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ALL INDIA RANKS IN JEE-ADVANCED 2022

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JEE MAIN (JAN) 2023 (25-01-2023-Session-2)

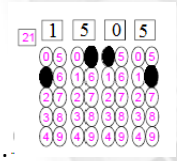
MATHEMATICS, PHYSICS & CHEMISTRY

Jee-Main_25-JAN-2023_Shift-02

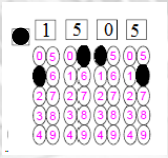
IMPORTANT INSTRUCTION:

1. Immediately fill in the Admission number on this page of the Test Booklet with **Blue/Black** Ball Point **Pen** only.
2. The candidates should not write their Admission Number anywhere (except in the specified space) on the Test Booklet/ Answer Sheet.
3. The test is of **3 hours** duration.
4. The Test Booklet consists of 90 questions. The maximum marks are **300.**
5. There are **three** parts in the question paper 1, 2, 3 consisting of **Physics, Chemistry** and **Mathematics** having **30 questions** in each subject and subject having **two sections.**
   (I) Section –I contains 20 **multiple choice** questions with only one correct option.
   Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.
   (II) Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only, if more than 5 questions attempted, First 5 attempted questions will be considered.

■ The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i, e. If answer is above **10** and less than **10.5** round off is **10** and If answer is from **10.5** and less than **11** round off is **11).**
To cancel any attempted question bubble on the question number box.
For example: To cancel attempted question 21. Bubble on 21 as shown below



Question Answered for Marking



Question Cancelled for Marking

Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases.

6. Use **Blue / Black Point Pen only** for writing particulars / marking responses on the Answer Sheet. **Use of pencil is strictly prohibited.**
7. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electron device etc, except the Identity Card inside the examination hall.
8. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
9. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Hall. **However, the candidate are allowed to take away this Test Booklet with them.**
10. **Do not fold of make any stray marks on the Answer Sheet**

PHYSICS

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

01. Every planet revolves around the sun in an elliptical orbit

A. The force acting on a planet is inversely proportional to square of distance from sun

B. Force acting on planet is inversely proportional to product of the masses of the planet and the sun

C. The Centripetal force acting on the planet is directed away from the sun

D. The square of time period of revolution of planet around sun is directly proportional to cube of semi – major axis of elliptical orbit

1) C and D Only **2)** A and D only **3)** B and C only **4)** A and C only

Key 2

Ans: $F = \frac{GmM}{r^2}$

Force acting on a planet is inversely proportional to square of distance from sun (Statement A)

The Square of time period of revolution of planet around sun is directly proportional to cube of semif major axis of elliptical orbit (statement D)

02. A body of mass is taken from earth surface to the height h equal to twice the radius of earth (R_e), the increase in potential energy will be (g = acceleration due to gravity on the surface of Earth)

1) $\frac{2}{3}mgR_e$ **2)** $3mgR_e$ **3)** $\frac{1}{3}mgR_e$ **4)** $\frac{1}{2}mgR_e$

Key 1

Sol $\frac{-GmM}{3R}-\left(\frac{-GmM}{R}\right)=\frac{2}{3}\frac{GmM}{R}.\frac{R}{R}=\frac{2}{3}mgR$

03. Match List I with List II

List I	List II
A. Gauss's law in Electrostatics	I. $\oint \vec{E}.\vec{dl} = -\frac{d\phi_B}{dt}$
B. Faraday' law	II. $\oint \vec{B}.\vec{dA} = 0$
C. Gauss's law in Magnetism	III. $\oint \vec{B}.\vec{dl} = \mu_0 i_c + \mu_0 \in_0 \frac{d\phi_E}{dt}$
D. Ampere –Maxwell Law	IV. $\oint \vec{E}.\vec{ds} = \frac{q}{\in_0}$

Choose the correct answer from the options given below

1. A – I, B – II, C – III, D - IV **2)** A – IV, B – I, C – II, D - III

3) A – II, B – III, C – IV, D -I **4)** A – III, B – IV, C – I, D - II

Key 2

Sol: Gauss law in electrostatic $\oint \vec{E}.\overrightarrow{ds} = \frac{q}{\varepsilon_0}$

Faradays law $\oint \vec{E}.\overrightarrow{dl} = \frac{d\phi_0}{dt}$

Gauss law in magnetism $\oint \vec{B}.\overrightarrow{dA} = 0$

Ampere Maxwell law $\oint \vec{B}.\overrightarrow{dl} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$

04) The distance travelled by a particle is related to time t as $x = 4t^2$. The body of the particle at t = 5s is

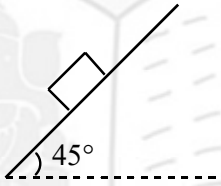
1) $40ms^{-1}$ **2)** $25ms^{-1}$ **3)** $8ms^{-1}$ **4)** $20ms^{-1}$

Key 1

Sol: $x = 4t^2 \Rightarrow v = \frac{dx}{dt} = 8t$

Velocity at t = 5sec is v = 8x 5 = 40m/s

05. Consider a block kept on an inclined plane (inclined at 45°) as shown in the figure. If the force required to just push it up the incline is 2 times the force required to just prevent it from sliding down, the coefficient of friction between the block and inclined plane (μ) is equal to



1) 0.50 **2)** 0.33 **3)** 0.25 **4)** 0.60

Key 2

Sol: Let F_1 be the force required to just prevent it from sliding $F_1 = mg\sin\theta - \mu mg\cos\theta$

Let F_2 be the minimum force to push it up the incline

$F_2 = mg\sin\theta + \mu mg\cos\theta$

$F_2 = 2F_1$ $\quad mg\sin\theta + \mu mg\cos\theta = 2(mg\sin\theta - \mu mg\cos\theta)$

$1 + \mu = 2(1 - \mu)$ $\quad \mu = \frac{1}{3}$

06. The light rays from an object have been reflected towards an observer from a standard flat mirror the image observed by the observer are:

A. Real **B.** Erect **C.** Smaller in size then object **D.** Laterally inverted

Choose the most appropriate answer from the options given below

1. A and D only **2)** A,C and D only **3)** B and D only **4)** B and C only

Key 3

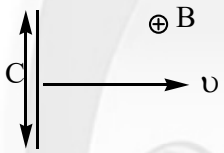
Sol: For real object virtual and erect image formed by a plane mirror

07. A wire of length 1m moving with velocity 8m/s at right angles to a magnetic field of 2T. The magnitude of induced emf, between the ends of wire will be

1) 20V **2)** 16V **3)** 8V **4)** 12V

Key 2

Sol:



$\text{enf} = B l \upsilon = 16 \text{ volt}$

08. The resistance of a wire is 5Ω. Its new resistance in ohm if stretched to 5 times of its original length will be

1) 625 **2)** 25 **3)** 125 **4)** 5

Key

Sol: Incase of stretching $R \alpha l^2$

$l' = 5l \Rightarrow R' = 25R = 125\Omega$

09. Two objects are projected with same velocity 'u' however at different angles α and β with the horizontal. If $\alpha + \beta = 90°$, the ratio of horizontal range of the first object to the 2nd object will be

1) 4:1 **2)** 1:1 **3)** 2:1 **4)** 1:2

Key 2

Sol: For two different angles of projection α and $90 - \alpha$, Range is same provided initial speed is unchanged

$$R = \frac{u^2 \sin 2\alpha}{2g} \left[\sin 2\alpha = \sin 2(90 - \alpha)\right]$$

10. A particle executes simple harmonic motion between x = -A and x = +A . If time taken by particle to go from x = 0 to A/2 is 2s; then time taken by particle in going from x = A/2 to A is

1) 3s **2)** 1)5s **3)** 4s **4)** 2s

Key 3

Sol: Time taken to move from near position to A/2 is 2 sec

$$\frac{\pi}{6}-\frac{1}{\omega}=2\sec$$

Time taken to move from A/2 to A = $\frac{\pi}{3}\frac{1}{\omega}=2\left(\frac{\pi}{6}\frac{1}{\omega}\right)=(2)(2)=4$

11. Match List I with List II

List I	List II
A. isothermal process	I. Work done by the gas decreases internal energy
B. Adiabatic process	II. No change in internal energy
C. Isochoric process	III. The heat absorbed goes partly to increase internal energy and partly to do work
D. Isobaric process	IV. No work is done on or by the gas

Choose the correct answer from the options given below

1. A – I, B- II, C – IV, D -III **2)** A – II, B-I, C – IV, D -III

3) A – II, B- II, C- III, D-IV **4)** A – I, B – II, C- III, D - IV

Key 2

Sol: **A.** In isothermal process, the temperature remains constant. So hence change in internal energy is zero. (internal energy $d\theta = nCv\Delta T$ here$(\Delta T = 0)$

B. In adiabatic process Q = 0 work done on it. And gas does work and temperature drops so hence, decrease in internal energy

C. In iso-choric process, volume is constant. So hence work done by the gas (or) on the gas equal to zero, we = $\int PdV \Rightarrow dV = 0 \Rightarrow w = 0$

D. In iso-baric process pressure is constant. So 1st law of thermodynamics $d\theta = du + P\int dv$

12. **Statement I:** When a Si sample is doped with Boron, it becomes P type and when doped by Arsenic it becomes N – type semiconductor such that P – type has excess holes and N – type has excess electrons

Statement II: When such P – type and N – type semi conductors, are fused to make a junction a current will automatically flow which can be detected with an exactly connected ammeter.

1) Both Statement I and Statement II are correct

2) Both Statement I and Statement II are incorrect

3) Statement I is correct but Statement II is incorrect

4) Statement I is incorrect but statement II is correct

Key 1

Sol: Statement I: Is correct and statement II is correct

i) When silicon has four velancy electrons and boron has three valency electrons. So these three valency electrons bond with four electrons of silicon atom taken. This creates a hole (or) valency. As a result p – type semi – conductor formed

ii) Where as arsenic has a penta valent, so dopping with silicons, they remains one free electrons. Thus n – type is formed

iii) p – side has excess of holes and n – type excess of electrons in the outer shells of electrically neutral. This allows electric current pakes through the junction. Only in one direction

13. Given below are two statements

Statement I: Stopping potential in photoelectric effect does not depend on the power of the light source

Statement II: For a given metal, the maximum kinetic energy of the photoelectron depends on the wavelength of the incident light

1)Both Statement I and Statement II are incorrect

2) Statement I is incorrect but Statement II is correct

3) Both Statement I and Statement II are correct

4) Statement I is correct but Statement II is incorrect

Key 3

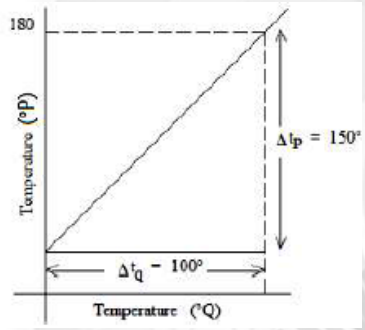
Sol: Stopping potential is Depends on frequency of hight, Doesn't depends on intensity of light. So hence I = P/A and power p = I(A)

