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ALL INDIA RANKS IN JEE-ADV 2022

JEE MAIN (JAN) 2023 (29-01-2023-Session-2)

MATHEMATICS, PHYSICS & CHEMISTRY

Sri Chaitanya IIT Academy.,India.

A.P T.S KARNATAKA TAMILNADU MAHARASTRA DELHI RANCHI

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ICON Central Office - Madhapur - Hyderabad

Jee-Main_Final_29-JAN-2023_Shift-02

IMPORTANT INSTRUCTION:

1. Immediately fill in the Admission number on this page of the Test Booklet with **Blue/Black** Ball Point **Pen** only.
2. The candidates should not write their Admission Number anywhere (except in the specified space) on the Test Booklet/ Answer Sheet.
3. The test is of **3 hours** duration.
4. The Test Booklet consists of 90 questions. The maximum marks are **300.**
5. There are **three** parts in the question paper 1, 2, 3 consisting of **Physics, Chemistry** and **Mathematics** having **30 questions** in each subject and subject having **two sections.**
   (I) Section –I contains 20 **multiple choice** questions with only one correct option.
   Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.
   (II) Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only, if more than 5 questions attempted, First 5 attempted questions will be considered.

■ The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i, e. If answer is above **10** and less than **10.5** round off is **10** and If answer is from **10.5** and less than **11** round off is **11).**
To cancel any attempted question bubble on the question number box.
For example: To cancel attempted question 21. Bubble on 21 as shown below

Question Answered for Marking **Question Cancelled for Marking**
Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases.

6. Use **Blue / Black Point Pen only** for writing particulars / marking responses on the Answer Sheet. **Use of pencil is strictly prohibited.**
7. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electron device etc, except the Identity Card inside the examination hall.
8. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
9. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Hall. **However, the candidate are allowed to take away this Test Booklet with them.**
10. **Do not fold of make any stray marks on the Answer Sheet**

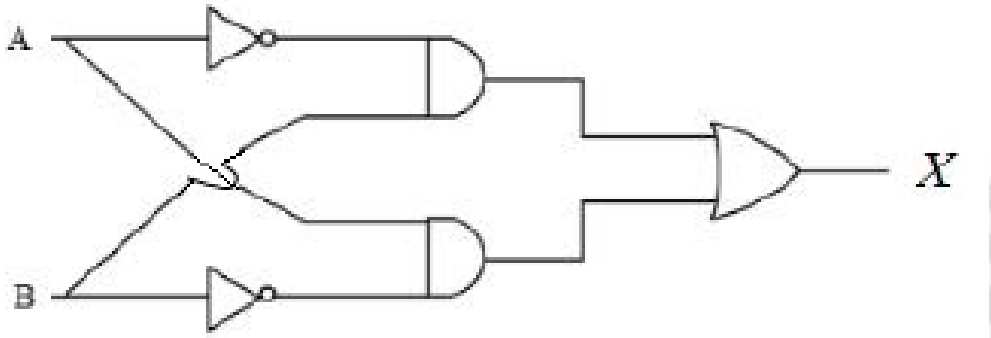
PHYSICS **Max Marks: 100**

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

01. For the given logic gates combination, the correct truth table will be



A)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

B)

A	B	X
0	0	1
0	1	0
1	0	1
1	1	0

C)

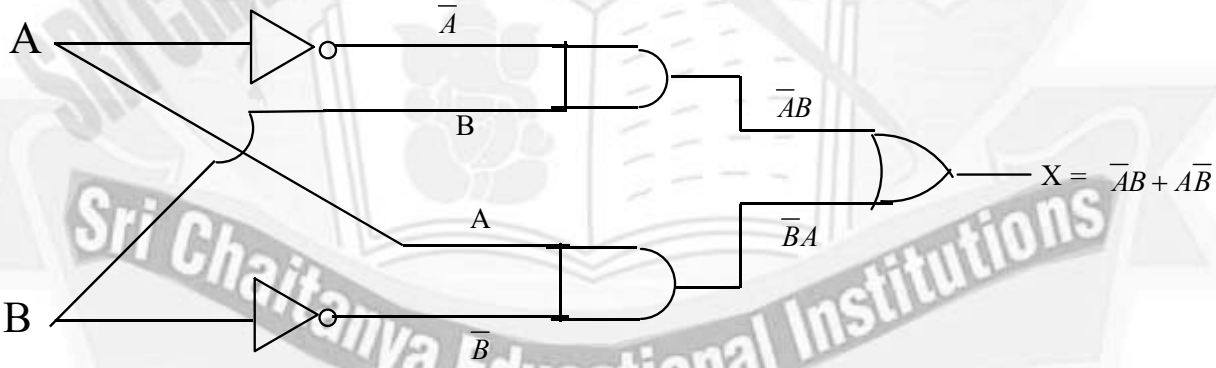
A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

D)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

Ans: A

Sol:



Exclusively OR gate or *XOR* gate

02. Identify the correct statements from the following

a. Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket is negative

b. Work done by gravitational force in lifting a bucket out of a well by a rope tied to the bucket is negative

c. Work done by friction on a body sliding down an inclined plane is positive

d. Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity in zero

e. Work done by the air resistance on an oscillating pendulum in negative

Choose the correct answer from the options given below

A) B and D only **B)** B, D and E only **C)** B and E only **D)**A and C only

Ans: C

Sol: In lifting bucket

Man does positive work

Gravitational force does negative work

Friction does negative work on block sliding down

Work done by applied force is non-zero due to friction force.

Air resistance does negative work on oscillating pendulum as it acts opposite to direction of motion.

03. With the help of potentiometer, we can determine the value of emf of a given cell. The Sensitively of the potentiometer is

a. Directly proportional to the length of the potentiometer wire
b. Directly proportional to the potential gradient of the wire
c. inversely proportional to the potential gradient of the wire
d. inversely proportional to the length of the potentiometer wire

Choose the correct option for the above statements

A) A and C only **B)** B and D only **C)** C only **D)**A only

Ans: A

Sol: $K = \frac{V_A - V_B}{l}$

Sensitivity of potentiometer wire $\alpha \ \frac{1}{K} \ \alpha \ \ l$

04. Heat energy of 184 kJ is given to ice of mass 600 g at -12^0C. Specific heat of ice is $2222.3J\ kg^{-1^0}C^{-1}$ and latent heat of ice in $336kJ/kg^{-1}$

a. Final temperature of system will be 0^0C

b. Final temperature of system will be greater than 0^0C

c. The final system will have a mixture of ice and water in the ratio of 5:1

d. The final system will have a mixture of ice and water in the ratio of 1:5

e. The final system will have water only

Choose the correct answer from the options given below:

A) A and C only **B)** A and E only **C)** B and D only **D)**A and D only

Ans: D

Sol: $Q_1 = ms\Delta t = 0.6\times 2222.3\times 12$

$= 16000.56J \approx 16KJ$

For complete melting

$Q_{required} = mL$

$= 0.6\times 336 = 201.6KJ$

All ice do not melt

Mass of ice melted $= \dfrac{184KJ - 16KJ}{336KJ}$

$= 0.5kg$

Final mixture has 0.5kg water and 0.1 kg ice at 0^0c

05. The ratio of de- Broglie wavelength of an α particle and a proton accelerated from rest by the same potential is $\dfrac{1}{\sqrt{m}}$, the value of m is

A) 2 **B)**16 **C)** 8 **D)**4

Ans: C

Sol: $\dfrac{\lambda\alpha}{\lambda p} = \dfrac{\dfrac{h}{\sqrt{2m_\alpha q_\alpha V}}}{\dfrac{h}{\sqrt{2m_p q_p V}}} = \sqrt{\dfrac{m_p}{m_\alpha}\dfrac{q_p}{q_\alpha}} = \sqrt{\dfrac{m}{4m}\dfrac{e}{2e}} = \dfrac{1}{\sqrt{8}} = \dfrac{1}{\sqrt{m}}$

$m = 8$

06. The time period of satellite of earth is 24 hours. If the separation between the earth and the satellite is decreased to one fourth of the previous value, then its new time period will become

A) 6 hours **B)** 12 hours **C)** 4 hours **D)** 3 hours

Ans: D

Sol: $T^2 \alpha R^3$

$$\left(\frac{T}{24}\right)^2 = \left(\frac{\frac{R}{4}}{R}\right)^3$$

$T = 3hr$

07. A fully loaded Boeing aircraft has a mass of $5.4\times10^5 kg$. Its total wing area is 500 m^2. It is in level flight with a speed of 1080 km/h. If the density of air ρ is 1.2 $kg\ m^{-3}$, the fractional increases in the speed of the air on the upper surface of the wing relative to the lower surface in percentage will be $\left(g = 10\ m/s^2\right)$

A) 6 **B)** 8 **C)** 10 **D)**16

Ans: C

Sol: $\Delta P \times A = mg$

$\Delta P \times 500 = 5.4\times10^5 \times 10$

$\Delta P = 108\times10^2\ pa$

$\frac{1}{2}\rho\left(V_2^2 - V_1^2\right) = \left(P_1 - P_2\right)$

$\frac{1}{2}(1.2)(V_2 + V_1)(V_2 - V_1) = 108\times10^2$

But $\frac{V_2 + V_1}{2} = V_{avg} = 1080\frac{km}{hr} = 1080\times\frac{5}{18} m/s = 300 m/s$

$(1.2)300(V_2 - V_1) = 108\times10^2$

$V_2 - V_1 = 30 m/s$

Fractional change$= \frac{V_2 - V_1}{V_{avg}} = \frac{30}{300} = \frac{1}{10}$

$\therefore \%increase = \frac{1}{10}\times100 = 10\%$

08. A square loop of area $25\ cm^2$ has a resistance of $10\ \Omega$. The loop is placed in uniform magnetic field of magnitude 40.0 T. The plane of loop is perpendicular to the magnetic field. The work done in pulling the loop out of the magnetic field slowly and uniformly in 1.0 sec, will be

A) $2.5\times10^{-3}J$ **B)** $5\times10^{-3}J$ **C)** $1.0\times10^{-3}J$ **D)** $1.0\times10^{-4}J$

Ans: C

Sol: Work done by agent

$= Power\times t$ $= i^2R\times t$

$=\left(\frac{BlV}{R}\right)^2 Rt$ $=\frac{B^2l^2V^2t}{R}=1\times10^{-3}J$

09. The equation of a circle is given by $x^2+y^2=a^2$, where a is the radius. If the equation is modified to change the origin other than $(0,0)$, then find out the correct dimensions of A and B in a new equation: $(x-At)^2+\left(y-\frac{t}{B}\right)^2=a^2$. The dimensions of t is given as $\left[T^{-1}\right]$

A) $A=\left[LT\right].B=\left[L^{-1}T^{-1}\right]$ **B)** $A=\left[L^{-1}T^{-1}\right],B=\left[LT\right]$

C) $A=\left[L^{-1}T^{-1}\right],B=\left[LT^{-1}\right]$ **D)** $A=\left[L^{-1}T\right],B=\left[LT^{-1}\right]$

Ans: A

Sol: Given $[t]=T^{-1}$

$[At]=[x]$ $\left[\frac{t}{B}\right]=[y]$ $[A]=LT$ $[B]=T^{-1}L^{-1}$

10. Given below are two statements:

Statement I: Electromagnetic waves are not deflected by electric and magnetic field.

Statement II: The amplitude of electric filed and the magnetic field in electromagnetic waves are related to each other as $E_0=\sqrt{\frac{\mu_0}{\varepsilon_0}}B_0$.

