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2025_Jee-Main_03-04-2025 (Shift-2)

MATHEMATICS **Max Marks: 100**

(SINGLE CORRECT ANSWER TYPE)

This section contains **20 Multiple Choice Questions.** Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

01. Let C be the circle of minimum area enclosing the ellipse $E:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with eccentricity $\frac{1}{2}$ and foci $(\pm 2,0)$. Let PQR be a variable triangle, whose vertex P is on the circle C and the side QR of length $2a$ is parallel to the major axis of E and contains the point of intersection of E with the negative $y-axis$. Then the maximum area of the triangle PQR is:

1) $8\left(2+\sqrt{3}\right)$ **2)** $8\left(3+\sqrt{2}\right)$ **3)** $6\left(3+\sqrt{2}\right)$ **4)** $6\left(2+\sqrt{3}\right)$

Key: 1

Sol: $2ae=4$

$ae=2$

$e=\frac{1}{2}\ \ a=4$

$b^2=a^2\left(1-e^2\right)=16\left(1-\frac{1}{4}\right)=12$

$b=2\sqrt{3}$

Circle of minimum area enclosing the ellipse radius $(r)=a=4$.

$y-$coordinate of Q,R is $=-y=-2\sqrt{3}$

The height of the Δle $h=r+b=4+2\sqrt{3}$

Area $=\frac{1}{2}bh$ $\quad b=2e=8$

$=\frac{1}{2}(8)\left(4+2\sqrt{3}\right)\ \ =8\left(2+\sqrt{3}\right)$

The maximum area of the $\Delta le=8\left(2+\sqrt{3}\right)$

02. If the probability that the random variable X takes the value x is given by $P(X=x)=k(x+1)3^{-x}, x=0,1,2,3,.....$ where k is a constant, then $P(X\geq 3)$ is equal to

1) $\frac{1}{9}$ **2)** $\frac{4}{9}$ **3)** $\frac{8}{27}$ **4)** $\frac{7}{27}$

Key: 1

Sol: $P(X=x)=k(x+1)3^{-x}$

$$\sum_{x=0}^{0} P(x=X)=1\Rightarrow k\Sigma\frac{(x+1)}{3^x}=1$$

$$k\left[1+2\frac{1}{3}+3\frac{1}{5}+.....\right]=1$$

$$K\left(1-\frac{1}{3}\right)^{-2}=1$$

$$K\left(\frac{9}{4}\right)=1\Rightarrow K=\frac{4}{9}$$

$$P|x\geq 3|=1-(px=01>p(x=1)+p(x=2))$$

$$=1-\frac{4}{9}\left(1+\frac{2}{3}+\frac{3}{9}\right)$$

$$=1-\frac{4}{9}\left(\frac{9+6+3}{9}\right)$$

$$=1-\frac{4}{9}(2)=\frac{1}{9}$$

03. The distance of the point $(7,10,11)$ from the line $\frac{x-4}{1}=\frac{y-4}{0}=\frac{z-2}{3}$ along the line $\frac{x-9}{2}=\frac{y-13}{3}=\frac{z-17}{6}$ is

1) 14 **2)** 18 **3)** 12 **4)** 16

Key: 1

Sol: $\frac{x-y}{2}=\frac{y-10}{3}=\frac{z-11}{6}=r$

$\theta(2r+7,3r+10,6r=11)$

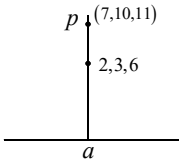
Lies on $\frac{x-4}{1}=\frac{y-4}{0}=\frac{z-2}{3}$

$$\frac{2r+3}{1}=\frac{3r+6}{0}=\frac{6r+8}{3}$$

$r=-2$

$Q(3,4,-1)$

$PQ=\sqrt{16+36+144}=\sqrt{196}=14$



04. Let $f:R\to R$ be a function defined by $f(x)=\left|\,|x+2|-2|x|\,\right|$. If m is the number of points of focal minima and n is the number of points of local maxima of f then $m+n$ is

1) 5 **2)** 4 **3)** 2 **4)** 3

Key: 4

Sol: $f(x)=\left|\,|x+2|-2|x|\,\right|$

$$f(x)=\begin{cases}\left|-(x+2)+2x\right| & x<-2\\ \left|+(x+2)+2x\right| & -2\le x<0\\ |x+2-2x| & x\ge 0\end{cases}$$

$$=\begin{cases}|x+2| & x<-2\\ |3x-2| & -2\le x<0\\ |2-x| & x\ge 0\end{cases}$$

$$=\begin{cases}-(x+2) & x<-2\\ -3x-2 & -2\le x<0\\ 2-x & 0\le x<2\\ x-2 & x\ge 2\end{cases}$$

Local minimum occur at $x=-\frac{2}{3}$ & $x=2$ $m=2$

Local maximum occur at $x=0$ $n=1$

$m+n=3$

05. The area of the region $\left\{(x,y):|x-y|\le y\le 4\sqrt{x}\right\}$ is

1) $\frac{1024}{3}$ **2)** $\frac{512}{3}$ **3)** 512 **4)** $\frac{2048}{3}$

Key: 1

Sol: $|x-y|\le y\le 4\sqrt{x}$

Consider $|x-y|=y$ & $y=4\sqrt{x}$

$x-y=\pm y$

$y=\frac{x}{2}$ *or* $x=0$

Solve $y=4\sqrt{x}$ & $y=\frac{x}{2}$

We get $x=0$ *or* $x=64$

Point of intersection $(0,0)$ & $(64,32)$

$$P\ A=\int_0^{64}\left(4\sqrt{x}-\frac{x}{2}\right)dx$$

$$=4\frac{2}{3}\left(x^{3/2}\right)_0^{64}-\left(\frac{x^2}{4}\right)_0^{64}$$

$$=\frac{8}{3}(512)-1024$$

$$=\frac{1024}{3}$$

06. Let the equation $x(x+2)(12-k)=2$ have equal roots. Then the distance of the point $\left(k,\frac{k}{2}\right)$ from the line $3x+4y+5=0$ is

1) 12 **2)** 15 **3)** $5\sqrt{3}$ **4)** $15\sqrt{5}$

Key: 2

Sol: $x(x+2)(12-k)=2$

$\Rightarrow \quad (12-k)x^2+2(12-k)x-2=0$

Equal roots

$\Rightarrow \quad b^2-400c=0$

$\Rightarrow \quad 4(12-k)^2-4(12-k)(-2)=0$

$\Rightarrow \quad k^2-26k+168=0$

$\Rightarrow \quad k=14 \quad (or)\,k=12$ not possible

$p(14,7)$

$3x+4y+5=0$

$$d=\frac{|3(14)+4(7)+5|}{\sqrt{3^2+4^2}}$$

$$=\frac{75}{5}=5$$

07. The sum $1+\frac{1+3}{2!}+\frac{1+3+5}{3!}+\frac{1+3+5+7}{4!}+.....$ upto ∞ terms, is equal to

1) $2e$ **2)** $6e$ **3)** $4e$ **4)** $3e$

Key: 1

Sol: $1+\frac{1+3}{2!}+\frac{1+3+5}{3!}+.....\infty$

$$=\sum_{n=1}^{\infty}\frac{n^2}{n!}$$

$$=\sum_{n=1}^{\infty}\frac{n}{(n-1)!}$$

$$=\sum_{n=1}^{\infty}\frac{(n-1)+1}{(n-1)!}$$

$$=\sum_{n=2}^{\infty}\frac{1}{(n-2)!}+\sum_{n=1}^{\infty}\frac{1}{(n-1)!}$$

$$=e+e\ =2e$$

08. If the domain of the function $f(x)=\log_7\left(1-\log_4\left(x^2-9x+18\right)\right)$ is $(\alpha,\beta)\cup(\gamma,\delta)$, then $\alpha+\beta+\gamma+\delta$ is equal to

