 MeV. The energy released if all the atoms in 120 g of pure ^{240}X undergo fission is _____$\times 10^{25} MeV$.(Given $N_A = 6\times 10^{23}$)

Ans: 6

Sol: NO.OF atoms = $\frac{120}{240}\times 6\times 10^{23}$ $= 3\times 10^{23}$

Energy released if all the atoms in 120 gm of pure $\times^{240}$ nuclei is = $3\times 10^{23}\times 200 MeV$

$= 6\times 10^{25} MeV$

24. A parallel plate capacitor with air between the plate has a capacitance of 15Pf. The separation between the plate becomes twice and the space between them is filled with a medium of dielectric constant 3.5 . Then the capacitance becomes $\frac{x}{4} pF$ .The value of x is _____

Ans : 105

Sol: $C = \frac{\in_0 A}{d} = 15PF$

$C^1 = \frac{K.\in_0 A}{d^1} = K.\frac{\in_0 A}{2d} = \frac{K}{2}[C]$

$C^1 = \frac{3.5}{2} = [15PF] = \frac{105}{4} P.F$

$\frac{x}{4} = \frac{105}{4}$ $\therefore x = 105$

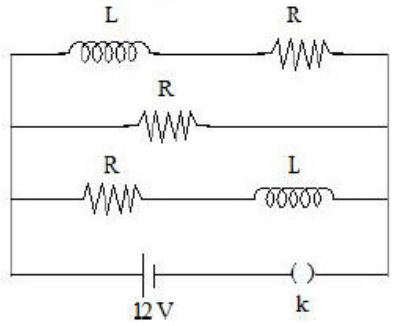
25. If a copper wire is stretched to increase its length by 20% .The percentage increase in resistance of the wire is _____%.

Ans : 44 %

Sol: $R = \frac{\delta l^2}{V} \Rightarrow R \alpha l^2$

$\frac{R_2}{R_1} = \left(\frac{l_2}{l_1}\right)^2 = \left(\frac{120}{100}\right)^2 = \frac{144}{100}\left(\frac{R_2}{R_1} - 1\right)\times 100 = \left(\frac{144}{100} - 1\right)\times 100$ $= 44\ \%$

26. Three identical resistors with resistance $R = 12\Omega$ and two identical inductors with self inductance L =5 Mh are connected to an ideal battery with emf of 12 V as shown in figure .The current through the battery long after the switch has been closed will be _____A.



Ans : 3 amp

Sol: $i = \frac{V}{R_{eff}} = \frac{12}{4} = 3amp$

Where $R_{eff} = \frac{12}{3} = 4\Omega$

27. A mass m attached to free end of a spring executes SHM with a period of 1s. If the mass is increased by 3 kg the period of oscillation increases by one second , the value of mass m is _____ kg.

Ans : m = 1

Sol: $T = 2\pi\sqrt{\frac{m}{k}}$ $T\alpha\sqrt{m}$ $\frac{T_2}{T_1} = \sqrt{\frac{m+3}{m}}$ $\frac{2}{1} = \sqrt{\frac{m+3}{m}}$

$\frac{4}{1} = \frac{m+3}{m}$ $4m = m+3$ $3m = 3$ m = 1

28. A convex lens of refractive index 1.5 and focal length 18cm in air is immersed in water .The change in focal length of the lens will be _____cm.

(Given refractive index of water = $\frac{4}{3}$)

Ans : 54 cm

Sol : $\left(\frac{\mu_g}{\mu_{air}} - 1\right) f_{air} = \left(\frac{\mu_g}{\mu_{liq}} - 1\right) f_{liq}$

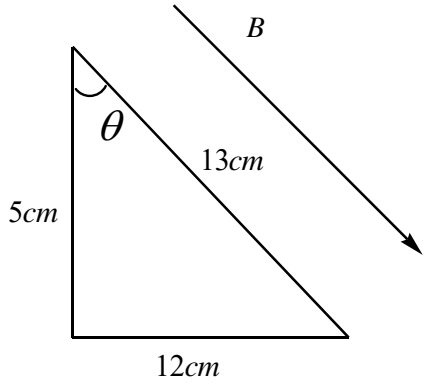
$\left(\frac{1.5}{1} - 1\right) 18 = \left(\frac{3/2}{4/3} - 1\right) f_{liq}$

$\therefore f_{liq} = 72cm$

$\Delta f = f_{liq} - f_{air}$

$= 72 - 18$ $\Delta f = 54cm$

29. A single turn current loop in the shape of a right angle triangle with sides 5 cm , 12 cm , 13 cm is carrying a current of 2 A. The loop is in a uniform magnetic field of magnitude 0.75 T whose direction is parallel to the current in the 13 cm side of the loop. The magnitude of the magnetic



force on the 5 cm side will be $\frac{x}{130}$N. The value of x is _____.

Ans : 9

Sol: $F = Bil\sin\theta$ $\frac{x}{130} = 0.75\times 2\times 5\times 10^{-2}\times\frac{12}{13}$

$\frac{x}{130} = \frac{3}{4}\times\frac{1}{10}\times\frac{12}{13}$

30. A uniform solid cylinder with radius R and length L has moment of inertia I_1, about the axis of the cylinder . A concentric solid cylinder of radius $R' = \frac{R}{2}$ and length $L' = \frac{L}{2}$ is carved out of the original cylinder. If I_2 is the moment of inertia of the carved out portion of the cylinder then $\frac{I_1}{I_2}$ = _____

(Both I_1 and I_2 are about the axis of the cylinder)

Ans : 32

Sol : $I_1 = \frac{mR^2}{2}$

$I_2 = \frac{m^1}{2}\left(\frac{R}{2}\right)^2$

$I_2 = \frac{m}{8}\times\frac{R^2}{8} = \frac{mR^2}{64}$

$\therefore \frac{I_1}{I_2} = \frac{mR^2}{2}\times\frac{64}{mR^2} = 32$

CHEMISTRY

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

31. The number of s-electrons present in an ion with 55 protons in its unipositive state is:

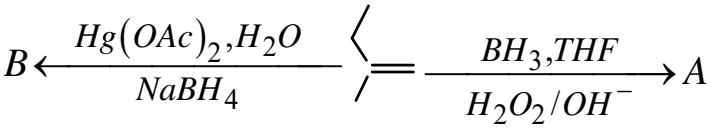
1) 10 **2)** 12 **3)** 8 **4)** 9

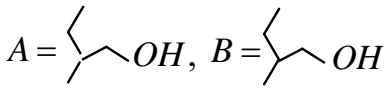
Ans: 1

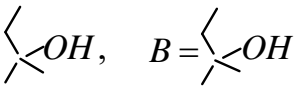
Sol: $CS^+ : 1S^2 2S^2 2P^6 3S^2 3P^6 4S^2 3d^{10} 4P^6 5S^2 4d^{10} 5p^6$

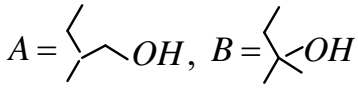
No of S-electrons 2+2+2+2+2=10

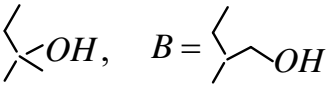
32. Find out the major products from the following reactions.



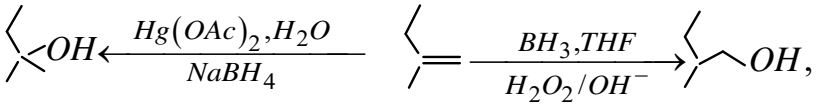
1) 

2) 

3) 

4) 

Ans: 3



Sol: Markonikoff addition Anti Markonikoff addition

33. $K_2Cr_2O_7$ paper acidified with dilute H_2SO_4 turns green when exposed to

1) Hydrogen sulphide **2)** Sulphur dioxide

3) Carbon dioxide **4)** Sulphur trioxide

Ans: 2

Sol: $K_2Cr_2O_7 + H_2SO_4 + SO_2 \rightarrow K_2SO_4 + Cr_2(SO_4)_3(Green) + H_2O$

34. Correct statement is:

1) An average human being consumes more food than air

2) An average human being consumes nearly 15 times more air than food

3) An average human being consumes 100 times more air than food

4) An average human being consumes equal amount of food and air

Ans: 2

Sol: An average human being consumes nearly 15 times more air than food

35. Given below are two statements:

Statement-I: Pure Aniline and other arylamines are usually colourless.