In the light of the above statements, choose the correct answer from the options given below

A) Statement I is false but statement II is true

B) Statement I is true but statement II is false

C) Both Statement I and statement II are false

D) Both Statement I and statement II are true

Ans: B

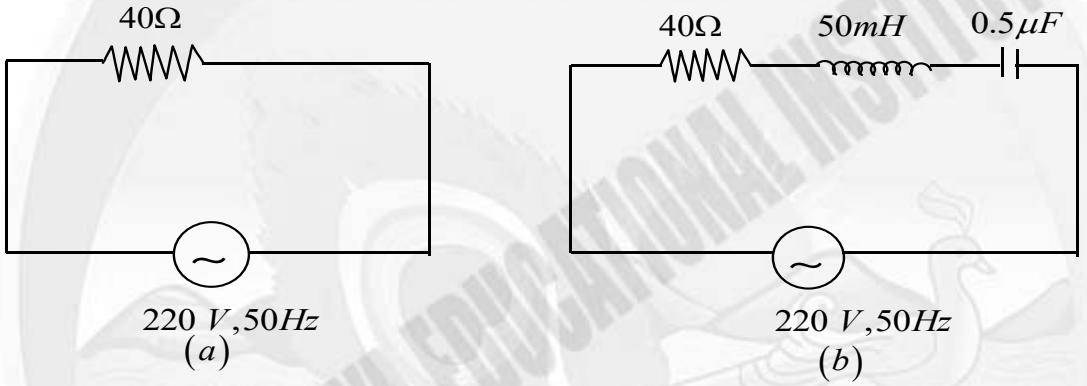
Sol: EM waves do not have charge as they are photons hence not deflected by electric and magnetic fields

$$E_0 = B_0 C$$

$$E_0 = \frac{B_0}{\sqrt{\mu_0 \in_0}}$$

Hence statement II is false

11. For the given figures, choose the correct options



A) The rms current in circuit **(B)** can never be larger than that in **(A)**

B) The rms current in circuit **(B)** can be larger than that in **(A)**

C) The rms current in figure **(A)** is always equal to that in figure **(B)**

D) At resonance, current in **(B)** is less than that in **(A)**

Ans: A

Sol: At resonance RMS current in both is same

In other cases, RMS in **(A)** is more than **(B)**

12. A scientist is observing a bacteria through a compound microscope. For better analysis and to improve its resolving power he should. (Select the best option)

A) Decrease the focal length of the eye piece.

B) Increase the refractive index of the medium between the object and objective lens.

C) Decrease the diameter of the objective lens.

D) Increase the wave length of the light.

Ans: B

Sol: $R.P = \dfrac{\mu \sin\alpha}{0.61\lambda}$ for compound microscope

Resolving power will increase by increasing refractive index of the medium between the object and objective lens, by increasing diameter of the objective lens and by decreasing the wave length of the light .a

13. Substance A has atomic mass number 16 and half-life of 1 day. Another substance B has atomic mass number 32 and half-life of $\dfrac{1}{2}$ day. If both A and B simultaneously start undergo radio activity at the same time with initial mass 320g each, how many total atoms of A and B combined would be left after 2 days.

A) 3.38×10^{24} **B)** 1.69×10^{24} **C)** 6.76×10^{24} **D)** 6.76×10^{23}

Ans: A

Sol: $N_A + N_B = \left[\dfrac{\dfrac{320}{2^2}+\dfrac{320}{2^4}}{16\quad 32}\right] N_A$

$= \left(5+\dfrac{5}{8}\right)\times N_A \qquad = 33.879\times10^{23} \qquad = 3.38\times10^{24}$

14. The time taken by an object to slide down 45^0 rough inclined plane is n times as it takes to slide down a perfectly smooth 45^0 incline plane. The coefficient of kinetic friction between the object and the incline plane is.

A) $\sqrt{\dfrac{1}{1-n^2}}$ **B)** $1+\dfrac{1}{n^2}$ **C)** $\sqrt{1-\dfrac{1}{n^2}}$ **D)** $1-\dfrac{1}{n^2}$

Ans: D

Sol: $l = \dfrac{1}{2} g\left(\sin\theta - \mu\cos\theta\right)t_1^2 = \dfrac{g}{2\sqrt{2}}\left(1-\mu\right)t_1^2 .....(i)$

$l = \dfrac{1}{2} g\sin\theta t_2^2 = \dfrac{g}{2\sqrt{2}} t_2^2 .....(ii)$

From equation (i) & (ii)

$\left(1-\mu\right)t_1^2 = t_2^2 \qquad as\, t_1 = nt_2$

So $n^2 = \dfrac{1}{1-\mu} \qquad \Rightarrow \mu = 1-\dfrac{1}{n^2}$

15. At $300K$, the rms speed of oxygen molecules is $\sqrt{\frac{\alpha+5}{\alpha}}$ times to that of its average speed in the gas. Then the value of α will be (used $\pi=\frac{22}{7}$)

A) 32 **B)**27 **C)** 24 **D)**28

Ans: D

Sol: $\sqrt{\frac{3RT}{M}}=\sqrt{\frac{\alpha+5}{\alpha}}\sqrt{\frac{8RT}{\pi M}}$

$3=\left(\frac{\alpha+5}{\alpha}\right)\frac{8}{\left(\frac{22}{7}\right)}\Rightarrow\frac{\alpha+5}{\alpha}=\frac{33}{28}\Rightarrow\alpha=28$

16. The electric current in a circular coil of four turns produces a magnetic induction 32 T at its center. The coil is unwound and is rewound into a circular coil of single turn, the magnetic induction at the center of the coil by the same current will be:

A) 4 T **B)** 16 T **C)** 8 T **D)** 2 T

Ans: D

Sol: As $B=\frac{\mu_0 Ni}{2R}=\frac{\mu_0 i\pi}{(2\pi R)N}N^2=\frac{\mu_0 i\pi}{l}N^2$

$B=B_o N^2$

$32=B_o\times(4)^2$

$B_o=2T$

17. The modulation index for an A.M wave having maximum and minimum peak – to peak Voltages of 14 mV and 6mV respectively is

A) 0.2 **B)** 0.4 **C)** 0.6 **D)** 1.4

Ans: B

Sol: modulation index, $m=\frac{A_{max}-A_{min}}{A_{max}+A_{min}}$

$\Rightarrow m=\frac{14-6}{14+6}=\frac{8}{20}=0.4$

Hence correct Answer is (2)

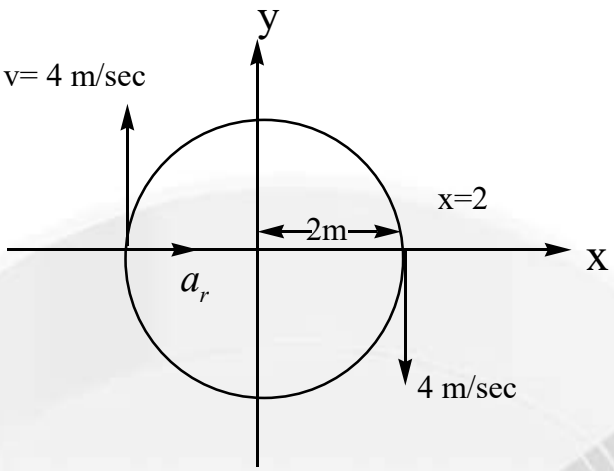
18. An object moves at a constant speed along a circular path in a horizontal plane with center at the origin. When the object is at $x=+2$ m, its velocity is $-4\hat{j}\ m/s$. The object's velocity (v) and acceleration (**A)** at $x=-2$ m will be

A) $v=4\hat{j}\ m/s, a=8\hat{i}\ m/s^2$ **B)** $v=-4\hat{j}\ m/s, a=8\hat{i}\ m/s^2$

C) $v=-4\hat{i}\ m/s, a=-8\hat{j}\ m/s^2$ **D)** $v=4\hat{i}\ m/s, a=8\hat{j}\ m/s^2$

Ans: A

Sol:



$$a_r = \frac{v^2}{R} = \frac{4^2}{2} = 8$$

19. A force acts for 20 s on a body of mass 20 kg starting from rest, after which the force Ceases and then body describes 50 m in the next 10 s. The value of force will be:

A) 5 N **B)** 40 N **C)** 20 N **D)** 10 N

Ans: A

Sol: Let say applied force is F then

Impulse, J = F $\Delta t = mu \Rightarrow u = \frac{F \times 20}{20} = F$

As $s = ut \Rightarrow 50 = F \times 10 \Rightarrow F = 5N$

20. A point charge $2 \times 10^{-2} C$ is moved from P to S in a uniform electric field of 30 NC^{-1} directed along Positive x- axis .If coordinates of P and S are (1,2,0) m and (0,0,0) m respectively, the work done by electric field will be

A) – 1200 mJ **B)** 600 mJ **C)**-600 mJ **D)** 1200 mJ

Ans: C

Sol: Work done, $W = \vec{F}.\vec{S}$ where

$$\vec{F} = q\vec{E} = 2 \times 10^{-2} \times 30\hat{i} = 0.6\hat{i}$$

And $\vec{S} = \vec{r_2} - \vec{r_1} = 0 - \left(\hat{i} + 2\hat{j}\right) = -\left(\hat{i} + 2\hat{j}\right)$

So $W = \vec{F}.\vec{S} = 0.6\hat{i}.\left(-\hat{i} - 2\hat{j}\right) = -0.6J = -600mJ$

(NUMERICAL VALUE TYPE)

Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i, e. If answer is above **10** and less than **10.5** round off is **10** andIf answer is from **10.5** and less than **11** round off is **11).**

Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases.

21. A null point is found at 200 cm in potentiometer when cell in secondary circuit is shunted by 5Ω.When a resistance of 15Ω is used for shunting , null point moves to 300cm.The internal resistance of the cell is _____Ω.

Ans: 5

Sol: as $r = R\left(\frac{l_0}{l} - 1\right)$

When $R = 5\Omega, l = 200\,cm$

So $r = 5\left(\frac{l_0}{200} - 1\right)$ ..... (i)

When $R = 15\Omega, l = 300\,cm$

So $r = 15\left(\frac{l_0}{300} - 1\right)$ ..... (ii)

From equation (i) and (ii)

$$5\left(\frac{l_0}{200} - 1\right) = 15\left(\frac{l_0}{300} - 1\right)$$

$$\Rightarrow \frac{l_0}{200} - 1 = \frac{3l_0}{300} - 3$$

$$\Rightarrow \frac{l_0}{100} - \frac{l_0}{200} = 2 \Rightarrow \frac{l_0}{200} = 2$$

So from Eqn (i)

$$r = 5\left(\frac{l_0}{200} - 1\right) = 5(2-1) = 5\Omega$$

22. Unpolarised light is incident on the boundary between two dielectric media , whose dielectric constants are 2.8 (medium-1) and 6.8 (medium -2), respectively .To satisfy the condition ,so that the reflected and refracted rays are perpendicular to each other , the angle of incidence should be $\tan^{-1}\left(1 + \frac{10}{\theta}\right)^{\frac{1}{2}}$ the value of θ is ___

(Given for dielectric media $\mu_r = 1$)

Ans: 7

Sol: Angle of incidence should be equal to Brewester's angle(θ_B). as $\theta_B = \tan^{-1}\left(\frac{\mu_2}{\mu_1}\right)$

As $C^1 = \frac{1}{\sqrt{\mu_0 \in_r}} = \frac{C}{\mu}$

$$\Rightarrow \frac{C}{\mu_1} = \frac{1}{\sqrt{\mu_0 K_1 \in_0}} \quad and \quad \frac{C}{\mu_2} = \frac{1}{\sqrt{\mu_0 K_2 \in_0}}$$

$$\Rightarrow \frac{\mu_2}{\mu_1} = \sqrt{\frac{K_2}{K_1}} \Rightarrow \theta_B = \tan^{-1}\left(\frac{6.8}{2.8}\right)^{1/2}$$

$$\theta_B = \tan^{-1}\left(1 + \frac{4}{2.8}\right)^{1/2} = \tan^{-1}\left(1 + \frac{10}{7}\right)^{1/2}$$

