



# PERFECT 100 PERCENTILERS

## JEE MAIN SESSION 1 JAN 2025

65 Students Secured 100 Percentiles\*



**100**  
Percentile

**D RUTVIK SAI**  
APP.No: 250310156772



**100**  
Percentile

**SHIVEN TOSHWAL**  
APP.No: 250310391420\*



**100**  
Percentile

**BHAVESH JAYANTHI**  
APP.No: 250310269939



**100**  
Percentile

**PRANAYA SAI MUKESH**  
APP.No: 250310608114



**100**  
Percentile

**SAI SRI RAM SATVIK**  
APP.No: 250310255926




**100**  
Percentile

**UJJWAL KESARI**  
APP.No: 250310008860\*



**100**  
Percentile

















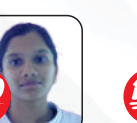









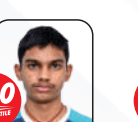

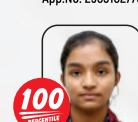


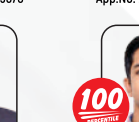
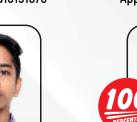
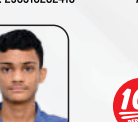



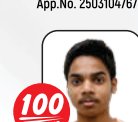


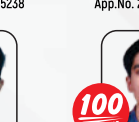
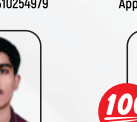

**DHINESH GOMATHI**  
APP.No: 250310496782



**100**  
Percentile

**ADEEB ALI ISLAM**  
APP.No: 250310004843\*

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 <p><b>100</b> Percentile</p> <p><b>KUSHAL N</b> App.No. 250310413022</p>	 <p><b>100</b> Percentile</p> <p><b>ALLABHYA PAREEK</b> App.No. 250310277087</p>	 <p><b>100</b> Percentile</p> <p><b>N SRIVATHSAV</b> App.No. 250310969414</p>	 <p><b>100</b> Percentile</p> <p><b>MADDU ROHIT</b> App.No. 250310415670</p>	 <p><b>100</b> Percentile</p> <p><b>K HAMSHINI</b> App.No. 250310191070</p>	 <p><b>100</b> Percentile</p> <p><b>M LAKSHMANA SEPUR</b> App.No. 250310282416</p>	 <p><b>100</b> Percentile</p> <p><b>AMIT PRASAD</b> App.No. 250310242521</p>	 <p><b>100</b> Percentile</p> <p><b>KANNEBOYENA SATVIK</b> App.No. 250310480858</p>	 <p><b>100</b> Percentile</p> <p><b>VANNALE ADEEP</b> App.No. 250310669118</p>
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Congratulations to Students, Parents & Staff

#TransformingYourDreamsIntoReality

\*in One or More Subjects



# Sri Chaitanya IIT Academy., India.

A.P, TELANGANA, KARNATAKA, TAMILNADU, MAHARASHTRA, DELHI, RANCHI

*A right Choice for the Real Aspirant*

**ICON Central Office – Madhapur – Hyderabad**

**02-04-2025\_Jee-Main\_Shift-2**

## MATHEMATICS

**Max Marks: 100**

### (SINGLE CORRECT ANSWER TYPE)

This section contains **20 Multiple Choice Questions**. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

**Marking scheme: +4 for correct answer, 0 if not attempted and -1 in all other cases.**

1. Let the area of the triangle formed by a straight line  $L: x + by + c = 0$  with co-ordinate axes be 48 square units. If the perpendicular drawn from the origin to the line  $L$  makes an angle of  $45^\circ$  with the positive  $x$ -axis, then the value of  $b^2 + c^2$  is
1. 93                      2. 83                      3. 97                      4. 90

**Key 3**

**Sol**  $\frac{c^2}{2|b|} = 48 \Rightarrow c^2 = 96|b| \dots (1)$

$$-\frac{1}{b} = 1 \Rightarrow b = -1 \Rightarrow c^2 = 96$$

$$b^2 + c^2 = 97$$

2. Let  $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$ ,  $\vec{b} = 3\hat{i} + 2\hat{j} + 5\hat{k}$  and a vector  $\vec{c}$  be such that  $(\vec{a} - \vec{c}) \times \vec{b} = -18\hat{i} - 3\hat{j} + 12\hat{k}$  and  $\vec{a} \cdot \vec{c} = 3$ . If  $\vec{b} \times \vec{c} = \vec{d}$ , then  $|\vec{a} \cdot \vec{d}|$  is equal to
1. 18                      2. 15                      3. 9                      4. 12

**Key 2**

**Sol**  $(\vec{a} - \vec{c}) \times \vec{b} = -18\hat{i} - 3\hat{j} + 12\hat{k}$   
 $\Rightarrow \vec{a} \times \vec{b} + \vec{b} \times \vec{c} = -18\hat{i} - 3\hat{j} + 12\hat{k} \dots (1)$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 3 & 2 & 5 \end{vmatrix} = \hat{i}(-17) - \hat{j}(7) + \hat{k}(13)$$

$$\text{From } \dots (1) \Rightarrow \vec{b} \times \vec{c} = (-18\hat{i} - 3\hat{j} + 12\hat{k}) - (-17\hat{i} - 7\hat{j} + 13\hat{k})$$

$$= -\hat{i} + 4\hat{j} - \hat{k} = \vec{d}$$

$$\Rightarrow \vec{d} = -\hat{i} + 4\hat{j} - \hat{k}$$

$$\vec{a} \cdot \vec{d} = -2 - 12 - 1 = -15$$

$$|\vec{a} \cdot \vec{d}| = 15$$

3. If the mean and the variance of 6, 4, a, 8, b, 12, 10, 13 are 9 and 9.25 respectively, then  $a + b + ab$  is equal to

1. 100

2. 105

3. 103

4. 106

**Key 3**

$$\text{Sol mean} = \frac{\sum x_i}{n} = \frac{6+4+a+8+b+12+10+13}{8} = \frac{53+a+b}{8} = 9$$

$$\Rightarrow 53 + a + b = 72 \Rightarrow a + b = 19$$

$$\text{Variance} = \frac{\sum x_i^2}{n} - \left( \frac{\sum x_i}{n} \right)^2 \Rightarrow 9.25 = \frac{36+16+a^2+64+b^2+144+100+169}{8} - 81$$

$$\Rightarrow 9.25 \times 8 + 81 \times 8 = a^2 + b^2 + 64 + 144 + 100 + 36 + 16 + 169 \Rightarrow a^2 + b^2 = 193$$

$$ab = \frac{(a+b)^2 - (a^2 + b^2)}{2} = 84$$

$$a + b + ab = 19 + 84 = 103$$

4. Let  $A = \{1, 2, 3, \dots, 100\}$  and  $R$  be a relation on  $A$  such that  $R = \{(a, b) : a = 2b + 1\}$ . Let  $(a_1, a_2), (a_2, a_3), (a_3, a_4), \dots, (a_k, a_{k-1})$  be a sequence of  $k$  elements of  $R$  such that the second entry of an ordered pair is equal to the first entry of the next ordered pair. Then the largest integer  $k$ , for which such a sequence exists, is equal to

1. 6

2. 5

3. 8

4. 7

**Key 2**

$$\text{Sol } a = 2b + 1$$

**Required relation**  $\{(95, 47), (47, 23), (23, 11), (11, 5), (5, 2)\} \Rightarrow \text{max. of } k = 5$

5. The number of terms of an A.P is even, the sum of all odd terms is 24, the sum of all the even terms is 30 and the last term exceeds the first by  $\frac{21}{2}$ . Then the number of terms

which are integer in the A.P is:

1. 4

2. 8

3. 6

4. 10

**Key 1**

**Sol** Let  $2n$  be the number of terms ( $n$  even,  $n$  odd)

Let  $a$  be the 1<sup>st</sup> term,

$2d$  is common difference

$a, a + 2d, a + 4d, \dots$

given sum of all odd terms = 24

$$\Rightarrow \frac{n}{2}(2a + (n-1)2d) = 24$$

$$\Rightarrow n[a + (n-1)d] = 24 \rightarrow (1)$$

sum of all even terms is = 30

$$\Rightarrow \frac{n}{2}[2(a + d) + (n-1)2d] = 30$$

$$\Rightarrow n(a + d + (n-1)d) = 30$$

$$\Rightarrow 24 + nd = 30 \Rightarrow nd = 6 \dots (2)$$

And last term exceeds the 1<sup>st</sup> term by  $\frac{21}{2}$

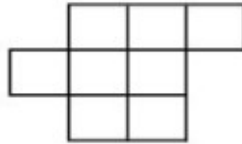
$$\Rightarrow a + (2n - 1)d - a = \frac{21}{2} \Rightarrow 2nd - d = \frac{21}{2} \Rightarrow 12 - d = \frac{21}{2}$$

$$\Rightarrow d = \frac{3}{2}$$

$$\text{and } nd = 6$$

$$\Rightarrow n \times \frac{3}{2} = 6 \Rightarrow n = 4$$

6. The number of ways, in which the letters A, B, C, D, E can be placed in the 8 boxes of the figure below so that no row remains empty and at most one letter can be placed in a box is



1. 5760

2. 5880

3. 840

4. 960

**Key 1****Sol** A, B, C, D, E No of ways =

$$\left( {}^3C_3 \times {}^3C_1 \times {}^2C_1 + {}^3C_1 \times {}^3C_3 \times {}^2C_1 + {}^3C_2 \times {}^3C_2 \times {}^2C_1 + {}^3C_2 \times {}^3C_1 \times {}^3C_2 + {}^3C_1 \times {}^3C_2 \times {}^2C_2 \right) 5!$$

$$(6 + 6 + 18 + 9 + 9) 120 \Rightarrow 48 \times 120 = 5760$$

7. Let A be a  $3 \times 3$  real matrix such that  $A^2(A - 2I) - 4(A - I) = O$ , where I and O are the identity and null matrices, respectively. If  $A^5 = \alpha A^2 + \beta A + \gamma I$ , where  $\alpha, \beta$  and  $\gamma$  are real constants, then  $\alpha + \beta + \gamma$  is equal to

