Sri Chaitanya Educational Institutions | Infinity Learn

38 Years of Excellence 1986-2023

The BEST Results in India!
Dominant! Unparalleled!

ALL INDIA RANKS IN JEE-ADV 2022

D JOHN JOSEPH | H.T.NO: 6036441*

JEE MAIN (JAN) 2023 (30-01-2023-Session-1)

MATHEMATICS, PHYSICS & CHEMISTRY

Sri Chaitanya IIT Academy.,India.

✪ A.P ✪ T.S ✪ KARNATAKA ✪ TAMILNADU ✪ MAHARASTRA ✪ DELHI ✪ RANCHI

A right Choice for the Real Aspirant

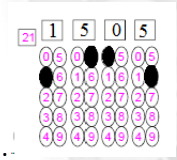
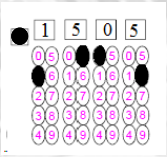
ICON Central Office - Madhapur - Hyderabad

Jee-Main_30-JAN-2023_Shift-01

IMPORTANT INSTRUCTION:

1. Immediately fill in the Admission number on this page of the Test Booklet with **Blue/Black** Ball Point **Pen** only.
2. The candidates should not write their Admission Number anywhere (except in the specified space) on the Test Booklet/ Answer Sheet.
3. The test is of **3 hours** duration.
4. The Test Booklet consists of 90 questions. The maximum marks are **300.**
5. There are **three** parts in the question paper 1, 2, 3 consisting of **Physics, Chemistry** and **Mathematics** having **30 questions** in each subject and subject having **two sections.**
   (I) Section –I contains 20 **multiple choice** questions with only one correct option.
   Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.
   (II) Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only, if more than 5 questions attempted, First 5 attempted questions will be considered.

■ The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i, e. If answer is above **10** and less than **10.5** round off is **10** and If answer is from **10.5** and less than **11** round off is **11).**
To cancel any attempted question bubble on the question number box.
For example: To cancel attempted question 21. Bubble on 21 as shown below

Question Answered for Marking **Question Cancelled for Marking**
Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases.

6. Use **Blue / Black Point Pen only** for writing particulars / marking responses on the Answer Sheet. **Use of pencil is strictly prohibited.**
7. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electron device etc, except the Identity Card inside the examination hall.
8. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
9. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Hall. **However, the candidate are allowed to take away this Test Booklet with them.**
10. **Do not fold of make any stray marks on the Answer Sheet**

PHYSICS

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. Speed of electron in Bohr's 7th orbit for hydrogen atom is $3.6\times10^6 m/s$. The corresponding speed of the electron in 3rd orbit, in m/s is:

1) $\left(3.6\times10^6\right)$ **2)** $\left(7.5\times10^6\right)$ **3)** $\left(1.8\times10^6\right)$ **4)** $\left(8.4\times10^6\right)$

ANS 4

SOL Speed of electron in orbit $V=2.18\times10^6\frac{Z}{n}m/s$

$$3.6\times10^6=\frac{2.18\times10^6}{7}Z$$

$$\frac{7}{3}\times3.6\times10^6=\frac{2.18\times10^6 Z}{3}$$

$$8.4\times10^6 m/s$$

2. A person has been using spectacles of power -1.0 dioptre for distant vision and a separate reading glass of power 2.0 dioptres. What is the least distance of distance of distinct vision for this person:

1) 10 cm **2)** 50 cm **3)** 30 cm **4)** 40 cm

ANS 2

SOL Focal length of convex lens (reading glass)

$$f=\frac{100}{2}=50\text{ cm}$$

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow\frac{1}{-x}-\frac{1}{-25}=\frac{1}{50}$$

Near point of defect eye x=50 cm

3. The charge flowing in a conductor charge with time as $Q(t)=\alpha t-\beta t^2+\gamma t^3$. Where α,β and γ are constants. Minimum value of current is:

1) $\beta-\frac{\alpha^2}{3\gamma}$ **2)** $\alpha-\frac{\beta^2}{3\gamma}$ **3)** $\alpha-\frac{\gamma^2}{3\beta}$ **4)** $\alpha-\frac{3\beta^2}{\gamma}$

ANS 2

SOL

$$Q(t) = \alpha t - \beta t^2 + \gamma t^3$$

$$I = \alpha - 2\beta t + 3\gamma t^2$$

For I $\rightarrow$ min So I min$= \alpha - \frac{\beta^2}{3\gamma}$

$$\frac{dI}{dt} = 0 \Rightarrow t = \frac{\beta}{3\gamma}$$

4. Electric field in a certain region region is given by $\vec{E}=\left(\frac{A}{x^2}\hat{i}+\frac{B}{y^3}\hat{j}\right)$. The SI unit of A and B are:

1) Nm^2C^{-1}; Nm^3C^{-1} **2)** Nm^2C; Nm^3C

3) Nm^3C^{-1}; Nm^2C^{-1} **4)** Nm^3C; Nm^2C

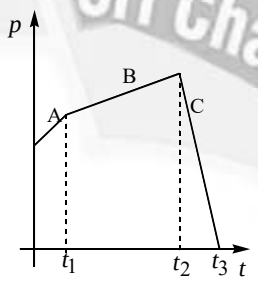
ANS 1

$$\frac{A}{x^2} = N/C$$

SOL $A = N\ m^2/C$

$$\frac{B}{y^3} = N/C \Rightarrow B = N\ m^3/C$$

5. The figure represents the momentum time (p-t) curve for a particle moving along an axis under the influence of the force. Identify the regions on the graph where the magnitude of the force is maximum and minimum respectively? *If* $(t_3 - t_2) < t_1$



1)c and b **2)**b and c **3)**c and a **4)**a and b

ANS 1

SOL $F = \frac{dp}{dt}$ = slope of p-t curve.

6. If The gravitational field in the space is given as $\left(-k / r^2\right)$ Taking the reference point to be at r=2 cm with gravitational potential V=10J/kg. Find the gravitational potential at r=3 cm in SI unit (Given, that K=6 Jcm/kg)

1)11 **2)**10 **3)**9 **4)**12

ANS 1

SOL

$E=\frac{k}{r^2}=-\frac{dv}{dr}$

$\Rightarrow v=\frac{-6}{r}+c$

$10=-\frac{6}{2}+c$

$c=13$

At r=3 cm

$V=-\frac{6}{3}+13=11$ J/kg

7. A ball of mass 200 g rests on vertical post of height 20m. A bullet of mass 10 g, travelling in horizontal direction, hits the centre of the ball. After collision both travels independently. The ball hits the ground at a distance 30 m and the bullet at a distance of 120 m from the foot of the post. The value of initial velocity of the bullet will be (if g=10 m / s^2):

1)120 m/s **2)**60 m/s **3)**360 m/s **4)**400 m/s

ANS 3

$t=\sqrt{\frac{2h}{g}}=2\,\text{sec}$

SOL $V_{\text{bullet}}=60$ m/s

$V_{ball}=15$ m/s

So using conservation of linear momentum $10V=200\times15+10\times60$

V=360 m/s

8. Two isolated metallic solid spheres of radii R and 2R are charged such that both have same charge density σ. The spheres are then connected by a thin conducting wire. If the new charge density of the bigger sphere is σ'. The radio $\frac{\sigma'}{\sigma}$ is:

1) $\frac{9}{4}$ **2)** $\frac{4}{3}$ **3)** $\frac{5}{6}$ **4)** $\frac{5}{3}$

ANS 3

SOL After spheres are connected by a metal wire, their potentials will become same

$$V=\frac{1}{4\pi \in 0}\frac{\sigma\left(4\pi r^2\right)}{r}=\text{constant} \Rightarrow \sigma r=\text{constant}$$

$$\sigma'(2R)=\sigma_2(R)$$

$$\Rightarrow \sigma_2=2\sigma'$$

Using charge conservation

$$\sigma 4\pi R^2+\sigma\left(4\pi(2R)^2\right)=2\sigma'\left(4\pi R^2\right)+\sigma' 4\pi(2R)^2$$

$$\sigma'/\sigma=5/6$$

9. A sinusoidal carrier voltage is amplitude modulated. The resultant amplitude modulated wave has maximum and minimum amplitude of 120 V and 80 V respectively. The amplitude of each sideband is:

1)15 V **2)**10 V **3)**5 V **4)**20 V

ANS 2

SOL

$$A_c-A_m=80$$

$$A_c+A_m=120$$

$$A_c=100 \text{ v}$$

$$m=\frac{A_m}{A_c}=\frac{40}{200}=\frac{1}{5}$$

Amplitude of side band

$$=m\frac{A_c}{2}$$

$$\frac{1}{5}(50)=10 \text{ v}$$

10. In a series LR circuit with $X_L=R$, power factor is P_1. If a capacitor of capacitance C with $X_C=X_L$ is added to the circuit the power factor becomes P_2. The ratio of P_1 to P_2 will be:

1)1 : 2 **2)**1 : $\sqrt{2}$ **3)**1 : 3 **4)**1 : 1

ANS 2

SOL

$$P_1 = \cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + R^2}} = \frac{1}{\sqrt{2}}$$

$$P_2 = \cos\phi = \perp \left(\because X_L = X_C\right)$$

$$\Rightarrow \frac{P_1}{P_2} = \frac{1}{\sqrt{2}}$$

11. The magnetic moments associated with two closely wound circular coils A and B of radius $r_A = 10$ cm and $r_B = 20$ cm respectively are equal if :(Where N_A, I_A and N_B, I_B are number of turn and current of A and respectively)

1) $N_A = 2N_B$ **2)** $4N_A I_A = N_B I_B$ **3)** $2N_A I_A = N_B I_B$ **4)** $N_A I_A = 4N_B I_B$

ANS 4

$$M_A = M_B$$

SOL $N_A I_A \pi (10)^2 = N_B I_B \pi (20)^2$

$$\Rightarrow N_A I_A = 4N_B I_B$$

12. Choose the correct relationship between poisson ratio (σ), bulk modulus (K) and modulus of rigidity (η) of a given solid object:

1) $\sigma = \frac{3K - 2\eta}{6K + 2\eta}$ **2)** $\sigma = \frac{6K - 2\eta}{3K - 2\eta}$ **3)** $\sigma = \frac{6K + 2\eta}{3K - 2\eta}$ **4)** $\sigma = \frac{3K + 2\eta}{6K + 2\eta}$

ANS 1

SOL

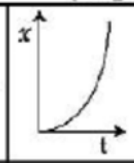
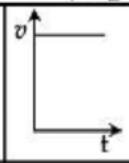
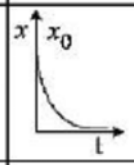
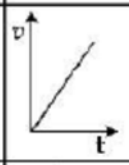
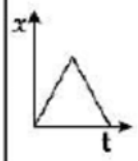
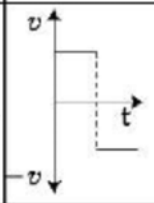
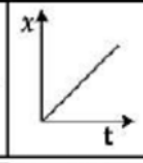
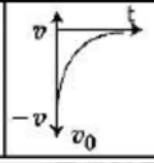
$$Y = 2\eta(1 + 5)$$

$$Y = 3k(1 - 2\sigma)$$

$$2\eta(1 + \sigma) = 3k(1 - 2\sigma)$$

$$\sigma = \frac{3k - 2\eta}{2\eta + 6k}$$

13. Match Column-I with Column-II

Column-I (x-t graphs)		Column-II (v-t graphs)	
A.		I.	
B.		II.	
C.		III.	
D.		IV.	