So hence power photo – electric effect doesn't depends on power of a digit source. St-I correct

The maximum kinetic energy of photo electrons is given by $KE = h\nu - \omega_0$ here υ = frequency of incident light radiation

So kinetic energy depends on frequency and wave length

14. The graph between two temperature scales P and Q is shown in the figure. Between upper fixed point and lower fixed point there are 150 equal divisions of scale P and 100 divisions on scale Q. The relationship for conversion between the two scales is given by



1) $\frac{t_Q}{150} = \frac{t_p - 180}{100}$ **2** $\frac{t_Q}{100} = \frac{t_p - 30}{150}$ **3)** $\frac{t_P}{100} = \frac{t_Q - 180}{150}$ **4)** $\frac{t_P}{180} = \frac{t_Q - 40}{100}$

Key 2

Sol: **from graph** $t_q = 0, t_A = 30°$

$$\frac{t_p = 30°}{180° - 30°} = \frac{t_q = 0°}{100 - 0} \Rightarrow \frac{t_p - 30}{150} = \frac{t_q}{100}$$

15. Match List I with List II

List I	List II
A. Troposphere	I. Approximate 65-75 km over Earth's surface
B. E – Part of Stratosphere	II. Approximate 300 km over Earth's surface
C. F_2 – Part of Thermosphere	III. Approximate 10 km over Earth's surface
D. D- Part of Stratosphere	IV. 100 km over Earth Surface

Choose the correct answer from the options given below

1. A – III, B – II, C – I, D - IV **2)** A – III, B – IV, C – II,

3) A – I, B – II, C – IV, D - III **4)** A – I, B – IV, C – III, D -II

Key 2

Sol: A. Tropo sphere range nearly 6 to 12 cm from the earth surface

B. c-part of stratosphere 100km

C. F2 – part o approximate 300km of earth surface in night time and 250-400 km. During day time

D. D – part is 65-75 km above

16. For a moving coil galvanometer the deflection in the coil is 0.05 rad when a current of 10mA is passed through it. If the torsional constant of suspension wire is $4.0\times10^{-5}\,\text{N m rad}^{-1}$, the magnitude field is 0.01T and the number of turns in the coil is 200, the area of each turn (in cm^2) is

1) 1)5 **2)** 1)0 **3)** 0.5 **4)** 2)0

Key 2

Sol: $CO = BIAN$

$$4\times10^{-5}\times0.05 = 0.01\times10\times10^{-3}\times A\times200$$

$$A = 1.0\times10^{-4}\,\text{m}^2$$

$$A = 1.0\,\text{cm}^2$$

17. According to law of equipartition of energy the molar specific heat of a diatomic gas at constant volume where the molecule has one additional vibrational mode is

1) 7/2 R **2)** 9/2 R **3)** 5/2 R **4)** 3/2 R

Key 1

Sol: $C_V = \frac{1}{n}\frac{dU}{dT} \Rightarrow \frac{1}{n}\frac{d}{dT}\left[\frac{5}{2}nRT + nRT\right] = \frac{7R}{2}$

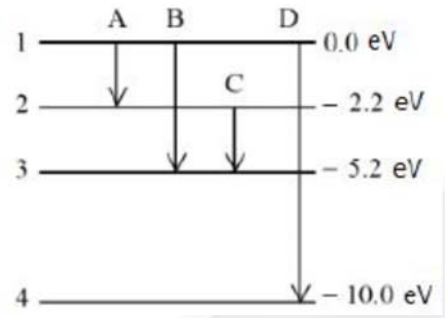
18. A point charge of $10\mu C$ is placed at the origin. At what location on the X-axis should a point charge of $40\mu C$ be placed so that the net electric field is zero at x = 2cm on the X-axis?

1) x = 4cm **2)** x= 6cm **3)** x =- 4cm **4)** x = 8cm

Key 2

Sol: Null point distance $x = \frac{d}{\sqrt{\frac{q_2}{q_1}}+1}\,2 = \frac{d}{\sqrt{\frac{40}{10}}+1} = \frac{d}{3} \Rightarrow d = 6\text{cm},$

19. The energy levels of an atom is shown in figure



Which one of these transitions will result in the emission of a photon of wavelength 124)1 cm? (Given $h = 6.62 \times 10^{-34}$ Js)

1. C **2)** D **3)** B **4)** A

Key 2

Sol: $E = \frac{1241}{\lambda(nm)} ev \Rightarrow \frac{1241}{124.1} e.v \Rightarrow E = 10.0 eV$

20. Match List I with List II

List I	List II
A. Young's Modulus (Y)	I. $\left[ML^{-1}T^{-1}\right]$
B. Coefficient of Viscosity (η)	II. $\left[ML^{2}T^{-1}\right]$
C. Plank's Constant (h)	III. $\left[ML^{-1}T^{-2}\right]$
D. Work function (ϕ)	IV. $\left[ML^{2}T^{-2}\right]$

Chooser the correct answer from the options given below

1. A - III, B – I, C – II, D- IV **2)** A – I, B – III, C – IV,

3) A – II, B – III, C – IV, D - I **4)** A – I, B – II, C – III, D -IV

Key 1

Sol: $y = \frac{F}{A} = \frac{MLT^{-2}}{L^2} = \left[ML^{-1}T^{-2}\right]$

$\eta = \frac{F}{A\frac{dv}{dx}} = \left[ML^{-1}T^{-1}\right]$

$h = ET = \left[ML^{2}T^{-1}\right]$ $\omega = \left[ML^{2}T^{-2}\right]$

(NUMERICAL VALUE TYPE)

Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i, e. If answer is above **10** and less than **10.5** round off is **10** andIf answer is from **10.5** and less than **11** round off is **11**).
Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases

21. A Train blowing a whistle of frequency 320Hz approaches an observer standing on the platform at a speed of 66 m/s. The frequency observed by the observer will be (given speed of sound = 330ms^{-1})……. Hz

Key **400**

Sol: $f' = f\left(\frac{v}{v - v_3}\right) = 320\left(\frac{330}{330 - 66}\right) = 400\text{Hz}$

22. A body of mass 1kg collides head on elastically with a stationary body of mass 3kg. After collision, the smaller body reverse its direction of motion and moves with a speed of 2m/s. The initial speed of the smaller body before collision is……..ms^{-1}.

Key **4**

Sol:

$m_1 = 1\text{kg} \qquad m_2 = 3\text{kg}$

$u_1 = 0 \qquad u_2 = 0$

$v_1 = -2\text{m/s}$

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$$

$$-2 = \left(\frac{1-3}{4}\right)u \Rightarrow u = 4\text{m/s}$$

23. A spherical drop of liquid splits into 1000 identical spherical drops. If u_i is the surface energy of the original drop and u_f is the total surface energy of the resulting drops, the (ignoring evaporation) $\frac{u_f}{u_i} = \left(\frac{10}{x}\right)$. Then value of x is……

Key **10**

Sol: $v_{big} = n\,v_{small} \Rightarrow R^3 = nr^3 \Rightarrow R = n^{1/3}r$

R = 10r

$u_1 = 4\pi R^2 T$

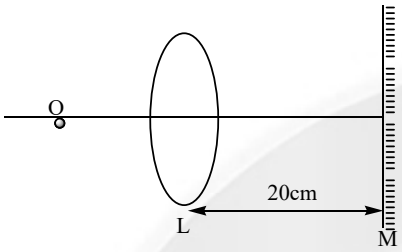
$u_2 = n4\pi r^2 T$

$$\frac{u_f}{u_1} = \frac{nr^2}{R^2} = n\left(\frac{r}{R}\right)^2$$

$$\frac{u_f}{u_i} = \frac{n}{n^{2/3}} = n^{1/3}$$

$$\frac{u_f}{u_i} = 10$$

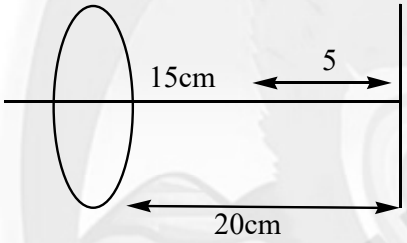
24. An object is placed on the principal axis of convex lens of focal length 10cm as shown. A plane mirror is placed on the other side of lens at a distance of 20cm. The image produced by the plane mirror is 5cm inside the mirror. The distance of the object from the lens is……cm



Key 30

Sol: Image distance due lens = 20 – 5 = 15cm

We have $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$



$$\frac{1}{10}=\frac{1}{15}+\frac{1}{x}\Rightarrow\frac{1}{10}-\frac{1}{15}=\frac{1}{x}$$

x=30cm

25. If a solid sphere of mass 5kg and a disc of mass 4kg have the same radius. Then the ratio of moment of inertia of the disc about a tangent in its plane to the moment of inertia of the sphere about its tangent will be x/7. Then the value of x is……..

Key 5

Sol: For disc about a tangent in its plane $I_d=\frac{5}{4}m_1R^2=\frac{5}{4}\times4R^2=5R^2$

For solid sphere about tangent $I_b=\frac{7}{5}m_2R^2=\frac{7}{5}\times5R^2=7R^2$ $\quad\frac{I_d}{I_s}=\frac{5}{7}=5$

26. A capacitor has capacitance 5μF when it's parallel plates are separated by air medium of thickness d. A slab of material of dielectric constant 1.5 having area equal to that of plates but thickness d/2 is…… μF

Key 6

Sol: $C'=\frac{\in_0 A}{d-t\left(1-\frac{1}{k}\right)}\Rightarrow C'=\frac{\in_0 A}{d-\frac{d}{2}\left(1-\frac{2}{3}\right)}$

$$C'=\frac{6\in_0 A}{5d}=\frac{6C}{5}\Rightarrow C'=\frac{6}{5}\times5=6\mu F$$

27. Two long parallel wires carrying currents 8A and 15A in opposite directions are placed at a distance of 7cm from each other. A point P is at equidistant from both the wires such that the lines joining the point P to the wires are perpendicular to each other. The magnitude of magnetic field at P is ……..$\times 10^{-6}$T

Key 68

Sol: $B = \sqrt{B_1^2 + B_2^2} \Rightarrow B = \frac{\mu_0}{2\pi d}\sqrt{I_1^2 + I_2^2} = \frac{2\times 10^{-7}\times\sqrt{2}}{7\times 10^{-2}}\times 17$

$= 68\times 10^{-6}$T

28. A nucleus disintegrates into two smaller parts, which have their velocities in the ratio 3:2. The ratio of their nuclear sizes will be $\left(\frac{x}{3}\right)^{1/3}$. The value of x is

Key 2

Sol: $P = mV = \cos t \Rightarrow m\alpha\frac{1}{V}$

$A\alpha\frac{1}{V} \Rightarrow R^3\alpha\frac{1}{V} \Rightarrow R\alpha\frac{1}{V^{1/3}}$

$\frac{R_1}{R_2} = \left(\frac{V_2}{V_1}\right)^{1/3} \Rightarrow \frac{R_1}{R_2} = \left(\frac{2}{3}\right)^{1/3}$

29. A Series LCR circuit is connected to an AC source of 220V , 50Hz. The circuit contains a resistance R = 80Ω an inductor of inductive resistance $X_L = 130\Omega$. The power factor of circuit is $\frac{x}{10}$ The value of x is…..