23. A car is moving on a circular path of radius 600 m such that the magnitudes of the tangential acceleration and centripetal acceleration are equal .The time taken by the car to complete first quarter of revolution ,if it is moving with an initial speed of 54 km/hr is $t\left(1-e^{-\pi/2}\right)s$. .The value of t is ____

Ans: 40

Sol: as $a_r = a_t \Rightarrow R\omega^2 = R\alpha$

$$\Rightarrow \alpha = \frac{d\omega}{dt} = \omega^2 \Rightarrow \frac{d\omega}{\omega^2} = dt$$

$$\Rightarrow \int_{\omega_0}^{\omega} \frac{d\omega}{\omega^2} = t \quad \text{, where } \omega_0 = \frac{v_0}{R} = \frac{15}{600} = \frac{1}{40} \text{ Rad/sec}$$

$$\frac{1}{\omega_0} - \frac{1}{\omega} = t \Rightarrow \omega = \frac{1}{40-t} = \frac{dq}{dt} \qquad \Rightarrow \int_0^{\pi/2} d\theta = \int_0^t \frac{dt}{40-t} \Rightarrow \frac{\pi}{2} = \left[-\ln(40-t)\right]_0^t$$

$$\Rightarrow \frac{\pi}{2} = \ln\frac{40}{40-t} \Rightarrow 1 - \frac{t}{40} = e^{-\pi/2} \Rightarrow t = 40\left(1-e^{-\pi/2}\right)$$

24. An inductor of inductance $2\,\mu$H is connected in series with a resistance , a variable capacitor and an AC source of frequency 7 kHz. The value of capacitance for which maximum current is drawn into the circuit is $\frac{1}{x}F$, where the value of x is _____(Take $\pi = \frac{22}{7}$)

Ans: 3872

Sol: For maximum currenct, $X_C = X_L$

$$\Rightarrow \omega = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{\omega^2 L} \qquad \Rightarrow C = \frac{1}{\left(2\times\frac{22}{7}\times 7\times 10^3\right)^2 \times 2\times 10^{-6}} \qquad = \frac{1}{4\times 22^2 \times 2} = \frac{1}{3872}$$

25. a particle of mass 250 g executes a simple harmonic motion under a periodic force $F = (-25x)$N. The particle attains a maximum speed of 4 m/s during its oscillation . The amplitude of the motion is _____cm.

Ans: 40

Sol: Given, F = -25 x

Compare with F = - Kx

K = 25 N/m

Angular frequency of oscillation $\omega = \sqrt{\frac{K}{M}}$

$$\omega = \sqrt{\frac{25}{0.25}} = 10 \text{ rad/s}$$

Maximum speed of particle $V_{\max} = A\omega$

$4 = A(10)$ $\qquad A = 0.4m = 40\,cm$

26. When two resistances R_1 and R_2 connected in series and introduced into the left gap of a meter bridge and a resistance of 10 Ω is introduced into the right gap, a null point is found at 60 cm from left side. When R_1 and R_2 are connected in parallel and introduced into the left gap, a resistance of 3Ω is introduced into the right -gap to get null point at 40 cm from left end .The product of R_1R_2 is____Ω^2

Ans: 30

Sol: Balancing condition in meter bridge is $\frac{R}{S}=\frac{l}{100-l}$

R = Resistance in left gap
S = Resistance in right gap
l = balancing length

When R,R_2 are in series, $l=60\,cm, S=10\,\Omega$

$$\frac{R_1+R_2}{S}=\frac{60}{40}$$

$$\frac{R_1+R_2}{10}=\frac{3}{2}$$

$$R_1+R_2=15 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

When R,R_2 are in parallel, $l=40\,cm, S=3\,\Omega$

$$\frac{\left(R_1R_2/R_1+R_2\right)}{3}=\frac{40}{60}$$

$$\frac{R_1+R_2}{10}=\frac{3}{2}$$

$$R_1R_2=2\left(R_1+R_2\right)$$

$$\therefore R_1R_2=2(15)=30\Omega^2$$

27. In an experiment of measuring the refractive index of a glass slab using travelling microscope in physics lab , a student measures real thickness of the glass slab as 5.25 mm and apparent thickness of the glass slab as 5.00 mm .Travelling microscope has 20 divisions in one cm on main scale and 50 divisions on Vernier scale is equal to 49 divisions on main scale .The estimated uncertainty in the measurement of refractive index of the slab is $\frac{x}{10}\times10^{-3}$, where x is ____

Ans: 41

Sol: Refractive index of slab $\mu=\frac{\text{Real thicknessof slab}}{\text{Apparent thickness of slab}}$

$$\mu=\frac{d_r}{d_a}=\frac{5.25}{5.00}=1.05$$

$$\frac{\Delta\mu}{\mu}=\frac{\Delta d_r}{d_r}+\frac{\Delta d_a}{d_a}$$

Least count error in measurement of thickness is
L.C=1M.S.D-1V.S.D

$$=\frac{1}{20}\text{cm}-\frac{49}{50}\times\frac{1}{20}\text{cm}$$

$$=\frac{1}{1000}cm$$

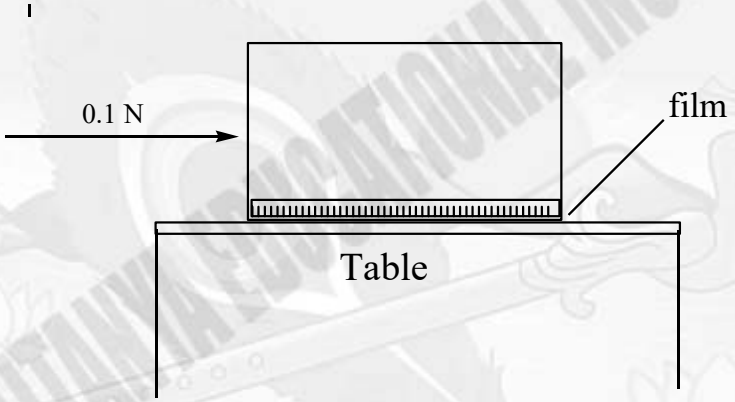
L.C = 0.01 mm

$$\frac{\Delta\mu}{\mu}=\frac{0.01}{5.25}+\frac{0.01}{5.00}$$

$$\Delta\mu=(0.0039)(1.05)=0.0041$$

$$\Delta\mu=\frac{41}{10}\times10^{-3}$$

28. A metal block of base area $0.20\,m^2$ is placed on a table, as shown in figure. A liquid film of thickness 0.25 mm is inserted between the block and the table .The block is pushed by a horizontal force of 0.1 N and moves with a constant speed .If the viscosity of the liquid is 5.0×10^{-3} Pl, the speed of block is_____$\times10^{-3}\ m/s$



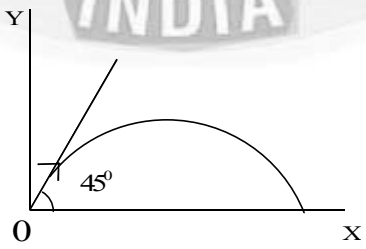
Ans: 25

Sol: From Newton's formula $F=\eta A\frac{dv}{dx}$

$$0.1=5\times10^{-3}\times\frac{(0.20)v}{25\times10^{-5}}$$

$$v=25\times10^{-3}\,m/s$$

29. A particle of mass 100g is projected at time $t=0$ with a speed 20 ms^{-1} at an angle 45^0 to the horizontal as given in the figure. The magnitude of the angular momentum of the particle about the point at time t=2 s is found to be $\sqrt{K}\ kg\ m^2/s$ .The value of K is __ (Take g=10m/s^2)



Ans: 800

Sol: $m = 100\ g = 0.1\ kg$

After time 't', position vector of particle is

$\vec{r} = x\hat{i} + y\hat{j}$

$\vec{r} = (u\cos\theta)t\ \hat{i} + \left(u\sin\theta t - \frac{1}{2}gt^2\right)\hat{j}$

Velocity of particle is

$\vec{V} = V_x\hat{i} + V_y\hat{j}$

$\vec{V} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$

Angular momentum of particle

$\vec{L} = m(\vec{r}\times\vec{v})$

$\vec{L} = m\left[u^2\sin\theta\cos\theta t - u^2\sin\theta\cos\theta t + \frac{1}{2}gt^2u\cos\theta\right]\hat{k}$

$|\vec{L}| = \frac{1}{2}mgt^2u\cos\theta$

$|\vec{L}| = \frac{1}{2}\times 0.1\times 10\times 4\times 20\times\frac{1}{\sqrt{2}} = \frac{40}{\sqrt{2}} = 20\sqrt{2}$

$|\vec{L}| = \sqrt{800}\ kg\ m^2/s$

30. For a charged spherical ball, electrostatic potential inside the ball varies with r as $V = 2ar^2 + b$ Here, a and b are constant and r is the distance from the center .The volume charge density inside the ball is $-\lambda a\varepsilon$ .The value of λ is _____ (ε = permittivity of the medium)

Ans: 12

Sol: $V = 2ar^2 + b$

Electric field $E = -\frac{dV}{dr} = -4ar$

From gauss law $\oint\vec{E}.\vec{ds} = \frac{q}{\in_0}$

$E4\pi r^2 = \frac{\rho\left(\frac{4}{3}\pi r^3\right)}{\in_0}$

$E = \frac{\rho r}{3\in_0}$

$-4ar = \frac{\rho r}{3\in_0} \qquad \Rightarrow \rho = -12a\in_0$

CHEMISTRY

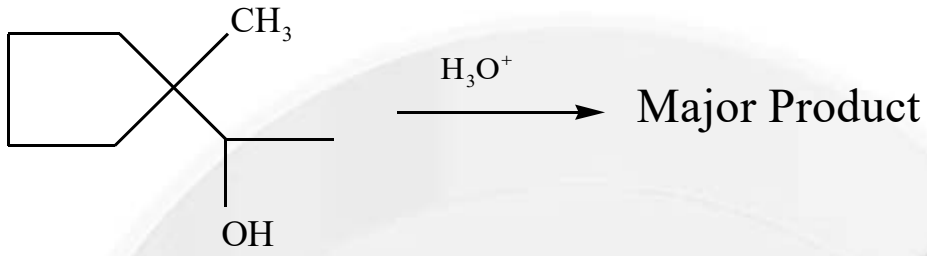
Max Marks: 100

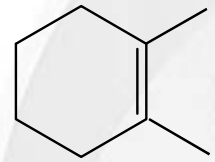
(SINGLE CORRECT ANSWER TYPE)

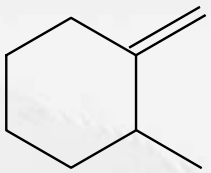
This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

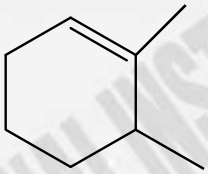
Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

31. Find the major product for the following reaction.



A) 

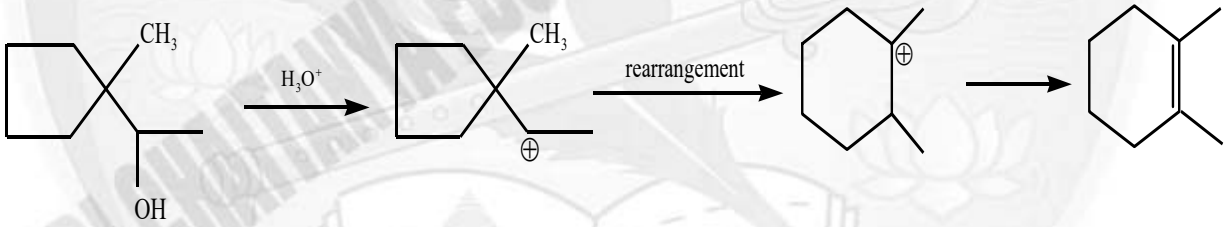
B) 