Choose the correct answer from the options given blow:

1)A-II, B-IV, C-III, D-I **2)**A-I, B-II, C-III, D-IV

3)A-II, B-III, C-IV, D-I **4)**A-I, B-III, C-IV, D-II

ANS 1

SOL **Conceptual**

14. The height of liquid column raised in a capillary tube of certain radius when dipped in liquid A vertically is 5 cm. If the tube is dipped in a similar manner in another liquid B of surface tension and density double the values of liquid A, the height column raised in liquid B would be _____ m.

1)0.10 **2)**0.05 **3)**0.5 **4)**0.20

ANS 2

SOL

$$h=\frac{2T\cos\theta}{r\rho g}$$

$$h'=\frac{2(2T)\cos\theta}{r(2\rho)g}=h=5\text{ cm}$$

15. A small object at rest, absorbs a light pulse of power 20mW and duration 300ns.

Assuming speed of light as 3×10^{8} m/s, the momentum of the object becomes equal to:

1) 0.5×10^{-17} kg m/s **2)** 2×10^{-17} kg m/s

3) 3×10^{-17} kg m/s **4)** 1×10^{-17} kg m/s

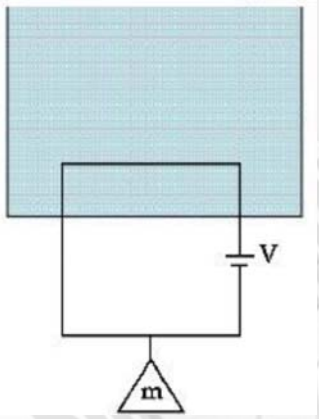
ANS 2

SOL P=momentum of photon $=\frac{E}{C}=\frac{Pt}{C}=2\times10^{-17}kg$ m/s

$\therefore$ momentum of object =P=$2\times10^{-17}kg$ m/s

16. A massless square loop, of wire of resistance 10Ω, supporting a mass of 1 g, hangs vertically with one of its sides in a uniform magnetic field of $10^{3}G$, directed outwards in the shaded region. A dc voltage V is applied to the loop. For what value of V, the magnetic force will exactly balance the weight of the supporting mass of 1g?

(If sides of the loop=10 cm, g=10 ms^{-2})



1) 1 V **2)** $\frac{1}{10}$ V **3)** 100 V **4)** 10 V

ANS 4

SOL

1 Gauss=$10^{-4}T$

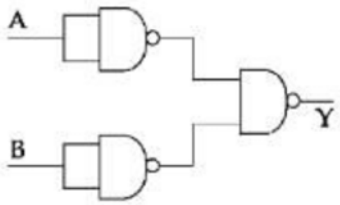
$\Rightarrow B=10^{-1}T$

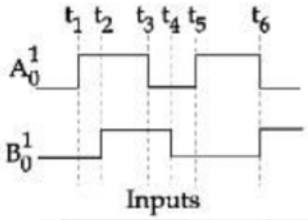
BIL=mg [in equilibrium]

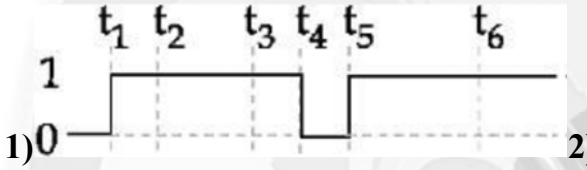
$10^{-1}\left(\frac{V}{10}\right)(0.1)=10^{-3}\times10$

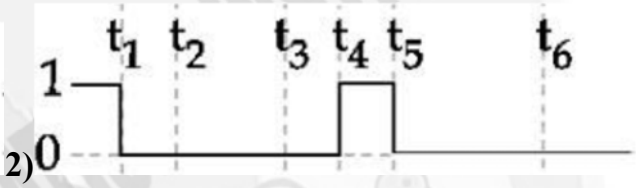
V=10 V

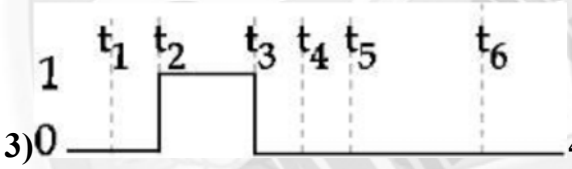
17. The output waveform of the given logical circuit for the following input A and B as shown below, is:

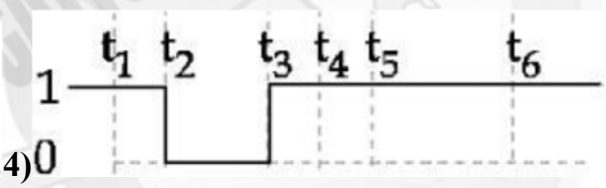












ANS 1

SOL $Y=\overline{\overline{A}\,\overline{B}}=\overline{\overline{A}}+\overline{\overline{B}}=A+B$ So gate is OR gate

18. The pressure (P) and temperature (T) relationship of an obeys the =constant. The volume expansion PT^2 coefficient of the gas will be:

1) $\frac{3}{T^3}$ **2)** $\frac{3}{T^2}$ **3)** $\frac{3}{T}$ **4)** $3T^2$

ANS 3

SOL

PT^2=constant

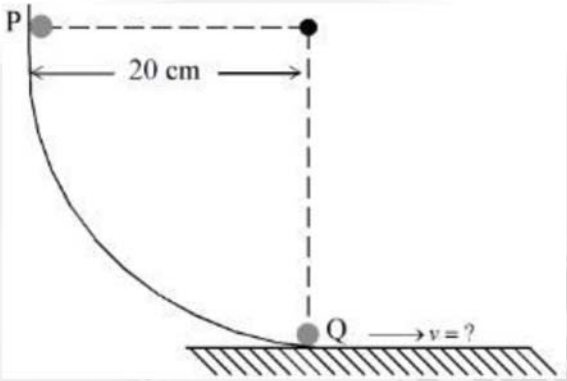
$\frac{nRT}{V}T^2$=constant

$\Rightarrow V=KT^3$

$\gamma=\frac{1}{V}\frac{dv}{dt}=\frac{3}{T}$

19. As per the given figure, a small ball P slides down the quadrant of a circle and hits the other ball Q of equal mass which is initially at rest. Neglecting the effect of friction and assume the collision to be elastic, the velocity of ball Q after collision will be:

($g=10\ m/s^2$)



1)0 **2)**4 m/s **3)**0.25 m/s **4)**2 m/s

ANS 4

SOL $u=\sqrt{2gh}=\sqrt{2\times10\times0.2}=2$ m/s $\therefore$ e=1 & masses are same

V=2 m/s.

20. Heat is given to an ideal gas in an isothermal process.

A. Internal energy of the gas will decrease

B. Internal energy of the gas will increase

C. Internal energy of the gas will not change

D. The gas will do positive work

E. The gas will do negative work

1)C and D only **2)**C and E only

3)A and E only **4)**B and D only

ANS 1

SOL T=constant

$\Rightarrow \Delta U = nC_V \Delta T = 0$

$\Delta\theta = \Delta U + W$

$\because \Delta\theta = +ve$

$\Rightarrow w = +ve$

(NUMERICAL VALUE TYPE)

Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i, e. If answer is above **10** and less than **10.5** round off is **10** andIf answer is from **10.5** and less than **11** round off is **11).**
Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases

21. In an experiment for estimating the value of focal length of converging mirror image of an object placed at 40 cm from the pole of the mirror is formed at distance 120cm from the pole of the mirror. These distances are measured with a modified scale in which there are 20 small divisions in 1cm. The value of error in measurement of focal length of the mirror is $\frac{1}{K}cm$. The value of K is _____

ANS 32

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{f}=\frac{1}{-120}+\frac{1}{-40}$$
$$\Rightarrow f=-30cm$$

SOL $\Delta u=\Delta v=\frac{1}{20}cm$

$$\frac{\Delta u}{v^2}+\frac{\Delta v}{u^2}=\frac{\Delta f}{f^2}\Rightarrow\Delta f=\frac{1}{32cm}$$
$$k=32$$

22. The general displacement of a simple harmonic oscillator is $x=A\sin\omega t$. Let T be its time period. The slope of its potential energy (U)- time(t) curve will be maximum when $t=\frac{T}{\beta}$. The value of β