1. 76

2. 12

3. 20

4. 4

**Key 2**

$$\text{Sol } A^2(A - 2I) - 4(A - I) = 0 \Rightarrow A^3 - 2A^2 - 4A + 4I = 0$$

$$\Rightarrow A^4 - 2A^3 - 4A^2 + 4A = 0 \Rightarrow A^5 - 2A^4 - 4A^3 + 4A^2 = 0$$

$$A^5 - 2(2A^3 + 4A^2 - 4A) - 4A^3 + 4A^2 = 0 \Rightarrow A^5 - 8A^3 - 4A^2 + 8A = 0$$

$$\Rightarrow A^5 - 8[2A^2 + 4A - 4I] - 4A^2 + 8A = 0 \Rightarrow A^5 = 20A^2 + 24A - 32I$$

$$\text{Given } A^5 = \alpha A^2 + \beta A + \gamma I \Rightarrow \alpha + \beta + \gamma = 20 + 24 - 32 = 12$$

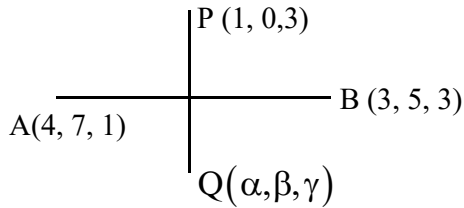
8. If the image of the point P (1, 0, 3) in the line joining the points A (4, 7, 1) and B (3, 5, 3) is

Q ( $\alpha, \beta, \gamma$ ), then  $\alpha + \beta + \gamma$  is equal to1.  $\frac{46}{3}$ 

2. 18

3. 13

4.  $\frac{47}{3}$ **Key 1****Sol** Q( $\alpha, \beta, \gamma$ ) is image of P (1, 0, 3)



Equation of line  $\overline{AB}$  is  $\frac{x-4}{1} = \frac{y-7}{2} = \frac{z-1}{-2} = k(\text{say})$

Point on line =  $(k+4, 2k+7, -2k+1)$

Dr of  $\overline{AB} = (1, 2, -2)$

Dr's of  $\overline{PQ} = (k+3, 2k+7, -2k-2)$

Since  $\overline{AB} \perp \overline{PQ}$ ,  $k+3+4k+14+4k+4=0 \Rightarrow 9k=-21$

$$\Rightarrow k = \frac{-21}{9}$$

Point Q =  $(2k+7, 4k+14, -4k-1)$   
 $= (\alpha, \beta, \gamma)$

$$\text{Now } \alpha + \beta + \gamma = 2k + 20 \Rightarrow \frac{-42}{9} + 20 = \frac{-42 + 180}{9} = \frac{46}{3}$$

9. Let the point P of the focal chord PQ of the parabola  $y^2 = 16x$  be  $(1, -4)$ . If the focus of the parabola divides the chord PQ in the ratio  $m : n$ ,  $\gcd(m, n) = 1$ . Then  $m^2 + n^2$  is equal to

1. 10                      2. 37                      3. 17                      4. 26

Key 3

Sol Given parabola  $y^2 = 16x$

$$P = (1, -4)$$

$$\text{Focus } S = (4, 0)$$

Since  $\overline{PQ}$  is focal chord  $t_1 t_2 = -1$

$$(1, -4) = (at_1^2, 2at_1) = (4t_1^2, 8t_1), \text{ but } a = 4 \Rightarrow 8t_1 = -4 \Rightarrow t_1 = \frac{-1}{2}$$

$$\text{And } t_2 = 2$$

$$Q(at_2^2, 2at_2) = (16, 16)$$

$$\text{Ratio } m : n = 1 - 4 : 4 - 16 = -3 : -12 = 1 : 4$$

$$m^2 + n^2 = 17$$

10. The line  $L_1$  is parallel to the vector  $\vec{a} = -3\hat{i} + 2\hat{j} + 4\hat{k}$  and passes through the point  $(7, 6, 2)$  and the line  $L_2$  is parallel to the vector  $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$  and passes through the point  $(5, 3, 4)$ . The shortest distance between the lines  $L_1$  and  $L_2$  is

1.  $\frac{23}{\sqrt{38}}$                       2.  $\frac{23}{\sqrt{57}}$                       3.  $\frac{21}{\sqrt{57}}$                       4.  $\frac{21}{\sqrt{38}}$

Key 2

Sol

$$\vec{c} = (x_1 y_1 z_1) = (7, 6, 2) \quad \vec{a} = (-3, 2, 4)$$

$$\vec{d} = (x_2 y_2 z_2) = (5, 3, 4) \quad \vec{b} = (2, 1, 3)$$

$$\vec{c} - \vec{d} = (2, 3, -2)$$

$$\Rightarrow \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & 4 \\ 2 & 1 & 3 \end{vmatrix} = \hat{i}(2) + \hat{j}(17) - 7\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{4 + 289 + 49} = 3\sqrt{38}$$

$$SD = \frac{(\vec{c} - \vec{d}) \cdot (\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|} = \frac{(2\hat{i} + 3\hat{j} - 2\hat{k}) \cdot (2\hat{i} + 17\hat{j} - 7\hat{k})}{3\sqrt{38}} = \frac{69}{3\sqrt{38}} = \frac{23}{\sqrt{38}}$$

11. If  $\sum_{r=0}^{10} \left( \frac{10^{r+1} - 1}{10^r} \right) \cdot {}^{11}C_{r+1} = \frac{\alpha^{11} - 11^{11}}{10^{10}}$ , then  $\alpha$  is equal to

1. 24

2. 15

3. 20

4. 11

Key 3

Sol 
$$\sum_{r=0}^{10} \left( \frac{10^{r+1} - 1}{10^r} \right) \cdot {}^{11}C_{r+1} = \sum_{r=0}^{10} (10 - 10^{-r}) \cdot {}^{11}C_{r+1}$$

$$= 10 \sum_{r=0}^{10} {}^{11}C_{r+1} - 10 \sum_{r=0}^{10} {}^{11}C_{r+1} 10^{-(r+1)}$$

$$= 10(2^{11} - 1) - 10 \left( (1 + 10^{-1})^{11} - 1 \right)$$

$$= 10 \cdot 2^{11} - 10 - 10 \left( \frac{11}{10} \right)^{11} + 10$$

$$= 10 \cdot 2^{11} - \frac{11^{11}}{10^{10}} = \frac{(20)^{11} - 11^{11}}{10^{11}}$$

$$\Rightarrow \alpha = 20$$

12. Let  $f : [1, \infty) \rightarrow [2, \infty)$  be a differentiable function. If  $10 \int_1^x f(t) dt = 5xf(x) - x^5 - 9$  for all  $x \geq 1$

, then the value of  $f(3)$  is

1. 26

2. 18

3. 22

4. 32

Key 4

Sol 
$$10 \int_1^x f(t) dt = 5xf(x) - x^5 - 9, x \geq 1$$

$$\Rightarrow 10f(x) = 5(xf'(x) + f(x)) - 5x^4$$

$$5x \cdot \frac{dy}{dx} - 5y = 5x^4 \Rightarrow \frac{dy}{dx} - \frac{1}{x} \cdot y = x^3 \Rightarrow \text{I.F.} = e^{\int \frac{-1}{x} dx} = \frac{1}{x}$$

$$y \cdot \frac{1}{x} = \int x^3 \cdot \frac{1}{x} dx \Rightarrow \frac{y}{x} = \frac{x^3}{3} + c$$

$$\text{put } x = 1, y = 2$$

$$\Rightarrow 2 = \frac{1}{3} + C \Rightarrow C = \frac{5}{3}$$

$$\frac{y}{3} = 9 + \frac{5}{3} \Rightarrow y = 27 + 5 = 32$$

13. Given three identical bags each containing 10 ball, whose colours are as follows:

	Red	Blue	Green
Bag I	3	2	5
Bag II	4	3	3
Bag III	5	1	4

A person chooses a bag at random and takes out a ball. If the ball is Red, the probability that I is from bag I is p and if the ball is Green, the probability that it is from bag III is q,

then the value of  $\left(\frac{1}{p} + \frac{1}{q}\right)$  is

1. 6

2. 8

3. 7

4. 9

**Key 3**

**Sol**  $P(I) = P(II) = P(III) = \frac{1}{3} \Rightarrow p = P\left(\frac{I}{R}\right) = \frac{P(I)P(R/I)}{P(R)}$

$$\frac{\frac{1}{3} \cdot \frac{3}{10}}{\frac{1}{3} \left( \frac{3}{10} + \frac{4}{10} + \frac{5}{10} \right)} = \frac{3}{12} = \frac{1}{4}$$

$$q = P\left(\frac{III}{G}\right) = \frac{P(III)P(G/III)}{P(G)} \Rightarrow \frac{\frac{1}{3} \cdot \frac{4}{10}}{\frac{1}{3} \left( \frac{5}{10} + \frac{3}{10} + \frac{4}{10} \right)} = \frac{4}{12} = \frac{1}{3}$$

$$\frac{1}{p} + \frac{1}{q} = 7$$

14.  $4 \int_0^1 \left( \frac{1}{\sqrt{3+x^2} + \sqrt{1+x^2}} \right) dx - 3 \log_e(\sqrt{3})$  is equal to

1.  $2 - \sqrt{2} - \log_e(1 + \sqrt{2})$

2.  $2 - \sqrt{2} + \log_e(1 + \sqrt{2})$

3.  $2 + \sqrt{2} + \log_e(1 + \sqrt{2})$

4.  $2 + \sqrt{2} - \log_e(1 + \sqrt{2})$

**Key 2**



**Sol**  $4 \int_0^1 \frac{\sqrt{3+x^2} - \sqrt{1+x^2}}{(3+x^2) - (1+x^2)} dx - 3 \log_e \sqrt{3}$

$$2 \int_0^1 \left( \sqrt{3+x^2} - \sqrt{2+x^2} \right) dx - 3 \log_e \sqrt{3}$$

$$\left( \because \int \sqrt{a^2 + x^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log \left( x + \sqrt{x^2 + a^2} \right) \right)$$

$$2 + 3 \cdot \log \left( \frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}} \right) - \left( \sqrt{2} + \log(1 + \sqrt{2}) \right) - 3 \log \sqrt{3}$$

$$= 2 - \sqrt{2} - \log(1 + \sqrt{2})$$

- 15.** Let (a, b) be the point of intersection of the curve  $x^2 = 2y$  and the straight line  $y - 2x - 6 = 0$  in the second quadrant. Then the integral  $I = \int_a^b \frac{9x^2}{1+5^x} dx$  is equal to