C) 

D) 

Ans: A

Sol:



32. An indicator 'X' is used for studying the effect of variation in concentration of iodide on the rate of reaction of iodide ion with H_2O_2 at room temp. The indicator 'X' forms blue colored complex with compound 'A' present in the solution. The indicator 'X' and compound 'A' respectively are

A) Starch and iodine **B)** Methyl orange and H_2O_2

C) Starch and H_2O_2 **D)** Methyl orange and iodine

Ans: A

Sol: Iodide ion is oxidized to I_2 Which gives blue color with starch indicator

33. Match List I with List II

List I	List II
A. Van't Hoff factor,i	I. Cryoscopic constant
B. Kf	II. Isotonic solutions
C. solutions with same Osmotic pressure	III. $\frac{\text{Normal molar mass}}{\text{Abnormal molar mass}}$
D. Azeotropes	IV. Solution with same composition of vapour above it

Choose the correct answer from the options given below:

A) A – III, B –II, C – I, D – IV

B)A – III, B –I, C – II, D – IV

C) A – I, B –III, C – II, D – IV

D)A – III, B –I, C – IV, D – II

Ans: B

Sol: A. Van't Hoff factor, i $\rightarrow \frac{\text{Normal molar mass}}{\text{Abnormal molar mass}}$

B. Kf $\rightarrow$ I. Cryoscopic constant

C. solutions with same Osmotic pressure $\rightarrow$ II. Isotonic solutions

D. Azeotropes $\rightarrow$ IV. Solution with same composition of vapour above it

34. Given below are two statements:

Statements I : Nickel is being used as the catalyst for producing syn gas and edible fats.

Statements II : Silicon forms both electron rich and electron deficient hydrides.

In the light of the above statements, choose the most appropriate answer from the options given below:

A) Statement I is incorrect but statement II is correct

B)Both the statements I and II are correct

C) Statement I is correct but statement II is incorrect

D)Both the statements I and II are incorrect

Ans: C

Sol: Statement I is correct but statement II is incorrect

Silicon can form only electron precise hydride

35. According to MO theory the bond orders for O_2^{2-}, CO and NO^+ respectively, are

A) 1,3 and 3 **B)**2,3 and 3 **C)** 1,2 and 3 **D)**1,3 and 2

Ans: A

Sol: Bond order of O_2^{2-} is 1

Bond order of CO is 3

Bond order of NO^+ is 3

36. Given below are two statements:

Statements I : The decrease in first ionization enthalpy from B to Al is much larger than that from Al to Ga.

Statements II : The d orbitals in Ga are completely filled.

In the light of the above statements, choose the most appropriate answers from the options given below

A) Statement I is incorrect but statement II is correct

B)Both the statements I and II are incorrect

C) Both the statements I and II are correct

D)Statement I is correct but statement II is incorrect

Ans: C

Sol: Both the statements I and II are correct

37. Reaction of propanamide with $Br_2 / KOH(aq)$ produces:

A) Ethylnitrile **B)**Propanenitrile **C)** Propylamine **D)**Ethylamine

Ans: D

Sol: $CH_3CH_2CONH_2 + Br_2/KOH \rightarrow CH_3CH_2NH_2$

38. A doctor prescribed the drug Equanil to a patient. The patient was likely to have symptoms of which disease?

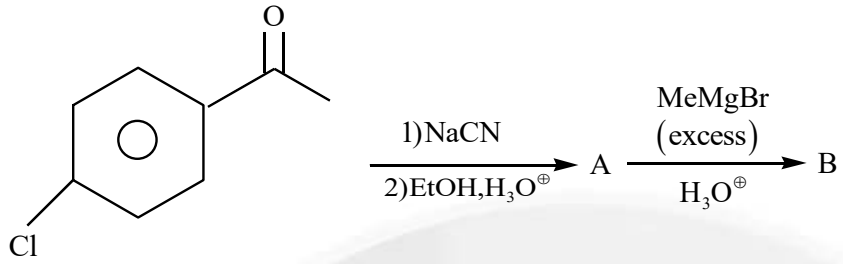
A) Depression and hypertension **B)**Anxiety and stress

C) Hyperacidity **D)**Stomach ulcers

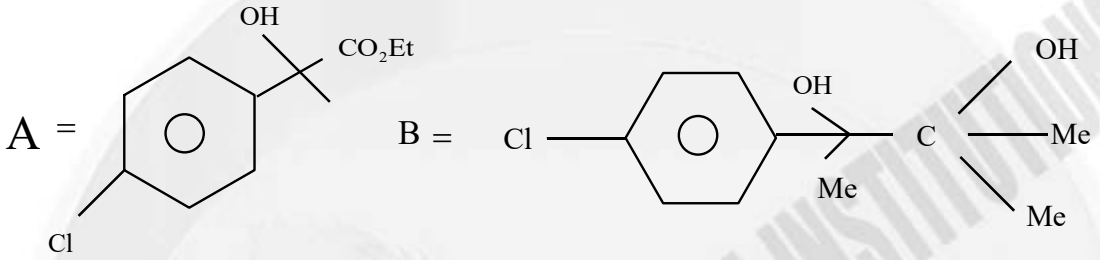
Ans: A

Sol: drug Equanil is used for Depression and hyertension

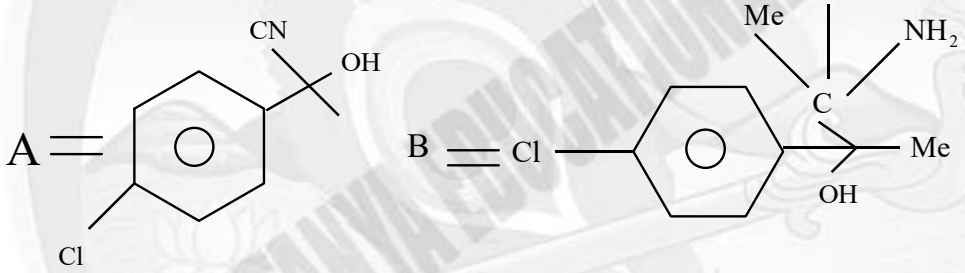
39. Find out the major products from the following reaction sequence.



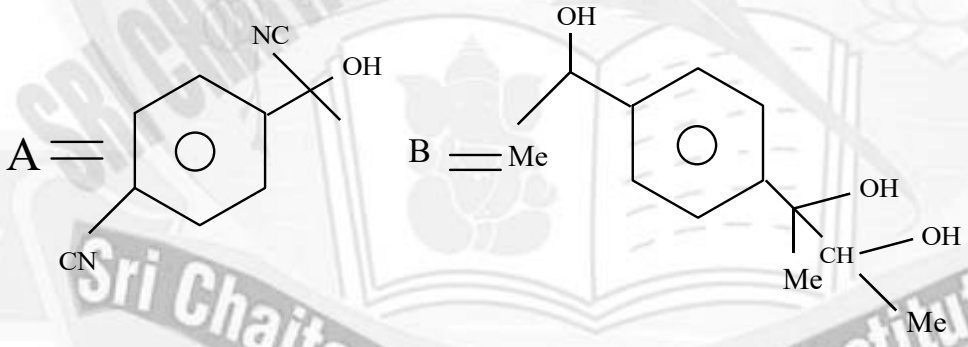
A)



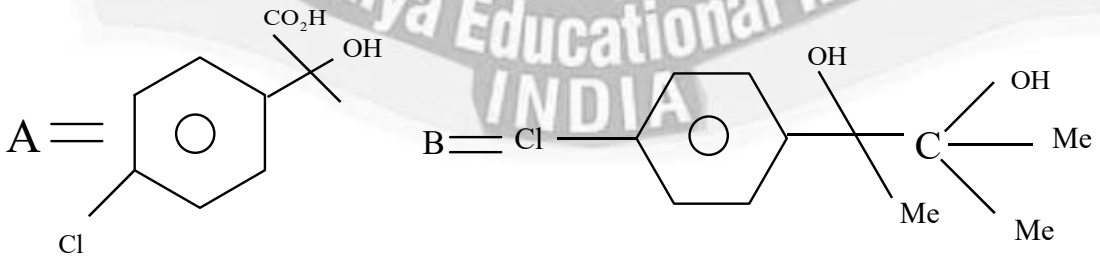
B)



C)

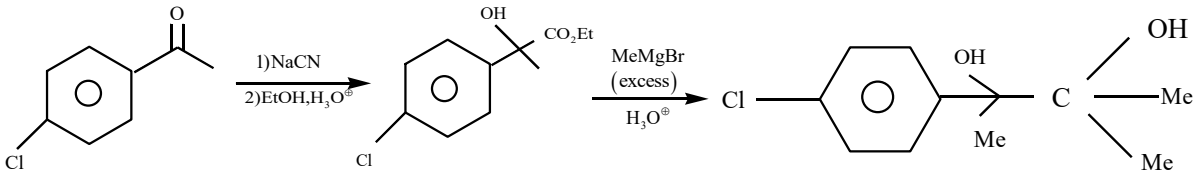


D)



Ans: A

Sol:



40. Match **List I** with **List II**

List I	List II
A. Osmosis	I. Solvent molecules pass through semi permeable Membrane towards solvent side
B. Reverse osmosis	II. Movement of charged colloidal particles under the influence of applied electric potential towards oppositely charged electrodes.
C. Electro osmosis	III. Solvent molecules pass through semi permeable membrane towards solution side.
D. Electrophoresis	IV. Dispersion medium move in an electric field.

Choose the correct answer from the option given below:

A) A – I, B –III, C – IV, D – II

B) A – III, B –I, C – IV, D – II

C) A – I, B –III, C – II, D – IV

D) A – III, B –I, C – II, D – IV

Ans: B

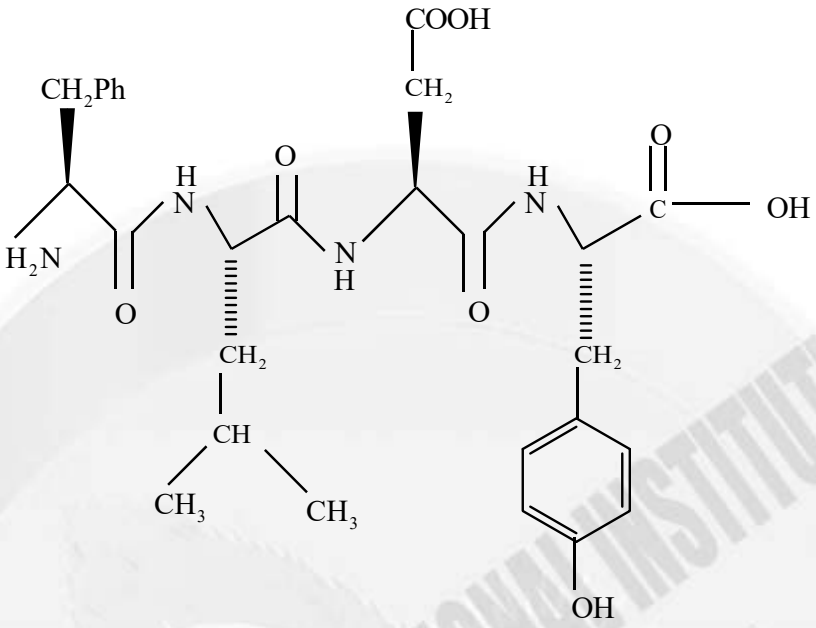
Sol: A. Osmosis → III. Solvent molecules pass through semi permeable membrane towards solution side.