ANS 8

SOL $$\frac{d^2\left(x^2\right)}{dt^2}=0\left[\because\frac{d^2U}{dt^2}=0\right]$$

$$\frac{d^2\left(x^2\right)}{dt^2}=0\left[\because \frac{d^2U}{dt^2}=0\right]$$

$$\Rightarrow A^2\frac{d}{dt}\left(2\sin\omega t\cos\omega t\right)\omega=0$$

$$\Rightarrow \text{w}\cos\left(2\omega t\right)=0$$

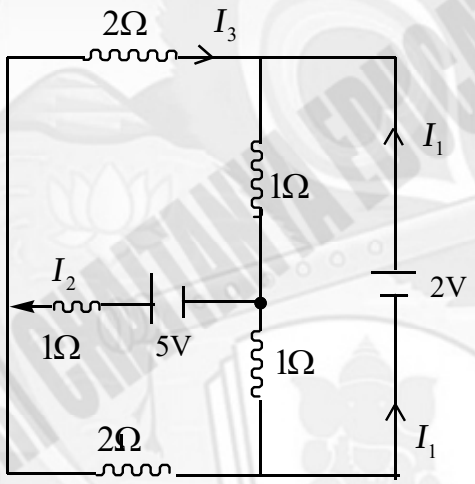
$$\Rightarrow 2\omega t=\pi/2$$

$$\Rightarrow t=\frac{\pi}{4\omega}=\frac{\pi}{4\left(\frac{2\pi}{T}\right)}$$

$$=\frac{T}{8}$$

$$\Rightarrow \beta=8$$

23. In the following circuit the magnitude of current I_1 is_____A



ANS 0

$$I_2=0$$

SOL From symmetry $I_1\frac{2}{\left(\frac{2}{3}+\frac{2}{3}\right)}=\left(\frac{3}{2}\right)A$

24. A thin uniform rod of length 2m. cross sectional area 'A' and density 'd' is rotated about an axis passing through the center and perpendicular to its length with angular velocity ω .If value of ω in terms of its rotational kinetic energy E is $\sqrt{\frac{\alpha E}{Ad}}$ then value of α is ____

ANS 3

SOL

$$E=\frac{1}{2}\left[\frac{dA(2)(2)^2}{12}\right]w^2$$

$$w=\sqrt{\frac{3E}{dA}}\Rightarrow\alpha=3$$

25. A capacitor of capacitance $900\mu F$ is charged by a $100V$ battery. The capacitor is disconnected from the battery and connected to another uncharged identical capacitor such that one plate of uncharged capacitor connected to positive plate and another plate of uncharged capacitor connected to negative plate of the charged capacitor. The loss of energy in this process is measured as $x=10^{-2}J$. The value of x is ____

ANS 225

SOL

$$V_i=\frac{1}{2}\times9\times10^{-4}\times(100)^2$$

$$=4.5\text{ J}$$

$$\text{U}_f=\frac{1}{2}\left(2\times910^{-4}\right)\times(50)^2\left[\because V_f=50\text{ V}\right]$$

$\frac{9}{4}$ J=2.25 J

loss of enegy $=\text{U}_i-V_f$

$=2.25\text{ J}=225\times10^{-2}$ J

26. A point of light is placed at the center of curvature of a hemispherical surface. The source emits a power of 24W. The radius of curvature of hemisphere is 10 cm and the inner surface is completely reflecting. The force on the hemisphere due to the light falling on it is__ $10^{-8}N$

ANS 4

SOL $dF = \dfrac{2\left(\dfrac{P}{4\pi R^2}\right)}{C} 2\pi R \sin\theta \ \mathrm{R}d\theta$

$= \dfrac{P}{C}\sin\theta d\theta$

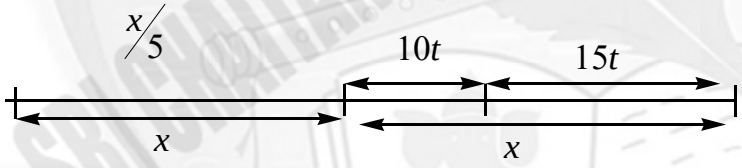
$$F_{net} = \int dF = \frac{P}{2C}\int_0^{\pi/2} \sin(2\theta)\, d\theta$$

$= \dfrac{P}{2C} = \dfrac{24}{6}\times 10^{-8} = 4\times 10^{-8} N$

27. A horse rider covers half the distance with 5 m/s speed. The remaining part of the distance was travelled with speed 10m/s for half the time and with speed 15m/s for other of the time. The mean speed of the rider averaged over the whole-time motion is $\dfrac{x}{7} m/s.$ The value of x is _____

ANS 50

SOL



$25t = x \Rightarrow t = x/25$

$$\langle v \rangle = \frac{\text{distance}}{time} = \frac{2x}{\left(x/5\right) + 2t}$$

$$= \frac{2x}{\left(x/5\right) + \left(2x/25\right)} = \frac{50}{7} \text{ m/s}$$

28. In a screw gauge there are 100 divisions on the circular scale and the main scale moves by 0.5mm on a complete rotation of the circular. The zero of circular scale lies 6 divisions below the line of graduation. When two studs are brought in contact with each other. When a wire is placed between the studs, 4liner scale divisions are clearly visible while 46^{th} division the circular scale coincides with the reference line. The diameter of the wire is_____ $10^{-2}mm$

ANS 220

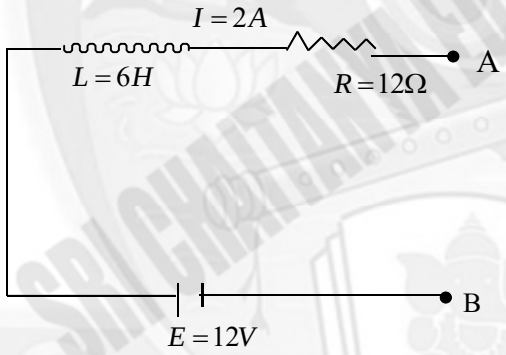
SOL LC= $\frac{0.5}{100}$ =0.005 mm Zero error=6 × LC

=0.03 mm Observed reading=LSR+CSR

=4 × 0.5 mm+46 × LC=2 mm+0.230 mm

=2.230 mm Actual reading=2.200 mm

29.

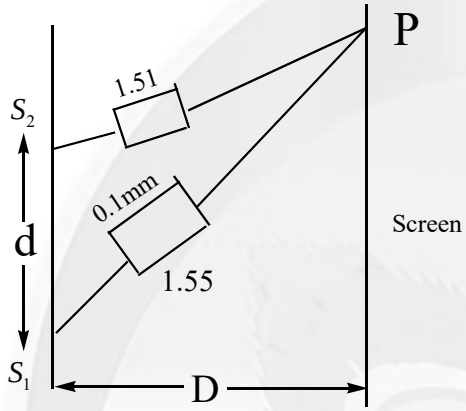


As per the given figure if $\frac{df}{dt}=-1A/s$ then the value of V_{AB} at this instant will be_____V.

ANS 30

SOL $V_A - V_B = 12\times 2+6(-1)+12$ 30 V

30. In Young's double slit experiment two slits S_1 and S_2 are 'd' distance apart and the separation from slits to screen is D (as shown in figure). Now if two transparent slabs of equal thickness 0.1mm but refractive index 1.51 and 1.55 are introduced in the path of beam $(\lambda = 4000A)$ from S_1 and S_2 respectively. The central bright fringe spot will shift by _____ number of fringes



ANS 10

$$\text{shift} = \left[(1.55-1)(0.1)-(1.51-1)(0.1)\right]\frac{D}{d}$$

$$= (0.004)\frac{D}{d} \text{ mm}$$

SOL $\therefore$ number of tringes shifted

$$= \frac{0.004\frac{D}{d}}{\lambda\frac{D}{d}} = 10$$

CHEMISTRY

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

31. Amongst the following compounds, which one is an Antacid?

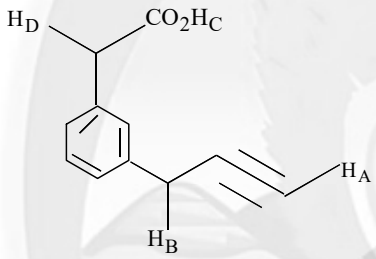
1)Brompheniramine **2)**Meprobamate

3)Ranitidine **4)**Terfenadine

ANS 3

SOL Ranitidine is used as antacid among the given Compounds.

32. What is the correct order of acidity of the protons marked A-D in the given compounds?



1) $H_D > H_C > H_B > H_A$ **2)** $H_C > H_A > H_D > H_B$

3) $H_C > H_D > H_A > H_B$ **4)** $H_C > H_D > H_B > H_A$

ANS 4

SOL Stability of conjugate base is

$H_C > H_D > H_B > H_A$ So acidity order is

$H_C > H_D > H_B > H_A$

33. For OF_2 molecule consider the following:

A. Number of lone pairs on oxygen is 2

B. FOF angle is less than 104.5^0

C. Oxidation state of O is -2.

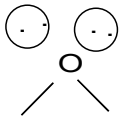
D. Molecule is bent 'v' shaped

E. Molecular geometry is linear.

1)A,B,D only **2)**A,C,D only

3)C,D,E only **4)**B,E,A only

ANS **1**



SOL Bond angle = 103^0.It is 'V' Shaped

34. Given below are two statements: one is labelled as **Assertion (1)** and the other is labelled as **Reason(R)**.

Assertion (1): In expensive scientific instruments. Silica gel is kept in watch-glasses or in semipermeable membrane bags.

Reason(R): Silica gel adsorbs moisture from air via adsorption, thus protects the instrument from water corrosion (rusting) and / or prevents malfunctioning.

In the light of the above statements, choose the correct answer from the options given below:

1) (**1**) is true but (R) is false

2) Both (**1**) and (R) are true and (R) is the correct explanation of (**1**)

3)is false but (R) is true

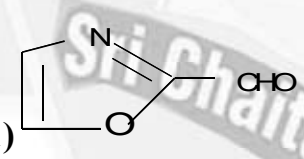
4)Both (**1**) and (R) are true but (R) is not the correct explanation of (**1**)

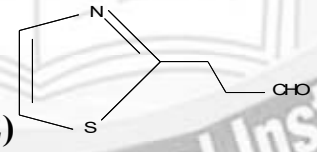
ANS 2

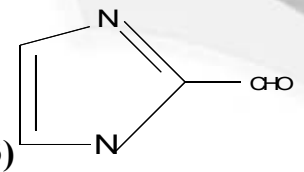
SOL Silica gel being nonreactive and porous can be used for adsorption on its surface

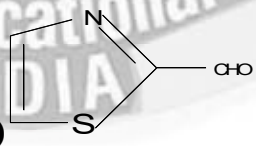
35. Which of the following compounds would give the following set of qualitative analysis?