1. 27                      2. 21                      3. 18                      4. 24

**Key**

**Sol**  $x^2 = 2y \dots (1); y = 2x + 6 \dots (2)$

solve (1), (2)  $x^2 = 2(2x + 6) \Rightarrow x^2 - 4x - 12 = 0 \Rightarrow (x - 6)(x + 2) = 0$

$\Rightarrow x = -2, 6$

$x = -2, \Rightarrow y = 2 \quad (a, b) = (-2, 2) \in Q_2$

$$I = \int_a^b \frac{9x^2}{1+5^x} dx = \int_{-2}^2 \frac{9x^2}{1+5^x} dx \dots (1)$$

King's rule

$$I = \int_{-2}^2 \frac{9x^2}{1+5^{-x}} dx \dots (2)$$

$$(1) + (2) \Rightarrow 2I = \int_{-2}^2 9x^2 dx = 2 \int_0^2 9x^2 dx$$

$$I = 9 \cdot \left( \frac{x^3}{3} \right)_0^2 = 24$$

- 16.** If  $\theta \in \left[ -\frac{7\pi}{6}, \frac{4\pi}{3} \right]$ , then the number of solutions of  $\sqrt{3} \operatorname{cosec}^2 \theta - 2(\sqrt{3} - 1) \operatorname{cosec} \theta - 4 = 0$  is equal to

1. 8                      2. 6                      3. 7                      4. 10

**Key** 2



**Sol**  $\sqrt{3} \operatorname{cosec}^2 \theta - 2(\sqrt{3} - 1) \operatorname{cosec} \theta - 420$

$$\operatorname{cosec} \theta = \frac{2(\sqrt{3} - 1) \pm \sqrt{4(\sqrt{3} - 1)^2 + 4\sqrt{3} \cdot 4}}{2\sqrt{3}}$$

$$= \frac{2(\sqrt{3} - 1) \pm 2\sqrt{4 - 2\sqrt{3} + 4\sqrt{3}}}{2\sqrt{3}}$$

$$= \frac{\sqrt{3} - 1 \pm (\sqrt{3} + 1)}{\sqrt{3}} = 2, \frac{-2}{\sqrt{3}}$$

$$\operatorname{cosec} \theta = 2, \quad \operatorname{cosec} \theta = \frac{-2}{\sqrt{3}}$$

$$\sin \theta = \frac{1}{2} \quad \sin \theta = -\frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{-7\pi}{6} \quad \theta = \frac{4\pi}{3}, \frac{-\pi}{3}, \frac{-2\pi}{3}$$

No of solutions 6

**17.** If the domain of the function  $f(x) = \frac{1}{\sqrt{10+3x-x^2}} + \frac{1}{\sqrt{x+|x|}}$  is (a, b) then  $(1+a)^2 + b^2$  is

equal to

1. 30

2. 25

3. 29

4. 26

**Key** 4

**Sol**  $10 + 3x - x^2 > 0$  and  $x + |x| > 0$

$$x^2 - 3x - 10 < 0 \text{ and } x > 0$$

$$(x+2)(x-5) < 0$$

$$x \in (-2, 5)$$

$$\text{But } x > 0 \Rightarrow x \in (0, 5) = (a, b)$$

$$(1+a)^2 + b^2 = 1 + 25 = 26$$

**18.** If the length of the minor axis of an ellipse is equal to one fourth of the distance between the foci, then the eccentricity of the ellipse is

1.  $\frac{\sqrt{5}}{7}$

2.  $\frac{3}{\sqrt{19}}$

3.  $\frac{\sqrt{3}}{16}$

4.  $\frac{4}{\sqrt{17}}$

**Key** 4

**Sol**  $2b = \frac{1}{4}(2ae) \Rightarrow 4b = ae \Rightarrow 16b^2 = a^2 e^2$

$$\Rightarrow 16a^2(1 - e^2) = a^2 e^2 \Rightarrow 16 - 16e^2 = e^2 \Rightarrow e^2 = \frac{16}{17}$$

$$\Rightarrow e = \frac{4}{\sqrt{17}}$$

19. If the system of equations  $2x + \lambda y + 3z = 5$ ;  $3x + 2y - z = 7$ ;  $4x + 4y + \mu z = 9$  has infinitely many solutions, then  $(\lambda^2 + \mu^2)$  is equal to

1. 30

2. 22

3. 18

4. 26

Key 4

Sol 
$$\begin{bmatrix} 2 & \lambda & 3 & 5 \\ 3 & 2 & -1 & 7 \\ 4 & 5 & \mu & 9 \end{bmatrix} \Rightarrow \begin{vmatrix} 2 & \lambda & 5 \\ 3 & 2 & 7 \\ 4 & 5 & 9 \end{vmatrix} = 0 \Rightarrow \lambda = -1$$

$$\begin{vmatrix} 2 & 3 & 5 \\ 3 & -1 & 7 \\ 4 & \mu & 9 \end{vmatrix} = 0 \Rightarrow \mu = -5$$

$$\lambda^2 + \mu^2 = 26$$

20. If  $\lim_{x \rightarrow 0} \frac{\cos(2x) + a \cos(4x) - b}{x^4}$  is finite then  $(a+b)$  is equal to

1.  $\frac{3}{4}$ 

2. 0

3. -1

4.  $\frac{1}{2}$ 

Key 1

Sol 
$$\lim_{x \rightarrow 0} \frac{\cos 2x + a \cos 4x - b}{x^4}$$
  
 $1 + a - b = 0 \dots (1)$

$$\lim_{x \rightarrow 0} \frac{-2 \sin 2x - 4a \sin 4x}{4x^3} = 1 \quad \lim_{x \rightarrow 0} \frac{-4 \cos 2x - 16a \cos 4}{12x^2} \Rightarrow 4 + 16a = 0$$

$$\Rightarrow a = \frac{-1}{4}, b = 1 \Rightarrow a + b = \frac{3}{4}$$

### SECTION-II (NUMERICAL VALUE TYPE)

This section contains 5 Numerical Value Type Questions. The Answer should be within 0 to 9999. If the Answer is in Decimal then round off to the Nearest Integer value (Example i.e. If answer is above 10 and less than 10.5 round off is 10 and If answer is from 10.5 and less than 11 round off is 11).

Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases..

21. If the sum of the first 10 terms of the series  $\frac{4.1}{1+4.1^4} + \frac{4.2}{1+4.2^4} + \frac{4.3}{1+4.3^4} + \dots$  is  $\frac{m}{n}$ , where  $\gcd(m,n)=1$ , then  $m+n$  is equal to

key 441

sol 
$$S_n = \sum T_n = \sum_{n=1}^{10} \frac{4n}{1+4n^4} = \sum_{n=1}^{10} \frac{4n}{1+2 \cdot 2n^2 + (2n^2)^2 - 4n^2}$$

$$\sum \frac{4n}{(1+2n^2) - (2n)^2} \Rightarrow \sum \frac{4n}{(1+2n+2n^2)(1-2n+2n^2)}$$

$$\sum_{n=1}^{10} \left( \frac{1}{1-2n+2n^2} - \frac{1}{1+2n+2n^2} \right) = \frac{220}{221} \Rightarrow m+n = 441$$

22. Let  $y = y(x)$  be the solution of the differential equation

$\frac{dy}{dx} + 2y \sec^2 x = 2 \sec^2 x + 3 \tan x \cdot \sec^2 x$  such that  $y(0) = \frac{5}{4}$ . Then  $12 \left( y\left(\frac{\pi}{4}\right) - e^{-2} \right)$  is equal to.....

**Key 21**

**Sol**  $\frac{dy}{dx} + 2y \sec^2 x = 2 \sec^2 x + 3 \tan x \cdot \sec^2 x$

$$I.F = e^{\int P dx} = e^{\int 2 \sec^2 x dx} = e^{2 \tan x}$$

solution is .....  $y e^{2 \tan x} = \int (2 \sec^2 x + 3 \tan x \sec^2 x) e^{2 \tan x} dx$  (put,  $\tan x = t$ )

$$\int (2 + 3t) e^{2t} dt$$

$$e^{2t} + 3 \left( t \cdot \frac{e^{2t}}{2} - \int \frac{e^{2t}}{2} dt \right) = e^{2t} + \frac{3}{2} t e^{2t} - \frac{3}{4} e^{2t} + C$$

$$= e^{2 \tan x} + \frac{3 \tan x}{2} e^{2 \tan x} - \frac{3}{4} e^{2 \tan x} + C$$

$$\text{Put } x = 0, y = \frac{5}{4}$$

$$\frac{5}{4} = 1 + 0 - \frac{3}{4} + C \Rightarrow C = 1$$

$$y e^{2 \tan x} = e^{2 \tan x} + \frac{3 \tan x}{2} e^{2 \tan x} - \frac{3}{4} e^{2 \tan x} + 1$$

$$12 \left( y \left( \frac{\pi}{4} \right) - e^{-2} \right) = 21$$

23. If the set of all  $a \in \mathbb{R} - \{1\}$ , for which the roots of the equation  $(1-a)x^2 + 2(a-3)x + 9 = 0$  are positive is  $(-\infty, -\alpha] \cup [\beta, \gamma)$ , then  $2\alpha + \beta + \gamma$  is equal to.....

**Key 7**

**Sol**  $(1-a)x^2 + 2(a-3)x + 9 = 0$

$$\Delta \geq 0$$

$$a \in (-\infty, -3] \cup [0, \infty) \quad \text{and } \alpha\beta > 0 \text{ and } \alpha + \beta > 0$$

$$a \in (-\infty, 3] \cup (0, \infty) \quad \text{and } a < 1$$

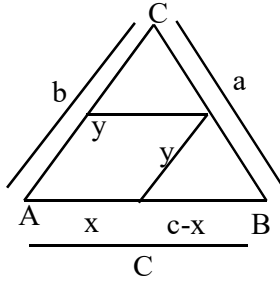
$$\Rightarrow a \in (-\infty, -3] \cup [0, 1]$$

$$2\alpha + \beta + \gamma = 7$$

24. Let  $A(4, -2)$ ,  $B(1, 1)$  and  $C(9, -3)$  be the vertices of a triangle ABC. Then the maximum area of the parallelogram AFDE, formed with vertices D, E and F on the sides BC, CA and AB of the triangle ABC respectively is.....