Statement-II: Arylamines get coloured on storage due to atmospheric reduction

In the light of the above statements, choose the most appropriate answer from the options given below:

1) Statement I is incorrect but Statement II is correct

2) Both Statement I and Statement II are correct

3) Statement I is correct but Statement II is incorrect

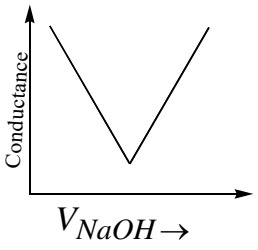
4) Both Statement I and Statement II are incorrect

Ans: 3

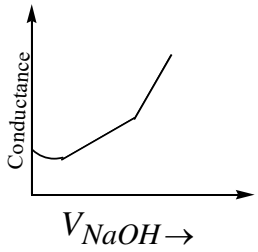
Sol: Pure Aniline and other arylamines are usually colourless. But they become coloured on storage due to atmospheric oxygen. Aniline develops a yellow to brown colour due to this reason

36. Choose the correct representation of conductometric titration of benzoic acid vs sodium hydroxide.

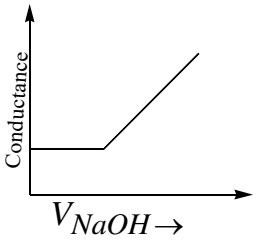
1)



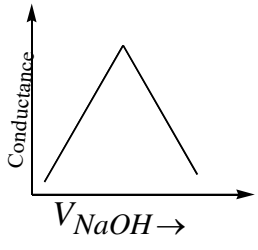
2)



3)



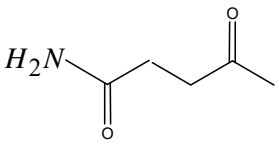
4)

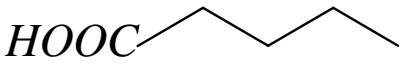


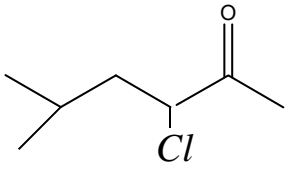
Ans: 2

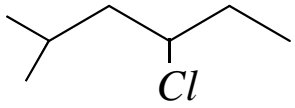
Sol: Benzoic acid is a weak acid, initially conductance low, by addition of strong base NaOH Conductance increases, after reacting end point conductance increased by strong base NaOH

37. Given below are two statements:

Statement I:  Under clemmensen reduction conditions will give



Statement II:  Under Wolff-Kishner reduction condition will give



In the light of the above statements, choose the correct answer from the options given below:

1) Both Statement I and Statement II are true

2) Statement I is false but Statement II is true

3) Both Statement I and Statement II are false

4) Statement I is true but Statement II is false

Ans: 2

Sol: In clemmensen reduction aldehyde and keto groups are reduced to corresponding hydrocarbon in presence of Zn/Hg

38. What is the number of unpaired electron(s) in the highest occupied molecular orbital of the following species $N_2; N_2^+; O_2; O_2^+$?

1) 2, 1, 2, 1 **2)** 2, 1, 0, 1 **3)** 0, 1, 0, 1 **4)** 0, 1, 2, 1

Ans: 4

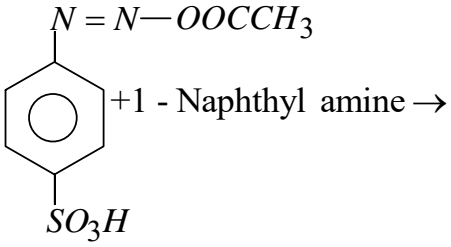
Sol: $N_2 : \sigma_{1s}^2 < \sigma_{1s}^{*2} < \sigma_{2s}^2 < \sigma_{2s}^{*2} < \pi_{2p_x}^2 = \pi_{2p_y}^2 < \sigma_{2p_z}^2 \rightarrow 0$

$N_2^+ : \sigma_{1s}^2 < \sigma_{1s}^{*2} < \sigma_{2s}^2 < \sigma_{2s}^{*2} < \pi_{2p_x}^2 = \pi_{2p_y}^2 < \sigma_{2p_z}^1 \rightarrow 1$

$O_2 : \sigma_{1s}^2 < \sigma_{1s}^{*2} < \sigma_{2s}^2 < \sigma_{2s}^{*2} < \sigma 2p_z^2 < \pi_{2p_x}^2 = \pi_{2p_y}^2 < \pi_{2p_x}^{*1} = \pi_{2p_y}^{*1} \rightarrow 2$

$O_2^+ : \sigma_{1s}^2 < \sigma_{1s}^{*2} < \sigma_{2s}^2 < \sigma_{2s}^{*2} < \sigma 2p_z^2 < \pi_{2p_x}^2 = \pi_{2p_y}^2 < \pi_{2p_x}^{*1} = \pi_{2p_y}^{*0} \rightarrow 1$

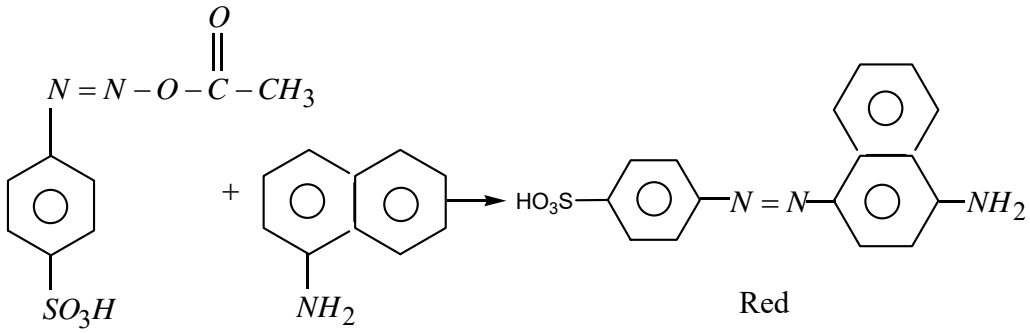
39. Choose the correct colour of the product for the following reaction.



1) Red **2)** Yellow **3)** Blue **4)** White

Ans : 1

Sol :



40. In which of the following reactions the hydrogen peroxide acts as a reducing agent ?

1) $PbS + 4H_2O_2 \rightarrow PbSO_4 + 4H_2O$ **2)** $Mn^{2+} + H_2O_2 \rightarrow Mn^{4+} + 2OH^-$

3) $2Fe^{2+} + H_2O_2 \rightarrow 2Fe^{3+} + 2OH^-$ **4)** $HOCl + H_2O_2 \rightarrow H_3O^+ + Cl^- + O_2$

Ans : 4

Sol : $PbS + 4H_2O_2 \rightarrow PbSO_4 + 4H_2O$ H_2O_2 act as oxidising agent

$Mn^{2+} + H_2O_2 \rightarrow Mn^{4+} + 2OH^-$ H_2O_2 act as oxidising agent

$2Fe^{2+} + H_2O_2 \rightarrow 2Fe^{3+} + 2OH^-$ H_2O_2 act as oxidising agent

$HOCl + H_2O_2 \rightarrow H_3O^+ + Cl^- + O_2$ H_2O_2 act as reducing agent

41. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: Benzene is more stable than hypothetical cyclohexatriene

Reason R: The delocalized π electron cloud is attracted more strongly by nuclei of carbon atoms.

In the light of the above statements, choose the correct answer from the options given below:

1) Both A and R are correct but R is NOT the correct explanation of A

2) A is true but R is false

3) Both A and R are correct and R is the correct explanation of A

4) A is false but R is true

Ans : 3

Sol : A: Benzene is more stable than hypothetical cyclohexatriene (Correct)

R: The delocalized π electron cloud is attracted more strongly by nuclei of carbon atoms. (Correct)

42. Match List I with List II

List I Type		List II Name	
A.	Antifertility drug	I.	Norethindrone
B.	Tranquilizer	II.	Meprobomate
C.	Antihistamine	III.	Seldane
D.	Antibiotic	IV.	Ampicillin

Choose the correct Answer from the options given below:

1) A-I, B-II, C-III, D-IV **2)** A-I, B-III, C-II, D-IV

3) A-IV, B-III, C-II, D-I **4)** A-II, B-I, C-III, D-IV

Ans : 1

Sol : Antifertility drug – Norethindrone

Tranquilizer – Meprobomate

Antihistamine – Seldane

Antibiotic - Ampicillin

43. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: Beryllium has less negative value of reduction potential compared to the other alkaline earth metals.