1. Fehling's test : positive
2. Na fusion extract upon treatment with sodium nitroprusside gives a blood red colour but not prussian blue

1) 

2) 

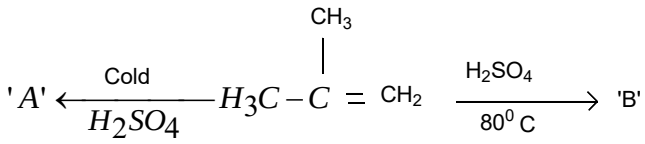
3) 

4) 

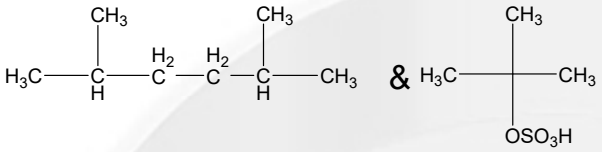
ANS 2

SOL Compound 2 gives positive Fehlings test.

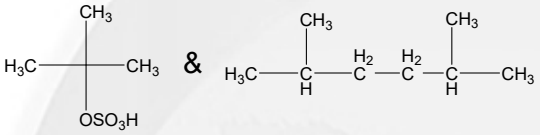
36. The major products 'A' and 'B' ,respectively, are



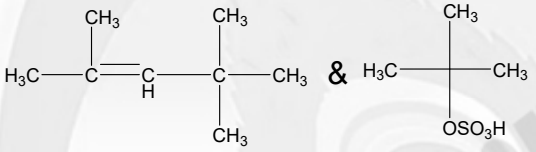
1)



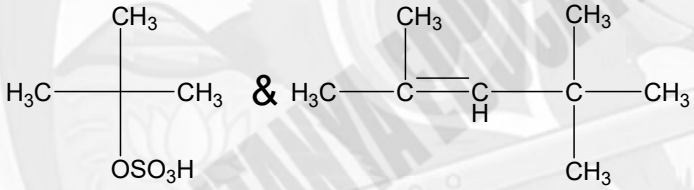
2)



3)

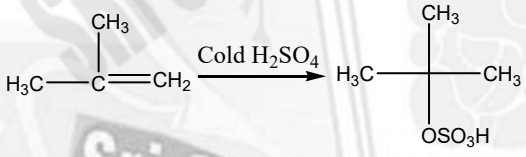


4)

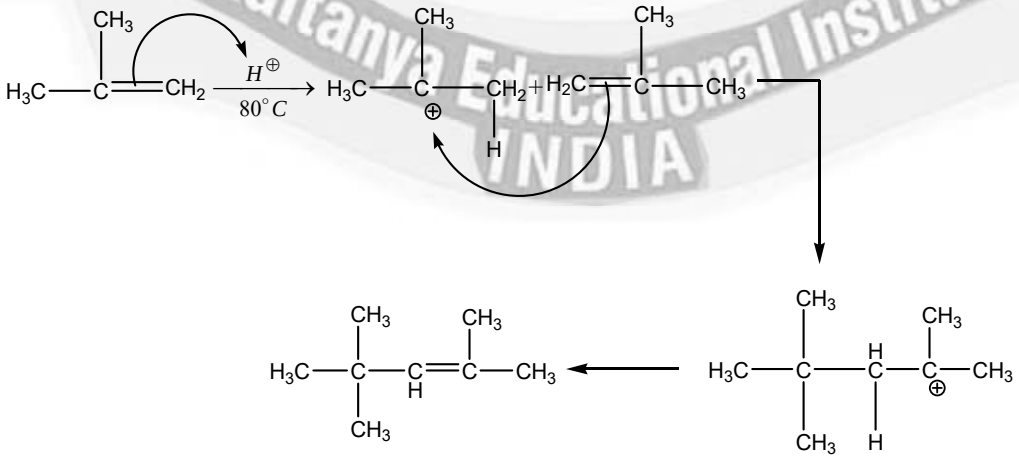


ANS 4

SOL Compound-A



Compound -B



37. Match list I with list II

List I (molecules/ions)		List II (no.of lone pairs of e^- on central atom)	
A.	IF_7	I.	Three
B.	ICl_4^-	II.	One
C.	XeF_6	III.	Two
D.	XeF_2	IV.	Zero

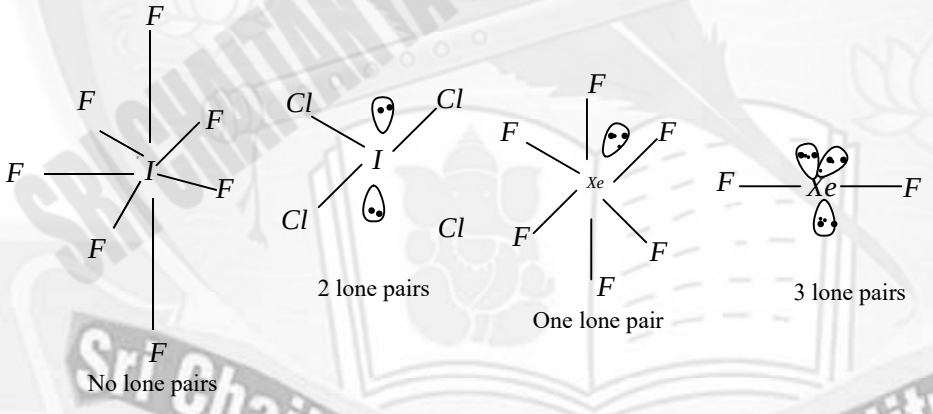
Choose the correct answer from the options given below:

1) A-IV,B-III,C-II,D-I **2)** A-II,B-I,C-IV,D-III

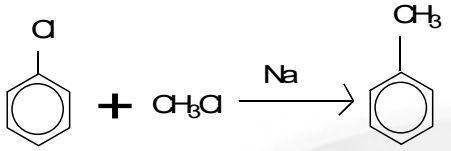
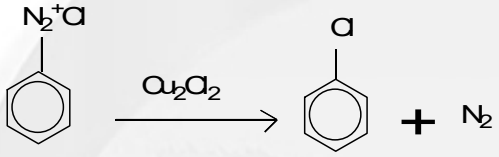
3) A-IV,B-I,C-II,D-III **4)** A-II,B-III,C-IV,D-I

ANS 1

SOL



38. Match List I With List II

List I		List II	
A.		I.	Fitting Reaction
B.		II.	Wurtz Fittig Reaction
C.		III.	Finkelstein Reaction
D.	$C_2H_5Cl+NaI \longrightarrow C_2H_5I+NaCl$	IV.	Sandmeyer Reaction

Choose the correct answer from the options given below:

1)A-II,B-I,C-IV,D-III **2)**A-IV,B-II,C-III,D-I

3)A-III,B-II,C-IV,D-I **4)**A-II,B-I,C-III,D-IV

ANS 4

SOL A is Wurtz Fittig Reaction

B is Fitting Reaction

C is Finkelstein Reaction

D is Sandmeyer Reaction

39. Given below are two statements: one is labelled as **Assertion (A)** and the other is labelled as **Reason(R)**.

Assertion (A): ketoses give seliwanoff 's test faster than aldoses.

Reason(R): ketoses undergo β elimination followed by formation of furfural. In the light of the above statements, choose the correct answer from the options given below:

A)(A)is false but (R) is true

2)Both (A) and (R) are true but (R) is not the correct explanation of (A)

3)Both (A) and (R) are true and (R) is the correct explanation of (A)

4)(2) is false but (R) is false

ANS 3

SOL Seliwanoff's reagent is a mixture of resorcinol and ConCHCl. This test is used to distinguish Ketoses from other Sugars. Only ketoses can produce furfurals which from coloured complexes.

40. The alkaline earth metal sulphate(s) which are readily soluble in water is/are

A. $BeSO_4$ B. $MgSO_4$ C. $CaSO_4$ D. $SrSO_4$ E. $BaSO_4$

Choose the correct answer from the options given below:

1)B and C **2)**A and B **3)**B only **4)**A only

ANS 2

SOL Solubility of group- II sulphates decreases from top to bottom.$BeSO_4$ and $MgSO_4$ are readily soluble in water.

41. Lithium aluminum hydride can be prepared from the reaction of

1)LiCl,Al and H_2 **2)**LiH and Al_2Cl_6

3)LiH and Al $(OH)_3$ **4)**LiCl and Al_2H_6

ANS 2

SOL $4LiH + AlCl_3 \rightarrow LiAlH_4 + 3LiCl$

42. Match list I with list II

list I (atomic number)		List II (Block of periodic table)	
A.	37	I.	p-block
B.	78	II.	d-block
C.	52	III.	f-block
D.	65	IV.	s-block

choose the correct answer from the options give below:

1)A-II,B-IV,C-I,D-III **2)**A-I,B-III,C-IV,D-II

3)A-IV,B-II,C-I,D-III **4)**A-IV,B-III,C-II,D-I

ANS 3

SOL $37 \rightarrow Rb \rightarrow s-block\ 78 \rightarrow Pt \rightarrow d-block\ 52 \rightarrow Te \rightarrow p-block\ 65 \rightarrow Tb \rightarrow f-block$

43. Which of the following is correct order of ligand field strength?

1) $NH_3 < en < CO < S^{2-} < C_2O_4^{2-}$ **2)** $S^{2-} < C_2O_4^{2-} < NH_3 < en < CO$

3) $CO < en < NH_3 < C_2O_4^{2-} < S^{2-}$ **4)** $S^{2-} < NH_3 < en < CO < C_2O_4^{2-}$

ANS 2

SOL Correct order of strength as per spectrochemical series is

$S^{2-} < C_2O_4^{2-} < NH_3 < en < CO$

44. In the extraction of copper, its sulphide ore is heated in a reverberatory furnace after mixing with silica to :

1)Remove FeO as $FeSiO_3$

2)Separate CuO as $CuSiO_3$

3)Decrease the temperature needed for roasting of Cu_2S

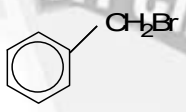
4)Remove calcium as $CaSiO_3$

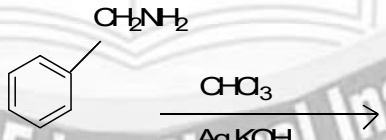
ANS 1

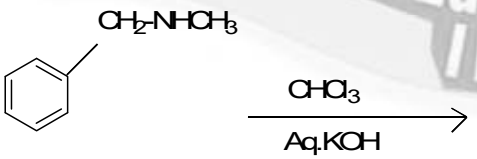
SOL: SiO_2 behaves as flux and reacts with FeO form slag

$FeO + SiO_2 \rightarrow FeSiO_3$

45. Benzyl isocyanide can be obtained by :

A. 