**Key 3**

**Sol**  $\frac{c-x}{c} = \frac{y}{b} \Rightarrow y = \frac{b}{c}(c-x)$



**Area of parallelogram =  $xy \sin A$**

$$= x \frac{b}{a}(c-x) \sin A = \frac{b}{c}(cx - x^2) \sin A$$

$$\text{Max} = \frac{b}{c} \frac{4(-1)(0) - c^2}{4(-1)} \cdot \sin A = \frac{bc}{4} \sin A = \frac{1}{2} \left( \frac{1}{2} bc \sin A \right) = \frac{1}{2} \cdot \frac{1}{2} \begin{vmatrix} 4 & -2 & 1 \\ 1 & 1 & 1 \\ 9 & -3 & 1 \end{vmatrix} = 3$$

**25.** If  $y = \cos\left(\frac{\pi}{3} + \cos^{-1} \frac{x}{2}\right)$ , then  $(x-y)^2 + 3y^2$  is equal to

**Key 3**

**Sol**  $y = \cos\left(\frac{\pi}{3} + \cos^{-1} \frac{x}{2}\right) \Rightarrow \cos^{-1} y = \frac{\pi}{3} - \cos^{-1} \frac{x}{2}$

$$\cos^{-1} y - \cos^{-1} \frac{x}{2} = \frac{\pi}{3} \Rightarrow \cos^{-1} \left( \frac{yx}{2} + \sqrt{1-y^2} \sqrt{1-\frac{x^2}{4}} \right) = \frac{\pi}{3}$$

$$\frac{xy}{2} + \sqrt{1-y^2} \sqrt{1-\frac{x^2}{4}} = \frac{1}{2}$$

$$\Rightarrow \sqrt{1-y^2} \sqrt{4-x^2} = xy - 1$$

square on both side

$$(x-y)^2 + 3y^2 = 3$$

## PHYSICS

Max Marks: 100

## SECTION-I (SINGLE CORRECT ANSWER TYPE)

This section contains **20 Multiple Choice Questions**. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

**Marking scheme: +4 for correct answer, 0 if not attempted and -1 in all other cases.**

26. A solenoid having area  $A$  and length ' $l$ ' is filled with a material having relative permeability 2. The magnetic energy stored in the solenoid is

1.  $\frac{B^2 A l}{4\mu_0}$       2.  $\frac{B^2 A l}{\mu_0}$       3.  $\frac{B^2 A l}{2\mu_0}$       4.  $B^2 A l$

Key 1

**Sol**  $\frac{du}{dv} = \frac{B^2}{2\mu} \Rightarrow du = \frac{B^2}{2\mu} \times dv$

$$\Rightarrow U = \frac{B^2}{2\mu_r \mu_0} \times A l \Rightarrow U = \frac{B^2 A l}{2 \times 2\mu_0} \Rightarrow U = \frac{B^2 A l}{4\mu_0}$$

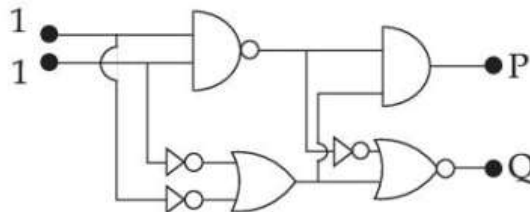
27. Consider a circular loop that is uniformly charged and has a radius  $a\sqrt{2}$ . Find the position along the positive  $z$ -axis of the Cartesian coordinate system where the electric field is maximum if the ring was assumed to be placed in  $xy$  plane at the origin

1.  $a/\sqrt{2}$       2.  $a/2$       3. 0      4.  $a$

Key 4

**Sol**  $E = \frac{kQz}{(R^2 + z^2)^{3/2}}$  then  $\frac{dE}{dz} = 0$ , then we get  $z = \frac{R}{\sqrt{2}}$ , So,  $z = \frac{\sqrt{2}a}{\sqrt{2}} \Rightarrow z = a$

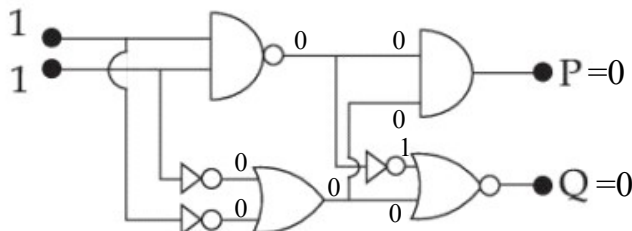
28. In the digital circuit shown in the figure, for the given inputs the  $P$  and  $Q$  values are:



1.  $P=0, Q=1$       2.  $P=0, Q=0$       3.  $P=1, Q=1$       4.  $P=1, Q=0$

Key 2

**Sol**  $P=0, Q=0$



29. If  $\mu_0$  and  $\epsilon_0$  are the permeability and permittivity of free space, respectively, then the dimension of  $\left(\frac{1}{\mu_0 \epsilon_0}\right)$  is

1.  $T^2/L^2$       2.  $L/T^2$       3.  $T^2/L$       4.  $L^2/T^2$

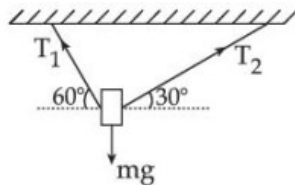
Key 4

**Sol** we know that  $C = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \Rightarrow c^2 = \frac{1}{\mu_0 \epsilon_0}$

But  $c$  = velocity of light

$$c = (LT^{-1}) \therefore C^2 = L^2 / T^2$$

- 30.** A body of mass 1kg is suspended with the help of two strings making angles as shown in figure. Magnitudes of tensions  $T_1$  and  $T_2$  respectively, are (in N) (Take acceleration due to gravity  $10 \text{ m/s}^2$ )



1.  $5\sqrt{3}, 5$

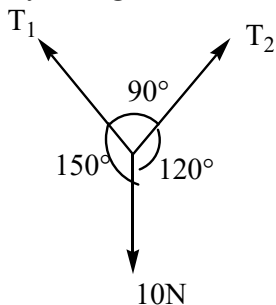
2.  $5, 5\sqrt{3}$

3.  $5, 5$

4.  $5\sqrt{3}, 5\sqrt{3}$

**Key 1**

**Sol** By using Lami's theorem



$$\frac{T_1}{\sin 120^\circ} = \frac{T_2}{\sin 150^\circ} = \frac{10}{\sin 90^\circ} \Rightarrow T_1 = 5\sqrt{3}; T_2 = 5$$

- 31.** Given a charge  $q$ , current  $I$  and permeability of vacuum  $\mu_0$ . Which of the following quality has the dimension of momentum?

1.  $q\mu_0 I$

2.  $qI / \mu_0$

3.  $q\mu_0 / I$

4.  $q^2 \mu_0 I$

**Key 1**

**Sol** Velocity of particle

$$B = \frac{\mu_0 i}{L}; L = \frac{mv}{qB}; q\mu_0 i = BLq$$

$$P = q\mu_0 I$$

- 32.** Assuming the validity of Bohr's atomic model for hydrogen like ions the radius  $\text{Li}^{++}$  ion in its ground state is given by  $\frac{1}{X} a_0$  where  $X = \dots\dots\dots$  (Where  $a_0$  is the first Bohrs radius)

1. 3

2. 1

3. 2

4. 9

**Key 1**

**Sol** Radius of  $n$ th Bohr orbit  $R = \frac{n^2}{Z} a_0 \Rightarrow n = 1, Z = 3$  (lithium)

$$R = \frac{1}{3} a_0$$

33. In a moving coil galvanometer, two moving coils  $M_1$  and  $M_2$  have the following particulars:

$$R_1 = 5\Omega, N_1 = 15, A_1 = 3.6 \times 10^{-3} \text{ m}^2, B_1 = 0.25 \text{ T}$$

$$R_2 = 7\Omega, N_2 = 21, A_2 = 1.8 \times 10^{-3} \text{ m}^2, B_2 = 0.50 \text{ T}$$

Assuming that torsional constant of the springs are same for both coils, what will be the ratio of voltage sensitivity of  $M_1$  and  $M_2$ ?

1. 1 : 4                      2. 1 : 2                      3. 1 : 3                      4. 1 : 1

**Key 4**

**Sol Torque**  $NiBA = C\theta \Rightarrow N\left(\frac{V}{R}\right)BA = C\theta \Rightarrow \frac{\theta}{V} = \frac{NBA}{CR}$

$$\frac{(\text{Voltage sensitivity})_1}{(\text{Voltage sensitivity})_2} = \frac{\frac{N_1 B_1 A_1}{C \times R_1}}{\frac{N_2 B_2 A_2}{C R_2}} = \frac{N_1 B_1 A_1 \times R_2}{N_2 B_2 A_2 R_1} = 1 : 1$$

34. **Match List – I with List - II**

**List I**

A. Heat capacity of body

B. Specific heat capacity of body

C. Latent heat

D. thermal conductivity

**List II**

I.  $\text{J kg}^{-1}$

II.  $\text{J K}^{-1}$

III.  $\text{J kg}^{-1} \text{K}^{-1}$

IV.  $\text{J m}^{-1} \text{K}^{-1} \text{s}^{-1}$

Choose the correct answer from the options given below

1. A – IV, B – III, C – II, D – I                      2. A – III, B – I, C – II, D – IV  
3. A – III, B – IV, C – I, D – II                      4. A – II, B – III, C – I, D – IV

**Key 4**

**Sol** (a) Heat capacity  $H = ms\Delta T \Rightarrow ms = \text{Heat capacity} = \text{J} / \text{K}$

(b) Specific heat capacity  $= S = \frac{H}{m\Delta T} \Rightarrow \frac{\text{J}}{\text{K} - \text{kg}}$

(c) Latent heat  $Q = mL \Rightarrow L = \frac{Q}{m} \Rightarrow \text{J} / \text{kg}$

(d)  $\frac{a}{t} = \frac{kA(\Delta\theta)}{\ell} \Rightarrow k = \frac{Q\ell}{A \times t \times \Delta\theta} \Rightarrow \frac{\text{J} - \text{m}}{\text{m}^2 \times \text{T} \times \text{K}} = \text{J m}^{-1} \text{T}^{-1} \text{K}^{-1}$

35. Two water drops each of the radius 'r' coalesce to form a bigger drop. If T is the surface tension, the surface energy released in this process is