Reason R: Beryllium has large hydration energy due to small size of Be^{2+} but relatively large value of atomization enthalpy

In the light of the above statements, choose the most appropriate answer from the options given below

1) A is correct but R is not correct

2) A is not correct but R is correct

3) Both A and R are correct but R is NOT the correct explanation of A

4) Both A and R are correct and R is the correct explanation of A

Ans: 3

Sol: A: Beryllium(-1.97) has less negative value of reduction potential compared to the other alkaline earth metals. (Correct)

R: Beryllium has large hydration energy due to small size of Be^{2+} but relatively large value of atomization enthalpy (Correct)

44. Which one amongst the following are good oxidizing agents?

A. Sm^{2+}

B. Ce^{2+}

C. Ce^{4+}

D. Tb^{4+}

Choose the most appropriate answer from the options given below:

1) C and D only **2)** A and B only **3)** C only **4)** D only

Ans: 1

Sol: Ce^{+4} & Tb^{+4} are good oxidizing agents

45. Identify the correct statements about alkali metals.

A. The order of standard reduction potential $\left(M^{+}|M\right)$ for alkali metal ions is

$Na > Rb > Li$.

B. CsI is highly soluble in water.

C. Lithium carbonate is highly stable to heat.

D. Potassium dissolved in concentrated liquid ammonia is blue in colour and paramagnetic.

E. All the alkali metal hydrides are ionic solids.

Choose the correct answer from the options given below.

1) A, B, D only **2)** A and E only **3)** C and E only **4)** A, B and E only

Ans: 2

Sol: (A) SRP of Li = **-3)**04

SRP of Na = **-2)**715

SRP of Rb = **-2.**9

(B) CsI is least soluble

(C) Li_2CO_3 is not stable to heat

(E) All the alkali metal hydrides are ionic and having high melting points.

46. A student has studied the decomposition of a gas AB_3 at $25°C$. He obtained the following data.

P(mm Hg)	50	100	200	400
Relative $t_{1/2}(s)$	4	2	1	0.5

The order of the reactions is

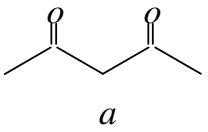
1) 1 **2)** 2 **3)** 0.5 **4)** 0 (zero)

Ans: 2

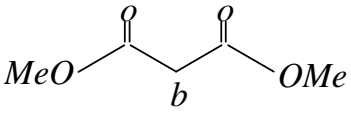
Sol: $\frac{(t_{1/2})_1}{(t_{1/2})_2} = \left(\frac{a_2}{a_1}\right)^{n-1} \qquad \frac{4}{2} = \left(\frac{100}{50}\right)^{n-1}$

$2 = 2^{n-1} \qquad \Rightarrow 2^{n-1} = 2^1 \qquad \Rightarrow n-1 = 1 \qquad \Rightarrow n = 2$

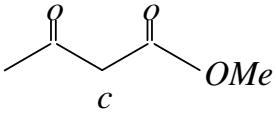
47. Which will undergo deprotonation most readily in basic medium?



a



b



c

1) Both a and c **2)** a only **3)** b only **4)** c only

Ans: 2

Sol: Due to presence of two electron withdrawing carbonyl group. Methylene protons are very much acidic

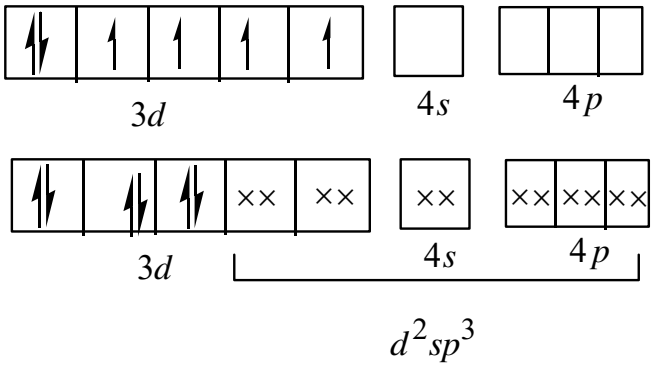
48. The hybridization and magnetic behaviour of cobalt ion in $\left[Co(NH_3)_6\right]^{3+}$ complex, respectively is

1) Sp^3d^2 and diamagnetic **2)** Sp^3d^2 and paramagnetic

3) d^2sp^3 and diamagnetic **4)** d^2sp^3 and paramagnetic

Ans: 3

Sol: $\left[Co(NH_3)_6\right]^{3+}$ $NH_3 = strong\ ligand.$ $CO^{+3}:[Ar]3d^6 4s^0$



d^2sp^3

Diamagnetic [absence of unpaired electrons]

49. The metal which is extracted by oxidation and subsequent reduction form its ore is:

1) Fe **2)** Cu **3)** Ag **4)** Al

Ans: 2

Sol: In the extraction of 'Cu' from sulphide ore

Cu_2S converted into Cu_2O [Oxidation]

Cu_2O converted into Cu [reduction]

$2Cu_2S + 3O_2 \rightarrow 2Cu_2O + 2SO_2$

$Cu_2S + 2Cu_2O \rightarrow 6Cu + SO_2$

50. Which of the following cannot be explained by crystal field theory ?

1) Magnetic properties of transition metal complexes

2) Colour of metal complexes

3) Stability of metal complexes

4) The order of spectrochemical series

Ans: 4

Sol: CFSE explain properties of transition – metal complexes including thir colour, magnetism structure stability and reactivity

(NUMERICAL VALUE TYPE)

Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i, e. If answer is above **10** and less than **10.5** round off is **10** andIf answer is from **10.5** and less than **11** round off is **11).**

Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases

51. Total number of tripeptides possible by mixing of valine and proline is_____

Ans: 8

Sol: No. of Tripeptides = $2^3 = 8$

52. The number of statements which are the characteristics of physisorption is_____

A. It is highly specific in nature

B. Enthalpy of adsorption is high

C. It decreases with increases in temperature

D. it results into unimolecular layer

E. No activation energy is needed

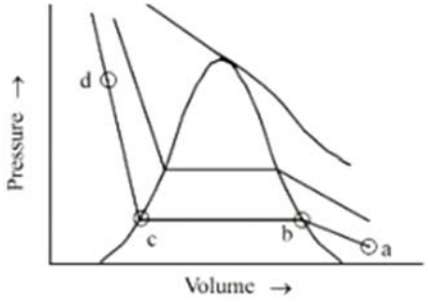
Ans: 2

Sol: C, E (2)

Statement A, B, D are for chemical adsorption

Statement C, E are for physical adsorption

53. The number of statements, which are correct with respect to the compression of carbon dioxide from point (a) in the Andrews isotherm from the following is_____



1. Carbon dioxide remains as a gas upto point (b)
2. Liquid carbon dioxide appears at point (c)
3. Liquid and gaseous carbon dioxide coexist between points (b) and (c)
4. As the volume decreases from (b) to (c), the amount of liquid decreases

Ans: 3

Sol: A, B, C [3]

CO_2 exist as a gas from a to b line

At point 'C' Liquification stats

From $b \rightarrow c$ both liquid & gases CO_2 coexist

54. the number of units, which are used to express concentration of solution from the following is_____

Mass percent, Mole, Mole fraction, Molarity, ppm, Molality

Ans: 5

Sol: 5 Concentration terms: molarity, molality, mole fraction, mass percent, ppm

55. If the pKa of lactic acid is 5, then the pH of 0.005 M calcium lactate solution at $25^{\circ}C$ is_____$\times 10^{-1}\left(\text{Nearest integer}\right)$

Lactic acid $CH_3-\underset{\underset{OH}{|}}{\overset{\overset{H}{|}}{C}}-COOH$

Ans: 85

Sol:

$$pH = 7+\frac{1}{2}p^{Ka}+\frac{1}{2}\log c \quad = 7+\frac{1}{2}\times 5+\frac{1}{2}\left[\log\left(5\times 10^{-3}\right)\right]$$

$$= 7+2.5+\frac{1}{2}\left[-3+0.6990\right] \quad = 7+2.5+\frac{1}{2}\left[-2.3010\right]$$

$$= 9.5-1.15 \quad = 8.35 \quad = 84\times 10^{-1}$$

56. The total pressure observed by mixing two liquids A and B is 350 mm Hg when their mole fractions are 0.7 and 0.3 respectively.

The total pressure becomes 410 mm Hg if the mole fractions are changed to 0.2 and 0.8 Respectively for A and B. The vapour pressure of pure A is_____mm Hg. (Nearest integer)

Consider the liquids and solutions behave ideally.