Key 8

Sol: $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} \Rightarrow \cos\phi = \frac{80}{\sqrt{80^2 + (X_L - X_C)^2}}$

$\cos\phi = \frac{8}{10}$

30. Two cells are connected between points A and B as shown. Cell 1 has emf of 12V and internal resistance of 3Ω. Cell 2 has emf of 6V, and internal resistance of 6Ω. An external resistor R of 4Ω is connected across A and B. The current flowing through R will be…….A

Key 1

Sol: $\Sigma_{eff} = \frac{\Sigma\frac{\varepsilon}{r}}{\Sigma\frac{1}{r}} = \frac{\frac{12}{3} - \frac{6}{6}}{\frac{1}{3} + \frac{1}{6}} = \frac{3}{1/2} = 6V$ $\qquad r_{eff} = \frac{3\times 6}{3+6} = 2\Omega \Rightarrow I = \frac{\Sigma_{eff}}{r_{eff} + R} = \frac{6}{2+4} = 1A$

CHEMISTRY

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

31. When the hydrogen ion concentration $[H^+]$ changes by a factor of 1000, the value of Ph of the solution

1) increases by 2 units **2)** Increases by 1000 units

3) decreases by 2 units **3)** Decreases by 3 units

Key 4

Sol: $PH_1 = -\log\left(H^+\right)_1$

If I will take $\left(H^+\right)_1 = 10^{-4}$ then $PH_1 = 4$

$PH_2 - \log\left(H^+\right)_2 = -\log\left(10^{-4} \times 10^3\right) = -\log 10^{-1} = \mathbf{1}$

PH decreases by 3 units

32. Which one among the following metals is the weakest reducing agent ?

1) Li **2)** Na **3)** K **4)** Rb

Key 2

Sol: : Among alkali metals Li in powerful radius agent because of its high enthalpy of hydration and Na is weak reducing agent

The reducing power depends on enthalpy of fusion enthalpy of vaporization enthalpy of ionization and enthalpy of hydration

33. Match List I with List II

LIST I		**LIST II**	
A	Cobalt catalyst	**I**	(H_2+C1_2) production
B	Syngas	**II**	Water gas production
C	Nickel catalyst	**III**	Coal gasification
D	Brine solution	**IV**	Methanol production

Choose the correct answer from the option given below :

1) A-IV; B-I; C-II; D-III **2)** A-IV; B-III; C-I; D-II

3) A-II; B-III; C-IV; D-I **4)** A-IV; B-III; C-II; D-I

Key 4

Sol: Memory based

34. What is the mass ration of ethylene glycol ($C_2H_6O_2$, molar mass = 62 g/mol) required for making 500 g for 0.25 molal aqueous solution and 250 Ml of 0.25 molal aqueous solution ?

1) 1:1 **2)** 2:1 **3)** 3:1 **4)** 1:2

Key 2

Sol: $\frac{m_1}{m_2} = \frac{n_1 / w_1}{n_2 / w_2} \Rightarrow \frac{0.25}{0.25} = \frac{n_1}{n_2} \times \frac{250}{500}$

$\frac{n_1}{n_2} = \frac{2}{1} = \frac{(\text{mass of solute})_1}{(\text{mass of solute})_2}$

35. Given below are two statements, one is labeled as **Assertion A** and the other is labeled as **Reason R.**

Assertion A : The alkali metals and their salts impart characteristic colour to reducing flame.

Reason R : Alkali metals can be detected using flame tests.

In the light of the above statements, choose the most appropriate answer from the options given below

1) A is correct but R is not correct

2). Both A and R are correct and R is the correct explanation of A

3) Both A and R are correct but R is NOT the correct explanation of A

4) A is not correct but R is correct

Key 4

Sol: The alkalimetas and their salts impart characteristic colour to an oxidizing flame. This is because that heat from the flame exits the outermost orbital electron cause back to the ground state, they emit the approved energy in the form of light in visible region

36. Which of the following represents the correct order of metallic character of the given elements ?

1) Be < Si < Mg < K **2)** K < Mg < Be < Si

3) Si < Be < Mg < K **4)** Be < Si < K < Mg

Key 3

Sol: Metallic nature in groups TOP to bottom increases

Metallic nature in periods left to right decreases

37. Match List I with List II

LIST I		**LIST II**	
A	Aniline	**I**	**3)**25
B	Ethanamine	**II**	**3)**00
C	N-Ethylethanamine	**III**	9.38
D	N, N-Diethylethanamine	**IV**	**3)**29

Choose the correct answer from the option given below :

1) A-III; B-II; C-I; D-IV **2)** A-I; B-IV; C-II; D-III

3) A-III; B-IV; C-II; D-I **4)** A-III; B-II; C-IV; D-I

Key 2

Sol: Aliphatic amines are more basic than aromatic

$2° > 3° > 1°$ basicity order $NH(Et)_2 > N(Et)_3 > EtNH_2$

38. A chloride salt solution acidified with dil. HNO_3 given a curdy white precipitate, [A], on addition of $AgNO_{3)}$ [A] on treatment with NH_4OH gives a clear solution, B. A and B are respectively

1) AgC1 & $[Ag(NH_{3)2}]C1$ **2)** AgC1 & $(NH_{4)}[Ag(OH)_2$

3) $H[AgC1_3]$ & $(NH_{4)}[Ag(OH)_2]$ **4)** $H[AgC1_3]$ & $[Ag(NH_{3)2}]C1$

Key 1

Sol: $AgNO_3$ and $[Ag(NH_3)_2]Cl$

$$AgNO_3 + NaCl \rightarrow AgCl + NaNO_3$$

$$AgCl \xrightarrow{NaOH} [Ag(NH_3)_2]Cl$$

39. Given below are two statements, one is labellaed as **Assertion A** and the other is labelled as **Reason R**

Assertion A : Carbon forms two important oxides – CO and CO_2, CO is neutral whereas CO_2 is acidic in nature

Reason R : CO_2 can combine with water in a limited way to form carbonic acid, while CO is sparingly soluble in water

1) Both A and R are correct and R is the correct explanation of A

2 A is not correct but R is correct

3) A is correct but R is not correct

4) Both A and R are correct but R is NOT the correct explanation of A

Key: 1

Both A and R correct and R is the correct explanation of A. CO. neutral CO_2 acidic

$$CO_2 + H_2O \rightarrow H_2CO_3 \rightleftharpoons H^+ + HCO_3^{2-}$$

CO sparingly soluble salt

40. **A.** Ammonium salts produce haze in atmosphere

B. Ozone gets produced when atmospheric oxygen reacts with chlorine radicals

C. Polychlorinated biphenyl is act as cleansing solvents.

D. 'Blue baby' syndrome occurs due to the presence of excess of sulphate ions in water.

Choose the correct answer from the options given below :

1) A and C only **2)** A, B and C only **3)** B and C only **4)** A and D only

Key: 1

Ammonium salts produce have in atmosphere

Polychlorinated biphenyls acts as clensing agent in case of metals

41. **Match List I with List II**

LIST I Coordination entity		**LIST II Wavelength of light absorbed in nm**	
A	$[CoC1(NH_3)_5]^{2+}$	**I**	310
B	$[Co(NH_3)_6]^{3+}$	**II**	475
C	$[Co(CN)_6]^{3-}$	**III**	535
D	$[Cu(H2O)_4]^{2+}$	**IV**	600

Choose the correct answer from the options given below

1) A-II; B-III; C-IV; D-I **2)** A-III; B-I; C-II; D-IV

3) A-IV; B-I; C-III; D-II **4)** A-III; B-II; C-I; D-IV

Key: 4

Based on NCERT Part I (XII) page no 259

$\left[CoCl(NH_3)_5\right]^{2+}$ 535

$[Co(NH_3)_6]^{+3}$ 475

$\left[CO(CN)_6\right]^{-3}$ 310

$[Cu(H_2O)_4]^{2+}$ 600

correct answer is 4

42. **Statement I :** Dipole moment is a vector quantity and by convention it is depicted by a small arrow with tail on the negative centre and head pointing towards the positive centre.

Statement II : The crossed arrow of the dipole moment symbolizes the direction of the shift of charges in the molecules.

In the light of the above statements, choose the most appropriate answer from the options given below:

1) Statement I is incorrect but Statement II is correct

2) Both Statement I and Statement II are correct

3) Statement I is correct but Statement II is incorrect

4) Both Statement I and Statement II are incorrect

Key: 3

Sol: small arrow is pointing from negative centre to positive centre but the cross arrow is opposite

43. Potassium dichromate acts as a strong oxidizing agent in acidic solution. During this process, the oxidation state changes from

1) +2 to +1 **2) +6 to +3** **3) +3 to +1** **4) +6 to +2**

Key: 2

Sol:

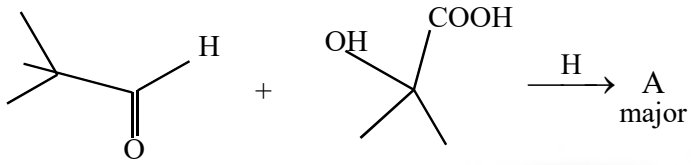
$Cr_2O_4^{2-} \rightarrow 2Cr^{+3}$

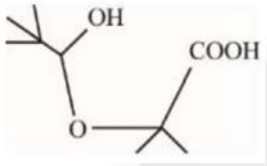
0.5 of Cr in $Cr_2O_7^{2-}$ $2x + 7(-2) = -2$

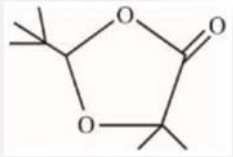
$2x = +12 \Rightarrow x = +6$

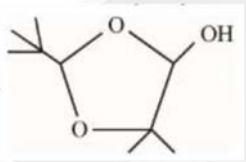
0.5 g Cr in Cr^{+3} in $+3$

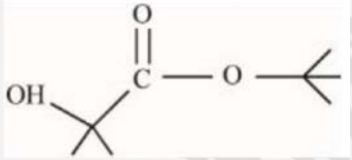
44. A in the given reaction is



1) 

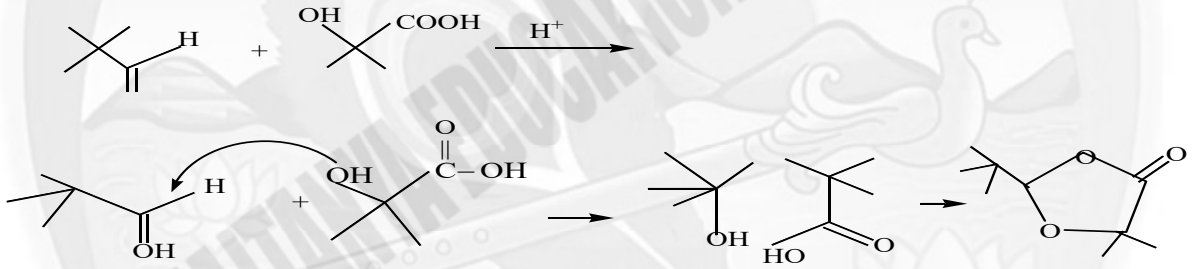
2) 