1.  $4\pi r^2 T [\sqrt{2} - 1]$     2.  $4\pi r^2 T [1 + \sqrt{2}]$     3.  $4\pi r^2 T [2 - 2^{1/3}]$     4.  $4\pi r^2 T [2 - 2^{2/3}]$

**Key 4**

**Sol** Surface energy  $w = T(\Delta A) \Rightarrow w = T(4\pi r_2^2 - 4\pi r_1^2)$

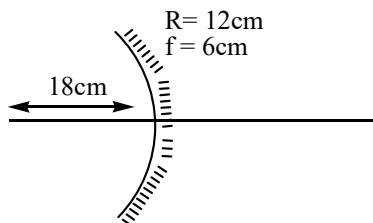
$$w = 4\pi T(r_2^2 - r_1^2) \Rightarrow w = 4\pi T(R^2 - r^2)$$



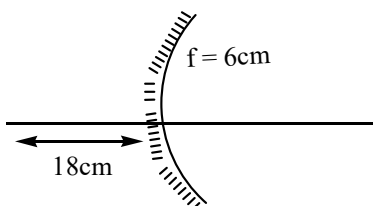
where  $R = (2)^{1/3} r$

36. Two identical objects are placed in front of convex mirror and concave mirror having same radii of curvature of 12cm. at same distance of 18cm from the respective mirrors. The ratio of sizes of the image formed by convex mirror and by concave mirror is
1. 3                      2. 2                      3. 1/2                      4. 1/3

Key 3  
Sol



$$h_{i1} = m \times h_0 = \frac{f}{f - u} \times h_0 = \frac{6}{6 + 18} \times h_0 = \frac{h_0}{4}$$



$$h_{i2} = m \times h_0 = \frac{f}{f - u} \times h_0 = \frac{-6}{-6 + 18} \times h_0 = \frac{h_0}{2}$$

37. A sinusoidal wave of wavelength 7.5cm travels a distance of 1.2cm along the x-direction in 0.3sec. The crest P is at  $x = 0$  at  $t = 0$  sec and maximum displacement of the wave is 2cm. Which equation correctly represents this wave?
1.  $y = 2 \sin(0.83x - 3.5t)$  cm                      2.  $y = 2 \cos(3.35x - 0.83t)$  cm  
3.  $y = 2 \cos(0.83x - 3.35t)$  cm                      4.  $y = 2 \cos(0.13x - 0.5t)$  cm

Key 3  
Sol

$$y = A \cos(kx - \omega t)$$

But A – max. Displacement 2cm  $k = \frac{2\pi}{\lambda} = \frac{2 \times 3.14}{7.5} = \frac{4.28}{7.5} = 0.83$

$$\omega = vk = \frac{1.2}{0.3} \times 0.83 = 3.35$$

38. Given below are two statements; One is labelled as assertion (A) and the other is labelled as Reason (R)

**Assertion (A):** Net dipole moment of a polar linear isotropic dielectric substances is not zero even in the absence of an external electric field

**Reason (R):** In absence of an external electric field, the different permanent dipoles of a polar dielectric substance are oriented in random directions

In the light of the above statements choose the most appropriate answer from the options given below

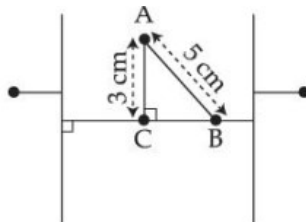
1. Both A and R are correct but R is not the correct explanation of A  
2. A is correct but R is not correct  
3. Both A and R are correct and R is the correct explanation of A

4. A is not correct but R is correct

**Key 4**

**Sol** Polar linear isotropic di-electric has permanent dipole moment in the absence of electric field is zero, so assertion - wrong, reason - correct

**39.** Two large plane parallel conducting plates are kept 10cm apart as shown in figure. The potential difference between them is  $V$ . The potential difference between the point A and B (shown in the figure) is



1.  $\frac{1}{4}V$

2.  $\frac{3}{4}V$

3.  $\frac{2}{5}V$

4.  $1V$

**Key 3**

**Sol** From the definition of potential gradient  $v = E(d)$

$$CB = 4\text{ cm}$$

$$v \propto d \Rightarrow \frac{v}{10} = \frac{v_{AB}}{4} \Rightarrow v_{AB} = \frac{2}{5}v$$

**40.** Energy released when two deuterons ( ${}_1\text{H}^2$ ) fuse to form a helium nucleus ( ${}_2\text{He}^4$ ) is:  
(Given: Binding energy per nucleon of  ${}_1\text{H}^2 = 1.1\text{ MeV}$  and binding energy per nucleon of  ${}_2\text{He}^4 = 7.0\text{ MeV}$ )

1.  $26.8\text{ MeV}$

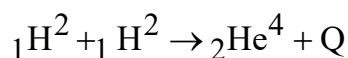
2.  $23.6\text{ MeV}$

3.  $5.9\text{ MeV}$

4.  $8.1\text{ MeV}$

**Key 2**

**Sol**  $\left(\frac{\text{B.E.}}{A}\right)_{\text{H}} = 1.1\text{ MeV} \Rightarrow (\text{B.E.})_{\text{H}} = 1.1 \times 2 = 2.2$



$$(\text{BE})_{\text{He}} - 2(\text{BE})_{\text{H}} = Q$$

$$\left(\frac{\text{BE}}{A}\right)_{\text{He}} = 7\text{ MeV} \Rightarrow (\text{BE})_{\text{He}} = 7 \times 4 = 8\text{ MeV}$$

**41.** A bi-convex lens has radius of curvature of both the surfaces same as  $\frac{1}{6}\text{ cm}$ . If the lens is required to be replaced by another convex lens having different radii of curvatures on both sides ( $R_1 \neq R_2$ ). Without any change in lens power then possible combination of  $R_1$  and  $R_2$  is

1.  $\frac{1}{3}\text{ cm}$  and  $\frac{1}{3}\text{ cm}$     2.  $\frac{1}{6}\text{ cm}$  and  $\frac{1}{9}\text{ cm}$     3.  $\frac{1}{5}\text{ cm}$  and  $\frac{1}{7}\text{ cm}$     4.  $\frac{1}{3}\text{ cm}$  and  $\frac{1}{7}\text{ cm}$

**Key 3**

**Sol** By using lens makers formula  $\frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$

In case (1)  $\frac{1}{f} = (\mu - 1) \left( \frac{2}{\frac{1}{6}} \right) = (\mu - 1) \times 12$

In case (2)  $\frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$

$$\frac{1}{R_1} + \frac{1}{R_2} = 12$$

42. The moment of inertia of a circular ring of mass  $M$  and diameter  $r$  about a tangential axis lying in the plane of the ring is

1.  $\frac{3}{8}Mr^2$       2.  $\frac{1}{2}Mr^2$       3.  $2Mr^2$       4.  $\frac{3}{2}Mr^2$

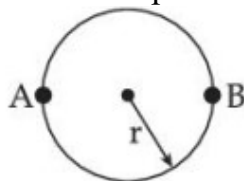
Key 1

Sol Here radius =  $r/2$

$$I_t = I_c + mx^2$$

$$= \frac{m \left( \frac{r}{2} \right)^2}{2} + m \left( \frac{r}{2} \right)^2 = \frac{3mr^2}{8}$$

43. A sportsman runs around a circular track of radius  $r$ . Such that he traverses the path ABAB. The distance travelled and displacement respectively are

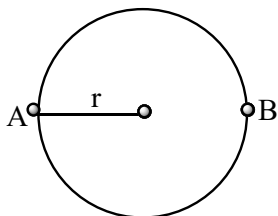


1.  $3\pi r, \pi r$       2.  $2r, 3\pi r$       3.  $\pi r, 3r$       4.  $3\pi r, 2r$

Key 4

Sol  $d = \text{length of the path} = 2\pi r + \pi r = 3\pi r$

$S = \text{shortest distance between A and B} = 2r$



44. An electron with mass  $m$  with an initial velocity  $(t = 0) \vec{v} = v_0 \hat{i}$  ( $v_0 > 0$ ) enters a magnetic field  $\vec{B} = B_0 \hat{j}$ . If the initial de-Broglie wavelength at  $t = 0$  is  $\lambda_0$  then its value after time 't' would be

1.  $\lambda_0$       2.  $\lambda_0 \sqrt{1 + \frac{e^2 B_0^2 t^2}{m^2}}$       3.  $\frac{\lambda_0}{\sqrt{1 + \frac{e^2 B_0^2 t^2}{m^2}}}$       4.  $\frac{\lambda_0}{\sqrt{1 - \frac{e^2 B_0^2 t^2}{m^2}}}$

**Key 1****Sol** When electron moves in a magnetic field  $F = q(\vec{v} \times \vec{B})$ So  $\vec{F}$  is  $\perp$  to both  $\vec{v}$  and  $\vec{B}$ , so work done is zero hence speed is constant

So de-Broglie wavelength does not change

**45.** Identify the characteristics of an adiabatic process in a monoatomic gas**A.** Internal energy is constant**B.** work done in the process is equal to the change in internal energy**C.** The product of temperature and volume is a constant**D.** The product of pressure and volume is a constant**E.** The work done to change the temperature from  $T_1$  and  $T_2$  is proportional to  $(T_2 - T_1)$ 

Choose the correct answer from the options given below

**1.** B, E only      **2.** B, D only      **3.** A, C, E only      **4.** A, C, D only**Key 1****Sol** Adiabatic process  $dQ = 0$ 

$$\Rightarrow du = -d\omega = -nC_v(T_2 - T_1)$$

**SECTION-II (NUMERICAL VALUE TYPE)**

This section contains **5 Numerical Value Type Questions**. The Answer should be within **0 to 9999**. If the Answer is in **Decimal** then round off to the **Nearest Integer** value (Example i.e. If answer is above **10** and less than **10.5** round off is **10** and If answer is from **10.5** and less than **11** round off is **11**).

**Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.**

**46.** A ray of light suffer minimum deviation when incident on a prism having angle of the prisms equal to  $60^\circ$ . The refractive index of the prism material is  $\sqrt{2}$ . The angle of incidence (in degrees) is.....