Ans: 314

Sol: $P = P_A^{\circ}X_A + P_B^{\circ}X_B$

$$350 = P_A^{\circ}\times 0.7 + P_B^{\circ}\times 0.3 \rightarrow (1)$$

$$410 = P_A^{\circ}\times 0.2 + P_B^{\circ}\times 0.8 \rightarrow (2)$$

$$(1)\times 8-3\times(2) \;:\; 2800 = 5.6\times P_A^{\circ}$$

$$1230 = 0.6\times P_A^{\circ}$$

$$=1570=5\times P_A^\circ \Rightarrow \frac{1570}{5}=314$$

57. maximum number of isomeric monochloro derivatives which can be obtained from 2,2,5,5-

Tetramethyl hexane by chlorinations is_____

Ans: 3

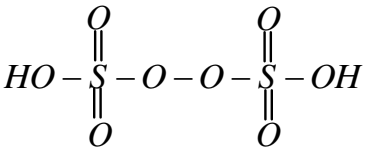
Sol: No of different carbon atoms in the molecule is 3

58. Sum of π - bonds present in peroxodisulphuric acid and pyrosulphuric acid is_____

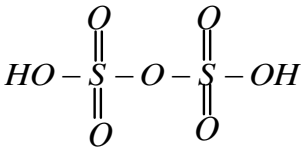
Ans: 8

Sol:

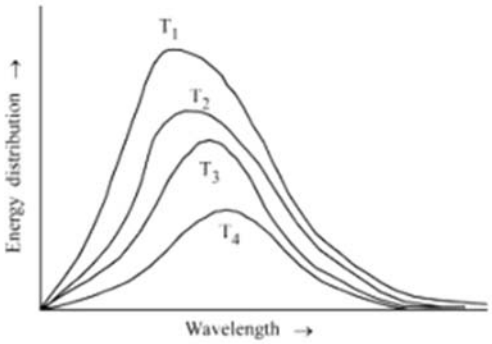
Peroxy disulphuric acid $H_2S_2O_8$:



Pyro sulphuric acid $H_2S_2O_7$:



59. Following figure shows spectrum of an ideal black body at four different temperatures. The number of correct statements from the following is_____



A. $T_4 > T_3 > T_2 > T_1$

B. The black body consists of particles performing simple harmonic motion.

C. the peak of the spectrum shifts to shorter wavelength as temperature increases.

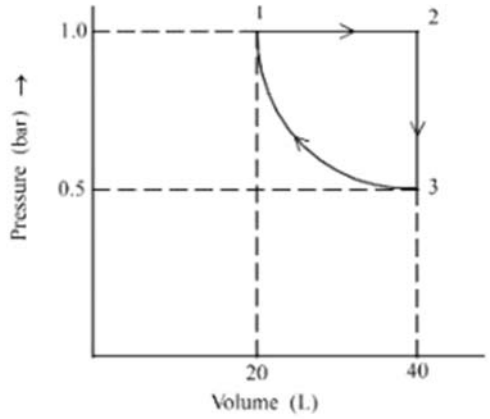
D. $\frac{T_1}{v_1}=\frac{T_2}{v_2}=\frac{T_3}{v_3} \neq$ constant

E. The given spectrum could be explained using quantisation of energy.

Ans: 2

Sol: C and E are correct

60. One mole of an ideal monoatomic gas is subjected to changes as shown in the graph. The magnitude of the work done (by the system or on the system) is_____ J (nearest integer)



Given: log 2 = 0.3

In 10 = 2.3

Ans: 620

Sol:

$$w:1\to 2:\text{isobaric}:w=-p\left(v_2-v_1\right)=-1\left(40-20\right)=-20Lbar$$

$$w:2\to 3:\text{isochoric}:w=-p\Delta v=0$$

$$w:3\to 1:\text{isothermal}:w=2.303nRT\log\frac{v_2}{v_1}$$

$$=-1\times 20\times \ell n\frac{20}{40}=-13.8$$

$$w=w_{1\to 2}+w_{2\to 3}+w_{3\to 1}$$

$$=-20+0+138=-6.2\,L\,bar=-620J$$

MATHEMATICS

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

61. The equation of the sides AB and AC a triangle ABC are $(\lambda+1)x-\lambda y=4$ and $\lambda x+(1-\lambda)y+\lambda=0$ respectively. Its vertex A is on the y-axis and its orthocentre is (1, **2**).The length of the tangent from the point C to the part of the parabola $y^2=6x$ in the first quadrant is

1) 2 **2)** $\sqrt{6}$ **3)** $2\sqrt{2}$ **4)** 4

Key:- 3

Sol:- Let $A=(0,y_1)$

$\lambda y_1=4$.....(1)

And $(1-\lambda)y_1=-\lambda$.....(2)

From (**1**) & (**2**)

$y_1=2$, A = (0, **2**), $\lambda=2$

$\overline{AB}=3x+2y=4,\ \overline{AC}=2x-y+2=0$

Let $C=[h,2h+2]$

$m_1\cdot m_2=-1 \quad \Rightarrow\left(\frac{-3}{2}\right)\left(\frac{2h}{h-1}\right)=-1 \quad \Rightarrow h=\frac{-1}{2}\Rightarrow C=\left(-\frac{1}{2},1\right)$

Equation of tangent is $y=mx+\frac{a}{m} \Rightarrow 1=m\left(\frac{-1}{2}\right)+\frac{3}{2m} \quad \Rightarrow m=1,-3$

$C=\left(\frac{-1}{2},1\right)$

$P=\left[\frac{a}{m^2},\frac{2a}{m}\right]=\left(\frac{3}{2},3\right)$

$CP=2\sqrt{2}$

62. Let $\vec{\alpha}=4\hat{i}+3\hat{j}+5\hat{k}$ and $\vec{\beta}=\hat{i}+2\hat{j}-4\hat{k}$ .Let β_1 be parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ be perpendicular to $\vec{\alpha}$. If $\vec{\beta}=\vec{\beta}_1+\vec{\beta}_2$, then the value of $5\vec{\beta}_2=(\hat{i}+\hat{j}+\hat{k})$ is

1)11 **2)** 6 **3)** 9 **4)** 7

Key:- 4

Sol:- Given $\vec{\beta}_1$ parallel to $\vec{\alpha} \Rightarrow \vec{\beta}_1 = t\vec{\alpha}$

$\vec{\beta}_2$ perpendicular to $\vec{\alpha} \Rightarrow \vec{\beta}_2 \cdot \vec{\alpha} = 0$

$\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$

Now do dot product with $\vec{\alpha}$ on both sides

$\vec{\alpha} \cdot \vec{\beta} = \vec{\beta}_1 \cdot \vec{\alpha} + \vec{\beta}_2 \vec{\alpha} \quad (\vec{\beta}_2 \cdot \vec{\alpha} = 0)$

$(4\hat{i} + 3\hat{j} + 5\hat{k})(\hat{i} + 2\hat{j} - 4\hat{k}) = (t\vec{\alpha}) \cdot \vec{\alpha} + 0 \quad (\vec{\beta}_1 = t\vec{\alpha})$

$4 + 6 - 20 = t(16 + 9 + 25) + 0 \qquad -10 = t(50) \Rightarrow t = \frac{-1}{5}$

$\vec{\beta}_1 = \frac{-1}{5}(4\hat{i} + 3\hat{j} + 5\hat{k})$

From $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$

$\vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1 = (i + 2j - 4k) - \left(\frac{-1}{5}(4i + 3j + 5k)\right)$

$\vec{\beta}_2 = \frac{1}{5}(9\hat{i} + 13\hat{j} - 15\hat{k}) \; 5\vec{\beta}_2 \cdot (\hat{i} + \hat{j} + \hat{k}) = 9 + 13 - 15 \qquad 5\vec{\beta}_2 \cdot (\hat{i} + \hat{j} + \hat{k}) = 7$

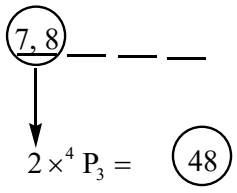
63. The number of integers greater than 7000 that can be formed using the digits 3, 5, 6, 7, 8 without repetition is

1) 48 **2)** 120 **3)** 168 **4)** 220

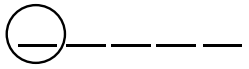
Key:- 3

Sol:- Given digits are 3, 5, 6, 7, 8

i) Four digits numbers starts with 7, 8



ii) Five digit numbers



$= {}^5P_4$

$= 120$

$= 120 + 48 = 168$

64. If the system of equations

$x+2y+3z=3$

$4x+3y-4z=4$

$8x+4y-\lambda z=9+\mu$

Has infinitely many solutions, then the ordered pair (λ,μ) is equal to

1) $\left(-\frac{72}{5},-\frac{21}{5}\right)$ **2)** $\left(-\frac{72}{5},\frac{21}{5}\right)$ **3)** $\left(\frac{72}{5},-\frac{21}{5}\right)$ **4)** $\left(\frac{72}{5},\frac{21}{5}\right)$

Key:- 3

Sol:- $x+2y+3z=3$

$4+3y-4z=4$

$8x+4y-\lambda z=9+\mu$ has infinitely solution $(\lambda,\mu)=$

$$\Delta=\begin{vmatrix}1 & 2 & 3\\ 4 & 3 & -4\\ 8 & 4 & -\lambda\end{vmatrix}=0\Rightarrow 5\lambda-72=0\Rightarrow\lambda=\frac{72}{5}$$

$$\Delta_3=\begin{vmatrix}1 & 2 & 3\\ 4 & 3 & 4\\ 8 & 4 & 9+\mu\end{vmatrix}=0\Rightarrow -5\mu-21=0\Rightarrow\lambda=\frac{-21}{5}$$