1) 17 **2)** 16 **3)** 15 **4)** 18

Key: 4

Sol: $f(x)=\log_7\left[1-\log_4\left(x^2-9x+18\right)\right]$

$1-\log_4\left(x^2-9x+18\right)>0$

$1>\log_4\left(x^2-9x+18\right)$

$4'>x^2-9x+18$

$\Rightarrow \quad x^2-9x+14<0$

$\Rightarrow \quad (x-2)(x-7)<0$

$\Rightarrow \quad 2<x<7 \qquad (1)$

$x^2-9x+18>0$

$\Rightarrow \quad (x-3)(x-6)>0$

$\Rightarrow \quad x<3(or)x>6 \qquad (2)$

From (1) and (2)

$x\in(2,3)\cup(6,7)$

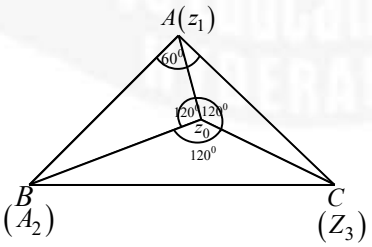
$\Rightarrow \quad \alpha+\beta+\gamma+\delta=2+3+6+7=18$

09. If $z_1,z_2,z_3\in\mathbb{C}$ are the vertices of an equilateral triangle, whose centroid is z_0, then $\sum_{k=1}^{3}(z_k-z_0)^2$ is equal to

1) 1 **2)** i **3)** $-i$ **4)** 0

Key: 4

Sol:



By rotation mechanism

$$\frac{z_1-z_0}{|z_1-z_0|}=\frac{z_3-z_0}{|z_3-z_0|}.e^{i\left(\frac{2\pi}{3}\right)}$$

Similarly

$$\frac{z_2-z_0}{|z_2-z_0|}=\frac{z_1-z_0}{|z_1-z_0|}.e^{i\left(\frac{2\pi}{3}\right)} \text{ and } \frac{z_3-z_0}{|z_3-z_0|}=\frac{z_2-z_0}{|z_2-z_0|}e^{i\left(\frac{2\pi}{3}\right)}$$

$$\Rightarrow \quad (z_1-z_0)^2+(z_2-z_0)^2+(z_3-z_0)^2=0$$

10. Line L_1 of slope 2 and line L_2 of slope $\frac{1}{2}$ intersect at the origin O. In the first quadrant, $P_1,P_2,.....P_{12}$ are 12 points on line L_1 and $Q_1,Q_2,.....,Q_9$ are 9 points on line L_2. Then the total number of triangles, that can be formed having vertices at three of the 22 points $O,P_1,P_2,.....P_{12}$, $Q_1,Q_2,.....Q_9$ is:

1) 1080 **2)** 1188 **3)** 1026 **4)** 1134

Key: 4

Sol: ${}^{22}C_3-{}^{10}C_3-{}^{13}C_3$

$=1134$ **(or)** ${}^{9}C_1\times{}^{13}C_2+{}^{12}C_1\times{}^{9}C_2$

11. The shortest distance between the curves $y^2=8x$ and $x^2+y^2+12y+35=0$ is:

1) $\sqrt{2}$ **2)** $3\sqrt{2}-1$ **3)** $2\sqrt{3}-1$ **4)** $2\sqrt{2}-1$

Key: 4

Sol: Let $P\left(at^2,2at\right)=\left(2t^2,4t\right)$

Slope of tangent of $y^2=8x$

$$2y\frac{dy}{dx}=8$$

$$\frac{dy}{dx}=\frac{4}{y}$$

Slope of normal $=\frac{-y}{4}$ at $p\left(2t^2,4t\right)$

Is $=\frac{-4t}{4}=-t$

Centre of circle $(0,-6)$ slope of $CP=\frac{4t+6}{2t^2-0}$

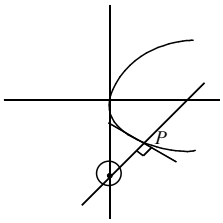
$$\therefore \quad \frac{4t+6}{2t^2}=-t$$

$$2t^3+4t+6=0$$

$t=-1\Rightarrow -2-4+6=0 \quad (t+1)\left(2t^2-2t+6\right)=0$

$\therefore \quad P(2,-4),C=(0,-6)$ then $PC=\sqrt{(-6+4)^2+4}=\sqrt{4+4}=\sqrt{8}$

$S.D=\sqrt{8}-\text{radius}=2\sqrt{2}-1$



12. The number of solutions of the equation
$\left(4-\sqrt{3}\right)\sin x-2\sqrt{3}\cos^2 x=-\frac{4}{1+\sqrt{3}}, x\in\left[-2\pi,\frac{5\pi}{2}\right]$ is

1) 5 **2)** 3 **3)**4 **4)** 6

Key: 1

Sol: $\left(4-\sqrt{3}\right)\sin x-2\sqrt{3}\left(1-\sin^2 x\right)=\frac{-4}{1+\sqrt{3}}$

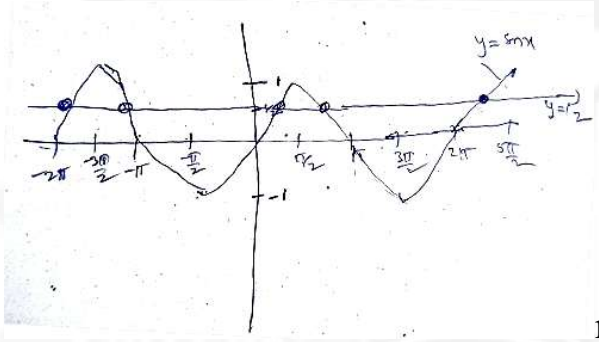
$\Rightarrow \quad 2\sqrt{3}\sin^2 x+\left(4-\sqrt{3}\right)\sin x-2\sqrt{3}+\frac{4}{1+\sqrt{3}}=0$

$\Rightarrow \quad 2\sqrt{3}\sin^2 x+\left(4-\sqrt{3}\right)\sin x+\frac{\left(-6-2\sqrt{3}+4\right)}{1+\sqrt{3}}=0$

$\Rightarrow \quad 2\sqrt{3}\sin^2 x+\left(4-\sqrt{3}\right)\sin x+\frac{-2\left(1+\sqrt{3}\right)}{1+\sqrt{3}}=0$

$\Rightarrow \quad 2\sqrt{3}\sin^2 x-\sqrt{3}\sin x+4\sin x-2=0$

$\sin x=\frac{1}{2}\sin x=\frac{\sqrt{3}}{-2}<-1$ not possible



no of solutions 5

13. Consider the line $x(3\lambda+1)+y(7\lambda+2)=17\lambda+5, \lambda$ being a parameter, all passing through a point P. One of these lines (say L) is farthest from the origin. If the distance of L from the point $(3,6)$ is d, then the value of d^2 is

1) 10 **2)** 20 **3)** 30 **4)** 15

Key: 2

Sol: Given $3\lambda x+x+7\lambda y+2y-17\lambda-5=0$

$(x+2y-5)+\lambda(3x+7y-17)=0$

This is in the from of $L_1+\lambda L_2=0$

$L_1=x+2y-5=0 \qquad (1)$

$L_2 = 3x + 7y - 17 = 0 \quad (2)$

Distance between $(3,6)$ *to* $x + 2y - 5 = 0$ **is d**

$$d = \left|\frac{3 + 12 - 5}{\sqrt{1+4}}\right| = \frac{10}{\sqrt{5}}$$

$d^2 = 20$

14. If the four distinct points $(4,6),(-1,5),(0,0)$ and $(k,3k)$ lie on a circle of radius r, then $10k + r^2$ is equal to:

1) 33 **2)** 35 **3)** 34 **4)** 32

Key: 2

Sol: equation of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$

Passing through the origin then $c = 0$

equation of the circle passing through the point $(4,6)$ then

$2g + 3f + 13 = 0$ ….. (i)

equation of the circle passing through the point $(-1,5)$

then $-g + 5f + 13 = 0$ ……. (ii)

solving (i) and (ii) then we get the circle $x^2 + y^2 - 4x - 6y = 0$

$(k,3k)$ then lies on the circle

$\therefore \quad k = \frac{11}{5}$

Radius of the circle $\sqrt{13}$

$\therefore \; 10k + r^2 = 35$

15. Each of the angles β and γ that a given line makes with the positive $y-$ and $z-$axes, respectively, is half of the angle that this line makes with the positive $x-$axes. Then the sum of all possible values of the angle β is

1) $\frac{\pi}{2}$ **2)** $\frac{3\pi}{4}$ **3)** π **4)** $\frac{3\pi}{2}$

Key: 2

Sol: Given that $\beta = \frac{\alpha}{2}, \gamma = \frac{\alpha}{2}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \quad \cos^2\alpha + 2\cos^2\frac{\alpha}{2} - 1 = 0$

$\Rightarrow \quad \cos^2\alpha + \cos\alpha = 0$

$\Rightarrow \quad \cos\alpha(\cos\alpha + 1) = 0$

$\Rightarrow \; \cos\alpha = 0, \quad \cos\alpha = -1$

$\alpha = \frac{\pi}{2} \qquad \alpha = \pi$

$\beta = \frac{\pi}{4}, \frac{\pi}{2}$ (possibilities)

Sum $= \frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$

16. Let f be a function such that $f(x) + 3f\left(\frac{24}{x}\right) = 4x, x \neq 0$. Then $f(3) + f(8)$ is equal to

1) 11 **2)** 12 **3)** 13 **4)** 10

Key: 2

Sol: Let f be a function such that

$f(x) + 3f\left(\frac{24}{x}\right) = 4x, x \neq 0$

Then $f(3) + f(8)$.

$f(x) + 3f\left(\frac{24}{x}\right) = 4x$, x replace by $\frac{24}{x}$

$f\left(\frac{24}{x}\right) + 3f(x) = \frac{96}{x}$

$3f\left(\frac{24}{x}\right) + 9f(x) = \frac{288}{x}$

$9f(x) + 3f\frac{(2y)}{x} = \frac{288}{x}$

$f(x) + 3f\frac{(2y)}{x} = 4x$

$- \quad - \quad -$

$8f(x) = \frac{288}{x} - \frac{4x}{2} = \frac{36}{x} - \frac{x}{2}$

$f(3) = \frac{36}{3} - \frac{3}{2} = \frac{12-3}{2} = \frac{21}{2}$

$f(8) = \frac{36}{8} - \frac{8}{2} = \frac{9}{2} - 4 = \frac{9-8}{2}$

$f(3) + f(8) = \frac{21}{2} + \frac{1}{2} = \frac{22}{2} = 11$

17. Let the Mean and Variance of five observations $x_1 = 1, x_2 = 3, x_3 = a, x_4 = 7$ and $x_5 = b, a > b$, be 5 and 10 respectively. Then the Variance of the observation $n + x_n, n = 1, 2, ...5$ is

1) 16 **2)** 17.4 **3)** 17 **4)** 16.4

Key: 1

Sol: Let the mean and variance of five observations $x_1 = 1, x_2 = 3, x_3 = a, x_4 = 7, x_5 = b, a > b$ be

5or 10 respectively then the variance of the observations $n+x_n, n=1,2,.....5$

Mean $\bar{x}=\frac{\Sigma x_i}{5}=5 \Rightarrow \Sigma x_i=25$

Value $\sigma^2=\frac{\Sigma x_i^2}{5}-(5)^2=10$

$\frac{\Sigma x_i^2}{5}=35 \Rightarrow \Sigma x_i^2=175$

$x_1+x_2+x_3+x_4+x_5=25 \quad 1+3+a+7+b=25 \quad a+b=14$

$x_1^2+x_2^2+x_3^2+x_4^2+x_5^2=175$

$1^2+3^2+a^2+7^2+\frac{b^2}{2}=175$

$a^2+b^2+59=17 \Rightarrow a^2+b^2=175-59=116$

$(a+b)^2=a^2+b^2+2ab \quad 196-116=2ab$

$(14)^2=116+2ab \qquad 80=2ab \Rightarrow ab=40$

$a+\frac{40}{a}=14 \Rightarrow a^2+40=14a$

$a^2-14a+40=0$

$a=10, a=4 \quad (a-10)(a-4)=0$

$b=14-a=14-10=4$

$n+x_n=1+x_1, 2+x_2, 3+x_3+4+x_4, 5+x_5$

$2,5,3+a,11,5+b$

$2,5,13,11,9$

Variance $\sigma^2=\frac{\Sigma x_i^2}{n}-\left(\frac{\Sigma x_i}{n}\right)^2$

Mean $=\frac{2+5+13+11+7}{5}=\frac{40}{5}=8$

$\frac{2^2+5^5+13^2+11^2+9^2}{5}=\frac{4+25+169+12+81}{5}=80$

Variance $=80-64=16$

18. Let $y=y(x)$ be the solution of the differential equation $\frac{dy}{dx}+3\left(\tan^2 x\right)y+3y=\sec^2 x, y(0)=\frac{1}{3}+e^3$. Then $y\left(\frac{\pi}{4}\right)$ is equal to

1) $\frac{2}{3}$ **2)** $\frac{4}{3}+e^3$ **3)** $\frac{4}{3}$ **4)** $\frac{2}{3}+e^3$

Key: 3

Sol: $\frac{dy}{dx}+\left(3\tan^2 x\right)y+3y=\sec^2 x$

$\frac{dy}{dx}+y\left(3\tan^2 x+3\right)=\sec^2$

$$\frac{dy}{dx}+y3\sec^2 x=\sec^2 x$$

$$I.F=e^{\int p(x)dx}=e^{3\int \sec^2 x\,dx}=e^{3\tan x}$$

$$G.S \qquad y.I.F=\int \sec^2 x.\ I.F\ dx$$

$$y.e^{3\tan x}=\int \sec^2 x e^{\tan x}dx$$

$$\tan x,t \Rightarrow \sec^2+dx,dt$$

$$\int e^{3t}dt=\frac{e^{3t}}{3}+c$$

$$e^{3\tan y}=\frac{e^{3\tan x}}{3}+c$$

$$x=0, y=\frac{1}{3}+C$$

$$\frac{1}{3}+e^3=\frac{1}{3}+c\Rightarrow c=e^3$$

$$ye^{3\tan x}=\frac{e^{3\tan x}}{3}+e^3$$

$$x=\frac{\pi}{4}$$

$$y\ e^3=\frac{e^3}{3}+e^3$$

$$y=\frac{4}{3}$$

19. Let $A=\{-2,-1,0,1,2,3\}$. Let R be a relation on A defined by xRy if and only if $y=\max\{x,1\}$. Let l be the number of elements in R. Let m and n be the minimum number of elements required to be added in R to make if reflexive and symmetric relations, respectively. Then $l+m+n$ is equal to