3) 

4) 

Key: 2

Sol:



45. Match List I with List II

LIST I Isomeric pairs		**LIST II Type of isomers**	
A	Propanamine and N-Methylethanamine	**I**	Metamers
B	Hexan-2-one and Hexan-3-one	**II**	Positional isomers
C	Ethanamide and Hydroxyethanimine	**III**	Functional isomers
D	o-nitrophenol and p-nitrophenol	**IV**	Tautomers

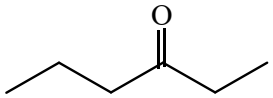
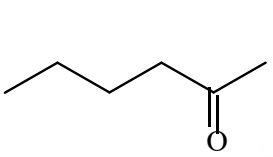
Choose the correct answer from the options given below

1) A-IV; B-III; C-I; D-II **2)** A-III; B-I; C-IV; D-II

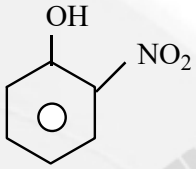
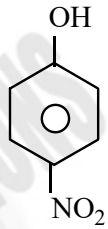
3) A-II; B-III; C-I; D-IV **4)** A-III; B-IV; C-I; D-II

Key: 2

Sol: 1° and 2° amines are F – I

Metamers


and


Ethanamide and hydroxyl ethanomine are tautomers are positional isomers

46. Given below are two statements, one is labeled as Assertion **A** and the other is labeled as **Reason R.**

Assertion A : Butylated hydroxyl anisole when added to butter increase its shelf life.

Reason R : Butylated hydroxyl anisole is more reactive towards oxygen than food

In the light of the above statements, choose the most appropriate answer from the option given below.

1) Both A and R are correct but R is NOT the correct explanation of A

2) Both A and R are correct and R is the correct explanation of A

3) A is not correct but R is correct**.**

4) A is correct but R is not correct

Key: 2

Sol: BHA help in food preservation by retarding the action of oxygen on food

47. The isomeric deuterated bromide with molecular formula C_4H^8DBr having two chiral carbon atoms is

1) 2 – Bromo – 1 – deuterobutane

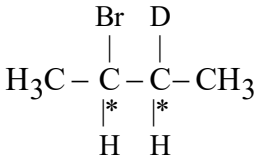
2) 2 - Bromo – 2 – deuterobutane

3) 2 – Bromo – 3 – deuterobutane

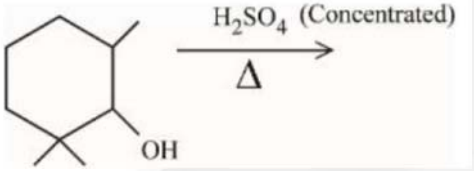
4) 2 – Bromo – 1 – deuteron – 2 – methylpropane

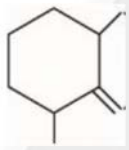
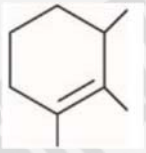
Key: 3

Ans



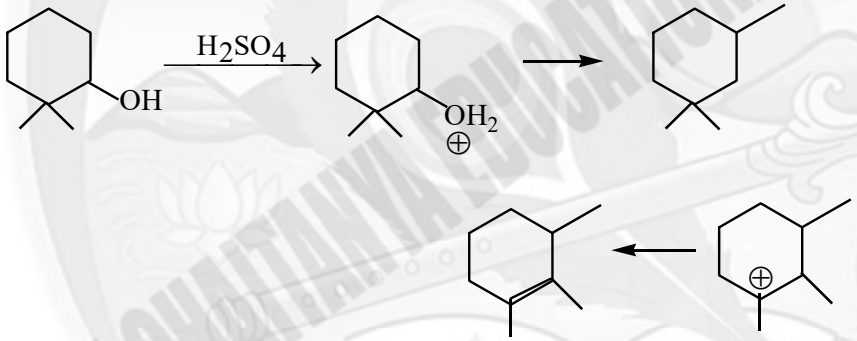
48. Find out the major product form the following reaction



1)  **2)**  **3)**  **4)** 

Key: 4

Sol:



49. Given below are two statements

Statement I : In froth floatation method a rotating paddle agitates the mixture to drive air out of it

Statement II : Iron pyrites are generally avoided for extraction of iron due to environmental reasons.

In the light of the above statements, choose the correct answer form the options given below

1) Both Statement I and Statement II are false

2) Statement I is false but Statement II is true

3) Statement I is true but Statement II is false

4) Both Statement I and Statement II are true

Key: 4

Sol: **Fact**

50. Match List I with List II

LIST I (Name of polymer)		**LIST II (Uses)**	
A	Glyptal	**I**	Flexible pipes
B	Neoprene	**II**	Synthetic wool
C	Acrilan	**III**	Paints and Lacquers
D	LDP	**IV**	Gaskets

Choose the correct answer from the options given below :

1) A-III; B-IV; C-I; D-II **2)** A-III; B-IV; C-II; D-I

3) A-III; B-I; C-IV; D-II **4)** A-III; B-II; C-IV; D-I

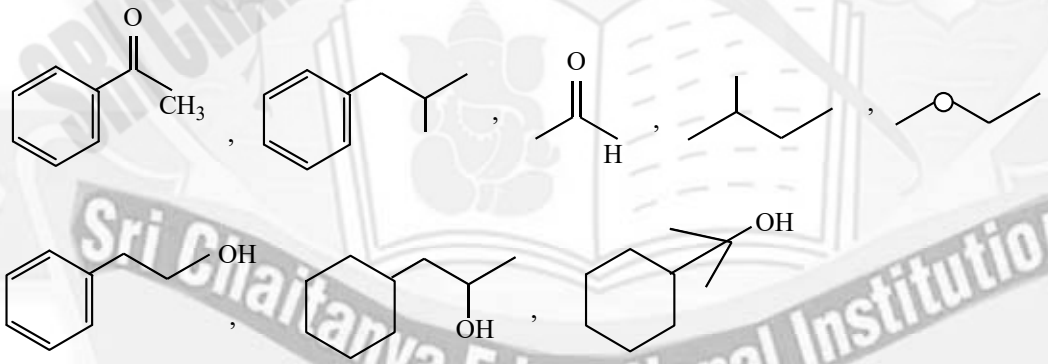
Key: **2**

Sol: use of polymers

(NUMERICAL VALUE TYPE)

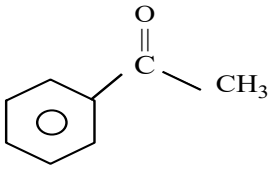
Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i, e. If answer is above **10** and less than **10.5** round off is **10** andIf answer is from **10.5** and less than **11** round off is **11).**
Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases

51. Number of compounds giving (i) red colouration with ceric ammonium nitrate and also (ii) positive iodoform test from the following is _____

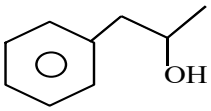


Key: **3**

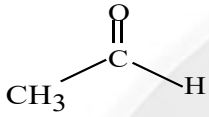
Sol:



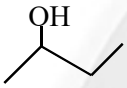
give haloform test but not CAN test



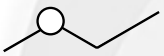
give haloform test and CAN test



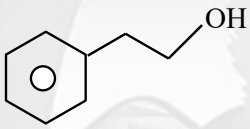
halo form test postive but not CAN test



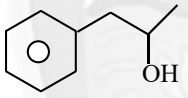
give haloform test and CAN test positive



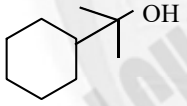
does not give haloform test and CAN test



give CAN Test but not haloform test



haloformtest + CAN test positive



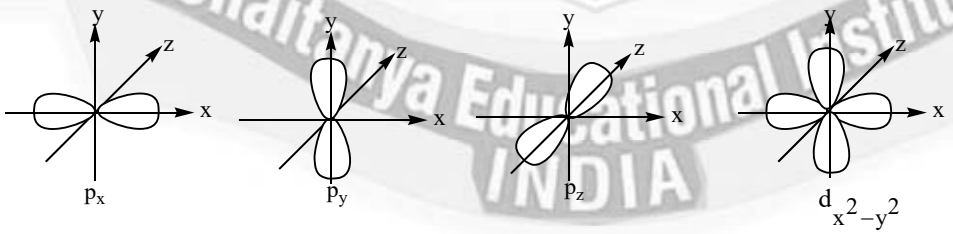
only can test

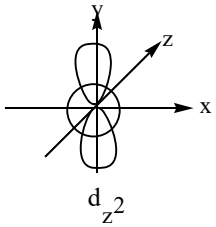
52. The number of given orbitals which have electron density along the axis is _____

P_x, P_y, P_z, d_{xy}, d_{yz} , d_z2, d_x2-y2

Key: 5

Sol: ***P_x, P_y, P_z, d_z2, d_x2-y2***





53. $Pt(s)\,|\,H_2(g)(1bar)\,|\,H^+(aq)(1M)\,||\,M^{3+}(aq), M^{2+}(aq)\,|\,Pt(s)$ The E_{cell} for the given cell is 0.1115V at 298 K when $\frac{[M^+(aq)]}{[M^{3+}(aq)]} = 10^a$ The value of a is …..

Given $E°M^{3+}/M^+ = 0.2V$ $\frac{2.303RT}{F} = 0.059V$

Key: 3

Sol:

$H_2 \rightarrow 2H^+ + 2e^-$

$M^{3+} 2e^- \rightarrow M^+$

$E^0_{cell} = E^0_C - E^0_A = 0.2 - 0 = 0.2$

$H_2 + M^{3+} \rightarrow 2H^+ + M^+$

$$E_{cell} = E^0_{cell} - \frac{0.0591}{2}\log\frac{[M^+][H^+]}{[M^{3+}]} = 0.2$$

$$0.1115 = 0.2 - \frac{0.059}{2}\log\frac{[M^+]}{[M^{3+}]}$$

$$-0.0885 = -0.0295\log\frac{[M^+]}{[M^{3+}]}$$

$$\log\frac{[M^+]}{[M^{3+}]} = 3 \Rightarrow \frac{[M^+]}{[M^{3+}]} = 10^3 = a = 3$$

54. Total number of moles of AgCl precipitated on addition of excess of AgNO3 to one mole each of the following complexes $[Co(NH_3)_4Cl_2)Cl$, $[Ni(H_2O)_6]Cl_2$, $[Pt(NH_3)_2Cl_2]$ and $[Pd(NH_3)_4]Cl_2$ is _____

Key: 5

Sol: $\left[Co(NH_3)_4Cl_2\right]Cl + AgNO_3 = 1 mole\ AgCl$

$\left[Ni(H_2O)_6\right]Cl_2 + AgNO_3 = 2 mole\ AgCl$

$\left[Pt(NH_3)_2Cl_2\right] + AgNO_3 = zero\ mole\ AgCl$

$\left[Pt(NH_3)_4\right]Cl_2 + AgNO_3 = 2 mole\ AgCl$

total number of moles of $AgCl = 5$

55. A first order reaction has the rate constant, k = 4.6 x $10^{-3}s^{-1}$. The number of **correct** statement/s from the following is/are _____Given : log 3 = 0.48