B. Reverse osmosis → I. Solvent molecules pass through semi permeable Membrane towards solvent side

C. Electro osmosis → IV. Dispersion medium move in an electric field.

D. Electrophoresis → II. Movement of charged colloidal particles under the influence of applied electric potential towards oppositely charged electrodes

41. Following tetrapeptide can be represented as



(F, L, D, Y, I, Q, P are one letter code for amino acids)

A) YQLF **B)**FLDY **C)** FIQY **D)**PLDY

Ans: B

Sol: Based on Hydrolysis (F, L, D, Y)

42. Match **List I** with **List II**

List I	List II
A. Elastomeric polymer	I. Urea formaldehyde resin
B. Fibre Polymer	II. Polystyrene
C. Thermosetting Polymer	III. Polyester
D. Thermoplastic Polymer	IV. Neoprene

Choose the correct answer from the options given below:

A) A – IV, B –I, C – III, D – II

B)A – II, B –I, C – IV, D – III

C) A – IV, B –III, C – I, D – II

D)A – II, B –III, C – I, D – IV

Ans: C

Sol . Elastomeric polymer – Neoprene

Fibre Polymer - Polyester

Thermosetting Polymer- Urea formaldehyde resin

Thermoplastic Polymer- Polystyrene

43. Correct order of spin only magnetic moment of the following complex ions is:(Given At.no Fe=26, Co=27)

A) $[FeF_6]^{-3} > [Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-}$

B) $[Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-} > [FeF_6]^{-3}$

C) $[CoF_6]^{3-} > [FeF_6]^{-3} > [Co(C_2O_4)_3]^{3-}$

D) $[FeF_6]^{-3} > [CoF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$

Ans : D

Sol: $[FeF_6]^{-3} > [CoF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$

44. The one giving maximum number of isomeric alkenes on dehydrohalogenation reaction is (excluding rearrangement)

A) 2 - Bromopropane **B)**2 - Bromopentane

C) 1 – Bromo – 2 - methylbutane **D)**2 – Bromo – 3, 3 - dimethylpentane

Ans : B

Sol:

Cis / Trans

45. The set of correct statements is:

(i) Manganese exhibits +7 oxidation state in its oxide.

(ii) Ruthenium and Osmium exhibit +8 oxidition in their oxides.

(iii) Sc shows +4 oxidition state which is oxidizing in nature.

(iv) Cr shows oxidizing nature in +6 oxidation state.

A) (i) and (iii) **B)**(ii), (iii) and (iv)

C) (i), (ii) and (iv) **D)**(ii) and (iii)

Ans : C

Sol: 1,2,4 Statements are correct.

46. The concentration of dissolved oxygen in water for growth of fish should be more than X ppm and Biochemical Oxygen demand in clean water should be less than Y ppm. X and Y in ppm are respectively.

A) $\begin{matrix} X & Y \\ 6 & 5 \end{matrix}$ **B)** $\begin{matrix} X & Y \\ 4 & 15 \end{matrix}$ **C)** $\begin{matrix} X & Y \\ 6 & 12 \end{matrix}$ **D)** $\begin{matrix} X & Y \\ 4 & 8 \end{matrix}$

Ans : A

Sol: $X = 6, Y = 5$

47. Which of the following relations are correct?

(A) $\Delta U = q + p\Delta V$

(B) $\Delta G = \Delta H - T\Delta S$

(C) $\Delta S = \frac{q_{rev}}{T}$

(D) $\Delta H = \Delta U - \Delta nRT$

Choose the most appropriate answer from the options given below

A) B and D only **B)**A and B only **C)**C and D only **D)**B and C only

Ans : D

Sol: $\Delta G = \Delta H - T\Delta S, \Delta S = \frac{q_{rev}}{T}$ *are correct*

48. A solution of CrO_5 in amyl alcohol has a _____ color.

A) Green **B)** Yellow **C)** Blue **D)** Orange-Red

Ans : C

Sol: CrO_5 in amyl alcohol exhibits blue colour

49. The major component of which of the following ore is sulphide based mineral?

A) Calamine **B)** Sphalerite **C)** Malachite **D)** Siderite

Ans : B

Sol: Calamine- $ZnCO_3$

Sphalerite - ZnS

Malachite - $CuCO_3.Cu(OH)_2$

Siderite - $FeCO_3$

50. When the hydrocarbon A undergoes combustion in the presence of air, it requires 9.5 equivalents of oxygen and produces 3 equivalent of water. What is the molecular formula of A?

A) C_6H_6 **B)** C_8H_6 **C)** C_9H_6 **D)** C_9H_9

Ans : B

Sol:

$$CxHy + \left(x + \frac{y}{4}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$$

$$9.5 \qquad\qquad 3$$

$$x + \frac{y}{4} = 9.5 \qquad y = 6$$

$$4x = 38 - 6 = 32$$

$$x = \frac{32}{4} = 8$$

$$\therefore Hydrocorbon\ is\ C_8H_6$$

(NUMERICAL VALUE TYPE)

Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i, e. If answer is above **10** and less than **10.5** round off is **10** andIf answer is from **10.5** and less than **11** round off is **11).**

Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases.

51. A metal M forms hexagonal close-packed structure. The total number of voids in 0.02 mol of it is _____ $\times 10^{21}$ (Nearest integer).

(Given $N_A = 6.02 \times 10^{23}$)

Ans: 36

Sol: mol of h.c.p = 0.02

$T.V = 2 \times 0.02 = 0.04$

$O.V = 0.02$

Total void in mol $= 0.06$

No of void $= 0.06 \times 6.02 \times 10^{23} = 36 \times 10^{21}$

52. Assume that the radius of the first Bohr orbit of hydrogen atom is 0.6 A. The radius of the third Bohr orbit of He^+ is _____ Picometer. (Nearest integer)

Ans: 270

Sol: $r = 0.6\frac{n^2}{z}A^0 = 270\,\text{pm}$

n = 3, z = 2

53. On heating, $LiNO_3$ gives how many compounds among the following? _____

$Li_2O, N_2, O_2, LiNO_2, NO_2$

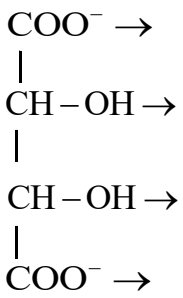
Ans: 3

Sol: $2LiNO_3 \longrightarrow Li_2O + 2NO_2 + \frac{1}{2}O_2$

54. The denticity of the ligand present in the Fehling's reagent is _____

Ans: 4

Sol:



55. When 0.01 mol of an organic compound containing 60% carbon was burnt completely, 4.4 g of CO_2 was produced. The molar mass of compound is _____ $g\,mol^{-1}$.

(Nearest integer)

Ans: 200

Sol: 0.01 mol compound = 0.1 mol carbon

1 mol compound = 10 mol carbon = 120 gram

60 gm carbon = 100 gm compound

120 gm carbon = 200 gm

56. Total number of acidic oxides among

$N_2O_3, NO_2, N_2O, Cl_2O_7, SO_2, CO, CaO, Na_2O$ and NO is _____

Ans: 4

Sol: $N_2O_3, NO_2, Cl_2O_7, SO_2$ are acidic

57. The equilibrium constant for the reaction $Zn(s) + Sn^{2+}(aq) \rightleftharpoons Zn^{2+}(aq) + Sn(s)$ is 1×10^{20} at $298\,K$. The magnitude of standard electrode potential of Sn/Sn^{2+} if $E^{o}_{Zn^{2+}/Zn} = -0.76\text{V}$. is _____ $\times10^{-2}$ V (Nearest integer).

Given: $\frac{2.303RT}{F} = 0.059V$

Ans: 17

Sol: $\text{Zn(s)} + \text{Sn}^{+2}(aq) \rightleftarrows \text{Zn}^{2+}(\text{aq}) + \text{Sn(s)}$

$K_c = 10^{20}$

Ecell = 0 at equilibrium

$\text{E}^{o}\text{cell} = \frac{0.059}{2}\log 10^{20}$

$\text{E}^{o}Sn^{2+}/sn - E^{o}Zn^{+2}/zn = 0.59$

$E^{o}Sn/Sn^{+2} = 17\times10^{-2}\text{V}$

58. At 298 K

$N_2(g)+3H_2(g)\rightleftharpoons 2NH_3(g), K_1=4\times10^5$

$N_2(g)+O_2(g)\rightleftharpoons 2NO(g), K_2=1.6\times10^{12}$

$H_2(g)+\frac{1}{2}O_2(g)\rightleftharpoons H_2O(g), K_3=1.0\times10^{-13}$

Based on above equilibria, the equilibrium constant of the reaction,

$2NH_3(g)+\frac{5}{2}O_2(g)\rightleftharpoons 2NO(g)+3H_2O(g)\, is$ _____$\times10^{-33}$.(Nearest integer)

Ans: 4

Sol: $K=\frac{1}{K_1}\times(K_3)^3\times K_2$

$=4\times10^{-33}$

59. The volume of HCl , containing $73\, gL^{-1}$,required to completely neutralise $NaOH$ obtained by reacting 0.69 g of metallic sodium with water, is _____ ml. (Nearest integer)

(Given: Molar Masses of Na, Cl, O, H are 23,35.5,16 and $1\, g\, mol^{-1}$ respectively).

Ans: 15

Sol: $Na+H_2O\longrightarrow NaOH+\frac{1}{2}H_2$

$\frac{0.69}{23}=0.03 \qquad 0.03$

Milli equivalent of NaOH = milli equivalent of HCl

$0.03\times10^3=2\times V(\text{ml})$

V=15ml

60. For the conversion of compound $A\rightarrow B$, the rate constant of the reaction was found to be $4.6\times10^{-5}\, L\, mol^{-1}s^{-1}$. The order of the reaction is _____

Ans: 2

Sol: Unit of rate constant is $L\, mol^{-1}s^{-1}$

MATHEMATICS

Max : Marks: 100

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can becorrect.
Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

61. If the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{1}$ and $\frac{x-a}{2}=\frac{y+2}{3}=\frac{z-3}{1}$ intersect at the point P, then the distance of the point P from the plane $z=a$ is :

A) 28 **B)**16 **C)** 22 **D)**10

Ans: A

Sol: Given lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{1}$ and $\frac{x-a}{2}=\frac{y+2}{3}=\frac{z-3}{1}$

Point of intersection be P=$(\lambda+1,2\lambda+2,\lambda-3)=(2k+a,3k-2,k+3)$

$\lambda+1=2k+a;2\lambda+2=3k-2;\lambda-3=k+3$

$\Rightarrow 2k+12+2=3k-2\Rightarrow k=16,\lambda=22$

$\Rightarrow a=-9,P=(23,46,19)$

Distance from P to z + 9=0 is 28

62. The value of the integral $\int_{1/2}^{2}\frac{\tan^{-1}x}{x}dx$ is equal to

A) $\pi\log_e 2$ **B)** $\frac{\pi}{4}\log_e 2$ **C)** $\frac{\pi}{2}\log_e 2$ **D)** $\frac{1}{2}\log_e 2$

Ans:C

Sol:

$$I=\int_{1/2}^{2}\frac{\tan^{-1}x}{x}dx \quad Put\ x=\frac{1}{t}$$

$$I=\int_{1/2}^{2}\tan^{-1}\left(\frac{1}{t}\right)\left(\frac{1}{t}\right)dt$$

$$\therefore 2I=\int_{1/2}^{2}\frac{\pi}{2}\frac{dt}{t}\Rightarrow I=\frac{\pi}{2}\ln 2$$