1) 11 **2)** 13 **3)** 14 **4)** 12

Key: 4

Sol: Find the elements of relation e

$x=-2, y=(-2,1)=1$, so $(-2,1)\in R$

$x=-1 \quad (-1,1)\in R$

$x=0 \quad (0,1)\in R$

$x=1 \quad (1,1)\in R$

$x=2 \quad (2,2)\in R$

$x=3 \quad (3,3)\in R$

$R=\{(-2,1)(-1,1)(0,1)(1,1)(2,2)(3,3)\}, L=6$

$A=\{-2,-1,0,1,2,3\}$

Already $((1,1),(2,2),(3,3))$

We added $(-2,-2),(-1,-1)\&(0,0)$, $m=3$

R be a symmetric $(a,b)\in R$

$R=\{(-2,1)(-1,1),(0,1),(1,1),(2,2)(3,3)\}$

We added $(1,-2),(1,-1),(1,0), n=3$

$n=3$

$\ell=6, m=3, n=3$

$\ell+m+n=12$

20. The integral $\int_0^{\pi}\frac{8x\,dx}{4\cos^2 x+\sin^2 x}$ is equal to

1) π^2 **2)** $\frac{3\pi^2}{2}$ **3)** $2\pi^2$ **4)** $4\pi^2$

Key: 3

Sol: $\int_0^{\pi}\frac{8x}{4\cos^2 x+\sin^2 x}dx$

$I=\int_0^{\pi}\frac{8(\pi-x)}{4(\cos(\pi-x))^2+(\sin(\pi-x))^2}dx$

$=\int_0^{\pi}\frac{8\pi}{4\cos^2 x+\sin^2 x}dx-\int_0^{\pi}\frac{8\pi}{4\cos^2 x+\sin^2 x}dx = I$

$2I=8\pi\int_0^{\pi}\frac{1}{4\cos^2 x+\sin^2 x}dx$

$I=8\pi\int_0^{\pi/2}\frac{\sec^2 x}{4+\tan^2 x}dx$

$=8\pi\int_0^{\infty}\frac{1}{4+t^2}dt$

$8\pi\frac{1}{2}\left(\tan\left(\frac{t}{2}\right)\right)_0^{\infty}$

$4\pi\frac{\pi}{2}=2\pi^2$

SECTION-II (NUMERICAL VALUE TYPE)

This section contains **5 Numerical Value Type Questions.** The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **Nearest Integer** value (Example i,e. If answer is above **10** and less than **10.5** round off is **10** and If answer is from **10.5** and less than **11** round off is **11).**

Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases..

21. If the equation of the hyperbola with foci $(4,2)$ and $(8,2)$ is $3x^2-y^2-\alpha x+\beta y+\gamma=0$, then $\alpha+\beta+\gamma$ is equal to_____.

Key:

Sol: e is not given problem is wrong

22. If $\lim\limits_{x\to 0}\left(\dfrac{\tan x}{x}\right)^{\frac{1}{x^2}} = p$, then $96\log_e p$ is equal to

Key: 32

Sol: $e^{\lim\limits_{x\to 0}\frac{1}{x^2}\frac{(\tan x - x)}{x}}$

$e^{\lim\limits_{x\to 0}\frac{Tanx - x}{x^3}}$

$e^{\lim\limits_{x\to 0}\frac{x+\frac{x^3}{3}+\frac{x^5}{25}+.....x}{x^3}}$

$e^{\lim\limits_{x\to 0}\frac{x^3\left(\frac{1}{3}+\frac{x^2}{25}\right)}{x^3}} = e^{\frac{1}{3}} = p$

$96\log e^{1/3} = 96.\dfrac{1}{3} = 32$

23. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b} = 3\hat{i} - 3\hat{j} + 3\hat{k}, \vec{c} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{d}$ be a vector such that $\vec{b}\times\vec{d} = \vec{c}\times\vec{d}$ and $\vec{a}.\vec{d} = 4$. Then $\left|\left(\vec{a}\times\vec{d}\right)\right|^2$ is equal to_____.

Key: 128

Sol: $b\times d = c\times d$

$(b\times d) - (c\times d) = 0$

$(b-c)\times d = 0$

$d = +\left(\overline{b} - \overline{c}\right)$

$\overline{d} = t\left(\hat{i} - 2\hat{j} + \hat{k}\right)$

$\overline{a}.\overline{d} = 4$

$t(1-4+1) = 4$

$-2t = 4$

$t = -2$

$\overline{d} = -2\left(\hat{i} - 2\hat{j} + \hat{k}\right)$

$\left|\vec{a}\times\vec{d}\right|^2 = (a.d)^2 + |a|^2|d|^2$

$= 4^2 + (1+4+1)4(4+1+1)$

$16 - 144 = 128$

24. Let $\left(1+x+x^2\right)^{10} = a_0 + a_1x + a_2x^2 + ... + a_{20}x^{20}$. If $\left(a_1 + a_3 + a_5 + ... + a_{19}\right) - 11a_2 = 121k$, then k is equal to_____

Key: 110

Sol: $\left(1+x+x^2\right)^{10}=a_0+a_1x+a_2x+.....+a_{20}x^{20}$ (1)

If $\left(a_1+a_3+a_5+.....+a_{19}\right)-11a_2=121K$

Put $x=1$

$3^{10}=a_0+a_1+a_2+.....+a_{20}$

Put $x=-1 \quad 1=1-a_1+a_2+.....+a_{20}$

$3^{10}-1=2\left(a_1+a_3+.....+a_{19}\right)$

Diff w.r. to 'x'

$10\left(1+x+x^2\right)^9\left(1+2x\right)=a_1+2a_2x+3a_3x^2+.....+20a_{20}x^{19}$

Again diff. w.r.to

$10.9\left(1+x+x^2\right)^8\left(1+2x\right)+10.2\left(1+x+x^2\right)^9=2a_2+x(.....)$

Put $x=0$

$90+20=2a_2$

$110=2a_2$

25. Let I be the identity matrix of order 3×3 and for the matrix $A=\begin{bmatrix}\lambda & 2 & 3\\ 4 & 5 & 6\\ 7 & -1 & 2\end{bmatrix}$, $|A|=-1$.

Let B be the inverse of the matrix $adj\left(Aadj\left(A^2\right)\right)$. Then $\left|\left(\lambda B+I\right)\right|$ is equal to_____

Key: Not an integer

Sol: $A=\begin{bmatrix}\lambda & 2 & 3\\ 4 & 5 & 6\\ 7 & -1 & 2\end{bmatrix}|A|=1$

$\lambda(10+6)-2(8-42)+3(-4-35)=-1$

$16\lambda+68-117=-1$

$16\lambda=50$

$\lambda=\frac{25}{4}$

PHYSICS

SECTION-I (SINGLE CORRECT ANSWER TYPE)

This section contains **20 Multiple Choice Questions.** Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

26. A monochromate light of frequency $5\times10^{14}\,Hz$ travelling through air, is incident on a medium of refractive index '2'. Wavelength of the refracted light will be:

1) 300 nm **2)** 500 nm **3)** 400 nm **4)** 600 nm

Key: 1

Sol: $f=5\times10^{14}\,Hz$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{5\times10^{14}} = 6\times10^{-7}\,m = 6000A^0$$

$$\lambda' = \frac{\lambda}{\mu} = \frac{600nm}{2} = 300nm$$

27. A particle moves along the $x-$axis and has its displacement x varying with time t according to the equation:

$$x = c_0\left(t^2-2\right)+c\left(t-2\right)^2$$

Where c_0 and c are constants of appropriate dimensions.

Then, which of the following statements is correct?

1) the initial velocity of the particle is 4c

2) the acceleration of the particle is $2\left(c+c_0\right)$

3) the acceleration of the particle is $2c$

4) the acceleration of the particle is $2c_0$

Key: 2

Sol: $V = \frac{dx}{dt} = c_0\left(2t\right)+c2\left(t-2\right)$

$$V = 2c_0t+2c\left(t-2\right)$$

$$V_{t=0} = -4c$$

$$\frac{dv}{dt} = a = 2c_0+2c$$

$$= 2\left(c_0+c\right)$$

28. Given below are two statements one is labeled as Assertion A and the other is labeled as Reason R

Assertion A: The Bohr model is applicable to hydrogen and hydrogen-like atoms only:

Reason R: The formulation of Bohr model does not include repulsive force between electrons. In the light of the above statements, choose the correct answer from the options given below:

1) Both A and R are true but R is NOT the correct explanation of A

2) Both A and R are true and R is the correct explanation of A

3) A is true but R is false

4) A is false but R is true

Key: 2

Sol: The Bohr's model only consider the electrostatic force between the nucleus and a single electron.