**Key: 45****Sol:** As prism is in minimum deviation position  $A = 2r$ 

$$r = \frac{A}{2} = 30^\circ$$

$$\mu = \frac{\sin i}{\sin r}$$

$$\sqrt{2} = \frac{\sin i}{\sin 30^\circ}$$

$$i = 45^\circ$$

**47.** A satellite of mass 1000kg is launched to revolve around the earth in an orbital at a height of 270 km from the earth's surface. Kinetic energy of the satellite in this orbit is.....

(Mass of earth =  $6 \times 10^{24}$  kg, Radius of earth =  $6.4 \times 10^6$  m, Gravitational constant =  $6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$ )

**Key: 3**

$$\text{Sol: K.E} = \frac{GMm}{2(R+h)} = 3 \times 10^{-10} \text{ J}$$

48. The length of a light string is 1.4m when the tension on it is 5N. If the tension increases to 7N, the length of the string is 1.56m. The original length of the string is .....m

**Key: 1**

**Sol:**  $F = K\Delta x$

$$5 = K(1.4 - \ell_0)$$

$$7 = K(1.56 - \ell_0)$$

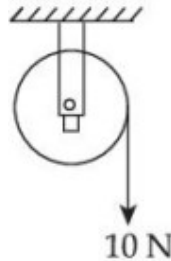
$$\text{Solving } \ell_0 = 1\text{m}$$

49. The internal energy of air in  $4\text{m} \times 3\text{m}$  sized room at 1 atmospheric pressure will be .....  
 $\times 10^6 \text{J}$   
 (Consider air as diatomic molecule)

**Key 12**

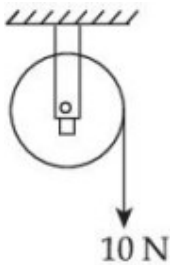
**Sol**  $U = \frac{f}{2} pV = \frac{5}{2} (10^5) (4)(4)(3) = 12 \times 10^6 \text{J}$

50. A wheel of radius 0.2m rotates freely about its center when a string that is wrapped over its rim is pulled by force of 10N as shown in figure. The established torque produces an angular acceleration of  $2\text{rad/s}^2$ . Moment of inertia of the wheel is..... $\text{kgm}^2$   
 (Acceleration due to gravity =  $10\text{m/s}^2$ )



**Key 1**

**Sol**  $r = 0.2, F = 10\text{N}; \alpha = 2\text{rad/sec}^2; I = ?$



$$\tau = I\alpha \Rightarrow 10(r) = I\alpha \Rightarrow 10 \times 0.2 = I \times 2 \Rightarrow I = 1$$

$$\text{So } n = 1$$

## CHEMISTRY

Max Marks: 100

## SECTION-I (SINGLE CORRECT ANSWER TYPE)

This section contains 20 Multiple Choice Questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 in all other cases.

51. In Dumas method for estimation of nitrogen 0.5 gram of an organic compound gave 60mL of nitrogen collected at 300K temperature and 715 mm Hg pressure. The percentage composition of nitrogen in the compound (Aqueous tension at 300 K = 15 mm Hg) is .....

1. 20.87                      2. 18.67                      3. 1.257                      4. 12.57

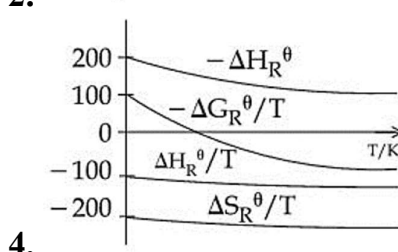
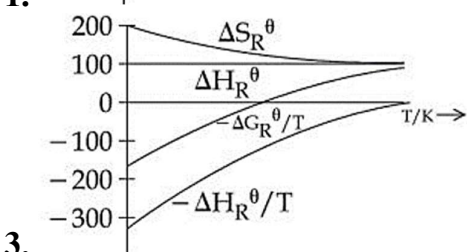
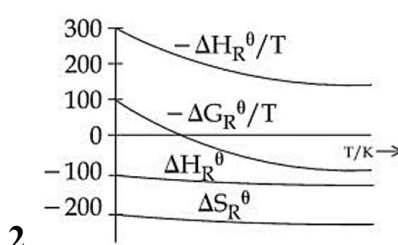
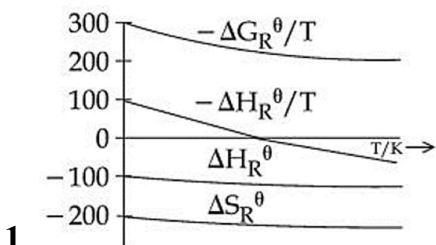
Key: 4

Sol:  $\%N = \frac{28}{22,400} \times \frac{V_{N_2}}{m_{O.C}} \times 100 \dots (1)$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \Rightarrow \frac{(715-15)60}{300} = \frac{760 \times v_2}{273} \Rightarrow v_2 = 50.3$$

From (1)  $\%N = \frac{28}{22400} \times \frac{50.3}{0.5} \times 100 = 12.57$

52. Which of the following graphs correctly represents the variation of thermodynamic properties of Haber's process?



Key: 2

Sol: Haber's process =  $\Delta H = -ve$ ;  $\Delta S = -ve$

$$\Delta G = \Delta H - T\Delta S \Rightarrow \Delta G < \Delta H$$

53. The nature of oxide ( $TeO_2$ ) and hydride ( $TeH_2$ ) formed by Te, respectively are:

1. Reducing and basic                      2. Oxidising and basic  
3. Oxidising and acidic                      4. Reducing and acidic

Key: 3

Sol:  $TeO_2$  oxidizing agent  $TeH_2$  is acidic

54. Arrange the following in order of magnitude of work done by the system / on the system at constant temperature

- a.  $|W_{\text{reversible}}|$  for expansion in infinite stages  
b.  $|W_{\text{irreversible}}|$  for expansion in single stage

c.  $|W_{\text{reversible}}|$  for compression in infinite stages

d.  $|W_{\text{irreversible}}|$  for compression in single stage

Choose the correct answer from the options given below

1.  $c = a > d > b$       2.  $d > c = a > b$       3.  $a > c > b > d$       4.  $a > b > c > d$

**Key: 2**

**Sol: a and c removable process  $d > c = a > b$**

$$|-W_{\text{rev}}| > |-W_{\text{irr}}|$$

$$a > b$$

$$W_{\text{irr}} > W_{\text{rev}}$$

$$d > c$$

**55.** Electronic configuration of four elements A, B, C and D are given below

- A.  $1s^2 2s^2 2p^3$       B.  $1s^2 2s^2 2p^4$       C.  $1s^2 2s^2 2p^5$       D.  $1s^2 2s^2 2p^2$

Which of the following is the correct order of increasing electronegativity (Pualing's scale)

1.  $A < C < B < D$       2.  $D < A < B < C$       3.  $A < B < C < D$       4.  $A < D < B < C$

**Key: 2**

**Sol: Electronegativity increases from  $C < N < O < F$**

**D    A    B    C**

**56.** The type of hybridization and the magnetic property of  $[\text{MnCl}_6]^{3-}$  are,

1.  $d^2 sp^3$ , paramagnetic with two unpaired electrons
2.  $sp^3 d^2$ , paramagnetic with four unpaired electrons
3.  $d^2 sp^3$  paramagnetic with four unpaired electrons
4.  $sp^3 d^2$ , paramagnetic with two unpaired electrons

**Key: 2**

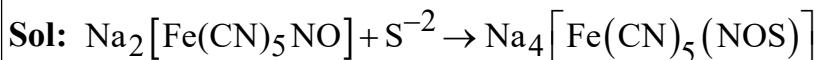
**Sol:  $\text{Cl}^-$  W.F.L**



**57.** Formation of  $\text{Na}_4[\text{Fe}(\text{CN})_5\text{NOS}]$  a purple coloured complex formed by addition of sodium nitroprusside in sodium carbonate extract of salt indicates the presence of

1. sulphide ion      2. Sulphite ion      3. Sulphate ion      4. Sodium ion

**Key: 1**



**58.** Consider the following chemical equilibrium of the gas phase reaction at a constant temperature :  $\text{A}(\text{g}) \rightleftharpoons \text{B}(\text{g}) + \text{C}(\text{g})$

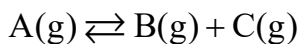
If  $p$  being the total pressure,  $K_p$  is the pressure equilibrium constant and  $\alpha$  is the degree of dissociation, then which of the following is true at equilibrium



1. When  $p$  increase  $\alpha$  decreases
2. When  $p$  increases  $\alpha$  increases
3. If  $K_p$  value is extremely high compared to  $p$ ,  $\alpha$  becomes much less than unity
4. If  $p$  values is extremely high compared to  $K_p$ ,  $\alpha \approx 1$

**Key: 1**

**Sol:**



$$K_p = \frac{\frac{\alpha^2}{(1+\alpha)^2} p^2}{\left(\frac{1-\alpha}{1+\alpha}\right) p} \Rightarrow K_p = \frac{\alpha^2}{1-\alpha^2} p$$

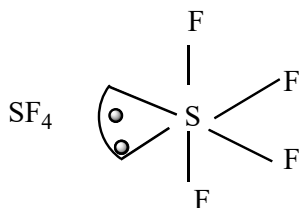
$p$  increases ' $\alpha$ ' decreases

59. Which among the following molecules (a) involved in  $sp^3d$  hybridization (b) has different bond lengths and (c) has lone pair of electrons on the central atom?