63. The set of all values of $t\in\mathbb{R}$,for which the matrix

$$\begin{bmatrix} e^{t} & e^{-t}(\sin t-2\cos t) & e^{-t}(-2\sin t-\cos t)\\ e^{t} & e^{-t}(2\sin t+\cos t) & e^{-t}(\sin t-2\cos t)\\ e^{t} & e^{-t}\cos t & e^{-t}\sin t \end{bmatrix}$$ is invertible, is

A) $\left\{k\pi+\frac{\pi}{4},k\in\mathbb{Z}\right\}$ **B)** $\mathbb{R}$ **C)** $\left\{(2k+1)\frac{\pi}{2},k\in\mathbb{Z}\right\}$ **D)** $\{k\pi,k\in\mathbb{Z}\}$

Ans: B

Sol: $e^{t}.e^{-t}.e^{-t}\begin{vmatrix}1 & \sin t-2\cos t & -2\sin t-\cos t\\ 1 & 2\sin t+\cos t & \sin t-2\cos t\\ 1 & \cos t & \sin t\end{vmatrix}\neq 0$

$1\left(2\sin^2 t+\sin t\cos t-\cos t\sin t+2\cos^2 t\right)$

$-1\left(\sin^2 t-2\sin t\cos t+2\sin t\cos t+\cos^2 t\right)$

$+1\left(\sin^2 t-4\sin t\cos t+4\cos^2 t+4\sin^2 t+\cos^2 t+4\sin t\cos t\right)\neq 0$

$=2-1+5=6\neq 0\ \ \forall t\in R$

64. The shortest distance between the lines $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$ and $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$ is

A) $3\sqrt{3}$ **B)** $4\sqrt{3}$ **C)** $2\sqrt{3}$ **D)** $5\sqrt{3}$

Ans:B

Sol: Given lines $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$ and $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$

$\vec{b_1}\times\vec{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -7 & 5\\ 2 & 1 & -3\end{vmatrix}=\hat{i}(16)-\hat{j}(-16)+\hat{k}(16)$

$\left(\vec{a_2}-\vec{a_1}\right).\left(\vec{b_1}\times\vec{b_2}\right)=\left(10\hat{j}+2\hat{k}\right)\left(\hat{i}(16)+\hat{j}(16)+\hat{k}(16)\right)$

$=16\times 12$

$\therefore S.D=\frac{16\times 12}{16\sqrt{3}}$

65. Let K be the sum of the coefficients of the odd powers of x in the expansion of $(1+x)^{99}$. Let a be middle term in the expansion of $\left(2+\frac{1}{\sqrt{2}}\right)^{200}$. If $\frac{^{200}C_{99}K}{a}=\frac{2^l m}{n}$, where m and n are odd numbers, then the ordered pair (l,n) is

A) $(50,51)$ **B)** $(50,101)$ **C)** $(51,99)$ **D)** $(51,101)$

Ans:B

Sol: $k=\frac{f(1)-f(-1)}{2}=\frac{2^{99}}{2}=2^{98}$

$$a = {}^{200}C_{100}(2)^{100}\left(\frac{1}{\sqrt{2}}\right)^{100} = {}^{200}C_{100}2^{50}$$

$$\frac{{}^{200}C_{99}.2^{98}}{{}^{200}C_{100}.2^{50}} = \frac{2^l m}{n} \Rightarrow \frac{100}{200-100+1}.2^{48} = \frac{2^l m}{n}$$

$$\Rightarrow \frac{100}{101}.2^{48} = \frac{2^l m}{n} \Rightarrow \frac{25.2^{50}}{101} = \frac{2^l m}{n} \Rightarrow l = 50, n = 101$$

66. The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is

A) 89 **B)**86 **C)** 84 **D)**79

Ans: A

Sol: Given words in alphabetical order GHOTU

Words starting with G____ 24
Words starting with H____ 24
Words starting with O____ 24
Words starting with TG____ 6
Words starting with TH ____ 6
Words starting with T O G ____ 2
Words starting with T O H ____ 2
Words starting with T O U G H _ 1
Rank of the word is **89**

67. The number of 3-digit numbers, that are divisible by either 3 or 4 but not divisible by 48 , is

A) 507 **B)**432 **C)** 400 **D)**472

Ans: B

Sol: No's divisible by 3 = 300
No's divisible by 4 =225
No's divisible by 12 = 75
No's divisible by 48 = 18
Total no's =300+225-75-18
Total no's=432

68. Let $S = \{w_1, w_2, ...\}$ be the sample space associated to a random experiment. Let $P(w_n) = \frac{P(w_{n-1})}{2}, n \geq 2.$, Let $A = \{2k+3l; k, l \in \mathbb{N}\}$ and $B = \{w_n : n \in A\}$. Then $P(B)$ is equal to

A) $\frac{3}{32}$ **B)** $\frac{1}{32}$ **C)** $\frac{3}{64}$ **D)** $\frac{1}{16}$

Ans:C

Sol: Let $P(w_1)=\alpha, P(w_2)=\frac{\alpha}{2}, P(w_3)=\frac{\alpha}{2^2}.....$

$$\sum P(w_i)=\alpha\left(1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}.....\right)=2\alpha=1\Rightarrow\alpha=\frac{1}{2}$$

$$A=\{5,6,7,8,.....\}\Rightarrow P(B)=\frac{1}{2^5}+\frac{1}{2^6}+\frac{1}{2^7}.....=\frac{3}{64}$$

69. Let R be a relation defined on $\mathbb{N}$ as aRb if $2a+3b$ is a multiple of $5, a,b\in\mathbb{N}$. Then R is

A) an equivalence relation **B)**transitive but not symmetric

C) not reflexive **D)**symmetric but not transitive

Ans: A

Sol: Let $(a,b)\in R$

$f(a,b)=2a+3b$

For Reflexive

$f(a,a)=2a+3a=5a$ *i,e divisible by* 5

$\Rightarrow(a,a)\in R$

For symmetric

$f(b,a)=2b+3a=5a+5b-(2a+3b)$

Divisible by 5 Divisible by 5

$f(b,a)$ is divisible by $5\Rightarrow(b,a)\in R$

For transitive

$f(a,b)=2a+3b$ is divisible by 5

$f(b,c)=2b+3c$ is divisible by 5

So, $2a+3c$ is divisible by 5

$\Rightarrow(a,c)\in R$

70. The statement $B\Rightarrow((\sim A)\vee B)$ is equivalent to

A) $B\Rightarrow(A\Rightarrow B)$ **B)** $A\Rightarrow(A\Leftrightarrow B)$

C) $B\Rightarrow((\sim A)\Rightarrow B)$ **D)** $A\Rightarrow((\sim A)\Rightarrow B)$

Ans: B

Sol:

A	$\sim A$	B	$(\sim AvB)$	$B\Rightarrow(\sim AvB)$	$A\Rightarrow B$	$B\Rightarrow(A\Rightarrow B)$
T	F	T	T	T	T	T
F	T	F	T	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T

71. Let $y=y(x)$ be the solution of the differential equation

$x(\log_e x)\frac{dy}{dx}+y=x^2\log_e x,(x>1)$. If $y(2)=2$ then $y(e)$ is equal to

A) $\frac{1+e^2}{4}$ **B)** $\frac{4+e^2}{4}$ **C)** $\frac{2+e^2}{2}$ **D)** $\frac{1+e^2}{2}$

Ans: B

Sol: $x\ln x\frac{dy}{dx}+y=x^2\ln x$

$\frac{dy}{dx}+\frac{y}{x\ln x}=x \qquad \therefore I.F=e^{\int\frac{1}{\sin x}dx}=e^{\ln|\ln x|}=\ln x$

$Sol\,is \quad y(\ln x)=\int x.\ln x dx$

$y\ln x=\ln x\left(\frac{x^2}{2}\right)-\frac{x^2}{4}+c$

$\because x=2,y=2\Rightarrow 2\ln 2=\ln 2(2)-1+c\Rightarrow c=1$

$For\,x=e,\quad y=\frac{e^2}{2}-\frac{e^2}{4}+1\Rightarrow y(e)=1+\frac{e^2}{4}$

72. The value of the integral $\int_1^2\left(\frac{t^4+1}{t^6+1}\right)dt$ is

A) $\tan^{-1}2-\frac{1}{3}\tan^{-1}8+\frac{\pi}{3}$ **B)** $\tan^{-1}\frac{1}{2}+\frac{1}{3}\tan^{-1}8-\frac{\pi}{3}$

C) $\tan^{-1}2+\frac{1}{3}\tan^{-1}8-\frac{\pi}{3}$ **D)** $\tan^{-1}\frac{1}{2}-\frac{1}{3}\tan^{-1}8+\frac{\pi}{3}$

Ans:C

Sol: $I=\int_1^2\frac{t^4+1}{t^6+1}dt=\int_1^2\frac{\left(t^4-t^2+1+t^2\right)}{\left(t^6+1\right)}dt$

$=\int_1^2\frac{t^4-t^2+1}{\left(t^6+1\right)}+\frac{\left(t^2+1\right)}{\left(t^6+1\right)}dt=\int_1^2\frac{dt}{t^2+1}+\frac{1}{3}\int_1^2\frac{3t^2dt}{\left(t^3\right)^2+1}$

$=\tan^{-1}t\Big]_1^2+\frac{1}{3}\tan^{-1}t^3\Big]_1^2$

$=\left(\tan^{-1}2-\tan^{-1}(1)\right)+\frac{1}{3}\left(\tan^{-1}(2)-\tan^{-1}(1)\right)$

$=\tan^{-1}2+\frac{1}{3}\tan^{-1}8-\frac{\pi}{3}$

73. The area of the region $A=\left\{(x,y):|\cos x-\sin x|\le y\le \sin x, 0\le x\le \frac{\pi}{2}\right\}$ is

A) $\sqrt{5}+2\sqrt{2}-4.5$ **B)** $\frac{3}{\sqrt{5}}-\frac{3}{\sqrt{2}}+1$

C) $\sqrt{5}-2\sqrt{2}+1$ **D)** $1-\frac{3}{\sqrt{2}}+\frac{4}{\sqrt{5}}$

Ans: C

Sol: $A=\int_{\theta}^{\pi/2}\left(\sin x-|\cos x-\sin x|\right)dx, where\ \tan\theta=\frac{1}{2}$

$=\int_{\theta}^{\pi/4}(\cos x-\sin x-\sin x)dx+\int_{\pi/4}^{\pi/2}\left(\sin x-(\sin x-\cos x)\right)dx$

$=2\cos x+\sin x\Big]_{\theta}^{\pi/4}+\sin x\Big]_{\pi/4}^{\pi/2}$

$=\sqrt{5}+1-2\sqrt{2}$

74. If the tangent at a point P on the parabola $y^2=3x$ is parallel to the line $x+2y=1$ and the tangents at the points Q and R on the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ are perpendicular to the line $x-y=2$, then the area of the triangle PQR is

A) $5\sqrt{3}$ **B)** $\frac{3}{2}\sqrt{5}$ **C)** $\frac{9}{\sqrt{5}}$ **D)** $3\sqrt{5}$

Ans: D

Sol: Slope of the tangent at $P(t)$ on the parabola $y^2=3x$ is parallel to the line $x+2y=1$ is $\frac{1}{t}=\frac{-1}{2}$

$\Rightarrow P=\left(at^2,2at\right)=(3,-3)$

Slope the tangents at the points Q and R on the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ are perpendicular to the line $x-y=2$ is $m=-1\Rightarrow c^2=a^2m^2+b^2\Rightarrow c=\pm\sqrt{5}$

$\Rightarrow Q=\left(\frac{4}{\sqrt{5}},\frac{1}{\sqrt{5}}\right)$ and $Q=\left(\frac{-4}{\sqrt{5}},\frac{-1}{\sqrt{5}}\right)$

Area of $\Delta PQR=3\sqrt{5}$ Squints.