B. 

C. 

D. 

choose the correct answer from the options given below:

1) A and B **2)** B and C **3)** Only B **4)** A and D

ANS A

SOL $PhCH_2 - Br \xrightarrow{AgCN} PhCH_2\text{-}NC$

$PhCH_2NH_2 \xrightarrow{CHCl_3+AqKOH} PhCH_2NC$

(isocyanide test)

46. Formation of photochemical among involves the following reaction in which a, b and c are respectively.

$No_2 \xrightarrow{h\nu} A + B$

$ii. B + O_2 \rightarrow C$

$iii. A + C \rightarrow NO_2 + O_2$

Choose the correct answer from the options given below:

1) $O, NO \& NO_3^-$ **2)** $O, N_2O \& NO$ **3)** $NO, O \& O_3$ **4)** $N, O_2 \& O_3$

ANS C

SOL $NO_2 \xrightarrow{h\nu} NO + (O)$; (A) (B)

$(O) + O_2 \rightarrow O_3$; (B) (C)

$NO + O_3 \rightarrow NO_2 + O_2$; (A) (C)

47. During the qualitative analysis of SO_3^{2-} using dilute H_2SO_4, SO_2 gas is evolved which turns $K_2Cr_2O_7$ solution(acidified with dilute H_2SO_4):

1)Black **2)**Green **3)**Red **4)**Blue

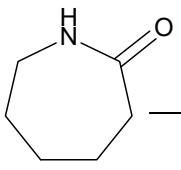
ANS B

SOL $Cr_2O_7^{-2} + 2SO_3^{-2} + 8H^+ \rightarrow 2Cr^{+3} + 2SO_4^{-2} + 4H_2O$

48. Caprolactam when heated at high temperature in presence of water, gives

1)Dacron **2)**Nylon6,6 **3)**Teflon **4)**Nylon6

ANS D

SOL  $\xrightarrow[H_2O]{\Delta}$ $[-\overset{O}{\overset{\|}{C}}-(CH_2)_5-NH-]_n$

Nylon6

49. To inhibit the growth of tumours, identify the compounds used from the following:

A.EDTA

B.Coordination Compounds of Pt

C.D-Penicillamine

D.Cis-Platin

Choose the correct answer from the option given below:

1)B and D only **2)**C and D only

3)A and C only **4)**A and B only

ANS D

SOL Platinum complexes like cisplatin, carboplatin are used in tumour treatment.

50. In the wet tests for identification of various cations by precipitation, which transition element cation doesn't belong to group IV in qualitative inorganic analysis?

1) Ni^{2+} **2)** Zn^{2+} **3)** Fe^{3+} **4)** Co^{2+}

ANS C

SOL Fe^{+3} belongs to III rd group

But $Zn^{+2}, Co^{+2}, Ni^{+2}, Mn^{+2}$ belong to IVth group of qualitative analysis.

(NUMERICAL VALUE TYPE)

Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i, e. If answer is above **10** and less than **10.5** round off is **10** andIf answer is from **10.5** and less than **11** round off is **11).**

Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases

51. A trisubstituted compound 'A' $C_{10}H_{12}O_2$ gives neutral $FeCl_3$ test positive. Treatment of compound 'A' with NaOH. and CH_3Br gives $C_{11}H_{14}O_2$,with hydroidic acid gives methyl iodide and with hot conc. NaOH gives a compound B. $C_{10}H_{12}O_2$ .compound 'A' also decolorizes alkaline $KMnO_4$.the number of π bonds present in the compound 'A' is

_____

ANS 4

SOL Compound A contains

$-OH, -OCH_3,$ $HC{=}C(H){-}CH_3$ Groups on benzene ring

52. Consider the cell

$$Pt_{(s)}\left|H_2(g,1\ atm)\right|\left|H^+(aq.1M)\right|\left|Fe^{3+}(aq),Fe^{2+}(aq)\right|Pt(s)$$

When the potential of the cell is 0.712 V at 298K,the ratio $\left[Fe^{2+}\right]/\left[Fe^{3+}\right]$ is _____

(Nearest integer)

Given : $Fe^{3+}+e^-=Fe^{2+},E^0Fe^{3+},Fe^{2+}$ $\qquad \frac{2.303RT}{F}=0.06$ V

$Pt=0.771$

ANS 10

SOL $\underset{1\text{ atm}}{\frac{1}{2}H_{2(g)}}+Fe^{+3}_{(aq)}\rightleftharpoons Fe^{+2}_{(aq)}+H^{\oplus}_{(aq)}$

$$E_{Cell}=E^0{}_{Cell}-\frac{0.06}{n}\log_{10}\frac{[products]}{[reactions]}$$

$$0.712=(0.771-0)-\frac{0.06}{1}\log_{10}\frac{\left[Fe^{+2}\right]\left[H^+\right]}{\left[Fe^{+3}\right]\left(P_{H_2}^{1/2}\right)}$$

$$0.712=0.771-0.06\log_{10}\frac{\left[Fe^{+2}\right]}{\left[Fe^{+3}\right]}$$

$$\frac{\left[Fe^{+2}\right]}{\left[Fe^{+3}\right]}=10$$

53. When 2 L of ideal gas expands isothermally into vacuum to a total volume of 6L,the change in internal energy is----J(nearest integer)

ANS 0

SOL For ideal gas, internal energy is function of temperature, but unaffected by presume (or) volumeSo $\Delta U=0\because\Delta T=0$ (it is isothermal process)

54. If compound A reacts with B following first order kinetics with rate constant 2.011×10^{-3} s^{-1}.The time taken by A (in seconds) to reduce from 7 g to 2 g will be ----.(nearest integer)log5=0.698,log=0.845,log2=0.301]

ANS 623 sec

SOL 1st order integral rate equation is

$t = \frac{2.303}{K}\log\frac{a}{a-x}$ Initial amount (a) = 7g

Remaining amount $(a-x) = 2g$ $K = 2.011\times10^{-3}\,\text{sec}^{-1}$

$t = \frac{2.303}{2.011\times10^{-3}}\log\frac{7}{2} = 623\,\text{seconds}$

55. The energy of one mole of photons of radiation of frequency 2×10^{12} Hz in J mol^{-1} is -----. (nearest integer)

[Given : h = 6.626×10^{-34} Js]

ANS 798

$E = Nh\nu$

SOL $= 6.022\times10^{23}\times6.626\times10^{-34}\times2\times10^{12}$

$= 798$ J/mole

56. A 300 ml bottle of soft drink has 0.2M CO_2 dissolved in it. Assuming CO_2 behaves as an ideal gas, the volume of the dissolved CO_2 at STP is ------- ml(nearest integer)

Given: At STP molar volume of an ideal gas is 22.7Lmol^{-1}

ANS 1362

SOL Moles of $CO_2 = MV$

$= 0.2\times300\times10^{-3}$

$= 6\times10^{-2}$

$1\ mole - 22700ml$

$6\times10^{-2}\ moles - ?$

$= 1362ml$

57. The number of electrons involved in the reduction of permanganate to manganese dioxide in acidic medium is _____.

ANS 3

SOL $\overset{+7}{Mn}O_4^{-1} \rightarrow \overset{+4}{Mn}O_2$

$MnO_4^{-1} + 4H^{\oplus} + 3e^- \rightarrow MnO_2 + 2H_2O$ Difference in oxidation state $= 7-4 = 3$

So number of elements involved $= 3e^-$

58. A solution containing 2g of a non–volatile solute in 20 g of water boils at 373.52 K. The molecular mass of the solute is _____ g mol^{-1}. (Nearest integer)

Given , water boils at 373 K. K_b for water = 0.52K kg mol^{-1}

ANS 100

SOL

$\Delta T_b = ik_b m \quad \because i = 1$

$0.52 = 1 \times 0.52 \times \frac{2}{mol.wt} \times \frac{1000}{20}$

$mol.wt = 100$

59. 600ml of 0.01M HCl is mixed with 400ml of 0.01 M H_2SO_4. The pH of the mixture is _____ $\times 10^{-2}$. (Nearest integer)

[Given log 2 = 0.30 log 3 = 0.48 log 5 = 0.69 log 7 = 0.84 log 11 = 1.04]

ANS 186

SOL

$H^{\oplus}(N) = \frac{N_1V_1 + N_2V_2}{Total\ volume} = \frac{0.01 \times 1 \times 600 + 0.01 \times 2 \times 400}{1000}$

$= \frac{14}{1000} = 14 \times 10^{-3}$ $pH = -\log 7 \times 2 \times 10^{-3}$

$= 3 - (\log 7 + \log 2) = 3 - (0.84 + 0.3)$

$= 1.86 = 186 \times 10^{-2}$

60. Some amount of dichloromethane (CH_2Cl_2) is added to 671.141 ml of chloroform $(CHCl_3)$ to prepare $2.6 \times 10^{-3} M$ solution of CH_2Cl_2 (DCM). The concentration of DCM is _____ ppm (by mass)

Given :