1.  $PF_5$
2.  $XeF_4$
3.  $SF_4$
4.  $XeF_2$

**Key: 3**

**Sol:**



60. Match List with list II

**List I**

(Purification technique)

- A. Distillation (simple)
- B. Fractional distillation
- C. Distillation under reduced pressure
- D. Steam distillation

**List II**

(Mixture of organic compounds)

- I. Diesel + petrol
- II. Aniline + Water
- III. Chloroform + Aniline
- IV. Glycerol + Spent-lyne

Choose the correct answer from the options given below

1. A – III, B – IV, C – II, D – I
2. A – II, B – IV, C – I, D – III
3. A – II, B – III, C – IV, D – I
4. A – III, B – I, C – IV, D – II

**Key: 4**

**Sol:** A. Distillation (simple) - Chloroform + Aniline (boiling point difference must be more than  $40^\circ\text{C}$ )

B. Fractional distillation - Diesel + petrol (boiling point difference must be less than  $40^\circ\text{C}$ )

C. Distillation under reduced pressure - Glycerol + Spent-lyne (This technique is used when the compound decomposed before its boiling point)

D. Steam distillation - Aniline + Water (hydrogen bonding)

61. Given below are two statements

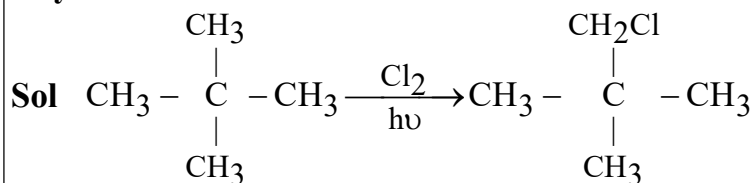
**Statement1:** Neopentane forms only one monosubstituted derivative

**Statement 2:** Melting point of neopentane is higher than n – pentane

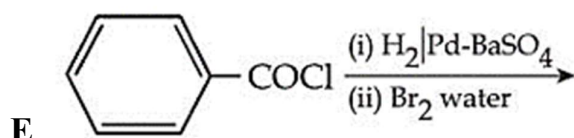
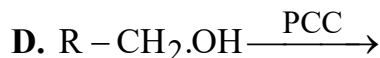
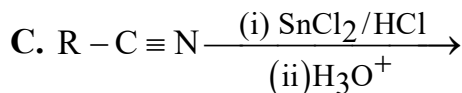
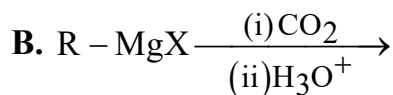
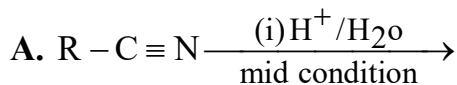
In the light of the above statements, choose the most appropriate answer from the options given below

- Both Statement I and Statement II are correct
- Both Statement I and Statement II are incorrect
- Statement I is correct but Statement II is incorrect
- Statement I is incorrect but Statement II is correct

**Key 1**



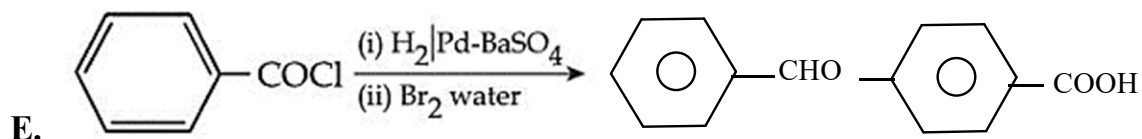
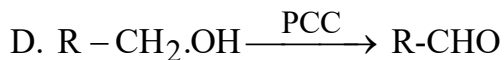
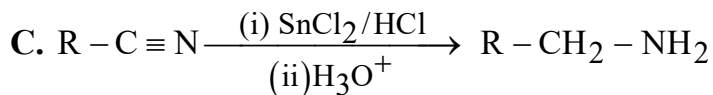
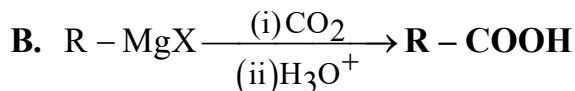
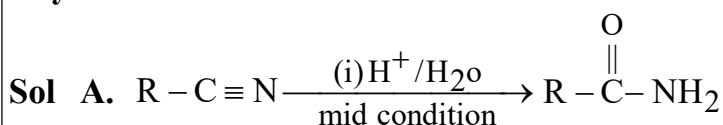
- 12 hydrogens are identical
  - Closed packing due to spherical shape will give more melting point
- 62.** Consider the following reactions. From these reactions which reaction will give carboxylic acid as major product?



Choose the correct answer from the options given below

1. B, C and E only    2. A, B and E only    3. A and D only    4. B and E only

**Key 4**



- 63.** When a concentrated solution of sulphanilic acid and 1-naphthylamine is treated with nitrous acid (273 K) and acidified with acetic acid, the mass (g) of 0.1 mole of product formed as (Given molar mass in  $\text{g mol}^{-1}$  H:1, C:12, N:14, O:16, S:32)

1. 33

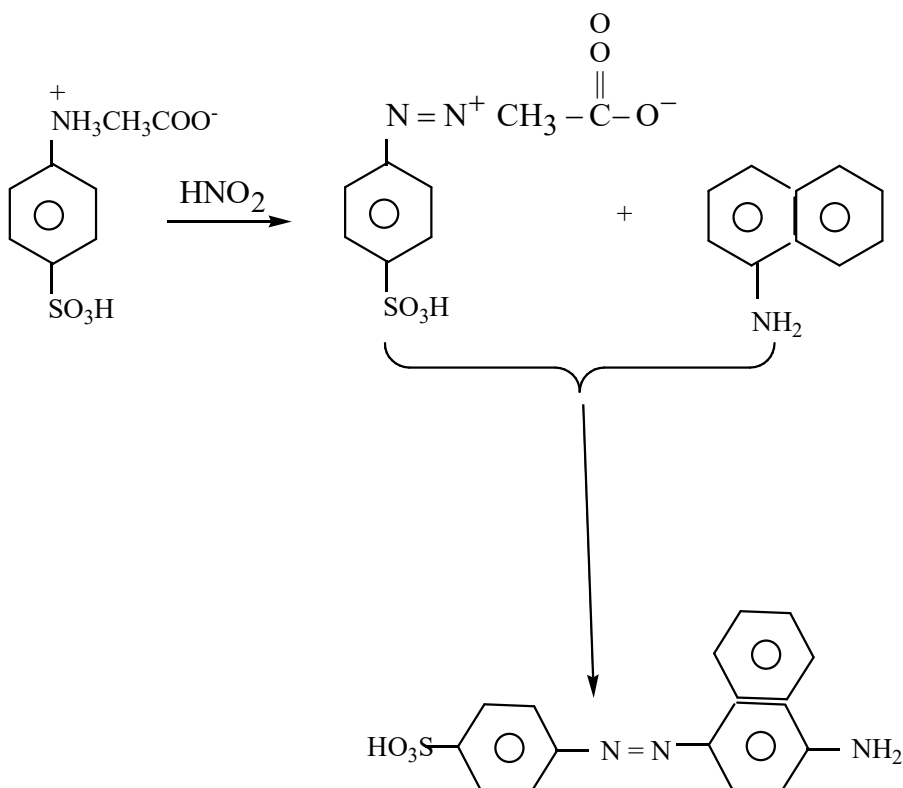
2. 330

3. 66

4. 343

Key 1

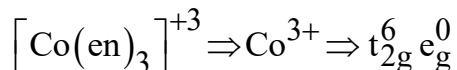
Sol

Molecular formula  $\text{C}_{16}\text{H}_{13}\text{N}_3\text{S}$ Molecular weight  $16 \times 12 + 13 + 3(14) + 3(16) + 32 = 329$ Weight of 0.1 mole of product is  $0.12 \times 329 = 32.9 = 33$ 

64. The d-orbital electronic configuration of the complex among  $[\text{Co}(\text{en})_3]^{3+}$ ,  $[\text{CoF}_6]^{3-}$ ,  $[\text{Mn}(\text{H}_2\text{O})_6]^{2-}$  and  $[\text{Zn}(\text{H}_2\text{O})_6]^{2+}$  that has the highest CFSE is
1.  $t_{2g}^4 e_g^2$
  2.  $t_{2g}^3 e_g^2$
  3.  $t_{2g}^6 e_g^4$
  4.  $t_{2g}^6 e_g^0$

Key 4

Sol en' strong field ligand

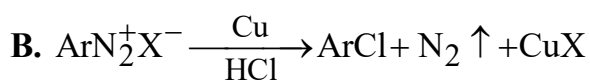
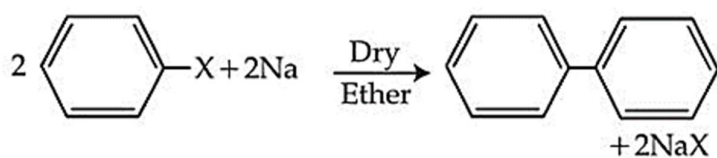


65. Match List I with List II

List I

(Reaction)

A.

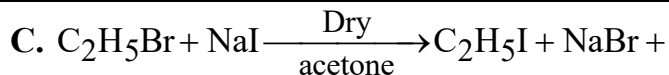


List II

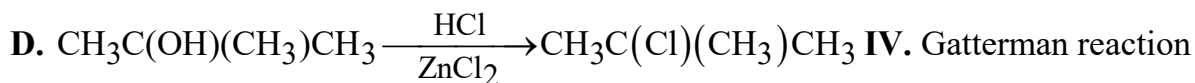
(Name of reaction)

I. Lucas reaction

II. Finkelstein reaction



III. Fittig reaction



1. A – IV, B – III, C – I, D – II

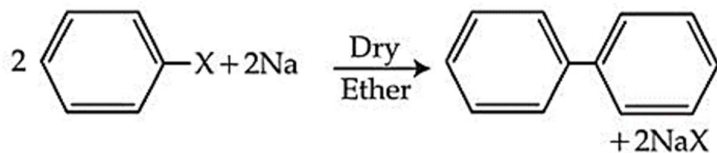
2. A – IV, B – III, C – I, D – II

3. A – III, B – IV, C – II, D – I

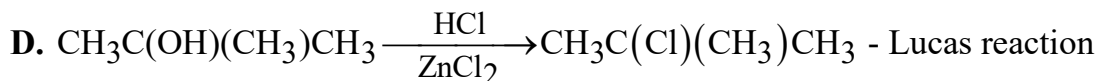
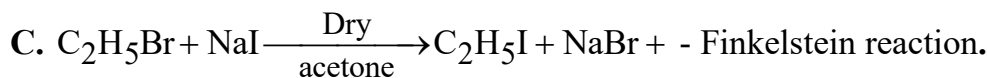
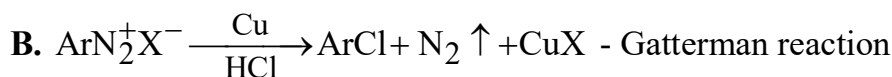
4. A – IV, B – I, C – II, D – III

Key 3

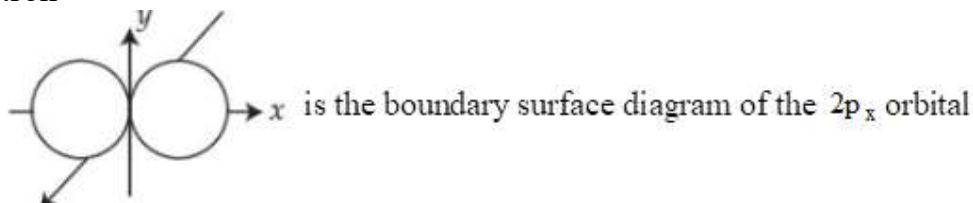
Sol A.