65. If $f(x)=x^3-x^2f^1(x)+xf^{11}(2)-f^{111}(3),\ x\in R$, then

1) $3f(1)+f(2)=f(3)$ **2)** $2f(0)-f(1)+f(3)=f(2)$

3) $f(3)-f(2)=f(1)$ **4)** $f(1)+f(2)+f(3)=f(0)$

Key:- 2

Sol:- Let $f(x)=x^3+bx^2+cx+d$

$f^1(x)=3x^2+2bx+c$

$f^{11}(x)=6x+2b$

$f^{111}(x)=6$

Given, $f(x)=x^3-x^2f^1(x)+xf^{11}(2)-f^{111}(3)$

$x^3+bx^2+cx+d=x^3-x^2(3+2b+c)+x(12+2b)-6$

By comparing coefficients

- b = 3 + 2b + c c = 12 + 2b

-3 = 3b + c .....**(1)** 2b – c = -12 .....**(2)**

Eq. (**1**) & (**2**)

$3b + c = -3$

$2b - c = -12$

5b = -15

b = -3, c = 6, d = -6

$f(x) = x^3 - 3x^2 + 6x - 6$

From option 2

2f(0) – f(1) + f(3) = f(2)

2(-6) – (-2) + 12 = 2

66. The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is

1) $-\frac{1}{2}(\sqrt{3}-i)$ **2)** $\frac{1}{2}(\sqrt{3}+i)$ **3)** $\frac{1}{2}(1-i\sqrt{3})$ **4)** $-\frac{1}{2}(1-i\sqrt{3})$

Key:- 1

Sol:- $E = \left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$

$$= \left[\frac{1+\cos\left(\frac{\pi}{2}-\frac{2\pi}{9}\right)+i\sin\left(\frac{\pi}{2}-\frac{2\pi}{9}\right)\cos\left(\frac{\pi}{4}-\frac{\pi}{9}\right)}{1+\cos\left(\frac{\pi}{4}-\frac{2\pi}{9}\right)+i2\sin\left(\frac{\pi}{4}-\frac{\pi}{9}\right)\cos\left(\frac{\pi}{4}-\frac{\pi}{9}\right)}\right]^3$$

$$= \left[\frac{2\cos\left(\frac{\pi}{4}-\frac{\pi}{9}\right)\left\{\cos\left(\frac{\pi}{4}-\frac{\pi}{9}\right)+i\sin\left(\frac{\pi}{4}-\frac{\pi}{9}\right)\right\}}{2\cos\left(\frac{\pi}{4}-\frac{\pi}{9}\right)\left\{\cos\left(\frac{\pi}{4}-\frac{\pi}{9}\right)-i\sin\left(\frac{\pi}{4}-\frac{\pi}{9}\right)\right\}}\right]$$

$$= \left[\text{cis}\left\{3\left(\frac{3\pi}{4}-\frac{\pi}{9}\right)\right\}-\left\{3\left(\frac{\pi}{4}-\frac{\pi}{9}\right)\right\}\right]$$

$$= \text{cis}\frac{5\pi}{6}+i\sin\frac{5\pi}{6}$$

$= -\frac{1}{2}(\sqrt{3} - i)$

67. The locus of the mid point of the chords of the circle $C_1 : (x-4)^2 + (y-5)^2 = 4$ which subtend an angle θ_1 at the centre of the circle C_1, is a circle of radius r_1. If $\theta_1 = \frac{\pi}{3}, \theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$, then θ_2 is equal to

1) $\frac{\pi}{4}$ **2)** $\frac{\pi}{6}$ **3)** $\frac{3\pi}{4}$ **4)** $\frac{\pi}{2}$

Key:- 1

Sol:- $\theta_1 = \frac{\pi}{3}, \theta_2 = \frac{2\pi}{3}$, $P(x_1, y_1)$ be the mid point of the chord AB

Then $\frac{\sqrt{(x-4)^2 + (y-5)^2}}{2} = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\Rightarrow r_1 = \sqrt{3}$

Similarly $r_3 = 1$

Given $r_1^2 = r_2^2 + r_3^2 \Rightarrow r_2 = \sqrt{2}$ $\therefore \theta = \frac{\pi}{4}$

68. Let y = y(x) be the solution of the differential equation $(x^2 - 3y^2)dx + 3xydy = 0$, y(**1**) =1. Then $6y^2(e)$ is equal to

1) $3e^2$ **2)** e^2 **3)** $\frac{3}{2}e^2$ **4)** $2e^2$

Key:- 2

Sol:- Put y = vx and variables separable then $6[y(e)]^2 = 2e^2$

69. Let A be a 3 x 3 matrix such that $|adj(adj(adjA))| = 12^4$ .Then $|A^{-1}adjA|$ is equal to

1) 12 **2)** $2\sqrt{3}$ **3)** 1 **4)** $\sqrt{6}$

Key:- 2

Sol:- $|A| = 2\sqrt{3}$

70. Let p and q be two statements. Then $\sim(p \wedge (p \Rightarrow \sim q))$ is equivalent to

1) $p \vee ((\sim p) \wedge q)$ **2)** $(\sim p) \vee q$ **3)** $p \vee (p \wedge q)$ **4)** $p \vee (p \wedge (\sim q))$

Key:- 2

Sol:- $\sim(p \wedge (p \Rightarrow \sim q)) =$

$= \sim p \vee \{\sim (p \Rightarrow \sim q)$

$= \sim p \vee q$

71. $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}dx$ is equal to

1) 2π **2)** $\frac{\pi}{2}$ **3)** $\frac{\pi}{3}$ **4)** $\frac{\pi}{6}$

Key:- 1

Sol:- $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{3^2-(2x)^2}}dx = 48\cdot\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}}$

$= 24\left(\sin^{-1}\left(\frac{2}{3}\cdot\frac{3\sqrt{3}}{4}\right) - \sin^{-1}\left(\frac{2}{3}\cdot\frac{3\sqrt{2}}{4}\right)\right)$

$= 24\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)$

$= 24\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = 24\left(\frac{\pi}{12}\right) = 2\pi$

72. If the foot of the perpendicular drawn from (1, 9, 7) to the line passing through the point (3, 2, 1) and parallel to the plane $x + 2y + z = 0$ and $3y - z = 3$ is (α,β,γ), then $\alpha+\beta+\gamma$ is equal to

1) – 1 **2)** 1 **3)** 3 **4)** 5

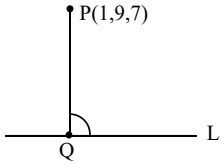
Key:- 4

Sol:- $L = \frac{x-3}{\ell} = \frac{y-2}{m} = \frac{z-1}{n}$

$\pi_1 = x + 2y + z = 0, \quad \pi_2 = 3y - z = 3$

$L // \pi_1 \Rightarrow \ell + 2m + n = 0$

$L // \pi_2 \Rightarrow 3m - n = 0$



2 1 1 2

3 -1 0 3

$\frac{\ell}{5}=\frac{m}{-1}=\frac{n}{-3}$

$\frac{x-3}{5}=\frac{y-2}{-1}=\frac{z-1}{-3}=t$

$Q=(5t+3,2-t,1-3t)$

d.r's of PQ = $(5t+2,-7-t,-6-3t)$

PQ perpendicular $\Rightarrow 5(5t+2)-1(-7-t)-3(-6-3t)$

$=25t+10+7+t+18+9t=0$

$35t=-35\Rightarrow t=-1$

$Q=(-2,3,4)=(\alpha,\beta,\gamma)$

$\alpha+\beta+\gamma=-2+3+4=7-2=5$

73. Let f(x) be a function such that $f(x+y)=f(x)\cdot f(y)$ for all $x,y\in N$. If $f(1)=3$ and $\sum_{k=1}^{n} f(k)=3279$ then the value of n is

1) 6 **2)** 8 **3)** 7 **4)** 9

Key:- 3

Sol:- $f(x)=a^x$ satisfies $f(x)+f(y)=f(x)\cdot f(y)$

$f(1)=a^1\Rightarrow 3=a \qquad \therefore f(x)=a^x=3^x$

$\sum_{k=1}^{n} f(x)=3279 \qquad 3+3^2+3^3+.....+3^n=3279$

$\frac{3(3^n-1)}{2}=3279 \qquad 3^n-1=2186\Rightarrow 3^n=2187=3^7\Rightarrow n=7$