Atomic mass: C = 12 H = 1

Cl = 35.5 Density of $CHCl_3$ = 1.49 g cm^{-3}

ANS 221

SOL wt of solution = $671.141(ml) \times 1.49\, g/ml$

$= 1000g$

$wt\ of\ \ solute = 2.6 \times 10^{-3} \times 85$

$(CH_2Cl_2) = 221 \times 10^{-3} g$

$= \frac{wt\ of\ \ solute(g)}{wt\ of\ \ solution(g)} \times 10^6\ ppm$

$= \frac{221 \times 10^{-3}}{10^3} \times 10^6 = 221 ppm$

MATHEMATICS

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

61. Among the statements:

1) nether (S1) nor (S2) is a tautology **2)**only (S1) is a tautology

3)both (S1) and (S2) are tautologies **4)**only (S2) is a tautology

ANS 3

SOL

S1: $\left(\left(\sim\left(P\vee q\right)\right)\vee r\right)\Leftrightarrow p\Rightarrow r$

$\left(\left(\sim p\wedge\sim q\right)\vee r\right)\Leftrightarrow p\Rightarrow r$

$\left(\left(\sim p\vee r\right)\wedge\left(\sim q\vee r\right)\right)\Leftrightarrow p\Rightarrow r$

$\left(\left(p\Rightarrow r\right)\wedge\left(q\Rightarrow r\right)\right)\Leftrightarrow p\Rightarrow r$

S1 is tautology

S2: $\left(p\vee q\Rightarrow r\right)\Leftrightarrow\left(p\Rightarrow r\right)\vee\left(q\Rightarrow r\right)$

$\left(\left(p\Rightarrow r\right)\wedge\left(q\Rightarrow r\right)\right)\Leftrightarrow\left(p\Rightarrow r\right)\vee\left(q\Rightarrow r\right)$

S2 is tautology

62. Let a unit vector $\widehat{OP}$ make angle α,β,γ with the positive directions of the co-ordinate axes OX, OY, OZ respectively, where $\beta\in\left(0,\frac{\pi}{2}\right)$. If $\widehat{OP}$ is perpendicular to the plane though points (1,2,3), (2,3,4) and (1,5,7), then which one of the following is true?

1) $\alpha\in\left(0,\frac{\pi}{2}\right)$ and $\gamma\in\left(\frac{\pi}{2},\pi\right)$ **2)** $\alpha\in\left(0,\frac{\pi}{2}\right)$ and $\gamma\in\left(0,\frac{\pi}{2}\right)$

3) $\alpha\in\left(\frac{\pi}{2},\pi\right)$ and $\gamma\in\left(0,\frac{\pi}{2}\right)$ **4)** $\alpha\in\left(\frac{\pi}{2},\pi\right)$ and $\gamma\in\left(\frac{\pi}{2},\pi\right)$

ANS 4

SOL Dc's of OP: $\cos\alpha,\cos\beta,\cos\gamma\ \ \left(\cos\beta>0\right)$

OP is parallel to the normal of the plane containing (1,2,3),(2,3,4),(1,5,7) is

Dr's of above normal: 1,-4,3 $\frac{\cos\alpha}{-1}=\frac{\cos\beta}{4}=\frac{\cos\gamma}{-3}$

$\Rightarrow\cos\alpha<0,\ \cos\gamma<0$

63. Let the solution curve y=y(x) of the differential equation

$$\frac{dy}{dx}-\frac{3x^5\tan^{-1}\left(x^3\right)}{\left(1+x^6\right)^{3/2}}y=2x\exp\left\{\frac{x^3-\tan^{-1}x^3}{\sqrt{\left(1+x^6\right)}}\right\}$$ pass through the origin. Then y(1) is

equal to:

1) $\exp\left(\frac{4-\pi}{4\sqrt{2}}\right)$ **2)** $\exp\left(\frac{\pi-4}{4\sqrt{2}}\right)$

3) $\exp\left(\frac{1-\pi}{4\sqrt{2}}\right)$ **4)** $\exp\left(\frac{4+\pi}{4\sqrt{2}}\right)$

ANS 1

SOL

$$\text{IF}=e^{\int\frac{-3x^5\tan^{-1}\left(x^3\right)}{\left(1+X^6\right)^{3/2}}dx}$$

$$=e^{\int\frac{-t.\tan t}{\sqrt{1+\tan^2 t}}dt}=e^{t\cos t-\sin t}$$

$$=e^{\int-t\sin t\,dt}=e^{\frac{t-\tan t}{\sec t}}=e^{\frac{\tan^{-1}x^3-x^3}{\sqrt{1+x^6}}}$$

$$y.e^{\frac{-\left(x^3-\tan^{-1}x^3\right)}{\sqrt{1+x^6}}}=\int 2x.e^{\frac{\left(x^3-\tan^{-1}x^3\right)}{\sqrt{1+x^6}}}e^{\frac{-\left(x^3-\tan^{-1}x^3\right)}{\sqrt{1+x^6}}}dx$$

$$x=0,y=0\Rightarrow c=0$$

$$\Rightarrow y(1)=e^{\frac{4-\pi}{4\sqrt{2}}}$$

64. Let $A=\begin{pmatrix}m & n\\ p & q\end{pmatrix}$, $d=|A|\neq 0$ and $|A-d(\text{Adj }A)|=0$. Then

1) $1+d^2=m^2+q^2$ **2)** $(1+d)^2=(m+q)^2$

3) $(1+d)^2=m^2+q^2$ **4)** $1+d^2=(m+q)^2$

ANS 2

SOL Here d=mq-pn

$$|A-d(\text{Adj } A)|=0$$

$$\Rightarrow\left|\begin{pmatrix} m & n \\ p & q \end{pmatrix}-d\begin{pmatrix} q & -n \\ -p & m \end{pmatrix}\right|=0$$

$$\Rightarrow\left|\begin{pmatrix} m-dq & n+dn \\ p+dp & q-dm \end{pmatrix}\right|=0$$

$$\Rightarrow (m-aq)(q-dm)-pn(1+d)^2=0$$

$$\Rightarrow mq-dm^2-dq^2+d^2qm=pn+pnd^2+2pnd$$

$$\Rightarrow d+d^3=d\left(m^2+q^2+2(mq-d)\right)$$

$$\Rightarrow d+d^3=d\left(m^2+q^2+2mq\right)-2d^2$$

$$\Rightarrow (1+d)^2=(m+q)^2$$

65. If P(h,k) be a point on the parabola $x=4y^2$, which is nearest to the point Q(0, 33), then the distance of P from the directrix of the parabola $y^2=4(x+y)$ is equal to:

1)2 **2)**4 **3)**8 **4)**6

ANS 4

$$y^2=\frac{x}{4}$$

$$4a=\frac{1}{4}\Rightarrow a=\frac{1}{16}$$

SOL normal at $\left(\frac{1}{16}t^2,\frac{1}{8}t\right)$

$$(0,33)\in y+xt=\frac{t^3}{16}+\frac{t}{8}$$

$$t^3+2t=528$$

t=8 is the only Solution

$$\therefore P=\left(\frac{64}{16},\frac{8}{8}\right)=(4,1)$$

Directrix of $(y-2)^2=4(x+1)$ *is* $x+1=-1\Rightarrow$ x+2=0

required distance=6

66. If the solution of the equation

$\log_{\cos x}\cot x+4\log_{\sin x}\tan x=1, x\in\left(0,\frac{\pi}{2}\right)$, is $\sin^{-1}\left(\frac{\alpha+\sqrt{\beta}}{2}\right)$, where α,β are integers,

then $\alpha+\beta$ is equal to:

1)4 **2)**6 **3)**5 **4)**3

ANS 1

SOL

$$\log_{\cos x}\frac{\cos x}{\sin x}+4.\log_{\sin x}\frac{\sin x}{\cos x}=1$$

$$\Rightarrow t+\frac{4}{t}=4\Rightarrow t=2 \text{ where } t=\log_{\cos x}^{\sin x}$$

$$\log_{\cos x}^{\sin x}=2\Rightarrow \sin x=\cos^2 x$$

$$\Rightarrow \sin x=\frac{-1+\sqrt{5}}{2}=\frac{\alpha+\sqrt{\beta}}{2}$$

$\alpha+\beta$=-1+5=4

67. If $\tan 15^\circ+\frac{1}{\tan 75^\circ}+\frac{1}{\tan 105^\circ}+\tan 195^\circ=2a$, then the value of $\left(a+\frac{1}{a}\right)$ is:

1)4 **2)**2 **3)** $5-\frac{3}{2}\sqrt{3}$ **4)** $4-2\sqrt{3}$

ANS 1

SOL

$$\tan 15^\circ+\cot 75^\circ+\cot 105^\circ+\tan 195^\circ=2a$$

$$\tan 15^\circ+\tan 15^\circ-\tan 15^\circ+\tan 15^\circ=2a$$

$$\frac{1}{a}=2-\sqrt{3}$$

$$a+\frac{1}{a}=2+\sqrt{3}+2-\sqrt{3}=4$$

68. The minimum number of elements that must be added to the relation R={(a,**b**),(b,**c**)} on the set {a,b,c} so that it becomes symmetric and transitive is:

1)4 **2)**5 **3)**7 **4)**3

ANS 3

SOL For transitive (a,**b**) (b,**c**) $\Rightarrow (a,c) \in R$

For symmetry $(b,a)(c,b)(c,a) \in R$

$(c,a) \in R, (a,c) \in R \Rightarrow (c,c) \in R$

$(a,b)(b,a) \in R \Rightarrow (a,a) \in R$

Total 7 elements to be added

69. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors and $\hat{n}$ is a unit vector perpendicular $\vec{c}$ to such that $\vec{a} = \alpha\vec{b} - \hat{n}, (\alpha \neq 0)$ and $\vec{b}.\vec{c} = 12$, then $\left|\vec{c} \times \left(\vec{a} \times \vec{b}\right)\right|$ is equal to:

1)9 **2)**12 **3)**6 **4)**15

ANS 2

SOL

$\hat{n} \perp \vec{c}$

$\vec{a} = \alpha\vec{b} - \hat{n}$

$\left|\vec{c} \times \left(\vec{a} \times \vec{b}\right)\right| = \left|\vec{c} \times \left(-\hat{n} \times \vec{b}\right)\right|$

$= \left|12\hat{n} - 0\vec{b}\right| = 12$

70. Suppose $f : R \to (0,\infty)$ be a differentiable function such that $f : R \to (0,\infty)$. If $f(3)320$, then $\sum_{n=0}^{5} f(n)$ is equal to:

1)6825 **2)**6875 **3)**6525 **4)**6575

ANS 1

SOL 5(x+y)=f(x).f(y)X=0,y=0 f(0)=5Differentiate partially wrt y

$5f'(x+y) = f(x).f'(y)$

$y = 0$ Integrating $5\log f(x) = f'(0).x + c$

$\frac{5f'(x)}{f(x)} = f'(0)$

$x=0$ $x=3$

$5\log 5=c$ $5\log\frac{f(3)}{5}=3f'(0)$

$5\log\frac{f(x)}{5}=xf'(0)$ $5\log 64=3f'(0)$

$f'(0)=\frac{5\times 6}{3}\log 2=10\log 2$ $5\log\frac{f(x)}{5}=(10\log 2)x$

$f(x)=5.4^x$

$$\sum_{n=0}^{5} f(n)=5\left(1+4+4^2+4^3+4^4+4^5\right)=6825$$

71. If an unbiased die, marked with -2,-1,0,1,2,3 on its faces, is thrown five times, then the probability that the product of the outcome is positive is:

1) $\frac{440}{2592}$ **2)** $\frac{521}{2592}$ **3)** $\frac{27}{288}$ **4)** $\frac{881}{2592}$

ANS 2

SOL product of the outcome is positive is:

(5+ve throws)+(3+ve,2-ve)+(1+,4-)

$$\left(\frac{3}{6}\right)^5+{}^5C_3\left(\frac{3}{6}\right)^3\left(\frac{2}{6}\right)^2+{}^5C_1\left(\frac{3}{6}\right)\left(\frac{2}{6}\right)^4\frac{521}{2592}$$

72. The coefficient of $x^{301}in(1+x)^{500}+x(1+x)^{499}+x^2(1+x)^{498}+.....+x^{500}$ is:

1) ${}^{501}C_{200}$ **2)** ${}^{500}C_{300}$ **3)** ${}^{500}C_{301}$ **4)** ${}^{501}C_{302}$

ANS 1

SOL

$$S=(1+x)^{500}+x(1+x)^{499}+x^2(1+x)^{498}+.....+x^{500}$$

$$\frac{S}{(1+x)^{500}}=1+\left(\frac{x}{1+x}\right)+\left(\frac{x}{1+x}\right)^2+.....+\left(\frac{x}{1+x}\right)^{500}$$

$$=\frac{1-\left(\frac{x}{1+x}\right)^{501}}{1-\frac{x}{1+x}}=(1+x)^{501}-x^{501}$$

$\therefore$ coeff of $x^{301=501}C_{301} = {}^{501}C_{200}$

73. If [t] denotes the greatest integer $\le t$, then the value of $\frac{3(e-1)}{e}\int_1^2 x^2 e^{[x]+\left[x^3\right]}dx$ is:

1) $e^8 - e$ **2)** $e^8 - 1$ **3)** $e^9 - e$ **4)** $e^7 - 1$

ANS 1

SOL $I=\int_1^2 x^2 e^{[x]+\left[x^3\right]}dx = \int_1^2 x^2 e^{1+\left[x^3\right]}dx$

$=\frac{e}{3}\int_1^2 e^{[t]}dt$ where $t=x^3$

$=\frac{e}{3}e\left(\frac{e^7-1}{e-1}\right)$

$\therefore$ Given DI$=\frac{3(e-1)}{e}.\frac{e^2}{3}\frac{\left(e^7-1\right)}{(e-1)} = e^8 - e$

74. If $a_n = \frac{-2}{4n^2-16n+15}$, then $a_1 + a_2 + ... + a_{25}$ is equal to:

1) $\frac{50}{141}$ **2)** $\frac{52}{147}$ **3)** $\frac{51}{144}$ **4)** $\frac{49}{138}$

ANS 1

SOL $a_n = \frac{-2}{4n^2-16n+15} = \frac{1}{2n-3} - \frac{1}{2n-5}$

$$\sum_{n=1}^{25} a_n = \left(\left(\frac{1}{-1}\right)-\left(\frac{1}{-3}\right)\right)+\left(\frac{1}{1}-\frac{1}{-1}\right)+\left(\frac{1}{3}-\frac{1}{1}\right)+.....+\left(\frac{1}{47}-\frac{1}{45}\right)$$

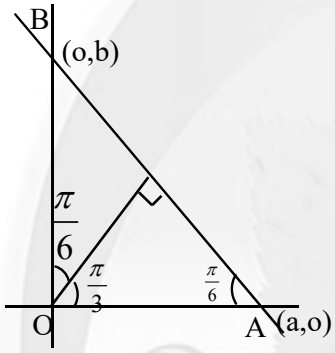
$=\frac{1}{47}+\frac{1}{3}=\frac{50}{141}$

75. A straight line cuts off the intercepts OA=a and OB=b on the positive directions of x-axis and y-axis respectively. If the perpendicular from origin O to this line makes an angle of $\frac{\pi}{6}$ with positive direction of y-axis and the area of ΔOAB is $\frac{98}{3}\sqrt{3}$, then a^2-b^2 is equal to:

1) $\frac{196}{3}$ **2)** 196 **3)** 98 **4)** $\frac{392}{3}$

ANS 4

SOL



$$\tan\frac{\pi}{6}=\frac{b}{a}\Rightarrow a=b\sqrt{3}$$

$$\Delta=\frac{98}{3}\sqrt{3}$$

$$\frac{1}{2}ab=\frac{98}{3}\sqrt{3}$$

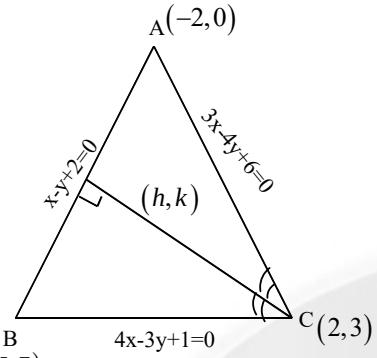
$$b^2=\frac{196}{3}$$

$$a^2-b^2=3b^2-b^2=2b^2=\frac{392}{3}$$

76. Let y=x+2, 4y=3x+6 and 3y=4x+1 be three tangent lines to the circle $(x-h)^2+(y-k)^2=r^2$. Then h+k is equal to:

1) $5\sqrt{2}$ **2)** 6 **3)** $5\left(1+\sqrt{2}\right)$ **4)** 5

ANS 4



SOL

$AB=\sqrt{49+49}=7\sqrt{2}$ $BC=\sqrt{9+16}=5$ In centre lines on x+y=5

$CA=\sqrt{16+9}=5$

77. The number of points on the curve $y=54x^5-135x^4-70x^3+180x^2+210x$ at which the normal lines are parallel to x+90y+2=0 is:

1)4 **2)**0 **3)**3 **4)**2

ANS 1

SOL $\frac{dy}{dx}=270x^4-540x^3-210x^2+360x+210$

$270x^4-540x^3-210x^2+360x+210=90$

Slope of tagent=90 $9x^4-18x^3-7x^2+12x+4=0$

$x=1,2$ are roots

$$1\left|\begin{array}{cccc|c} 9 & -18 & -7 & 12 & 4 \\ & 9 & -9 & -16 & -4 \end{array}\right.$$

$$2\left|\begin{array}{cccc|c} 9 & -9 & -16 & -4 & 0 \\ & 18 & 18 & 4 & \\ \hline 9 & 9 & 2 & & 0 \end{array}\right.$$

Four roots=4such normals

$9x^2+9x+2=0$

$\Delta=81-4.9.2>0$

78. The line l_1 passes thorugh the point (2,6,2) and is perpendicular to plane 2x+y-2z=10. Then the shortest distance between the l_1 and the line $\frac{x+1}{2}=\frac{y+4}{-3}=\frac{z}{2}$ is:

1)7 **2)** $\frac{13}{3}$ **3)**9 **4)** $\frac{19}{3}$

ANS 3

SOL $l_1 = \frac{x-2}{2} = \frac{y-6}{1} = \frac{z-2}{-2}$ S.D $= \frac{\begin{vmatrix} 3 & 10 & 2 \\ 2 & 1 & -2 \\ 2 & -3 & 2 \end{vmatrix}}{\sqrt{64+16+64}}$ $\left|\frac{-12-80-16}{12}\right| = \frac{108}{12} = 9$

79. Let the system of linear equations

x+y+kz=2 2x+3y-z=1 3x+4y+2z=k

Have infinitely many solutions. Then the system

(k+1)x+(2k-1)y=7

(2k+1)x+(k+5)y=10

has:

1)unique solutions satisfying x-y=1 **2)**unique solutions satisfying x+y=1

3)infinitely many solutions **4)**No solutions

ANS 2

SOL $\Delta = \begin{vmatrix} 1 & 1 & k \\ 2 & 3 & -1 \\ 3 & 4 & 2 \end{vmatrix} = 0 \Rightarrow 10 - 7 - k = 0 \Rightarrow k = 3$

For k=3, clearly addition of first 2 equations given the third one $\begin{matrix} 4x+5y=7 \\ 7x+8y=10 \end{matrix}$

$x=-2,\ y=3$

80. If the coefficient of x^{15} in the expansion of $\left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15}$ is equal to the coefficient of x^{-15} in the expansion of $\left(ax^{1/3} + \frac{1}{bx^3}\right)^{15}$, where a and b are positive real numbers, then for each such ordered pair (a,**b**):

1)ab=3 **2)**ab=1 **3)**a=3b **4)**a=b

ANS 2

SOL $^{15}C_r\left(ax^3\right)^{15-r}\left(\frac{1}{bx^{1/3}}\right)^r = {}^{15}C_r\frac{a^{15-r}}{b^6}x^{45-3r-\frac{r}{3}}$

$$45-3r-\frac{r}{3}=15\Rightarrow r=9$$

$$^{15}C_p\left(ax^{1/3}\right)^{15-p}\left(\frac{-1}{bx^3}\right)^p = {}^{15}C_p\frac{a^{15-p}}{(-b)^p}x^{5-\frac{p}{3}-3p}$$

$$5-\frac{p}{3}-3p=-15$$

$$\frac{10p}{3}=20\Rightarrow p=6$$

$$\therefore {}^{15}C_9\frac{a^6}{b^9} = {}^{15}C_6\frac{a^9}{(-b)^6}$$

$$(ab)^3=1 \Rightarrow ab=1$$

(NUMERICAL VALUE TYPE)

Section-II contains 10 **Numerical Value Type** questions. Attempt any 5 questions only. First 5 attempted questions will be considered if more than 5 questions attempted. The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **nearest Integer** value (Example i, e. If answer is above **10** and less than **10.5** round off is **10** andIf answer is from **10.5** and less than **11** round off is **11).**
Marking scheme: +4 for correct answer, 0 if **not attempt** and **-1** in all other cases

81. Let S={1,2,3,4,5,6}. Then the number of one-one functions $f:S\to P(S)$, where P(S) denote the power set of S, such that $f(n)\subset f(m)$ when n<m where is_____.