- Fittig reaction



66. Which of the following statements are true?

A. The subsidiary quantum number  $l$  describes the shape of the orbital occupied by the electron

B.

C. The + and – signs in the wave function of the  $2p_x$  orbital refer to chargeD. The wave function of  $2p_x$  orbital is zero everywhere in the xy plane

Choose the correct answer from the options given below

1. B and D only

2. A and B only

3. A, B, C only

4. C, D only

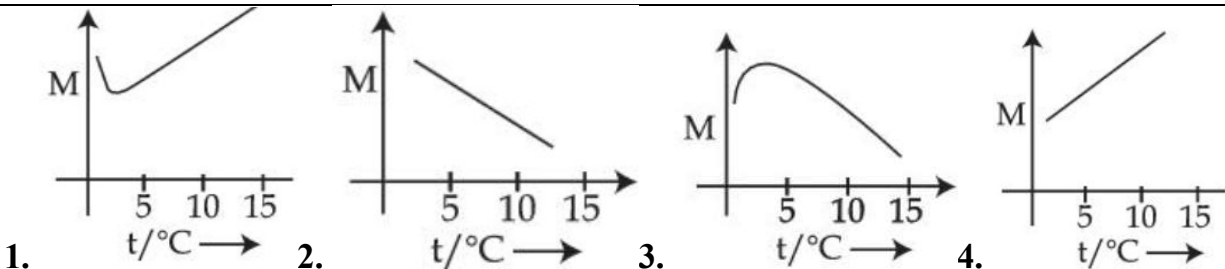
Key 2

Sol  $2p_x$  orbital modal plane is yz

+ve and –ve signs refer to constructive and destructive interference

67. 'x' g of NaCl is added to water in a beaker with a lid. The temperature of the system is raised from  $1^\circ\text{C}$  to  $25^\circ\text{C}$ . Which out of the following plots, is best suited for the change in the molarity (M) of the solution with respect to temperature?

[Consider the solubility of NaCl remains unchanged over the temperature range]

**Key 3**

**Sol** From  $0^{\circ}$  and  $4^{\circ}\text{C}$  volume of water decreases and then increases so, molarity from  $0^{\circ}\text{C}$  to  $4^{\circ}\text{C}$  increase and then decreases

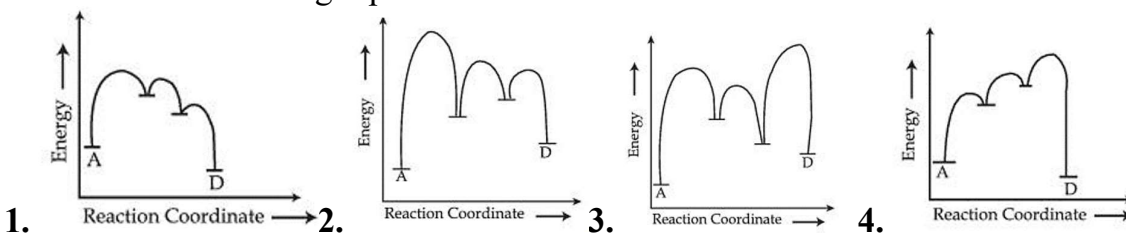
**68.** Reactant A converts to product D through the given mechanism (with the net evolution of heat)

$A \rightarrow B$  slow;  $\Delta H = +ve$

$B \rightarrow C$  fast;  $\Delta H = -Ve$

$C \rightarrow D$  fast;  $\Delta H = -Ve$

Which of the following represents the above reaction mechanism?

**Key 1**

**Sol** Overall reactions is exothermic and first step is slow

**69.** A tetrapeptide, 'x' on complete hydrolysis produced glycine (Gly), alanine (Ala), valine (Val), leucine (Leu) in equimolar proportion each. The number of tetrapeptides (sequences) possible involving each of these amino acids is

1. 8

2. 24

3. 32

4. 16

**Key 2**

**Sol** ABCD  
ABDC

BACD  
BADC

CABD  
CADB

DABC  
DACB

ADBC  
ADCB

BCAD  
BCBA

CBAD  
CBDA

DBAC  
DBCA

ACBD  
ACDB

BDAB  
BDAB

CDAB  
CDBA

DCAB  
DCBA

**70.** In 3, 3 –dimethylhex -1-en -4 – yne, there are .....  $sp^3$ , .....  $sp^2$  and .....  $sp$  hybridized carbon atoms respectively

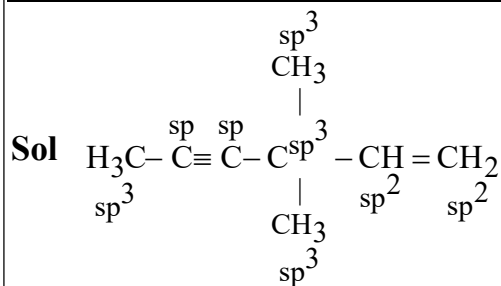
1. 3, 3, 2

2. 4, 2, 2

3. 2, 2, 4

4. 2, 4, 2

**Key 2**



### SECTION-II (NUMERICAL VALUE TYPE)

This section contains **5 Numerical Value Type Questions**. The Answer should be within **0 to 9999**. If the Answer is in **Decimal** then round off to the **Nearest Integer** value (Example i.e. If answer is above **10** and less than **10.5** round off is **10** and If answer is from **10.5** and less than **11** round off is **11**).

**Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.**

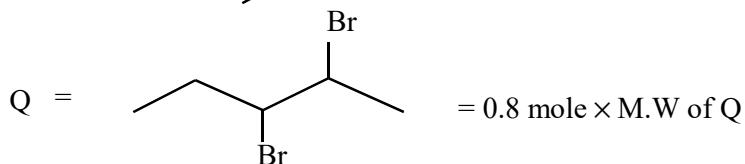
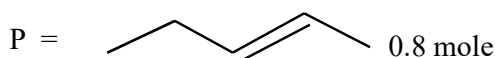
- 71.. Consider the below sequence of reactions. 151g of 2-bromopentane is made to react. Yield of major product P is 80% whereas Q is 100%  
Mass of product Q obtained is.....g.

(Given molar mass in  $\text{g mol}^{-1}$  H:1, C:12, O:16; Br:80)



**Key: 184**

**Sol:**



$$0.8 \times 230 = 184\text{gm}$$

72. 0.2% (w/v) solution of NaOH is measured to have resistivity  $870.0\text{m}\Omega\text{m}$ . The molar conductivity of the solution will be..... $\times 10^2\text{mSdm}^2\text{mol}^{-1}$  (Nearest integer)

**Key 230**

Sol  $0.2\left(\frac{w}{v}\right)\text{NaOH} \Rightarrow M = \frac{0.2}{40} \times \frac{1000}{100} = \frac{2}{40} = \frac{1}{20}$

$$\rho = 870\text{msm} \Rightarrow \rho = 870 \times 10^{-3}\text{sm} \Rightarrow k = \frac{1}{870 \times 10^{-3}}\text{s}^{-1}\text{m}^{-1}$$

$$\Lambda_m = ?$$

$$\Lambda_m = \frac{K}{1000M} = \frac{1}{870 \times 10^{-3} \times 1000 \times 0.05} = 0.2299\text{sm}^2\text{mol}^{-1} = 22.9 \times 10^{-2}\text{ms dm}^2\text{mol}^{-1}$$

73. When 1g each compounds AB and AB<sub>2</sub> are dissolved in 15g of water separately, they increased the boiling point of water by 2.7K and 1.5K respectively. The atomic mass of A (in amu) is..... $\times 10^{-1}$  (Nearest integer)

(Given: Molal boiling point elevation constant is  $0.5 \text{ K kg mol}^{-1}$ )

**Key 25**

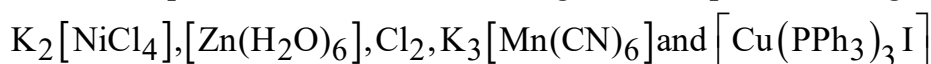
**Sol**  $2.7 = 0.5 \times \frac{1}{a+b} \times \frac{1000}{50}$

$$1.5 = 0.5 \times \frac{1}{a+2b} \times \frac{1000}{50}$$

$$a = 25$$

**74.** The spin only magnetic moment value of  $M^{n+}$  ion formed among Ni, Zn, Mn and Cu that has the least enthalpy of atomization is.....(in nearest integer)

Here n is equal to the number of diamagnetic complexes among



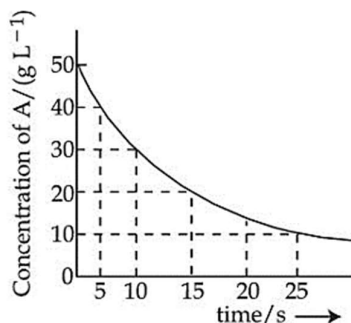
**Key 0**

**Sol** Zinc has least enthalpy of atomization more volatile

$Zn^{+2}$  state spin only magnetic moment is zero

**75.** For the reaction  $A \rightarrow B$  the following graph was obtained. The time required (in seconds) for the concentration of A to reduce to  $2.5 \text{ g L}^{-1}$  (if the initial concentration of A was  $50 \text{ g L}^{-1}$ ) is..... (Nearest integer) Given :

$$\log 2 = 0.3010$$



**Key 67 (According to shape of the graph)**

**Sol**  $k = \frac{2.303}{5} \log \frac{50}{40} \Rightarrow k = \frac{2.303}{5} \log \frac{5}{4}$

$$t = \frac{2.303}{k} \log \frac{50}{2.5} = \frac{2.303 \times 5}{2.303 \log \frac{5}{4}} \times \log 20 = 67 \text{ min}$$

**Key 30 (According to values given)**

$$50 - 10 = 8 \times 25 \Rightarrow K = \frac{32}{25} \Rightarrow 50 - 2.5 = \frac{40}{25} \times t$$

$$t = 30$$