74. The number of square matrices of order 5 with entries from the set {0, 1}, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1 is

1) 125 **2)** 150 **3)** 225 **4)** 120

Key:- 4

Sol:- $A=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}$

Number of such matrices = 5! = 120

75. The number of real solutions of the equation $3\left(x^2+\frac{1}{x^2}\right)-2\left(x+\frac{1}{x}\right)+5=0$ is

1) 4 **2)** 2 **3)** 3 **4)** 0

Key:- **4**

Sol:- $3\left[\left(x+\frac{1}{x}\right)^2-2\right]-2\left(x+\frac{1}{x}\right)+5=0$

Put $x+\frac{1}{x}=y$ $\quad 3(y^2-2)-2y+5=0$

$3y^2-2y-1=0$ $\quad 3y^2-2y+1=2\Rightarrow(3y-1)^2=2$

$3y-1=\pm\sqrt{2}$ $\quad 3\left(x+\frac{1}{x}\right)-1\pm\sqrt{2}=0$

$3(x^2+1)+x(-1\pm\sqrt{2})=0\Rightarrow 3x^2+(-1\pm\sqrt{2})x+3=0$

$3x^2-(1+\sqrt{2})x+3=0$ $\quad 3x^2+(\sqrt{2}-1)x+3=0$

$D_1=(1+\sqrt{2})^2 4(3)(3)<0$, $D_2=(\sqrt{2}-1)^2-4(3)(3)<0$

No, of real roots = 0

76. If $f(x)=\frac{2^{2x}}{2^{2x}+2}, x\in R$, then $f\left(\frac{1}{2023}\right)+f\left(\frac{2}{2023}\right)+...+f\left(\frac{2022}{2023}\right)$ is equal to

1) 2011 **2)** 2010 **3)** 1011 **4)** 1010

Key:- **3**

Sol:- $f(x)=\frac{2^{2x}}{2^{2x}+2}, x\in R$

$f(x)=\frac{4^x}{4^x+2}$ $\quad f(x)+f(1-x)=\frac{4^x}{4^x+2}+\frac{4^{1-x}}{4^{1-x}+2}$

$=\frac{4^x}{4^x+2}+\frac{4}{4+2\cdot 4^x}=\frac{4^x+2}{4^x+2}=1$ $\quad \Rightarrow f(x)+f(1-x)=1$

Now

$$f\left(\frac{1}{2023}\right)+f\left(\frac{2}{2023}\right)+...+f\left(\frac{2022}{2023}\right)$$

$$=f\left(\frac{1}{2023}\right)+f\left(\frac{2022}{2023}\right)+f\left(\frac{2}{2023}\right)+f\left(\frac{2021}{2023}\right)+...+f\left(\frac{1011}{2023}\right)+f\left(\frac{1012}{2023}\right)$$

= 1 + 1 + 1 + ... 1011 times = 1011

77. The set of all values of a for which $\lim\limits_{x\to a}([x-5]-[2x+2])=0$, where $[\alpha]$ denotes the greatest integer less than or equal to α is equal to

1) [-7.5, -6.5] **2)** (-7.5, -6.5) **3)** [-7.5, -6.5) **4)** (-7.5, -6.5)

Key:- 3

Sol:- $\lim\limits_{x\to a}([x-5]-[2x+2])=0$

$\Rightarrow [a-5]-[2a+2]=0$

$\Rightarrow [a-5]=[2a+2]$

Here $a\in[-7.5,-6.5)$

78. Let the plane containing the line of intersection of the plane P1: $x+(\lambda-4)y-4y+z=1$ and $P2: 2x+y+z=2$ pass through the point (0, 1, 0) and (1, 0, **1)**. Then the distance of the point $(2\lambda,\lambda,-\lambda)$ from the plane P2 is

1) $5\sqrt{6}$ **2)** $4\sqrt{6}$ **3)** $3\sqrt{6}$ **4)** $2\sqrt{6}$

Key:- 3

Sol:- $P_1: x+(\lambda+4)y+z-1=0$

$P_2: 2x+y+z-2=0$

Equation of the plane containing the line of intersection of the plane P1 and P2 is of the from $P_1+tP_2=0$

$\Rightarrow [x+(\lambda+4)y+z-1]+t[2x+y+z-2]=0.....(1)$

Eq. (**1)** pass through (0, 1, 0) & (1, 0, **1)**

i.e., $[(\lambda+4)+t]-1-2t=0$

$\Rightarrow \lambda-t+3=0.....(2)$

Also $(1+2t)+(1+t)-1-2t=0\Rightarrow t=-1....(3)$

Eq. **(2)** & **(3)** $\lambda+1+3=0\Rightarrow\lambda=-4$

Now $(2\lambda,\lambda,-\lambda)=(-8,-4,4)$

Then distance from (-8, -4, **4)** to P2 is

$$= \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{|2(-8) + (-4) + (4) - 2|}{\sqrt{4+1+1}} = \frac{|-16-4+4-2|}{\sqrt{6}} = \frac{18}{\sqrt{6}} = 3\sqrt{6}$$

79. Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$ be in A.P and $a_1 + a_3 = 10$. If the mean of these six numbers is $\frac{19}{2}$ and their variance is σ^2, then $8\sigma^2$ is equal to

1) 105 **2)** 200 **3)** 210 **4)** 220

Key:- 3

Sol:- Given $a_1, a_2, a_3, a_4, a_5, a_6$ are in A.P

Let $T_1 = a_1 = a$

Common difference =d

Also given $a_1 + a_3 = 10$

$\Rightarrow a + a + 2d = 10 \Rightarrow 2a + 2d = 10$.....(1)

Mean of six numbers is $\frac{19}{2}$

$$\Rightarrow \frac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6}{6} = \frac{19}{2}$$

$$\Rightarrow a + (a+d) + (a+2d) + (a+3d) + (a+4d) + (a+5d) = \frac{6 \times 19}{2}$$

$\Rightarrow 6a + 15d = 3 \times 19$

$\Rightarrow 2a + 5d = 19$.....(2)

Eq. (**1**) & (**2**) $3d = 9 \Rightarrow d = 3$

Also $2a + 2(3) = 10 \Rightarrow 2a = 4 \Rightarrow a = 2$

$\therefore$ six numbers are 2, 5, 8, 11, 14, 17

Wkt variance $= \sigma^2 = \frac{\sum xi^2}{n} - \mu^2$

$$= \frac{2^2 + 5^2 + 8^2 + 11^2 + 14^2 + 17^2}{6} - \left(\frac{19}{2}\right)^2$$

$$= \frac{4 + 25 + 64 + 121 + 196 + 289}{6} - \left(\frac{19}{2}\right)^2$$

$$= \frac{699}{6} - \frac{361}{4} \Rightarrow \frac{1398-1083}{12} = \frac{315}{12} = \frac{105}{4}$$

$$\sigma^2 = \frac{105}{4} \Rightarrow 8\sigma^2 = 8 \times \frac{105}{4} = 2 \times 105 = 210$$

80. If $({}^{30}C_1)^2 + 2({}^{30}C_2)^2 + 3({}^{30}C_3)^2 + \ldots\ldots + 30({}^{30}C_{30})^2 = \frac{\alpha 60!}{(30!)^2}$ then α is equal to

1) 30 **2)** 60 **3)** 10 **4)** 15

Key:- 4

Sol:- $({}^{30}C_1)^2 + 2({}^{30}C_2)^2 + 3({}^{30}C_3)^2 + \ldots. + 30({}^{30}C_{30})^2 = \alpha \cdot \frac{\lfloor 60}{(\lfloor 30)^2}$

Wkt $a \cdot C_0^{\,2} + (a+d)C_1^2 + (a+2d)C_2^2 + \ldots.. + (a+nd)C_n^2 = \frac{(2a+nd)}{2} . {}^{2n}C_n$

Now $0 \cdot ({}^{30}C_0)^2 + 1 \cdot ({}^{30}C_1)^2 + 2{}^{30}C_2)^2 + \ldots\ldots\ldots + 30 \cdot ({}^{30}C_{30})^2 - 0 \cdot ({}^{30}C_0)^2 = \alpha \cdot \frac{\lfloor 60}{(\lfloor 30)^2}$

$\Rightarrow \left(\frac{2(0)+30(1)}{2}\right) {}^{60}C_{30} - 0 = \alpha \cdot ({}^{60}C_{30})$ Here a = 0, d = 1

$\Rightarrow 15 \cdot {}^{60}C_{30} = \alpha \cdot {}^{60}C_{30} \Rightarrow \alpha = 15$

(NUMERICAL VALUE TYPE)

Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i, e. If answer is above **10** and less than **10.5** round off is **10** andIf answer is from **10.5** and less than **11** round off is **11).**
Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases

81. Let the sum of the coefficient of the first three terms in the expansion of $\left(x - \frac{3}{x^2}\right)^n, x \neq 0, n \in N$, be 376. Then the coefficient of x^4 is

Key:- 405

Sol:- Given binomial expansion is $\left(x - \frac{3}{x^2}\right)^n$

Sum of the coefficient of first 3 terms

${}^nC_0 - {}^nC_1(3) + {}^nC_2 \cdot 3^2 = 372$

$\Rightarrow 1 - 3n + \frac{n(n-1)}{1-2} \cdot 9 = 372$

$\Rightarrow 9n^2 - 15n - 750 = 0$

$\Rightarrow (n-10)(9n+75) = 0$

$\therefore n = 10$

Now $T_{2+1} = {}^{10}C_2 x^8 \left(-\frac{3}{x^2}\right)^2 = {}^{10}C_2 \times 9 \times x^4$

$\therefore$ Coefficient x^4 in $\left(x-\frac{3}{x^2}\right) = {}^{10}C_2 \times 9 = 405$

82. If the area of the region bounded by the curves $y^2-2y=-x, x+y=0$ is A, then 8A is equal to

Key:- 9/2 sq units

Sol:- Given curves are

$y^2-2y=-x$.....(1)

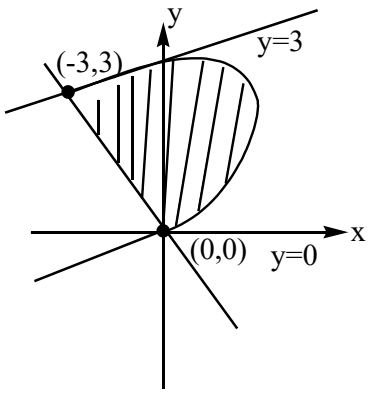
And y = -x .....(2)

From (1) & (2) $\Rightarrow y^2-2y=y$

$\Rightarrow y^2-3y=0 \Rightarrow y=0, y=3$

Point of intersections are A (0, 0) B (-3, 3)

Required area $A=\int_0^3 [2y-y^2-(y)]dy$



$=\int_0^3 (3y-y)^2 dy = \left(\frac{3y^2}{2}-\frac{y^3}{3}\right)_0^3 = \frac{9}{2}$ sq.units

83. Let $S=\{\theta\in[0,2\pi): \tan(\pi\cos\theta+\tan(\pi\sin\theta)=0\}$. Then $\sum_{\theta\in S}\sin^2\left(\theta+\frac{\pi}{4}\right)$ is equal to ___

Key:- 1

Sol:- $\tan(\pi\cos\theta) = -\tan(\pi\sin\theta)$

$= \tan(\pi-\pi\sin\theta)$

$\Rightarrow \pi\cos\theta+\pi\sin\theta=\pi \qquad \Rightarrow \cos\theta+\sin\theta=1$

$\Rightarrow \sin\left(\theta+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}} \quad \therefore \theta+\frac{\pi}{4}=n\pi+(-1)^n\frac{\pi}{4}, n\in Z$

$\theta_n = n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{4}, n\in Z$

$n=0 \Rightarrow \theta_0=0$

$n=1 \Rightarrow \theta_1=\pi-\frac{\pi}{4}-\frac{\pi}{4}=\frac{\pi}{2}$

$n=1 \Rightarrow \theta_1 = 2\pi + \frac{\pi}{4} - \frac{\pi}{4} = 2\pi \notin [0, 2\pi]$

$\therefore \theta = 0, \frac{\pi}{2} \in [0, 2\pi)$

$\sum_{\theta \in S} \sin^2\left(\theta + \frac{\pi}{4}\right) = \sin^2\left(0 + \frac{\pi}{4}\right) + \sin^2\left(\frac{\pi}{2} + \frac{\pi}{4}\right)$

$= \sin^2 \frac{\pi}{4} + \cos^2 \frac{\pi}{4} = 1$

84. Let $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}$, $\vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}$, $\vec{a} \cdot \vec{c} = 7$, $2\vec{b} \cdot \vec{c} + 43 = 0$, $\vec{a} \times \vec{c} = \vec{b} \times \vec{c}$ then $|\vec{a} \cdot \vec{b}|$ is equal to__

Key:- 25.79

Sol:- $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}$, $\Rightarrow \bar{c} = t(-2i + 7j + 2\lambda k)$

$\bar{a} \times \bar{c} = \bar{b} \times \bar{c}$ $\because \bar{a} \cdot \bar{c} = 7$

$\Rightarrow (\bar{a} - \bar{b}) \times \bar{c} = 0$ $\Rightarrow t(-2 + 14 + 2\lambda^2) = 7$

$\Rightarrow \bar{a} - \bar{b}, \bar{c}$ are parallel $\Rightarrow t(2\lambda^2 + 12) = 7$

$\Rightarrow \bar{c} = t(\bar{a} - \bar{b})$ $\Rightarrow \bar{c} = t(-2i + 7j + 2\lambda k)$, $\vec{b} = 3\hat{i} - 5\hat{j} - \lambda\bar{k}$,

$\because \bar{b} \cdot \bar{c} = \frac{-43}{2}$ $\Rightarrow t(-6 - 35 - 2\lambda^2) = -\frac{43}{2}$

$\left(\frac{7}{2\lambda^2 + 12}\right)(-41 - 2\lambda^2) = -\frac{43}{2}$ $\Rightarrow (287 + 14\lambda^2) = 43(\lambda^2 + 6)$

$287 + 14\lambda^2 = 43\lambda^2 + 258 \Rightarrow 29\lambda^2 = 29$ $\Rightarrow \lambda^2 = 1 \Rightarrow \lambda = 1$

Now $|\bar{a} \cdot \bar{b}| = |3 - 10 - \lambda^2| = |-7 - 1| = 8$

85. The equation of the sides AB, BC and CA of n triangle ABC are: $2x + y = 0$, $x + py = 21a$, $(a \neq 0)$ and $x - y = 3$ respectively. Let P(2, a) be the centroid of ΔABC. Then $(BC)^2$ is equal to

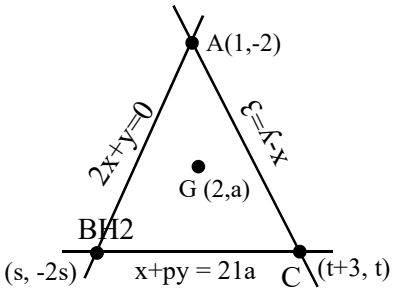
Key:- 122

Sol:- $2x + y = 0$ ------(1)

$x - y = 0$ -----(2)

$x + py = 21a$ -------(3)

solving (1) & (2) $\Rightarrow$ A (1, -2)



centriod of triangle ABC is

$\left(\frac{4 + s + t}{3}, \frac{-2 - 2s + t}{3}\right) = (2, a)$ $\Rightarrow s + t = 2$.....(4)

$-2s + t = 3a + 2$.....(5)

Solving (4) 7 95) we get $\Rightarrow s=-a,\ t=2+a$

$B(-a,2a);\ C(a+5,a+2)$

$\because$ B, C lies on x + py = 21a

$\Rightarrow -a+2ap=21a \Rightarrow P=11$

and $pa+2p=20a-5$

If p = 11 $\therefore 27=9a \Rightarrow a=3$

$\therefore B(-3,6),\ C(8,5),\ (BC)^2=(8+3)^2+(5-6)^2$

$\therefore$ Distance $(BC)^2=122$

86. If the shortest between the lines $\frac{x+\sqrt{6}}{2}=\frac{y-\sqrt{6}}{3}=\frac{z-\sqrt{6}}{4}$ and $\frac{x-\lambda}{3}=\frac{y-2\sqrt{6}}{4}=\frac{z+2\sqrt{6}}{5}$ is 6, then the square of sum of all possible values of λ is

Key:- 624

Sol:- Given lines are

$\frac{x+\sqrt{6}}{2}=\frac{y-\sqrt{6}}{3}=\frac{3-\sqrt{6}}{4}$ and $\frac{x-\lambda}{3}=\frac{y-2\sqrt{6}}{4}=\frac{3+2\sqrt{6}}{5}$

$\vec{a}=-\sqrt{6}\bar{i}+\sqrt{6}\bar{j}+\sqrt{6}\bar{k}$ $\qquad$ $\vec{c}=\lambda\bar{i}+2\sqrt{6}\bar{j}-2\sqrt{6}\bar{k}$

$\vec{b}=2\bar{i}+3\bar{j}+4\bar{k}$ $\qquad$ $\vec{d}=3\bar{i}+4\bar{j}+5\bar{k}$