A. Reaction completes in 1000 s.

B. The reaction has a half-life of 500 s.

C. The time required for 10% completion is 25 times the time required for 90% completion.

D. The degree of dissociation is equal to ($1-e^{-kt}$)

E. The rate and the rate constant have the same unit

Key: 2

Sol: **A.** $t = \frac{2.303}{K}\log\frac{a}{(a-x)} \Rightarrow t = \frac{2.303}{4.6\times10^{-3}} = \frac{2.303}{4.6}\times10^3 = 2500\,sec$

B. $t_{1/2} = \frac{0.693}{K} = \frac{0.693}{4.6}\times10^3 = \frac{693}{4.6} = 150\,sec$

C. $\frac{t_{90\%}}{t_{10\%}} = \frac{\frac{2.303}{K}\log\left(\frac{100}{10}\right)}{\frac{2.303}{k}\log\left(\frac{100}{10}\right)} = \frac{\log 10}{\log 10 - \log 9} = \frac{1}{0.04} = 25$

D. Arrhenious equation $K = A_0e^{-kt}$

$A \rightarrow B$

$a_0 \quad 0$

$a_0(1-\alpha)\ a_a\alpha$

$a_0(1-\alpha) = a.e^{-kt}$

$(1-\alpha) = e^{-kt}$

$\alpha = e^{1-kt}$

$= 1 - e^{-kt}$

56. Number of hydrogen atoms per molecules of a hydrocarbon A having 85.8% carbon is _____

(Given : molar mass of A = 84 g mol^{-1})

Key: 12

Sol: **% of C** $\frac{85.8}{12}=\frac{7.15}{7.15}=1$

%of H $\frac{14.2}{1}=\frac{14.2}{7.15}=2$

Empirical formula = CH_2

Molecular formula = $(EF)_n \Rightarrow n=\frac{MW}{EFW}=\frac{84}{14}=6$

$MF=(CH_2)_6=C_6H_{12}$

Number of hydrogen atoms = 12

57. The number of pairs of the solutions having the same value of the osmotic pressure from the following is _____

(Assume 100% ionization)

A. 0.500 M C_2H_5OH (aq) and 0.25 M KBr (aq)

B. 0.100 M $K_4[Fe(CN)_6]$ (aq) and 0.100 M $FeSO_4(NH_4)_2SO_4$ (aq)

C. 0.05 M $K_4[Fe(CN)_6]$ (aq) and 0.25 M NaCl (aq)

D. 0.15 M NaCl (aq) and 0.1 M $BaCl_2$ (aq)

E. 0.02 M $KCl.MgCl_2.6H_2O$ (aq) and 0.05 M KCl (aq

Key: 4

Sol: **(i)** $0.5M\ C_2H_5OH \rightarrow$ particle conc $= 0.5$

and $0.25nKBr \rightarrow 0.25X_2 = 0.5$

(ii) $0.1M\,K_4[Fe(CN)_6] \rightarrow 0.5$ and $0.1M\,FeSO_4(NH_4)SO_4 \rightarrow 0.5$

$\left(0.1Fe^{2+}+0.1M\,SO_4+0.2NH_4^{+}+0.1M\,SO_4^{-2}\right)$

(iii) $0.05\,K_4[Fe(CN)_6] \rightarrow 0.05\times 5=0.25$ and $0.25M\ NaCl \rightarrow 0.5$

(iv) $0.15\,M\,NaCl \rightarrow 0.30M$ and $0.1M\ BaCl_2 \rightarrow 0.304$

(v) $0.02\,KCl\ MgCl_2.6H_2O \rightarrow 0.1M$

$0.5\,M\,KCl \rightarrow 0.1M$

So except 3

1, 2, 3, 4 are correct

58. 28.0 L of CO_2 is produced on complete combustion of 16.8 L gaseous mixture of ethane and methane at 25^0C and 1 atm. Heat evolved during the combustion process is _____ kJ.

Given $\Delta H_c(CH_4) = -900 kJ\,mol^{-1}$; $\Delta H_C(C_2H_4) = -1400 kJ\,mol^{-1}$

Key: 925.22

Sol: Ethen + methane mixture

$$\underset{a\,L}{C_2H_4} + 3O_2 \rightarrow \underset{2a\,L}{2CO_2} + 2H_2O$$

$$\underset{(16.8-a)}{CH_4} + 2O_2 \rightarrow \underset{(16.8-a)L}{CO_2} + 2H_2O$$

Total volume of $CO_2 = (16.8 - a + 2a)L = 16.8 + aL = 28.026$

$a = 28.02 - 16.8 = 11.226$

$V_{C_2H_4} = 11.22L$ $\quad V_{CH_4} = 5.58L$

$n_{C_2H_4} = \frac{11.22}{22.4}$ $\quad n_{CH_4} = \frac{5.58}{224}$

$n_{C_2H_4} = 0.5008$ $\quad n_{CH_4} = 0.249$ mole

$\Delta_0 H\ CH_4 \Rightarrow 1$mole CH_4 gives 900KT

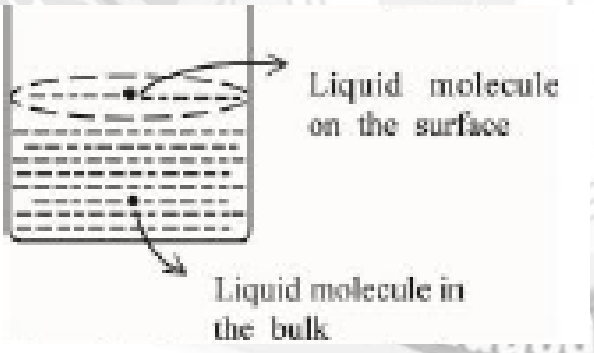
$\Delta_0 H\ C_2H_2 \Rightarrow 1$mole of C_2H_4 give $= 1400$kJ

total energy og combustion

$= 0.5008 \times 1400 + 0.249 \times 900$

$= 701.12 + 224.1 = 925.22$KJ

59. Based on the given figure, the number of **Correct** statement/s is/are _____



A. Surface tension is the outcome of equal attractive and repulsive forces action on the liquid molecule in bulk.

B. Surface tension is due to uneven forces acting on the molecules present on the surface.

C. The molecule in the bulk can never come to the liquid surface

D. The molecules on the surface are responsible for vapour pressure if the system is a closed system

Key: 2

Sol: BD otpions

60. The number of **incorrect** Statement/s from the following is are _____

A.Water vapours are adsorbed by anhydrous calcium chloride.

B. There is a decrease in surface energy during adsorption.

C.As the adsorption proceeds, ΔH becomes more and more negative.

D.Adsorption is accompanied by decrease in entropy of the system

Key: 1

Sol: Water vapour are absorbed by $CaCl_2$ not adsorbed during adsorption surface area a decreased

ΔH becomes –v e in adsorption

Entropy decrease during adsorption due to accumulation

Only given statement 1 is wrong in the above question

MATHEMATICS

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

61. If the function $f(x)=\begin{cases}\left(1+|\cos x|\right)\dfrac{\lambda}{|\cos x|}, & 0<x<\dfrac{\pi}{2}\\ \mu, & x=\dfrac{\pi}{2}\\ \dfrac{\cot 6x}{e^{\cot 4x}}, & \dfrac{\pi}{2}<x<\pi\end{cases}$ is continuous at

$x=\dfrac{\pi}{2}$, then $9\lambda+6\log_e\mu+\mu^6-e^{6\lambda}$ is equal to

1)10 **2) 8** **3) 11** **4)** $2e^4+8$

Key: 1

Sol: $f(x)=\begin{cases}\left(1+|\cos x|\right)\dfrac{\lambda}{|\cos x|}, & 0<x<\dfrac{\pi}{2}\\ \mu, & x=\dfrac{\pi}{2}\\ \dfrac{\cot 6x}{e^{\cot 4x}}, & \dfrac{\pi}{2}<x<\pi\end{cases}$ **is continuous**

$$\lim_{x\to\pi/2} f(x)=\lim_{x\to\frac{\pi}{2}}(1+\cos x)^{\lambda/\cos x}=1^{\alpha}\ \text{form}\ e^{\lim\limits_{x\to\pi/2}\frac{\lambda}{\cos x}(1+\cos x-1)}=e^{\lambda}$$

$$\lim_{x\to\frac{\pi+}{2}} f(x)=\lim_{x\to\frac{\pi}{2}} e^{\frac{\cot 6x}{\cot 4x}}=\lim_{x\to\frac{\pi}{2}} e^{\frac{\tan 4x}{\tan 6x}}$$

$$=e^{\lim\limits_{x\to\frac{\pi}{2}}\frac{\tan 4x}{\tan 6x}=\frac{0}{0}\text{form}}=e^{2/3}\ (\text{by using L.Hospital})$$

$f(x)$ **is continuous at** $x=\dfrac{\pi}{2}\Rightarrow \lim\limits_{x\to\frac{\pi^-}{2}} f(x)=\lim\limits_{x\to\frac{\pi+}{2}} f(x)=f(\pi/2)\Rightarrow e^{\lambda}=\mu=e^{2/3}$

$\Rightarrow \lambda=2/3,\mu=e^{2/3}$

Now $9\lambda+6\log_e\mu+\mu^6-e^{6\lambda}=9\times\dfrac{2}{3}+6\log_e e^{2/3}+\left(e^{2/3}\right)^6-e^{6\times 2/3}$ $\qquad 6+4+e^4-e^4=10$

62. The number of number strictly between 5000 and 10000 can be formed using the digits 1, 3, 5, 7, 9

Without repetition is

1. 6 **2)** 120 **3)** 12 **4)** 72

Key: 4

Sol:. No of numbers between 5000 and 10000 can be formed using the digits 1, 3, 5, 7, 9 wihtout reputation $\boxed{-}---$ The 1st place fill by 5 or 7 or 9 in 3 ways. Remaining 3 poles can be filled by other 4 digits in $4P_3$ ways

Required no of ways = $3\times {}^4P_3 = 3\times 4! = 72$

63. Let z be a complex number such that $\left|\frac{z-2i}{x+1}\right| = 2,\ z \neq -1$. Then z lies on the circle of radius 2 and centre

1) (0,0) **2)** (2,0) **3)** (0,2) **4)** (0, -2)

Key: 4

Sol: Z be a complex numbers such that $\left|\frac{z-2i}{z+i}\right| = 2,\ z \neq -i$

Let $z = x+iy, \Rightarrow |z-2i| = 2|z+i| \Rightarrow |x+i(y-2)| = 2|x+i(y+1)|$

$\Rightarrow x^2+(y-2)^2 = 4\left(x^2+(y+1)^2\right) \Rightarrow x^2+y^2+4y = 0$ Centre (0,-2)