29. The truth table corresponding to the circle given below is:

1)

A	B	C
0	0	0
0	1	0
1	0	1
1	1	1

2)

A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

3)

A	B	C
0	0	0
1	0	0
0	1	0
1	1	1

4)

A	B	C
0	0	1
1	0	0
0	1	0
1	1	0

Key: 1

Sol: input

A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

output

$C = A.(A + B)$

0

0

1

1

30. An electric bulb rated as 100 W-220 V is connected to an ac source of rms voltage 220 V. The peak value of current through the bulb is:

1) 2.2A **2)** 0.32 A **3)** 0.45 A **4)** 0.64 A

Key: 3

Sol: $P = 100w$

$V = 220v \quad R = \frac{V^2}{P} = \frac{220 \times 220}{100} = 484\Omega$

$E_0 = \sqrt{2}E_{rms} = 1.414 \times 220 = 311v$

$i = \frac{P}{E_{rms}} = \frac{100}{220} = 0.45A$

31. A block of mass 1 kg moving along x with speed $v_i = 10m/s$ enters a rough region ranging from $x = 0.1m$ to $x = 1.9m$. The retarding force acting on the block in this range is $F_r = -kxN$, with $k = 10N/m$. Then the final speed of the block as it crosses rough region is.

1) 4 m/s **2)** 10 m/s **3)** 6 m/s **4)** 8 m/s

Key: 4

Sol: $KE = \frac{1}{2}mv^2$

$$\omega = \int_{x_1}^{x_2} F\, d_x = \int_{x_1=0.1}^{x_2=1.9} \tan dx = -10\left[\frac{n^2}{2}\right]_{0.1}^{1.9} = -\frac{k}{2}\left[(1.9)^2 - (0.1)^2\right]$$

$$\omega = \frac{10}{5}[3.61 - 0.01] = 5 + 3.6 = 18.0\frac{3}{5}$$

$\omega = 1\ KE$

$$-18 = \frac{1}{2}m\left(v_2^2 - v_1^2\right)$$

$$-36 = 1\left(v_2^2 - 100\right)$$

$v_2^2 = 64 \quad v_2 = 8\ m/s$

32. A motor operating on 100V draws a current of 1A. If the efficiency of the motor is 91.6%, then the loss of power in units of cal/s is

1) 4 **2)** 8.4 **3)** 6.2 **4)** 2

Key: 2

Sol: $P_{in} = VI = 100 \times 1 = 1000$

$P_{out} = 4P_{in} = 91.6$

$1000 = 100 - 916$

$= 8.4\ m$

33. Which of one of the two slits in a Young's double slit interference experiment is half of the other slit. The ratio of the maximum to the minimum intensity in the interference pattern is:

1) 3:1 **2)** 9:1 **3)** $\left(2\sqrt{2}+1\right):\left(2\sqrt{2}-1\right)$ **4)** $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$

Key: 2

Sol: $\frac{I_{\max}}{I_{\min}}:\frac{\left(\frac{34}{2}\right)^2}{\left(\frac{4}{2}\right)^2}=\frac{9}{1}$

34. A particle is projected with velocity u so that its horizontal range is three times the maximum height attained by it. The horizontal range of the projectile is given as $\frac{mu^2}{25g}$, where value of n is: (g is acceleration due to gravity)

1) 12 **2)** 24 **3)** 18 **4)** 6

Key: 2

Sol: $\frac{4^2\sin 2\theta}{g}=\frac{34^2\sin^2\theta}{2g}$

$2\sin\theta\cos\theta=\frac{3}{2}\sin^2 6$

$\frac{4}{3}=\tan\theta$

$\cos\theta=\frac{3}{5}$

$R=\frac{4^2\sin\theta\cos\theta}{g}$

$=4^2\times\frac{2}{g}\times\frac{4}{5}\times\frac{3}{5}=\frac{244^2}{25g}$

35. Two cylindrical vessels of equal cross sectional area of $2m^2$ contain water upto heights $10m$ and 6m, respectively. If the vessels are connected at their bottom then the work done by the force of gravity is
(Density of water is $10^3 kg/m^3$ and $g=10\ m/s^2$)

1) $4\times10^4 J$ **2)** $8\times10^4 J$ **3)** $1\times10^5 J$ **4)** $6\times10^4 J$

Key: 2

Sol: $\omega=\frac{1}{4}\left(4_1-4_2\right)^2 gAP=\frac{1}{4}\times16\times10\times2\times10^3$

$=8\times10^4$

36. Using a battery, a 100 pF capacitor is charged to 60 V and then the battery is removed. After that, a second uncharged capacitor is connected to the first capacitor in parallel. If the final voltage across the second capacitor is 20 V, is capacitance is: (in pF)

1) 600 **2)** 200 **3)** 100 **4)** 400

Key: 2

Sol: $Q_i = C_1 V_1$

$= 100 \times 10^{-12} \times 60 = 6 \times 10^{-9} C$

$Q_{1f} = C_1 V_f = 100 \times 10^{-12} \times 20$

$= 2 \times 10^{-9} C$

$Q_{2f} = Q_i - Q_{1f} = 4 \times 10^{-9} C$

$C_2 = \frac{Q_{2f}}{V_f} = \frac{4 \times 10^{-9}}{20} = 200 pF$

37. In the resonance experiment, two air columns (closed at one end) of 100 cm and 120 cm long, give 15 beats per second when each one is sounding in the respective fundamental modes. The velocity of sound in the air column is:

1) 340 m/s **2)** 335 m/s **3)** 370 m/s **4)** 360 m/s

Key: 4

Sol: $\Delta n = m_1 - n_2$

$15 = \frac{v}{4\ell_1} - \frac{v}{4\ell_2}$

$15 = \frac{v}{4 \times 1} - \frac{v}{4 \times 1.2}$

$= \frac{v}{4}\left[1 - \frac{10}{12}\right]$

$15 = \frac{v}{4}\left[1 - \frac{5}{6}\right]$

$15 = \frac{v}{4} . \frac{1}{6}$

$v = 15 \times 24$

$= 360\ m/s$

38. A magnetic dipole experience a torque of $80\sqrt{3} Nm$ when placed in uniform magnetic field in such a way that dipole moment makes angle of 60^0 with magnetic field. The potential energy of the dipole is :

1) – 60 J **2)** $-40\sqrt{3} J$ **3)** $80 J$ **4)** $-80 J$

Key: 4

Sol: $\tau = MB \sin\theta$

$8\sqrt{3} = MB \frac{\sqrt{3}}{2}$

$MB = 160$

$P\sqrt{3} = -MB \cos\theta$

$= -160 \times \frac{1}{2}$

$= -80 J$

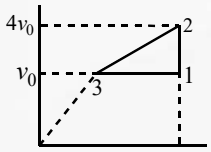
39. An ideal gas exists in a state with pressure P_0, volume V_0. It is isothermally expanded to 4 times of its initial volume (V_0), then isobarically compressed to its original volume. Finally the system is heated isochorically to bring it to its initial state. The amount of heat exchanged in this process is

1) $P_0V_0(\ln 2 - 0.25)$ **2)** $P_0V_0(2\ln 2 - 0.25)$

3) $P_0V_0(\ln 2 - 0.75)$ **4)** $P_0V_0(2\ln 2 - 0.75)$

Key: 4

Sol:



$P_1 = P_0 \quad V_1 = V_0 \quad T_1 = T_0$

$P_2 = P_2 \quad V_2 = 4V_0 \quad T_2 = T_0$

$P_3 = P_2 \quad V_3 = V_0 \quad T_3 = T_0$

$P_4 = P_4 \quad V_4 = V_0 \quad T_4 = T_0$

$P_0V_0 = P_2 4V_0$

$P_2 = \frac{P_0}{4}$

$du = 0(cyclic)$

$dQ = d\omega$

$= d\omega_{12} + d\omega_{23} + d\omega_{31}$

$= nRT_0 \log\frac{4v_0}{v_0} + P_2\left[\frac{v}{3} - \frac{v}{2}\right] + 0$

$= nRT_0 2\log 2 + \frac{P_0}{4}[v_0 - 4v_0]$

$= nRT_0 2\log 2 - \frac{3}{4}P_0V_0$

$= P_0V_0 2\log 2 - \frac{3}{4}P_0V_0$

$= P_0V_0[2\log 2 - 0.75]$

40. A solid steel ball of diameter 3.6 mm acquired terminal velocity $2.45\times10^{-2} m/s$ while falling under gravity through an oil of density $925 kg\, m^{-3}$. Take density of steel as $7825\ kg\ m^{-3}$ and g as $9.8\ m/s^2$. The viscosity of the oil in SI unit is

1) 1.68 **2)** 2.38 **3)** 2.18 **4)** 1.99

Key: 4

Sol: $v_t = \frac{2}{9}\frac{R^2}{\eta}g(\rho - \pi)$

$$2.45\times10^{-2}=\frac{2}{9}\times\frac{\left(1.8\times10^{-3}\right)^2}{\eta}[7825-925]10$$

$$\eta=\frac{2\left(1.8\times10^{-3}\right)^2[6900]\times9.8}{9\times2.45\times10^{-2}}$$

$$=\frac{6.48\times4900\times10^{-6}\times9.8}{22.05\times10^{-2}}=\frac{44712\times9.8}{22.05\times10^{-4}}$$

$$=19872\times10^{-4}$$

$$=1.9872$$

$$=1.99\ m/s$$

41. Pressure of an ideal gas, contained in a closed vessel, is increased by 0.4% when heated by 1^0C. Its initial temperature must be:

1) 250^0C **2)** 25^0C **3)** $250\ K$ **4)** $2500\ K$

Key: 3

Sol: Initial state $p_1v=nRT_1$ …….. (1)

Final state $p_2v=nRT_2$ ……..(2)

$p_2=p_1+0.004p_1=1.004p_1$ and $T_2=T_1+1$

$$(2)=\frac{p_2v}{p_1v}=\frac{nRT_2}{nRT_1}$$

$$(1)=\frac{p_2}{p_1}=\frac{T_2}{T_1}$$

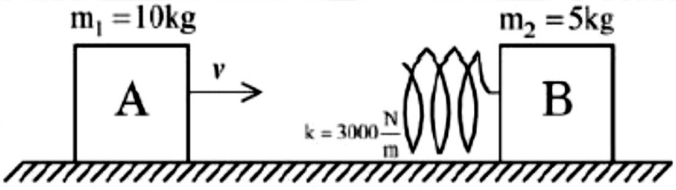
$$\frac{1.004p_1}{p_1}=\frac{T_1+1}{T_1}$$

$$1.004=\frac{T_1+1}{T_1}$$

$$1.004T_1-T_1=1$$

$$T_1=\frac{1}{0.004}=250K$$

42.



Consider two blocks A and B of masses $m_1=10\ kg$ and $m_2=5\ kg$ that are placed on a frictionless table. The block A moves with a constant speed $v=3m/s$ towards the block B kept at rest. A spring with spring constant $k=3000\ N/m$ is attached with the block B as shown in the figure. After the collision, suppose that the blocks A and B, along with the spring in constant compression state,

move together, then the compression in the spring is, (Neglect the mass of the spring)

1) 0.1 m **2)** 0.3 m **3)** 0.2 m **4)** 0.4 m

Key: 1

Sol: acc to L.C.M $m_1u_1+m_2u_2=(m_1+m_2)v$

$10\times3+5\times0=(10+5)v$

$v=2\ m/s$

Here $KE=$ elastic potential energy

$\frac{1}{2}(m_1+m_2)v^2=\frac{1}{2}kx^2$

(m here) $v^2=-Kx^2$

$x^2=\frac{(m_1+m_2)}{k}v^2$

$x=\sqrt{\frac{(m_1+m_2)}{K}}v^2$

$\therefore \quad \sqrt{\frac{(10+5)2^2}{3000}}=\sqrt{\frac{15\times4}{3000}}=\sqrt{\frac{60}{3000}}=\sqrt{0.02}=0.1414m$

$x\approx0.1m$

43. Match the List – I with List – II

List – I		List – II	
A	Boltmann constant	I	ML^2T^{-1}
B	Coefficient of viscosity	II	$MLT^{-3}K^{-1}$
C	Planck's constant	III	$ML^2T^{-2}K^{-1}$
D	Thermal conductivity	IV	$ML^{-1}T^{-1}$

Choose the correct answer from the options given below:

1) $A-III;B-II;C-I;D-IV$ **2)** $A-II;B-III;C-IV;D-I$

3) $A-III;B-IV;C-II;D-I$ **4)** $A-III;B-IV;C-I;D-II$

Key: 4

Sol: Boltzmann constant $K_B=\frac{pv}{NT}=ML^2T^{-2}K^{-1}$

Coefficient of viscosity $h=\frac{F}{A\frac{dv}{dx}}=ML^{-1}T^{-1}$

Plank's constant $h=\frac{E}{\gamma}=ML^2T^{-1}$

Thermal conductivity

$K=\frac{ad}{A(T_2-T_1)}=MLT^{-3}K^{-1}$

44. Given below are two statements: one is labeled as Assertion A and the other is labeled as Reason R

Assertion A: If oxygen ion $\left(O^{-2}\right)$ and Hydrogen ion $\left(H^{+}\right)$ enter normal to the magnetic field with equal momentum, then the path of O^{-2} ion has a smaller curvature than that of H^{+}.

Reason R: A proton with same linear momentum as an eletron will form a path of smaller radius of curvature on entering a uniform magnetic field perpendicularly.

In the light of the above statements, choose the correct answer from the options given below

1) A is true but R is false

2) Both A and R are true but R is NOT the correct explanation of A

3) Both A and R are true and R is the correct explanation of A

4) A is false but R is true

Key: 1

Sol: A) $F_m = F_c$

$$Bqv = \frac{mv^2}{2}$$

$$r = \frac{p}{Bq}$$

$$q_{0^{-2}} > q_{4^{+}}$$

$$r \propto \frac{1}{q}$$

B) charge of proton = charge of electron and $P;B$ are correct

$$r = \frac{P}{Bq}$$

r is same for proton and electron

45. Two monochromatic light beams have intensities in the ratio 1:9. An interference pattern is obtained by these beams. The ratio of the intensities of maximum to minimum is

1) 3:1 **2)** 9:1 **3)** 8:1 **4)** 4:1

Key: 4

Sol: Intensity $\times$ $amplitude^2$

$$I \times A^2$$

$$I_{\max} = \left(A_1 + A_2\right)^2$$

$$I_{\min} = \left(A_1 - A_2\right)^2$$

$$I \alpha A^2 \qquad \therefore \frac{I_1}{I_2} = \frac{A_1^2}{A_2^2}$$

$$\frac{1}{9}=\frac{A_1^2}{A_2^2}$$

$$\frac{1}{3}=\frac{A_1}{A_2}$$

$$\therefore \quad A_1=1, A_2=3$$

$$\text{Now } \frac{I_{\max}}{I_{\min}}=\frac{\left(A_1+A_2\right)^2}{\left(A_1-A_2\right)^2}$$

$$=\frac{16}{4}$$

$$\frac{I_{\max}}{I_{\min}}=\frac{4}{1}$$

SECTION-II (NUMERICAL VALUE TYPE)

This section contains **5 Numerical Value Type Questions.** The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **Nearest Integer** value (Example i,e. If answer is above **10** and less than **10.5** round off is **10** and If answer is from **10.5** and less than **11** round off is **11).**

Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.