$\vec{a}-\vec{c}=(-\sqrt{6}-\lambda)\bar{i}=\sqrt{6}\bar{j}+3\sqrt{6}\bar{k}$

$$[\vec{a}-\vec{c}\ \vec{b}\ \vec{d}]=\begin{vmatrix}-\sqrt{6}-\lambda & -\sqrt{6} & 3\sqrt{6}\\ 2 & 3 & 4\\ 3 & 4 & 5\end{vmatrix}=(-\sqrt{6}-\lambda)(15-16)+\sqrt{6}(10-12)+3\sqrt{6}(8-9)$$

$=\sqrt{6}+\lambda-2\sqrt{6}-3\sqrt{6}=\lambda-4\sqrt{6}$

$$\bar{b}\times\bar{d}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 2 & 3 & 4\\ 3 & 4 & 5\end{vmatrix}=\bar{i}(15-16)-\bar{j}(10-12)+\bar{k}(8-9)=-\bar{i}+2\bar{j}-\bar{k}\quad |\bar{b}\times\bar{d}|=\sqrt{1+4+1}=\sqrt{6}$$

Shortest distance is '6' $\qquad \frac{|[\bar{a}-\bar{c}\ \ \bar{b}\ \ \bar{d}]|}{|\bar{b}\times\bar{d}|}=6\Rightarrow|\lambda-4\sqrt{6}|=6\sqrt{6}$

$\lambda-4\sqrt{6}=\pm6\sqrt{6}\Rightarrow\lambda=10\sqrt{6},\lambda=-2\sqrt{6}$

Sum of squares of all possible values of λ is

$(10\sqrt{6})^2+(-2\sqrt{6})^2=600+24=624$

87. Let f be a differentiable function defined on $\left[0,\frac{\pi}{2}\right]$ such that f(x) > 0 and

$f(x)+\int_0^x f(t)\sqrt{1-(\log_e f(t))^2}\,dt=e,\ \forall x\in\left[0,\frac{\pi}{2}\right]$. Then $\left(6\log_e f\left(\frac{\pi}{6}\right)\right)^2$ is equal to _

Key:- 27

Sol:- $f(x)+\int_0^x f(t)\sqrt{1-(\log_e f(t))^2}\,dt=e$

By Leibnitz condition

$f^1(x)+f(x)\sqrt{1-(\log_e f(x))^2}(1)-0=0$ $\quad f^1(x)+f(x)\sqrt{1-(\log_e f(x))^2}=0$

$f^1(x)=-f(x)\sqrt{1-(\log_e f(x))^2}$ $\quad \frac{1}{\sqrt{1-(\log_e f(x))^2}}\frac{f^1(x)}{f(x)}=-1$

Integrate on both sides

$\int \frac{1}{\sqrt{1-(\log_e f(x))^2}}\frac{f^1(x)}{f(x)}dx=-\int dx$ $\quad \log_e f(x)=t$

Differentiate on both sides w.r.t to x $\quad \frac{f^1(x)}{f(x)}dx=dt$ $\quad \int \frac{1}{\sqrt{1-t^2}}dt-\int dx$

$\sin^{-1}(t)=-x+c$ $\quad \sin^{-1}(\log_e f(x))=-x+c.....(1)$

Put $x=0 \Rightarrow \sin^{-1}(\log_e f(0))=0+C$ $\quad \because f(0)=e$ $\quad \sin^{-1}(1)=c \Rightarrow c=\frac{\pi}{2}$

From (1)

$\sin^{-1}(\log_e f(x))=-x+\frac{\pi}{2}$ $\log_e f(x)=\sin\left(\frac{\pi}{2}-x\right)$ $\quad \log_e f(x)=\cos x$

Put $x=\frac{\pi}{6}$ $\quad \log_e f\left(\frac{\pi}{6}\right)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

$6\log_e f\left(\frac{\pi}{6}\right)=6\left(\frac{\sqrt{3}}{2}\right)=3\sqrt{3}$

S.O.B $\quad \left(6\log_e f\left(\frac{\pi}{6}\right)\right)^2=(3\sqrt{3})^2=27$

88. If $\frac{1^3+2^3+3^3+....\text{up to n terms}}{1.3+2.5+3.7+....\text{up to n terms}}=\frac{9}{5}$, then the value of n is

Key:- 5

Sol:- $\frac{1^3+2^3+3^3+..... \text{ up to n terms}}{1.3+2.5+3.5+.... \text{ up to n terms}}=\frac{9}{5}$

$1^3+2^3+3^3+..... \text{ up to n terms}=\frac{n^2(n+1)^2}{4}$

$1.3+2.5+3.5+.....\text{up to n terms}=\frac{n(n+1)(2n+1)}{3}+\frac{n(n+1)}{2}$

$t_n=n(2n+1)=2n^2+n$

$S_n=\sum n=\sum n^2+\sum n=\frac{2(n(n+1)(2n1)}{6}+\frac{n(n+1)}{2}$

$\frac{\frac{n^2(n+1)^2}{4}}{\frac{n(n+1)(2n+1)}{3}+\frac{n(n+1)}{2}}=\frac{9}{5}$ $\quad \frac{\frac{n(n+1)}{4}}{\frac{2n+1}{3}+\frac{1}{2}}=\frac{9}{5}$ $\quad \Rightarrow \frac{\frac{n^2+n}{4^2}}{\frac{4n+2+3}{6}}=\frac{9}{5}$

$$\frac{n^2+n}{2}\times\frac{3}{4n+5}=\frac{9}{5} \qquad 5n^2+5n=24n+30$$

$$5n^2-19n-30=0 \qquad 5n^2-25n+6n-30=0$$

$$5n(n-5)+6(n-5)=0$$

$$(5n+6)(n-5)=0\Rightarrow n=5$$

89. Three urns A, B and C contain 4 red, 6 block, 5 red, 5 block, and λ red, 4 block balls respectively. One of the urns is selected at random and a ball is drawn. If the ball drawn is red and the probability that it is drawn from urn C is 0.4, then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola $y^2 = \lambda x$ with one vertex at the vertex of the parabola is__

Key:- 432

Sol:-

Urn	Red	Black
A	4	6
B	5	5
C	λ	4

Let E be the event to set red ball from 'C'

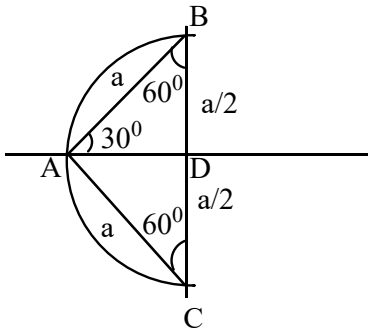
$$P(C/E) = \frac{P(C)\cdot P(E/C)}{P(A)P(E/A)+P(B)P(E/B)+P(C)P(E/C)}$$

Given $0.4 = \dfrac{\frac{1}{3}\times\frac{\lambda}{\lambda+4}}{\frac{1}{3}\left(\frac{4}{10}+\frac{5}{10}+\frac{\lambda}{\lambda+4}\right)}$

$$\frac{4}{10}\left(\frac{9}{10}+\frac{\lambda}{\lambda+4}\right)=\frac{\lambda}{\lambda+4}$$

$$76\lambda+144=100\lambda \Rightarrow 24\lambda=144 \Rightarrow \lambda=6$$

Given parabola $y^2 = \lambda x \Rightarrow y^2 = 6x$



$$\sin 30^0 = \frac{BD}{a} \Rightarrow BD = \frac{a}{2}$$

$$\cos 30^0 = \frac{AD}{a} \Rightarrow AD = \frac{\sqrt{3}}{2}a$$

$B = \left(\frac{\sqrt{3}}{2}a, \frac{a}{2}\right)$ lies on parabola $y^2 = 6x$

$$\frac{a^2}{4} = 6\left(\frac{\sqrt{3}a}{2}\right) \Rightarrow a = 12\sqrt{3}$$

$$a^2 = 144\times 3 \Rightarrow a^2 = 432$$

90. The minimum number of elements that must be added to the relation $R = \{a,b),(b,c),(b,d)\}$ on the set $\{a,b,c,d\}$ so that it is an equivalence relation is _____

Key:- 13

Sol:- Let $A = \{a,b,c,d\}$

Given $R = \{(a,b),(b,c),(b,d)\}$

We can made R as equivalence

We have to add order pairs (a,a), (b,b), (c,c), (d,d), (b,a), (c, b), (d, b), (a, c), (c, a), (c, d), (d, c)

(a, d), (d, a)

Then R = {(a,a), (b,b), (c,c), (d,d), (b,a), (c, b), (d, b), (a, c), (c, a), (c, d), (d, c), (a, d), (d, a)}

Then R is equivalence

Minimum number of ordered pairs = 13