64. $\sum_{k=0}^{6} {}^{51-k}C_3$ **is equal to**

1) ${}^{52}C_4 - {}^{45}C_4$ **2)** ${}^{52}C_3 - {}^{45}C_3$ **3)** ${}^{51}C_3 - {}^{45}C_3$ **4)** ${}^{51}C_4 - {}^{45}C_4$

Key: 1

Sol: $\sum_{k=0}^{6} {}^{(51-k)}C_3$

${}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{45}C_3$

Add and subtract by ${}^{45}C_4$ and **used the result** ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$$={}^{51}C_3+{}^{50}C_3+{}^{49}C_3+{}^{48}C_3+{}^{47}C_3+{}^{46}C_3+\left({}^{45}C_3+{}^{45}C_4\right)-{}^{45}C_4$$

$$={}^{51}C_3+{}^{50}C_3+{}^{49}C_3+{}^{48}C_3+{}^{47}C_3+\left({}^{46}C_3+{}^{46}C_4\right)-{}^{45}C_4$$

$$={}^{51}C_3+{}^{50}C_3+{}^{49}C_3+{}^{48}C_3+\left({}^{47}C_3+{}^{47}C_4\right)-{}^{45}C_4$$

$$={}^{51}C_3+{}^{50}C_3+{}^{49}C_3+\left({}^{48}C_3+{}^{48}C_4\right)-{}^{45}C_4$$

$$={}^{51}C_3+{}^{50}C_3+{}^{50}C_4-{}^{45}C_4$$

$$={}^{51}C_3+{}^{50}C_4-{}^{45}C_4$$

$$={}^{51}C_4-{}^{45}C_4$$

65. Let T and C respectively be the transverse and conjugate axes of the hyperbola $16x^2-y^2+64x+4y+44=0$. Then the area of the region above the parabola $x^2=y+4$ below the transverse axis T and on the right of the conjugate axis C is

1) $4\sqrt{6}-\frac{44}{3}$ **2)** $4\sqrt{6}+\frac{28}{3}$ **3)** $4\sqrt{6}+\frac{44}{3}$ **4)** $4\sqrt{6}-\frac{28}{3}$

Key: 2

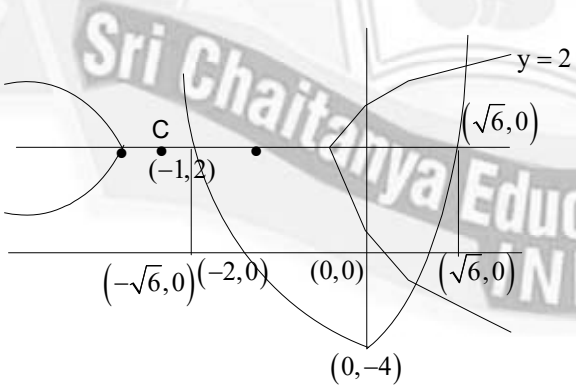
Sol: **Given hyperbola** $16x^2-y^2+64x+4y+44=0$

$$\Rightarrow\frac{(x+2)^2}{12}-\frac{(y-2)^2}{4^2}=1$$

Centre (-2, 2), $a^2=1^2, b^2=4^2$

Given parabola $x^2=y+4$

Required Area = $\int_{-\sqrt{6}}^{\sqrt{6}}(2-\left(x^2-4\right)dx-\int_{-\sqrt{6}}^{-2}\left(2-\left(x^2-4\right)\right)dx$



$$= 2\int_0^{\sqrt{6}} \left(6 - x^2\right)dx - \int_{\sqrt{6}}^{-2} \left(6 - x^2\right)dx$$

$$= 2\left[6x - \frac{x^3}{5}\right]_0^{\sqrt{6}} - \left[6x - \frac{x^3}{3}\right]^{-2}$$

$$= 2\left[6\sqrt{6} - \frac{6\sqrt{6}}{3}\right]\left(-12 + \frac{8}{3} + 6\sqrt{6} = 2\sqrt{6}\right)$$

$$= 8\sqrt{6} + \frac{28}{3} - 4\sqrt{6}$$

$$4\sqrt{6} + \frac{28}{3}$$

66. Let $a = -\hat{i} - \hat{j} + \hat{k}$, $\vec{a}.\vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{i} - \hat{j}$ then $\vec{a} - 6\vec{b}$ is equal to

1) $3\left(\hat{i} + \hat{j} + \hat{k}\right)$ **2)** $3\left(\hat{i} + \hat{j} - \hat{k}\right)$ **3)** $3\left(\hat{i} - \hat{j} + \hat{k}\right)$ **4)** $3\left(\hat{i} - \hat{j} - \hat{k}\right)$

Key: 1

Sol: $\vec{a} - \hat{i} - \hat{j} + \hat{k}$, $\vec{a}.\vec{b} = 1 \Rightarrow \vec{a} \times \vec{b} = \hat{i} - \hat{j}$ then $\vec{a} - 6\vec{b}$

Let $\vec{b} = a\hat{i} + b\hat{j} + c\hat{k} \Rightarrow \vec{a}.\vec{b} = 1$

$-a - b + c = 1$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ -1 & -1 & 1 \\ a & b & c \end{vmatrix}$$

$i = j = i(-c - b) - j(-c - a) + k(-b + c) - c - b = 1, -c - a = 1, -b + a = 0$

from $-2a + c = 1$

$-a + c = 1$ **a = -2/3**

$= 3a = 2$

From (2) -c + 2/3 =1

C – 1 -2/3 =1/3 C = -1/3

$\vec{b} = \frac{2i - 2j - k}{3} \Rightarrow \vec{a} - 6\vec{b} = \hat{i} - \hat{j} + k - 2\left(-2\hat{i} - 2j - k\right)$

$= i - j + k + 4i + 4j + 2k$

$= 3i + 3j + 3k = 3\left(\hat{i} + \hat{j} + \hat{k}\right)$

67. The integral $16\int_1^2 \frac{dx}{x^3\left(x^2+2\right)^2}$ is equal to

1) $\frac{11}{6}-\log_e 4$ **2)** $\frac{11}{6}+\log_e 4$ **3)** $\frac{11}{126}+\log_e 4$ **4)** $\frac{11}{126}-\log_e 4$

Key: 1

Sol: $\int_1^2 \frac{1}{x^7\left(x^2+2\right)^2}dx=\int_1^2 \frac{1}{x^7\left(1+\frac{2}{x^2}\right)}dx \Rightarrow -\frac{1}{4}\int_3^{3/2}\left(\frac{t-1}{2}\right)^2\frac{dt}{t^2}$

$\frac{1}{4}\int_{3/2}^{3}\frac{t^2+1-2t}{4}\frac{1}{t^2}dt \Rightarrow \frac{1}{16}\int_{3/2}^{3}1+\frac{t}{t^2}-\frac{2}{t}dt$

$\frac{1}{16}\left(3-\frac{1}{3}-2\log-\frac{3}{2}+\frac{2}{3}+2\log\frac{3}{2}\right)\Rightarrow\frac{1}{16}\left(\frac{3}{2}+\frac{1}{3}+2\log\frac{1}{2}\right)$

$\Rightarrow\frac{1}{16}\left(\frac{9+2}{6}+2\log t\right)$ $\quad 16I=\frac{11}{6}-\log 4$

68. If the four points, whose position vectors are

$3\hat{i}-4\hat{j}+2\hat{k},\hat{i}+2\hat{j}-\hat{k},-2\hat{i}-\hat{j}+3\hat{k}$ and $5\hat{i}-2\alpha\hat{j}+4\hat{k}$ are coplanar then α is equal to

1) $\frac{73}{17}$ **2)** $-\frac{73}{17}$ **3)** $\frac{107}{17}$ **4)** $-\frac{107}{17}$

Key: 1

Sol: $\overrightarrow{OA}=3\hat{i}-4\hat{j}+2\hat{k}$; $\overrightarrow{OB}=\hat{i}+2\hat{j}-\hat{k}$; $\overrightarrow{OC}=-2\hat{i}-\hat{j}+3\hat{k}$; $\overrightarrow{OD}=5\hat{i}-2\alpha\hat{j}+4\hat{k}$

$$\overrightarrow{AB}=\hat{i}+2\hat{j}-\hat{k}-3\hat{i}+4\hat{j}-2\hat{k}=-2\hat{i}+6\hat{j}-3\hat{k}$$

$\overrightarrow{AC}=-2\hat{i}-\hat{j}+3\hat{k}-3\hat{i}+4\hat{j}-2\hat{k}=-5\hat{i}+3\hat{j}-\hat{k}$

$\overrightarrow{AD}=5\hat{i}-2\alpha\hat{j}+4\hat{k}-3\hat{i}+4\hat{j}-2\hat{k}=-2\hat{i}+\left(-2\alpha+4\right)+2\hat{k}$

ABCD are coplanr $\left[\overrightarrow{AB}\ \overrightarrow{AC}\overrightarrow{AD}\right]=0\Rightarrow\begin{vmatrix}-2 & 6 & -3\\ -5 & 3 & 1\\ 2 & -2\alpha+4 & 2\end{vmatrix}=0$

$$-2(6+2\alpha-4)-6(-10-2)-3(10\alpha-20-6)$$

$$-2(2+2\alpha)-6(-12)-3(10\alpha-26)=0$$

$$=4-4\alpha+72-30\alpha+78=0$$

$$-34\alpha+146=0$$

$$-34\alpha=-146$$

$$\alpha=\frac{146}{34}=\frac{73}{17}$$

69. The number of functions $f:\{1,2,3,4\}\to\{a\in:Z\mid a\mid\leq 8\}$ satisfying

$f(n)+\frac{1}{n}f(n+1)=1,\forall n\in\{1,2,3\}$ is

1. 1 **2)** 3 **3)** 2 **4)** 4

Key: 3

Sol:

$$f:\{1,2,3,4\}\to\{a\in z,\ |a|\leq 8\}$$

$$f(n)+\frac{1}{2}f(n+1)=1,\ n\in\{1,2,3\}$$

$$\left.\begin{aligned}f(1)+\frac{1}{1}f(2)&=1\\ f(2)+\frac{1}{2}f(3)&=1\end{aligned}\right\}f(3)+\frac{1}{3}f\mid 4\mid=1$$

$$f(1)=\frac{1}{2}f(3)\qquad 2f(1)+\frac{1}{3}f\mid 4\mid=1$$

$$4f(1)+f(4)=2$$

$$f(3)=2f(1)$$

$$f(3)=\frac{f(1)+f(4)}{2}$$

$f(1),f(3),f(4)$ arein A

no of function 3

70. Let $\Delta,\nabla\in\{\wedge,\vee\}$ be such that $(p\to q)\Delta(p\nabla q)$ is tautology. Then

1) $\Delta=\wedge,\nabla=\wedge$ **2)** $\Delta=\wedge,\nabla=\vee$ **3)** $\Delta=\vee,\nabla=\vee$ **4)** $\Delta=\vee,\nabla=\wedge$

Key: 3

Sol: $(\sim p \vee q) \vee (p \vee q) \Rightarrow (\sim p \vee q) \vee (p \vee q)$ **true**