46. Two cells of emfs 1 V and 2V and internal resistances 2Ω and 1Ω, respectively, are connected in series with an external resistance of 6Ω. The total current in the circuit is I_1. Now the same two cells in parallel configuration are connected to same external resistance. In this case, the total current drawn is I_2. The value of $\frac{I_1}{I_2}$ is $\frac{x}{3}$. The value of x is_____.

Key: 4

Sol: $i_1=\frac{3}{2+1+6}=\frac{1}{3}A$

$$i_2=\frac{\left(\frac{5}{3}\right)}{\left(\frac{2}{3}+6\right)}=\frac{1}{4}A$$

$$\therefore \quad \frac{i_1}{i_2}=\frac{4}{3}$$

47. The excess pressure inside a soap bubble A in air is half the excess pressure inside another soap bubble B in air. If the volume of the bubble A is n times the volume of the bubble B, then, the value of n is_____.

Key: 8

Sol: $\frac{4T}{R_A}=\frac{1}{2}\left(\frac{4T}{R_B}\right)$

$$\therefore \quad R_A=2R_B$$

$$V_A=8V_B$$

48. A physical quantity C is related to four other quantities p,q,r and s as follows $C=\frac{pq^2}{r^3\sqrt{s}}$. The percentage errors in the measurement of p,q,r and s are 1%, 2%, 3% and 2%, respectively. The percentage error in the measurement of C will be_____ %.

Key: 15

Sol: $C=\frac{pq^2}{r^3\sqrt{s}}$

% error in $C=(1)(1)+(2)(2)+(3)(3)+\left(\frac{1}{2}\right)(2)=15$

49. Light from a point source in air falls on a spherical glass surface (refractive index, $\mu=1.5$ and radius of curvature = 50 cm). The image is formed at a distance of 200 cm from the glass surface inside the glass. The magnitude of distance of the light source from the glass surface is_____ m.

Key: 4

Sol: $\frac{1.5}{200}-\frac{1}{-x}=\frac{1.5-1}{50}$

$x=400\ cm$

50. An electron in the hydrogen atom initially in the fourth excited state makes a transition to n^{th} energy state by emitting a photon of energy 2.86 eV. The integer value of n will be_____.

Key: 2

Sol: $\frac{13.6}{n^2}-\frac{13.6}{25}=2.86eV$

$\therefore \quad n=2$

CHEMISTRY

Max Marks: 100

SECTION-I (SINGLE CORRECT ANSWER TYPE)

This section contains **20 Multiple Choice Questions.** Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

51. Consider the following statements related to temperature dependence of rate constants. Identify the correct statements.

A. The Arrhenius equation holds true only for an elementary homogenous reaction

B. The unit of A is same as that of k in Arrhenius equation

C. At a given temperature, a low activation energy means a fast reaction

D. A and Ea as used in Arrhenius equation depend on temperature

E. When $Ea >> RT$, A and Ea become interdependent

Choose the correct answer from the options given below:

1) A and B only **2)** B, D and E only **3)** A, C and D only **4)** B and C only

Key: 4

Sol: Since A and K must have the same units [as the exponential term is dimensionless]

At a given temperature a low activation energy means fast react correct

52. The standard cell potential $\left(E^{\Theta}_{cell}\right)$ of a fuel cell based on the oxidation of methanol in air that has been used to power television relay station is measured as 1.21 V. The standard half cell reduction potential for $O_2\left(E_{O_2/H_2O}\right)$is 1.229 V.

Choose the correct statement:

1) Oxygen is formed at the anode

2) The standard half -cell reduction potential for the reduction of $CO_2\left(E^{o}_{CO_2/CH_3OH}\right)$ is 19 mV

3) Reduction of methanol takes place at the cathode

4) Reactants are fed at one go to each electrode

Key: 1

Sol: In a methanol fuel cell methanol is oxidized at anode and oxygen is reduced at cathode.

Anode $CH_3OH + H_2O \rightarrow CO_2 + 6H^+ + 6C^-$

Cathode $\frac{3}{2}O_2 + 6H^+ + 6C^- \rightarrow 3H_2O$

$E^0_{cell} = E^0_{cathode} - E^0_{anode}$

$1.21 = 1.229 - E^0_{anode}$

$\Rightarrow \quad E^0_{anode} = 0.019V$

This matches the value given for $E^0_{CO_2/CH_3OH}$ conforming CH_3OH oxidation occur at anode.

53. Given below are two statements:

Statement I : When a system containing ice in equilibrium with water (liquid) is heated, heat is absorbed by the system and there is no change in the temperature of the system until whose ice gets melted.

Statement II: At melting point of ice, there is absorption of heat in order to overcome intermolecular forces of attraction within the molecules of water in ice and kinetic energy of molecules is not increased at melting point.

In the light of the above statements, choose the correct answer from the options given below

1) Statement I is false but Statement II is true

2) Both Statement I and Statement II are false

3) Statement I is true but Statement II is false

4) Both Statement I and Statement II are true

Key:4

Sol: The absorbed heat [latent heat] is used to break inter molecular bonds, not to increase K.E. (so temp remain constant)

54. Given below are two statements:

Statement I: CrO_3 is a stronger oxidizing agent than MoO_3

Statement II: $Cr(VI)$ is more stable than $Mo(VI)$

In the light of the above statements, choose the correct answer from the options given below

1) Statement I is false but Statement II is true

2) Both Statement I and Statement II are true

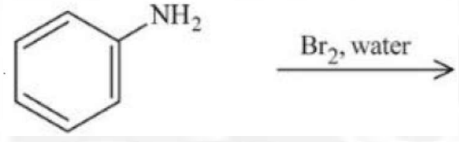
3) Both Statement I and Statement II are false

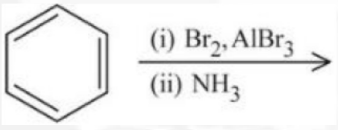
4) Statement I is true but Statement II is false

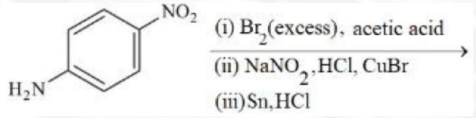
Key: 4

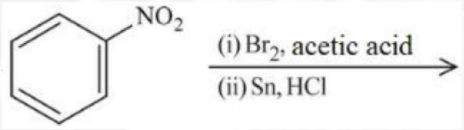
Sol: CrO_3 is stronger oxidizing agents than MoO_3. This is because Cr has a higher tendency to get reduced from $+6$ to lower oxidation state.