ANS 3240

SOL $f(1)\subset f(2)\subset f(3)\subset f(4)\subset f(5)\subset f(6)$
required number of ways

$${}^6C_2.4.3.2.1+6.{}^5C_2.3.2.1+6.5.{}^4C_2.2.1+6.5.4.{}^3C_2.1+6.5.4.3.{}^2C_2=3240$$

82. Let z=1+i and $z_1=\frac{1+i\bar{z}}{\bar{z}(1-z)+\frac{1}{z}}$. Then $\frac{12}{\pi}\arg(z_1)$ is equal to____.

ANS 9

SOL $z_1=\frac{1+i(1-i)}{1-i-2+\frac{1-i}{2}}=\frac{1+i+1}{\frac{-1}{2}-\frac{3i}{2}}=\frac{(2+i)2}{-(1+3i)}$

$$=\frac{2(2+i)(1-3i)}{-(1+9)}=\frac{-2}{10}\left((2+3)+i(1-6)\right)$$

$$=1-i$$

$$\frac{12}{\pi}\arg(z_1)=\frac{12}{\pi}\times\frac{3\pi}{4}=9$$

83. Let α be the area of the larger region bounded by the curve $y^2=8x$ and the lines y=x and x=2, which lies in the first quadrant. Then the value of 3α is equal to_____.

ANS 22

SOL

$$A_1=\int_0^2\left(2\sqrt{2}\sqrt{x}-x\right)dx=\left(2\sqrt{2}.\frac{2}{3}x^{3/2}-\frac{x^2}{2}\right)_0^2$$

$$=\frac{4\sqrt{2}}{3}2\sqrt{2}-2=\frac{16}{3}-2=\frac{10}{3}$$

$$A_2=\int_2^8\left(2\sqrt{2}\sqrt{x}-x\right)dx=\left(2\sqrt{2}.\frac{2}{3}x^{3/2}-\frac{x^2}{2}\right)_2^8$$

$$=\frac{4\sqrt{2}}{3}8\sqrt{8}-32-\frac{10}{3}$$

$$=\frac{128}{3}-32-\frac{10}{3}=\frac{22}{3}=\alpha$$

$$\therefore 3\alpha=22$$

84. $\lim\limits_{x\to0}\frac{48}{x^4}\int_0^x\frac{t^3}{t^6+1}dt$ is equal to_____.

ANS 12

SOL $\lim\limits_{x\to 0}\dfrac{48\int\limits_0^x \dfrac{t^3}{t^6+1}dt}{x^4}=\lim\limits_{x\to 0}\dfrac{48\dfrac{x^3}{x^6+1}}{4x^3}=12$

85. If the equation of the plane passing through the point(1,1,2) and perpendicular to the line x-3y+2z-1=0=4x-y+z is Ax+By+Cz=1, then 140 (C-B+A) is equal to_____.

ANS 15

SOL Dr's line: -3 2 1 -3

-1 1 4 -1-1,7,11Required plane:-(x-1)+7(y-1)+11(z-2)=0

-x+7y+11z=-1+7+22

$$\frac{-x}{28}+\frac{7y}{28}+\frac{11}{28}z=1$$

$$140\left(\frac{11}{28}-\frac{7}{28}-\frac{1}{28}\right)=15$$

86. If $\lambda_1<\lambda_2$ are two values of λ such that the angle between the planes $P_1:\vec{r}\left(3\hat{i}-5\hat{j}+\hat{k}\right)=7$ and $P_2:\vec{r}\left(\lambda\hat{i}+\hat{j}-3\hat{k}\right)=9$ is $\sin^{-1}\left(\dfrac{2\sqrt{6}}{5}\right)$, then the square of the length of perpendicular from the point to $\left(38\lambda_1,10\lambda_2,2\right)$ the plane P_1 is

ANS 315

SOL $\cos\theta=\dfrac{|3\lambda-5-3|}{\sqrt{35}\sqrt{\lambda^2+10}}=\dfrac{1}{5}$

$\Rightarrow\lambda=\dfrac{25}{19},5$

$Point=(50,50,2)$

$d^2=315$

87. Numbers of 4-digit numbers (the repeation of digits is allowed) which are made using the digits 1,2,3 and 5 and are divisible by 15, is equal to_____.

ANS 21

SOL Sum can be 9 or 12 or 15 or 18

When sum is 9 5 $\rightarrow$ 3ways When sum is 12 3 ways 3 ways3 ways $\rightarrow$9ways

When sum is 15 2 3 5 5 $\rightarrow$3!=6 waysWhen sum is 18

2 5 5 5 $\rightarrow$3 waysTotal=21 ways

88. The mean and variance of 7 observations are 8 and 16 respectively. If one observation 14 is omitted and a and b are respectively mean and variance of remaining 6 observation, than a+3b-5 is equal to_____.

ANS 37

SOL $x_1, x_2, ..., x_7$

$$\frac{\sum x_i}{7} = 8 \Rightarrow \sum x_i = 56$$

For 6 observation new mean $a = \frac{56-14}{6} = 7$

$$\frac{\sum x_i^2}{7} - 64 = 16 \Rightarrow \sum x_i^2 = 560$$

For 6 observation new mean $b = \frac{560-14^2}{6} - 49 = \frac{35}{3}$

$\therefore a + 3b - 5 = 7 + 35 - 5 = 37$

89. Let $\sum_{n=0}^{\infty} \frac{n^3((2n)!)+(2n-1)(n!)}{(n!)((2n)!)} = ae + \frac{b}{e} + c$, where

a, b, c $\in Z$ and e= $\sum_{n=0}^{\infty} \frac{1}{n!}$ Then $a^2 - b + c$ is equal to _____.

ANS 26

SOL $\sum\left(\frac{n^3}{n!}+\frac{2n-1}{(2n)!}\right)$

$$=\sum\frac{n^2}{(n-1)!}+\sum\frac{1}{(2n-1)!}-\sum\frac{1}{(2n)!}\rightarrow(1)$$

Consider $\sum\frac{n^2}{(n-1)!}=\sum\frac{(n-1)(n+1)+1}{(n-1)!}$

$$=\sum\frac{1}{(n-3)!}+3\sum\frac{1}{(n-2)!}+\sum\frac{1}{(n-1)!}$$

=e+3e+e=5e

$$e+\frac{1}{e}=\left(1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+.....\right)+\left(1-\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+.....\right)$$

$$=2\left(1+\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+.....\right)=2\sum\frac{1}{(2n)!}$$

$$e-\frac{1}{e}=2\left(\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+.....\right)=2\sum\frac{1}{(2n-1)!}$$

using in $(1)\Rightarrow 5e+\frac{e-\frac{1}{e}}{2}-\frac{e+\frac{1}{e}}{2}$

$$5e-\frac{1}{2e}-\frac{1}{2e}=5e-\frac{1}{e}=ae+\frac{b}{e}+c$$

$$a^2-b+c=25+1=26$$

Let $f^1(x)=\frac{3x+2}{2x+3}, x\in R-\left\{\frac{-3}{2}\right\}$

90. For $n\geq 2$, define $f^n(x)=f^1 of^{n-1}(x)$

If $f^5(x)=\frac{ax+b}{bx+a}, \gcd(a,b)=1,$ then a+b is equal to

ANS 3125

SOL $f^2(x)=\frac{3\left(\frac{3x+2}{2x+3}\right)+2}{3\left(\frac{3x+2}{2x+3}\right)+2}=\frac{9x+6+4x+6}{6x+4+6x+9}$

$\Rightarrow \frac{13x+12}{12x+13}$

$f^1(x)=\frac{px+q}{qx+p}$

$f^2(x)=\frac{p\left(\frac{px+q}{qx+p}\right)+q}{\frac{q(px+q)}{qx+p}+p}=\frac{\left(p^2+q^2\right)x+2pq}{2pqx+\left(p^2+q^2\right)x}=\frac{ax+b}{bx+a}\Rightarrow a+b=(p+q)^2$

$f^3(x)=\frac{\left(p^2+q^2\right)\left(\frac{px+q}{qx+p}\right)+2pq}{2pq\left(\frac{px+q}{qx+p}\right)+\left(p^2+q^2\right)\left(\frac{px+q}{qx+p}\right)}$

$=\frac{\left(p^3+3pq^2\right)x+\left(3p^2q+q^3\right)}{\left(3p^2q+q^3\right)x+\left(p^3+3pq^2\right)}=\frac{ax+b}{bx+a}=a+b=(p+q)^3$

$f^5(x)=\frac{ax+b}{a+bx}\Rightarrow a+b=(p+q)^5=5^5=3125$