71. Let N be the sum of the numbers appeared when two fair dice are rolled and let the probability that $N-2\sqrt{3N}, N+2$ are in geometric progression be $\frac{k}{48}$. Then the value of k is

1) 16 **2)** 2 **3)** 4 **4)** 8

Key: 4

Sol: **given** $N-2, \sqrt{3}N, N+2$ **are in G.P then** $\left(\sqrt{3}N\right)^2 = (N-2)(N+2) \Rightarrow 3N = N^2 - 4$

$\Rightarrow N^2 - 3N - 4 = 0 \rightarrow (N-4)(N+1) = 0$

$N = 4 \rightarrow \text{sum } 4 = (1,3)(3,1)(2,2)$

then $P(X=4) = \frac{3}{36} = \frac{1}{12} = \frac{4}{48}$

then $= k = 4$

72. Let the function $f(x) = 2x^3 - (2p-7)x^2 + 3(2p-9)x - 6$ have a maximum for some value of $x < 0$ and a minima for some value of $x > 0$. Then the set of all value of p is

1) $\left(\frac{9}{2}, \infty\right)$ **2)** $\left(0, \frac{9}{2}\right)$ **3)** $\left(-\frac{9}{2}, \frac{9}{2}\right)$ **4)** $\left(-\infty, \frac{9}{2}\right)$

Key: 4

Sol:. $f(x) = 2x^3 + (2p-7)x^2 + 3(2p-9)x - 6$

$f'(x) = 6x^2 + 2x(2p-7) + 3(2p-9)$

Since $\alpha\beta < 0 \rightarrow 3(2p-9) < 0 \ , 2\beta < 9 \Rightarrow p < \frac{9}{2}$

$p \in (-\infty, 9/2)$

73. The shortest distance between the lines x + 1 = 2y = -12z and x = y + 2 = 6z – 6 is

1) 5/2 **2)** 3 **3)** 3/2 **4)** 2

Key: 4

Sol:

$$\frac{x-(-1)}{1}=\frac{y-0}{1/2}=\frac{z-0}{-1/2} \text{ i.e } \frac{x+1}{-12}=\frac{y}{-6}=\frac{z}{1}\rightarrow\frac{x-(-1)}{-12}=\frac{y}{-6}=\frac{z}{1}$$

$$\text{also } x=y+2=6(z-1)\Rightarrow\frac{x}{6}=\frac{y+2}{6}=\frac{z-1}{1}$$

$$(x_1,y_1,z_1)=(-1,0,0),(x_2,y_2,z_2)=(0,-2,1)$$

$$(b_1,b_2,b_3)=(-12,-6,1),(c_1,c_2.c_3)=(6,6,1)$$

$$\text{S.D}=\frac{\left|\left[\vec{a}-\vec{c}\ \vec{b}\ \vec{d}\right]\right|}{\left|\vec{b}\times\vec{d}\right|}=\left[\bar{a}-\bar{c}\ \bar{b}\ \bar{d}\right]=\begin{vmatrix}-1 & 2 & -1\\ -12 & -6 & 1\\ 6 & 6 & 1\end{vmatrix}$$

$$=-(-6,-6)-2(-12-6)-1(-72+36)$$

$$=12+36+36=84$$

$$\vec{b}\times\vec{d}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -12 & -6 & 1\\ 6 & 6 & 1\end{vmatrix}=\vec{i}(-6-6)-\hat{j}(12-6)+\hat{k}(-72+36)$$

$$=-12\hat{i}+18\hat{j}-36\hat{k}$$

$$-6\left(2\hat{i}-3\hat{j}+6\hat{k}\right)$$

$$\left|\vec{b}\times\vec{d}\right|=6\sqrt{4+9+36}=6\times7\ \text{ S.D}=\frac{8-4}{6\times7}=2$$

74. Let $y=y(t)$ be a solution of the differential equation $\frac{dy}{dt}+\alpha y=\gamma e^{-\beta t}$ where $\alpha>0,\beta>0$ and $\gamma>0$ then $\lim\limits_{t\to\infty}y(t)$

1) is 1 **2)** Does not exist **3)** Is 0 **4)** Is -1

Key: 3

Sol: $\frac{dy}{dt}+\alpha y=\gamma e^{-\beta t}\Rightarrow IF=e^{\int\alpha\,dt}=e^{\alpha t}$

$$\text{sol is } y(IF)=\int Q(IF)dt+C\Rightarrow y\left(e^{\alpha t}\right)=\int\gamma e^{-\beta t}.e^{\alpha t}dt+C$$

$$y\left(e^{\alpha t}\right)=\int\gamma.e^{(\alpha.\beta)t}dt+C$$

$$y\left(e^{\alpha t}\right)=\gamma.\frac{e^{(\alpha+\beta)t}}{(\alpha-\beta)}+C$$

$$y = \gamma.\frac{e^{-\beta t}}{\alpha-\beta} + c.e^{-\alpha t}$$

$$\lim_{t\to 0} y(t) = \lim_{t\to\infty}\left(\frac{\gamma}{\alpha-\beta}.e^{-\beta^{+}} + c.e^{-\alpha t}\right) = 0$$

75. **Let** $A = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$, where $i = \sqrt{-1}$ **If** $M = A^{T}BA$, **then the inverse of the matrix** $AM^{2023}A^{T}$ **is**

1) $\begin{bmatrix} 1 & 0 \\ -2023i & 1 \end{bmatrix}$ **2)** $\begin{bmatrix} 1 & 0 \\ 2023i & 1 \end{bmatrix}$ **3)** $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$ **4)** $\begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

Key: 3

$$M^2 = \left(A^{T}BA\right)\left(A^{T}BA\right) = A^{T}B^2A$$

Sol: $AA^{T} = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ -\frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} = \begin{bmatrix} \frac{1}{10}+\frac{9}{10} & \frac{-3}{10}+\frac{3}{10} \\ \frac{-3}{10}+\frac{3}{10} & \frac{9}{10}+\frac{1}{10} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **Continuig like this**

$$M^3 = \left(A^{T}B^2A\right)\left(A^{T}BA\right) = A^{T}B^3A$$

76. If $f : R \to R$ be a function defined by $f(x) = \log_{\sqrt{m}}\left\{\sqrt{2}\left(\sin x - \cos x\right) + m - 2\right\}$ for some m, such that the range of f is [0,2] Then the value of m is ……

1) 5 **2)** 3 **3)** 4 **4)** 2

Key: 1

Sol: $f(x) = \log_{\sqrt{m}}\left(\sqrt{2}\left(\sin x - \cos x\right) + m - 2\right)$

$$\sqrt{2}\left(\sin x - \cos x\right) = \sqrt{2}\left[-\sqrt{2}, \sqrt{2}\right] = [-2,2]$$

$$f(x) = \log_{\sqrt{m}}\left([-2,2] + m - 2\right)$$

$$= \log_{\sqrt{m}}[-4+m] \in [0,z]$$

$$\log_{\sqrt{m}} = (-4+m) = 0 \Rightarrow -4+m = 1 \Rightarrow m = 5$$

77. The equation of two sides of a variable triangle are x = 0 and y = 3 and its third side is a tangent to the parabola $y^2 = 6x$. The locus of its circumcentre is

1) $4y^2 - 18y + 3x + 18 = 0$ **2)** $4y^2 + 18y + 3x + 18 = 0$

3) $4y^2 - 18y - 3x + 18 = 0$ **4)** $4y^2 - 18y - 3x - 18 = 0$

Key: 1

Sol: **L is tangent to** $y^2 = 6x$

$4a = 6 \Rightarrow a = \frac{3}{2}$

Tangent $y = mx + \frac{a}{m} \Rightarrow y = mx + \frac{3}{2m}$

$B = \left(0, \frac{3}{2m}\right), C = \left(\frac{3}{m} - \frac{3}{2m^2}, 3\right)$

$C = \left(\frac{3}{m} - \frac{3}{2m^2}, 3\right)$

Wt (x1, y1) be the circum centre of ΔABC

$x_1 = \frac{0 + \frac{3}{m} - \frac{3}{2m^2}}{2}, y_1 = \frac{\frac{3}{2m} + 3}{L}$

$x_1 = \frac{3}{2m} - \frac{3}{4m^2}$ $y_1 = \frac{3}{4m} + \frac{3}{2}$

$\Rightarrow x_1 y_1^2 - 18y_1 + 3x_1 + 18 = 0$

$\Rightarrow 4y^2 - 18y + 3x + 18 = 0$

78. The foot of perpendicular of the point (2, 0, 5) on the line $\frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1}$ is (α, β, γ)

Then which of the following is NOT correct?

1) $\frac{\alpha\beta}{\gamma} = \frac{4}{15}$ **2)** $\frac{\beta}{\gamma} = -5$ **3)** $\frac{\gamma}{\alpha} = -\frac{5}{8}$ **4)** $\frac{\alpha}{\beta} = -8$

Key: 2

Sol: $L \cong \frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1} (= \lambda)$

$G = (x, y, z) = (2\lambda - 1, 5\lambda + 1, -\lambda - 1)$

Dr's $\overrightarrow{PQ} = (2\lambda - 3, 5\lambda + 1, -\lambda - 6)$

$\overrightarrow{PQ} \perp L \Rightarrow 2(2\lambda - 3) + 5(5\lambda + 1) - 1(-\lambda - 6) = 0$

$4\lambda - 6 + 25\lambda + 5 + \lambda + 6 = 0$

$30\lambda = -5,\ \lambda = -\frac{1}{6}$

$Q = \left(\frac{-2}{6} - 1,\ \frac{-5}{6} + 1,\ \frac{1}{6} - 1\right)$

$Q = \left(\frac{-4}{3}, \frac{1}{6}, \frac{-5}{6}\right)$

1) $\frac{\alpha\beta}{\gamma} = \frac{\frac{-4}{3}\frac{\lambda 1}{r}}{\frac{-5}{t}} = \frac{14}{15}$ **correct**

2) $\frac{\beta}{\gamma} = \frac{\frac{1}{6}}{\frac{-5}{6}} = \frac{-1}{5}$ incorect givne $\frac{\beta}{\gamma} = -5$

3) $\frac{\gamma}{\alpha} = \frac{\frac{-5}{6}}{\frac{-4}{3}} = \frac{5}{6} \times \frac{3}{4} = \frac{5}{8}$ correct

79. Let A, B, C be 3 x 3 matrices such that A is symmetric and B and C are skew symmetric consider the statements

S1: $A^{13}B^{26} - B^{26}A^{13}$ is symmetric S2: $A^{26}C^{13} - C^{13}A^{26}$ is symmetric then

1) both S1 and S2 are false **2)** Both S1 and S2 are true

3) Only S2 is true **4)** Only S1 is true

Key: 3

Sol: $A^T = A,\ B^T = B,\ C^T = C$

$S_1 = \text{wt } M = A^{13}B^{26} - B^{26}A^{13}$

$M^T = \left(B^T\right)^{26}\left(A^T\right)^{13} - \left(A^T\right)^{13}\left(B^T\right)^{26}$