75. If $\vec{a}=\hat{i}+2\hat{k},\vec{b}=\hat{i}+\hat{j}+\hat{k},\vec{c}=7\hat{i}-3\hat{j}+4\hat{k},\vec{r}\times\vec{b}+\vec{b}\times\vec{c}=\vec{0}$ and $\vec{r}.\vec{a}=0$. Then $\vec{r}.\vec{c}$ is equal to

A) 36 **B)** 34 **C)** 30 **D)** 32

Ans: B

Sol: given $\vec{r}\times\vec{b}+\vec{b}\times\vec{c}=\vec{0}$

$\Rightarrow \vec{r}\times\vec{b}-\vec{c}\times\vec{b}=\vec{0}\Rightarrow\left(\vec{r}-\vec{c}\right)\times\vec{b}=\vec{0}$

$\Rightarrow\left(\vec{r}-\vec{c}\right)=\lambda\vec{b}\Rightarrow\vec{r}=\vec{c}-\lambda\vec{b}$

$\Rightarrow \vec{r}.\vec{a}=\vec{c}.\vec{a}-\lambda\vec{b}.\vec{a}=0\Rightarrow\lambda=5$

$\therefore \vec{r}.\vec{c}=\vec{c}.\vec{c}-5\vec{b}.\vec{c}=34$

76. The plane $2x-y+z=4$ intersects the line segment joining the points $A(a,-2,4)$ and $B(2,b,-3)$ at the point C in the ratio 2:1 and the distance of the point C from the origin is $\sqrt{5}$. If $ab<0$ and P is the point $(a-b,b,2b-a)$ then CP^2 is equal to

A) $\frac{17}{3}$ **B)** $\frac{97}{3}$ **C)** $\frac{73}{3}$ **D)** $\frac{16}{3}$

Ans: A

Sol: $2x-y+z=4,\quad A(a,-2,4)\ B(2,b,-3)$ Let $C(x,y,z)$

$x=\frac{4+a}{3}, y=\frac{2b-2}{3}, z=\frac{-6+4}{3}=\frac{-2}{3}$

$\Rightarrow 8+2a-2b+2-2=12 \quad \Rightarrow 2a-2b=4 \quad \Rightarrow a-b=2$

$OC=\sqrt{5}\Rightarrow\frac{(4+a)^2}{9}+\frac{(2b-2)^2}{9}+\frac{4}{9}=5$

$a^2+8a+16+4a^2-24a+36+4=45 \quad (\because b=a-2)$

$\Rightarrow 5a^2-16a+11=0$

$\Rightarrow (5a-11)(a-1)=0$

$\Rightarrow a=1\ \because ab<0\ \&\ b=-1$

$C\left(\frac{5}{3},-\frac{4}{3},-\frac{2}{3}\right)P(2,-1,-3)$

$CP^2=\left(2-\frac{5}{3}\right)^2+\left(\frac{4}{3}-1\right)^2+\left(3-\frac{2}{3}\right)^2\Rightarrow CP^2=\frac{1}{9}+\frac{1}{9}+\frac{49}{9}=\frac{51}{9}=\frac{17}{3}$

77. The set of all values of λ for which the equation $\cos^2 2x-2\sin^4 x-2\cos^2 x=\lambda$ has a real solution x, is

A) $\left[-2,-\frac{3}{2}\right]$ **B)** $\left[-\frac{3}{2},-1\right]$ **C)** $[-2,-1]$ **D)** $\left[-1,-\frac{1}{2}\right]$

Ans: B

Sol:

$$\cos^2 2x - 2\sin^4 x - 2\cos^2 x = \lambda$$

$$\Rightarrow \cos^2 2x - \frac{1}{2}(1-\cos 2x)^2 - (1+\cos 2x) = \lambda$$

$$\Rightarrow 2\cos^2 2x - \left(1+\cos^2 2x - 2\cos 2x\right) - 2 - 2\cos 2x = 2\lambda$$

$$\Rightarrow \cos^2 2x - 3 = 2\lambda$$

$$\therefore \lambda \in \left[-\frac{3}{2}, -1\right]$$

78. Consider a function $f:\mathbb{N}\to\mathbb{R}$, satisfying

$f(1)+2f(2)+3f(3)+.....+xf(x)=x(x+1)f(x); x\geq 2$ *with* $f(1)=1.$

Then $\frac{1}{f(2022)}+\frac{1}{f(2028)}$ is equal to

A) 8100 **B)**8000 **C)** 8200 **D)**8400

Ans: A

Sol: $f(1)+2f(2)+.....+xf(x)=x(x+1)f(x)$

$$f(1)+2f(2)=2.3f(2)\Rightarrow f(2)=\frac{1}{4}$$

$$f(1)+2f(2)+3f(3)=3.4f(3)\Rightarrow f(3)=\frac{1}{6}$$

$$\because f(2)=\frac{1}{2.2}, f(3)=\frac{1}{2.3}, ...f(n)=\frac{1}{2.n}$$

$$\frac{1}{f(2022)}+\frac{1}{f(2028)}=2\times 2022+2\times 2028=8100$$

79. Let $\vec{a}=4\hat{i}+3\hat{j}, \vec{b}=3\hat{i}-4\hat{j}+5\hat{k}$. If $\vec{c}$ is a vector such that $\vec{c}.\left(\vec{a}\times\vec{b}\right)+25=0,$ and $\vec{c}.\left(\hat{i}+\hat{j}+\hat{k}\right)=4,$ Projection of $\vec{c}$ *on* $\vec{a}$ is 1, then the projection of $\vec{c}$ *on* $\vec{b}$ equals

A) $\frac{1}{\sqrt{2}}$ **B)** $\frac{3}{\sqrt{2}}$ **C)** $\frac{5}{\sqrt{2}}$ **D)** $\frac{1}{5}$

Ans: C

Sol:

$$\bar{a}\times\bar{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\4&3&0\\3&-4&5\end{vmatrix}=15\hat{i}-20\hat{j}-25\hat{k}\Rightarrow\left|\bar{a}\times\bar{b}\right|^2=1250$$

$$\bar{c}=x\bar{a}+y\bar{b}+Z\left(\bar{a}\times\bar{b}\right)$$

$$\bar{c}.\bar{a}=5\Rightarrow 5=x\left|\bar{a}\right|^2=25x\therefore x=\frac{1}{5}$$

$$c.\left(a\times b\right)=z\left|\bar{a}\times\bar{b}\right|^2=1250z=-25\Rightarrow z=-\frac{1}{50}$$

$$\bar{c}=\frac{4}{5}\hat{i}+\frac{3}{5}\hat{j}+y\bar{b}+-\frac{1}{50}\left(15\hat{i}-20\hat{j}-25\hat{k}\right)$$

$$\bar{c}=\left(\frac{4}{5}+3y-\frac{3}{10}\right)\hat{i}+\left(\frac{3}{5}-4y+\frac{z}{5}\right)\hat{j}+\left(5y+\frac{1}{z}\right)\hat{k}$$

$$c\left(\hat{i}+\hat{j}+\hat{k}\right)=4\Rightarrow\frac{1}{2}+3y+1-4y+5y+\frac{1}{z}=4\Rightarrow 4y=2\Rightarrow y=\frac{1}{2}$$

$$\therefore\frac{\bar{c}.\bar{b}}{\left|\bar{b}\right|}=y\left|\bar{b}\right|=\frac{5}{\sqrt{2}}$$

80. Let f and g be twice differentiable functions on $\mathbb{R}$ such that

$$f''(x)=g''(x)+6x$$

$$f'(1)=4g'(1)-3=9$$

$$f(2)=3g(2)=12.$$

Then which of the following is **NOT** true?

A) $\left|f'(x)-g'(x)\right|<6\Rightarrow -1<x<1$ **B)** $g(-2)-f(-2)=20$

C) If $-1<x<2,$ then $\left|f(x)-g(x)\right|<8$

D)There exists $x_0\in(1,3/2)$ such that $f(x_0)=g(x_0)$

Ans: C

Sol: $f''(x)-g''(x)=6x\Rightarrow f'(x)-g'(x)=3x^2+C$

$$x=1\Rightarrow 9-3=3+C\therefore C=3$$

$$f'(x)-g'(x)=3x^2+3$$

$$f(x)-g(x)=x^3+3x+C_1$$

$$x=2\Rightarrow 12-4=8+6+C_1\Rightarrow C_1=-6$$

$$h(x)=f(x)-g(x)=x^3+3x-6$$

$$h'(x)=3x^2+3>0$$

$$for-1<x<2\Rightarrow h(-1)<h(x)<h(z)\Rightarrow h(x)<8$$

(NUMERICAL VALUE TYPE)

Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i, e. If answer is above **10** and less than **10.5** round off is **10** andIf answer is from **10.5** and less than **11** round off is **11).**
Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases.

81. If the equation of the normal to the curve $y=\frac{x-a}{(x+b)(x-2)}$ at the point $(1,-3)$ is $x-4y=13$, then the value of $a+b$ is equal to _____.

Ans: 4

Sol: $(1,-3)$ lie on $y=\frac{x-a}{(x+b)(x-2)} \Rightarrow a+3b+2=0-----I$

Slope of the tangent$=-4=\left(\frac{dy}{dx}\right)_{x=1}$

$\Rightarrow -4=\frac{(1+b)3-(1-a)(b)}{(1+b)^2}$

$\Rightarrow -4(1+b)^2=3(1+b)(1-a)-----II$

solve I and II

$\Rightarrow b=-3, a=7 \Rightarrow a+b=4$

82. The total number of 4-digit number whose greatest common divisor with 54 is 2, is _____.

Ans: 3000

Sol: $54=2.3^3$

N be the 4-digit number which should be divisible by2 but not by 3

number of numbers divisible 2=9000/2=4500

number of numbers divisible by 6= 9000/6=1500

Required 4-digit number=number of numbers divisible 2

– number of numbers divisible by 6

=4500-1500=3000

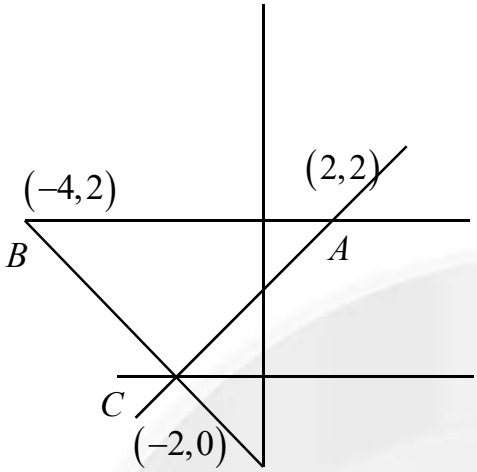
83. A triangle is formed by the tangents at the point $(2,2)$ on the curves $y^2=2x$ and $x^2+y^2=4x$, and the line $x+y+2=0$. If r is the radius of its circumcircle, then r^2 is equal to _____.

Ans: 10

Sol: $2y=x+2 \quad for\ y^2=2x$

$2x+2y=2(x+2)\ for\ x^2+y^2=4x$

$y=2 \qquad x+y+2=0$



To find circumcenter Find the perpendicular bisectors of AB & AC

For AB if is $x=-1$

For BC

$(x+4)^2+(y-2)^2=(x+2)^2+y^2$

$8x-4y+20=4x+4\Rightarrow 4x-4y+16=0\Rightarrow x-y+4=0$

$\therefore Centre(-1,3)\quad Radius\ r^2=10$

84. Let A be a symmetric matrix such that $|A|=2$ *and* $\begin{bmatrix}2 & 1\\ 3 & \frac{3}{2}\end{bmatrix}A=\begin{bmatrix}1 & 2\\ \alpha & \beta\end{bmatrix}$.

If the sum of the diagonal elements of A is s, then $\frac{\beta s}{\alpha^2}$ is equal to _____.

Ans: 5

Sol: $|A|=2\qquad \begin{bmatrix}2 & 1\\ 3 & \frac{3}{2}\end{bmatrix}A=\begin{bmatrix}1 & 2\\ \alpha & \beta\end{bmatrix}$

Det.on both sides $\Rightarrow \beta-2\alpha=0$

Let $A=\begin{bmatrix}a & b\\ b & c\end{bmatrix}\quad ac-b^2=2.....(i)$

$\begin{bmatrix}2 & 1\\ 3 & \frac{3}{2}\end{bmatrix}\begin{bmatrix}a & b\\ b & c\end{bmatrix}=\begin{bmatrix}1 & 2\\ \alpha & 2\alpha\end{bmatrix}$

$2a+b=1.....(ii)$ *and* $2b+c=2.....(iii)$

$3a+\frac{3b}{2}=\alpha\Rightarrow 3b+\frac{3c}{2}=2\alpha\Rightarrow 6a+3b=3b+\frac{3c}{2}$

$c=4a.....(iv)\quad$ *from* $(i)\ \&\ (ii)\ a=\frac{3}{4}$

$$b=-\frac{1}{2}.c=3S=a+c=\frac{15}{4},$$

$$\alpha=\frac{9}{4}+\left(-\frac{3}{4}\right)=\frac{6}{4}=\frac{3}{2}\Rightarrow\beta=3\quad\therefore\frac{\beta S}{\alpha^2}=5$$

85. Let $\{a_k\}$ *and* $\{b_k\}, k\in N,$ be two G.P.s with common ratios r_1 *and* r_2 respectively such that $a_1=b_1=4$ and $r_1<r_2.$ Let $c_k=a_k+b_k, k\in N.$ If $c_2=5$ *and* $c_3=\frac{13}{4}$ then $\sum_{k=1}^{\infty}c_k-\left(12a_6+8b_4\right)$ is equal to _____.

Ans: 9

Sol: $a_1=b_1=4;\quad c_2=a_1r_1+b_1r_2;\ c_2=4\left(r_1+r_2\right)=5$

$$c_3=4\left(r_1^2+r_2^2\right)=\frac{13}{4}$$

$$\left(r_1+r_2\right)^2-2r_1r_2=\frac{13}{16}\Rightarrow\frac{25}{16}-\frac{13}{16}=2r_1r_2$$

$$\Rightarrow 2r_1r_2=\frac{12}{16}=\frac{3}{4}\Rightarrow r_1=\frac{1}{2},r_2=\frac{3}{4}$$

$$\sum_{k=1}^{\infty}C_k=4\left(1+\frac{1}{2}+\frac{1}{2^2}+....\right)+4\left(1+\frac{3}{4}+\left(\frac{3}{4}\right)^2+...\right)$$

$$=8+4\times4=24$$

$$Ans=24-15=9$$

86. Let $a_1=b_1=1$ and $a_n=a_{n-1}+\left(n-1\right), b_n=b_{n-1}+a_{n-1},\ \forall\ n\geq2.$ If $S=\sum_{n=1}^{10}\frac{b_n}{2^n}$ and $T=\sum_{n=1}^{8}\frac{n}{2^{n-1}},$ then $2^7\left(2S-T\right)$ is equal to _____.

Ans: 461

Sol:

$$S=\sum_{n=1}^{10}\frac{b_n}{2^n}=\frac{b_1}{2}+\frac{b_2}{2^2}+.....+\frac{b_{10}}{2^{10}}$$

$$\frac{S}{2}=\frac{b_1}{2^2}+....+\frac{b_7}{2^{10}}+\frac{b_{10}}{2^{11}}\ on\ subtracting$$

$$\frac{S}{2}=\frac{b_1}{2}+\frac{b_2-b_1}{2^2}+\frac{b_3-b_2}{2^3}+.....+\frac{b_{10}-b_9}{2^{10}}-\frac{b_{10}}{2^{11}}$$

$$\frac{S}{2}=\frac{1}{2}+\frac{a_1}{2^2}+\frac{a_2}{2^3}+...+\frac{a_9}{2^{10}}-\frac{b_{10}}{2^{11}}$$

$$\frac{S}{4}=\frac{1}{4}+\frac{a_1}{2^3}+\frac{a_2}{2^4}+.....+\frac{a_8}{2^{10}}+\frac{a_9}{2^{11}}-\frac{b_{10}}{2^{12}}\ on\ subtracting$$

$$\frac{S}{4}=\frac{1}{4}+\frac{a_2-a_1}{2^3}+\frac{a_3-a_2}{2^4}+.....+\frac{a_9-a_8}{2^{10}}-\frac{b_{10}}{2^{11}}-\frac{a_9}{2^{11}}+\frac{b_{10}}{2^{12}}$$

$$S=1+\frac{a_2-a_1}{2}+\frac{a_3-a_2}{2^2}+....+\frac{a_9-a_8}{2^8}-\frac{a_9}{2^9}-\frac{b_{10}}{2^{10}}$$

$$S=1+\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+....+\frac{8}{2^8}-\frac{a_9}{2^9}-\frac{b_{10}}{2^{10}}$$

$$T=1+\frac{2}{2}+\frac{3}{2^2}+....+\frac{8}{2^7}$$

$$2S=2+\left(1+\frac{2}{2^1}+\frac{3}{2^2}+....+\frac{8}{2^7}\right)-\frac{a_9}{2^8}-\frac{b_{10}}{2^9}$$

$$2S-T=2-\frac{a_9}{2^8}-\frac{b_{10}}{2^9}$$

$$2^7\left(2S-T\right)=2^8-\frac{a_9}{2}-\frac{b_{10}}{4}.....(1)$$

$a_n-a_{n-1=n-1}\qquad a_1=1,a_2=2,a_3=4$

$a_k=ak^2+bk+c$

$a+b+c=1$

$4a+2b+c=2$

$9a+3b+c=4$

$$a=\frac{1}{2},b=-\frac{1}{2},c=1$$

$a_9=37$

$b_k = ak^3 + bk^2 + ck + d$

$a + b + c + d = 1$

$8a + 4b + 2c + d = 2$

$27a + 9b + 3c + d = 4$

$64a + 16b + 4c + d = 8$

$b_k = \frac{k^3}{6} - \frac{k^2}{2} + \frac{4}{3}k$

$b_{10} = 130$

Using a_9 & b_{10} in equation ….(1)

We get S=461

87. Let $\alpha = 8 - 14i$, $A = \left\{ z \in C : \frac{\alpha z - \overline{\alpha} \overline{z}}{z^2 - \left(\overline{z}\right)^2 - 112i} = 1 \right\}$ and $B = \left\{ z \in C : |z + 3i| = 4 \right\}$.

Then $\sum_{z \in A \cap B} \left(\text{Re}\, z - \text{Im}\, z \right)$ is equal to _____.

Ans: 14

Sol: Let $z = x + iy$ given $\alpha = 8 - 14i$

$\alpha z - \overline{\alpha} \overline{z} = z^2 - \overline{z}^2 - 112i \qquad z(\alpha - z) - \overline{z}\left(\overline{\alpha} - \overline{z}\right) = -112i$

$Im\left(z(\alpha - z)\right) = -\frac{112}{2} \qquad y(8 - x) - x(14 + y) = -56 \Rightarrow 8y - xy - 14x - xy = -56$

$\Rightarrow xy + 7x - 4y - 28 = 0$

$(x - 4)(y + 7) = 0$ ..... (1)

$x^2 + (y + 3)^2 = 16$ ..... (2)

Solving (1) and (2) we will get $(0, -7)$ and $(4, -3)$

$\sum_{z \in A \cap B} \left(\text{Re}(z) - \text{Im}(z) \right) = 4 + 10 = 14$

88. Let $X = \{11, 12, 13, ..., 40, 41\}$ and $Y = \{61, 62, 63, ..., 90, 91\}$ be the two sets of observations. If $\overline{x}$ *and* $\overline{y}$ are their respective means and σ^2 is the variance of all the observations in $X \cup Y$, then $\left| \overline{x} + \overline{y} - \sigma^2 \right|$ is equal to _____.

Ans: 603

Sol: $\bar{x}=\dfrac{\frac{31}{2}(11+41)}{31}=\dfrac{1}{2}\times 52=26 \quad \bar{y}=\dfrac{\frac{31}{2}(61+91)}{31}=\dfrac{1}{2}\times 152=76$

$$\sigma^2=\frac{\sum x_i^2+\sum y_i^2}{62}-\left(\frac{\sum x_i^2+\sum y_i^2}{62}\right)^2$$

$$=\frac{\left(11^2+12^2+....+41^2\right)+\left(61^2+62^2+....+91^2\right)}{62}-(51)^2$$

$$=\frac{\left(\frac{41.42.83}{63}-\frac{10.11.21}{6}\right)+\left(\frac{91.92.183}{6}-\frac{60.61.121}{6}\right)}{62}-(51)^2$$

$$=\frac{(41.7.83-11.35)+(91.46.61-10.61.121)}{62}-(51)^2$$

$$=\frac{23436+181536}{62}-(51)^2=3306-2601=705$$

$$\bar{x}+\bar{y}-\sigma^2=26+76-705=-603$$

89. Let $\alpha_1,\alpha_2,.....,\alpha_7$ be the roots of the equation $x^7+3x^5-13x^3-15x=0$ and $|\alpha_1|\geq|\alpha_2|\geq....\geq|\alpha_7|$. Then $\alpha_1\alpha_2-\alpha_3\alpha_4+\alpha_5\alpha_6$ is equal to _____

Ans: 9

Sol: $x=0, x^6+3x^4-13x^2-15=0$

Let $x^2=t\Rightarrow t^3+3t^2-13t-15=0$

$\Rightarrow x^2=t=-1,3,-5$

$\Rightarrow x=-i\sqrt{5},i\sqrt{5},\sqrt{3},-\sqrt{3},-i,-i$

$\Rightarrow \alpha_1=-i\sqrt{5},\alpha_2=i\sqrt{5},\alpha_3=\sqrt{3},\alpha_4=-\sqrt{3},\alpha_5=-i,\alpha_6=-i$

$\Rightarrow \alpha_1\alpha_2-\alpha_3\alpha_4+\alpha_5\alpha_6=5+3+1=9$

90. A circle with center $(2,3)$ and radius 4 intersects the line $x+y=3$ at the points P and Q. If the tangents at P and Q intersect at the point $S(\alpha,\beta)$, then $4\alpha-7\beta$ is equal to _____.

Ans: 11

Sol: The given line is the polar of $S(\alpha,\beta)$ w.r.t given circle.

Circle : $x^2+y^2-4x-6y-3=0$

Chord of contact : $\alpha x+\beta y-2(x+\alpha)-3(y+\beta)-3=0$

$\Rightarrow(\alpha-2)x+(\beta-3)y-(2\alpha+3\beta+3)=0$

But equation of chord of contact is $x+y-3=0$

Comparing the coefficients

$$\frac{\alpha-2}{1}=\frac{\beta-3}{1}=\frac{-(2\alpha+3\beta+3)}{3}\Rightarrow\alpha=-6,\beta=-5\Rightarrow 4\alpha-7\beta=11$$