$M^T = B^{26}A^{13} - A^{13}B^{26} = -M$

M is skew symmetric

S2 = $N = A^{26}C^{13} - C^{13}A^{26}$

$N^T = \left(C^T\right)^{13}\left(A^T\right)^{26} - \left(A^T\right)^{26}\left(C^T\right)^{13}$

$N^T = -C^{13}A^{26} + A^{26}C^{13}$

$N^T = N$

Nis symmetric

Only S2 is true

80. Let $f(x) - 2x^n + \lambda,\ \lambda \in R, n \in N$ and $f(4) = 133, f(5) = 255$. Then the sum of all the positive integer divisors of $\left(f(3) - f(2)\right)$ is

1) 60 **2)** 58 **3)** 61 **4)** 59

Key: 1

Sol: $f(x) = 2x^n + \lambda \Rightarrow f(x) = 133 \quad f(5) = 255$

$2(4)^n + \lambda = 133 \quad 2(5)^n + \lambda = 255$

$2(5)^n - 2(4)^n = 122$

$5^n - 4^n = 61 \rightarrow n = 3$

$f(3) - f(2) = 2(3)^n - 2(2)^n = 2(27) - 2(8) = 54 - 38$

Dividing 1, 2, 19, 38 sum =60

(NUMERICAL VALUE TYPE)

Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i, e. If answer is above **10** and less than **10.5** round off is **10** andIf answer is from **10.5** and less than **11** round off is **11**).
Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases

81. Point P (-3, 0), Q (9, 10) and R (a, 4) lie on a circle C with PR as its diameter. The tangents to C at the points Q and R intersect at the point S. If S lies on the line 2x – ky = 1 then k is equal to

Key: 3

Sol: **Equation of circles** $(x+3)(x-a)+(y-2)(y-4)=0$

$Q(9,10) \Rightarrow a = 13$

equation of circle $x^2 + y^2 - 10x - 6y - 31 = 0$

Equation of tangent at $Q: 4x + 7y - 106 = 0$.....(1)

Equation of tangent R = 8x + y – 108 = 0……(2)

From (1), (2) (x, y) $= \left(\frac{25}{2}, 8\right)$

$\left(\frac{25}{2}, 8\right) \Rightarrow 2x - ky = 1 \Rightarrow 25 - 8k = 1 \Rightarrow 8k = 24 \Rightarrow k = 3$

82. If the shortest distance between the line joining the points (1, 2, 3) and (2, 3, 4) and the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$ is α, then $28\alpha^2$ is equal to

Key: 18

Sol $L_i = \vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + t\left(\hat{i} + j + \hat{k}\right)$

$L_2 = \vec{r} = (i + 2j + 3k) + 5(2i - j)$

$$\vec{b} \times \vec{d} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & -1 & 0 \end{vmatrix} = i + 2j - 3k$$

$$\left[\vec{a} - \vec{c}\ \vec{b}\ \vec{d}\right] = \begin{vmatrix} 0 & 3 & 1 \\ 1 & 1 & 1 \\ 2 & -1 & 0 \end{vmatrix}$$ **=0 -3 (-2) +1(-3)**

$$S.D = \frac{\left[\vec{a} - \vec{c}.\ \vec{b} - \vec{d}\right]}{\left|\vec{b} \times \vec{d}\right|} = \frac{3}{\sqrt{14}}$$

$28a^2 = 28.\frac{9}{14} = 18$

83. Let $a \in R$ and let α, β be the roots of the equation $x^2 + 60^{1/4}x + a = 0$ If $\alpha^4 + \beta^4 = -30$ then the product of all possible values of a is ……..

Key: 45

Sol:

$x^2 + 60^{1/4}x + a = 0 \Rightarrow \alpha + \beta = -60^{1/4}, \alpha\beta = a$

$\alpha^2 + \beta^2 + 2\alpha\beta = \sqrt{60}$,

$\alpha^2 + b^2 = \sqrt{60} - 2a$

$\alpha^4 + \beta^4 + 2\alpha^2\beta^2 = 60 + 40^2 - 4\sqrt{60}a$

$\Rightarrow -30 + 2a^2 = 60 + 4a^2 - 4\sqrt{60}a$

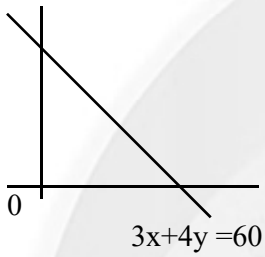
$\Rightarrow 2a^2 - 4\sqrt{60}a + 90 = 0$

Product of rots = 90/2 = 45

84. A triangle is formed by X-axis, Y-axis and the line 3x + 4y =60. Then the number of points P(a, b) which lie strictly inside the triangle where as is an integer and b is a multiple of a, is

Key: 31

Sol: Required points



$(1,1),(1,2),(1,3)(1,4).....(1,14)$

$(2,2),(2,4),(2,6),(2,8),(2,10),(2,12)$

$(3,3)(3,6)(3,9)(3,12)\ (4,4)(4,8)(5,5)(5,10)$

$(6,6),(7,7),(8,8)$

Total number of points 31

85. If m and n respectively are the number of positive and negative values of θ in the interval $[-\pi,\pi]$ that satisfy the equation $\cos 2\theta\cos\frac{\theta}{2}=\cos 3\theta\cos\frac{9\theta}{2}$ then mn is equal to ……

Key: 25

Sol

$$\cos 2\theta\cos\frac{\theta}{2}=\cos 3\theta\cos\frac{9\theta}{2},[-\pi,\pi]$$

$$2\cos 2\theta\cos\frac{\theta}{2}=2\cos 3\theta\cos\frac{9\theta}{2}$$

$$\cos\frac{5\theta}{2}+\cos\frac{3\theta}{2}=\cos\frac{15\theta}{2}+\cos\frac{3\theta}{2}$$

$$\Rightarrow\frac{15\theta}{2}=2n\pi\pm\frac{5\theta}{2}\Rightarrow 15\theta=4n\pi\pm 5\theta\Rightarrow\theta=\frac{n\pi}{5},\frac{2n\pi}{5}$$

$$+\text{ve sol}=\left\{\frac{\pi}{5},\frac{2\pi}{5},\frac{3\pi}{5},\frac{4\pi}{5},\pi\right\}$$

$$-\text{ve sol}=\left\{-\pi,\frac{-\pi}{5},\frac{-2\pi}{5},\frac{-3\pi}{5},-\frac{4\pi}{5}\right\}\text{mn}=25$$

86. 25% of the population are smokers A smokers has 27 times more chances to develop lung cancer than a non smoker. A person is diagnosed with lung cancer and the probability that this person is an smoker is k/10 . Then the value of k is ……..

Key: 9

Sol: Probability of a person being smoker P(S) = 25/100 = ¼

Probability of a person being non smokes $P(\overline{S}) = 1 - 1/4 = 3/4$

Probability of anon smoker diagnosed with cancer $P\left(C/\overline{S}\right) = P$

Probability of a smoker diagnosed with cancer P(C/S) = 27P

$$P\left(\frac{\text{Person is smoker}}{\text{Person diagnosed with cancer}}\right) = P\left(\frac{S}{C}\right) = \frac{P(S \cap C)}{P(C)}$$

$$\frac{P(S).P\left(\frac{C}{S}\right)}{P(S).P\left(\frac{C}{S}\right) + P\left(\overline{S}\right).P\left(\frac{C}{\overline{S}}\right)} = \frac{\frac{1}{4}.27P}{\frac{1}{4}27P + \frac{3}{4}P} = \frac{\frac{P}{4}.27}{\frac{P}{4}(27+3)} = \frac{27}{30} = \frac{9}{10}$$

87. If $\int_3^3 |\log_e x| dx = \frac{m}{n}\log_e\left(\frac{n^2}{e}\right)$, where m and n are coprime natural numbers then $m^2 + n^2 - 5$ is equal to

Key: 20

Sol: $\int_{1/3}^{3} |\ln x| dx = \int_{1/3}^{1} -\ln x dx + \int_1^3 \ln x \, dx = -\left(x\ln x - x\right)_{1/3}^{1} + \left(x\ln x - x\right)_1^3$

$$= -\left[(0-1) - \left(\frac{1}{3}\ln\frac{1}{3} - \frac{1}{3}\right)\right] + (3\ln 3 - 3) - (0-1)$$

$$= \frac{2}{3} + \frac{1}{3}\ln\frac{1}{3} + 3\ln 3 - 2$$

$$= 3\ln 3 - \frac{1}{3}\ln 3 - \frac{4}{3} = \frac{8}{3}\ln 3 - \frac{4}{3}$$

$$\frac{4}{3}(2\ln 3 - 1) = \frac{4}{3}\ln\left(3^2/e\right)$$

$m = 4, n = 3$ then $m^2 + n^2 - 5 = 20$

88. Suppose Anil's mother wants to give 5 whole fruits to Anil from a basket of 7 red apples, 5 white apples and 8 oranges. If in the selected 5 fruits at least 2 oranges, at least one red apple and at least one while apple must be given then the number of ways. Anils mother can offer 5 fruits to anile is ……

Key:

Sol:

case	7 red applies	5 white apples	8 oranges	Selection of 5 fruits
I	**1**	**1**	**3**	$^{7}C_1.^{5}C_1.^{8}C_3$
II	**1**	**2**	**2**	$^{7}C_1.^{5}C_2.^{8}C_3$
III	**2**	**1**	**2**	$^{7}C_2.^{5}C_1.^{8}C_3$

Total number of ways =

$$=^{7C}C_1.^{5}C_1.^{8}C_3+^{7}C_1.^{5}C_2.^{8}C_3+^{7}C_2.^{5}C_1.^{8}C_3$$

$$=7.5.56+7.10.28+21.5.28$$

$$=7.5.28(2+2+3)$$

$$=7\times5\times28\times7=6860$$

89. For the two positive a, b, if a, b and 1/18 are in a geometric progression while 1/a, 10 and 1/ b are in an arithmetic progression then 16a + 12b is equal to…….

Key: 3

Sol: $a,b,\frac{1}{18}$ are in GP $\Rightarrow b^2=\frac{a}{18}\Rightarrow a=18b^2$.....(1)

$\frac{1}{a},10,\frac{1}{b}$ are in AP $\Rightarrow \frac{1}{a}+\frac{1}{b}=20\Rightarrow \frac{1}{18b^2}+\frac{1}{b}=20$

$\Rightarrow 360b^2-18b-0\Rightarrow(12b-1)(30b+1)=0(-30,12)$

$\Rightarrow b=\frac{1}{12},b\frac{-1}{30}\ (b>0)$

From 1 $a=18.\frac{1}{144}=\frac{1}{8}\Rightarrow 16a+12b=2+1=3$

90. The remainder when $(2023)^{2023}$ is divided by 35 is……

Key: 7

Sol: $2023=-7(\text{mod }35)$

$(2023)^2=14(\text{mod }35)$

$(2023)^4=-14(\text{mod }35)$

$(2023)^{16}=-14(\text{mod }35)$

$(2023)^{2020}=-14(\text{mod }35)$