55. The sequence from the following that would result in giving predominantly 3, 4, 5- Tribromoaniline is:

1) 

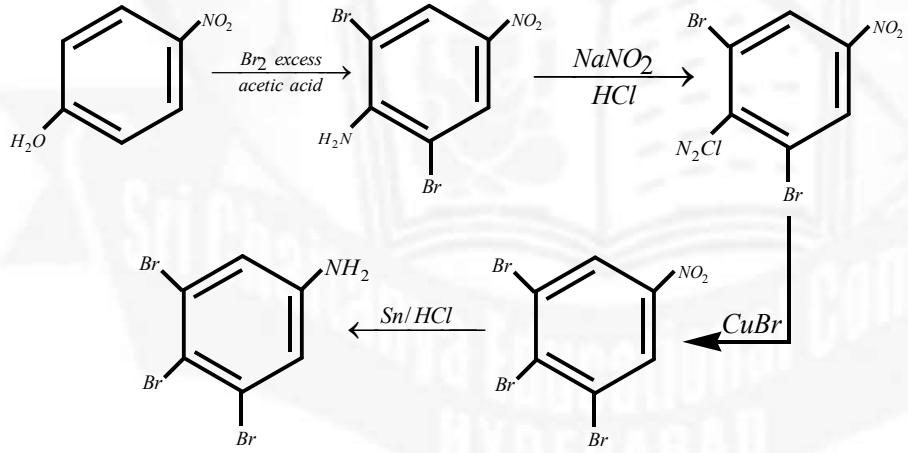
2) 

3) 

4) 

Key: 3

Sol:



56. Fat soluble vitamins are:

A. Vitamin B_1 B. Vitamin C C. Vitamin E D. Vitamin B_{12}

E. Vitamin K

Choose the correct answer from the options given below:

1) B & C only **2)** C & D only **3)** A & B only **4)** C & E only

Key: 4

Sol: Vitamins $\rightarrow B$ and $C \Rightarrow$ water soluble vitamins

Vitamins $\rightarrow A, D, E$ and K for soluble vitamins

57. The correct orders among the following are

Atomic radius : $B < Al < Ga < In < Tl$

Electronegativity: $Al < Ga < In < Tl < B$

Density : $Tl < In < Ga < Al < B$

1st Ionisation Energy : $In < Al < Ga < Tl < B$

Choose the correct answer from the options given below:

1) A and C only **2)** C and D only **3)** B and D only **4)** A and B only

Key: 3

Sol: Atomic radius $B < Ga < Al < In < T\ell$

Electronegativity $Al < Ga < In < T\ell < B$

Density $B < Al < Ga < In < T\ell$

1st I.E. $In < Al < Ga < T\ell < B$

58. $10mL$ of $2MNaOH$ solution is added to 20mL of $1\,M$ HCl solution kept in a beaker. Now, 10 mL of this mixture is poured into a volumetric flask of 100 mL containing 2 moles of HCl and made the volume upto the mark with distilled water. The solution in this flask is:

1) 20 M HCl solution **2)** 10 M HCl solution

3) 0.2 M NaCl solution **4)** Neutral solution

Key: 3

Sol: $NaOH + HCl \longrightarrow NaCl + H_2O$

$10 \times 2 \quad 20 \times 1$

$= 20\ mmol \quad = 20\ mmol \quad 20\ mmol$

$\therefore$ concentration of $NaCl = \dfrac{20}{100} = 2 \times 10^{-1}$

$= 0.2M$

59. Mass of magnesium required to produce 220 mL of hydrogen gas at STP on reaction with excess of dil. HCl is

Given: molar mass of Mg is $24\ g\ mol^{-1}$

1) 236 mg **2)** 2.444 g **3)** 0.24 mg **4)** 235.7 g

Key: 1

Sol: Balanced chemical equation is

$Mg + 2HCl \longrightarrow MgCl_2 + H_2(g)$

24 g---------- 22400 mL

? ---------- 220 mL

$x = \dfrac{24 \times 220}{22400} = 0.236g$

$= 236\ mg$

60. Given below are two statements:

Statement I: Hyper conjugation is not a permanent effect

Statement II: In general, greater the number of alkyl groups attached to a positively charged C-atom, greater is the hyper conjugation interaction and stabilizationof the cation

In the light of the above statements, choose the correct answer from the options given below

1) Both Statement I and Statement II are true

2) Both Statement I and Statement II are false

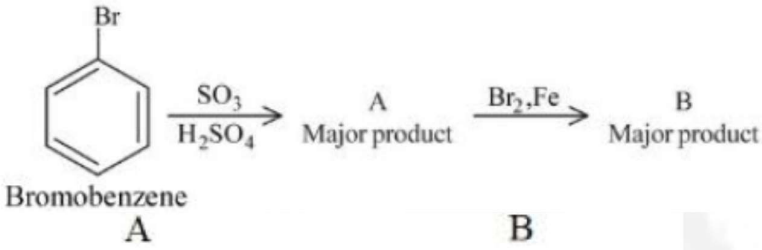
3) Statement I is false but Statement II is true

4) Statement I is true but Statement II is false

Key: 3

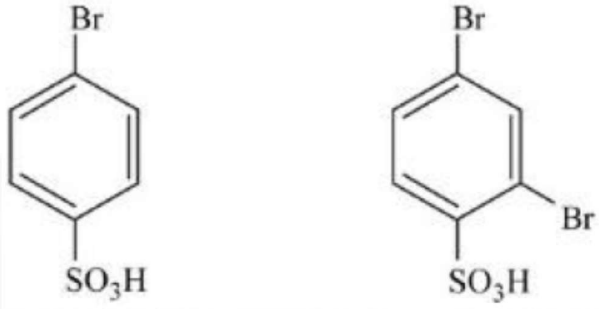
Sol: Hyper conjugation is a permanent effect and in general, greater the number of alkyl groups attached to a positively charged C-atom $\left(C^{+}\right)$, greater is the hyper conjugation and more stable is the carbocation.

61. In the following series of reactions identify the major products A & B respectively



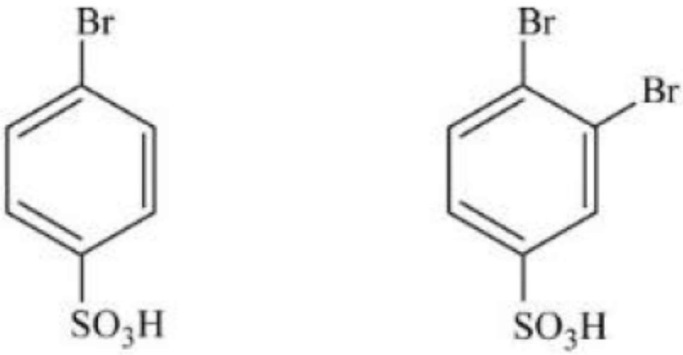
A B

1)

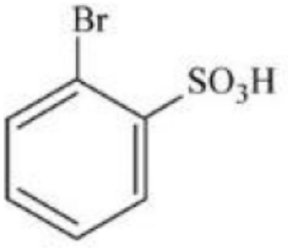


A B

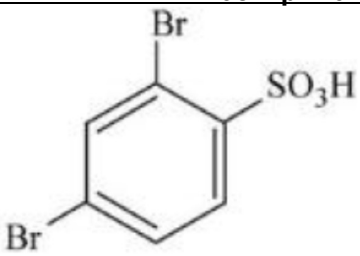
2)



3)

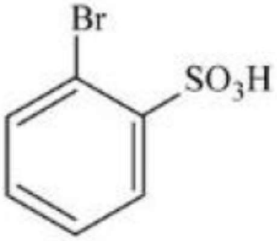


A



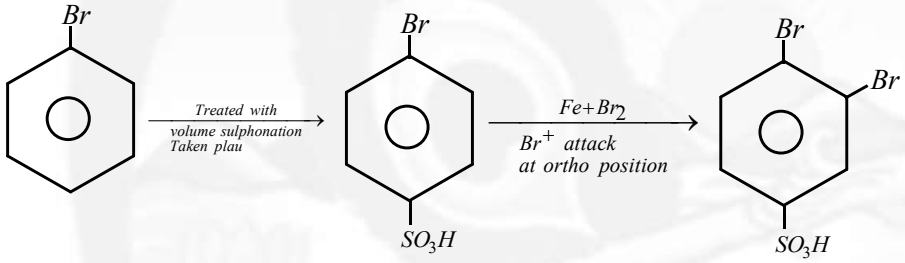
B

4)



Key: 2

Sol:



62. Given below are two statements:

Statement I: Wet cotton clothes made of cellulose based carbohydrate takes comparatively longer time to get dried than wet nylon polymer based clothes.

Statement II: Intermolecular hydrogen bonding with water molecule is more in nylon-based clothes than in the case of cotton clothes.

In the light of above statements, choose the correct answer from the options given below

1) Statement I is false but Statement II is true

2) Both Statement I and Statement II are false

3) Both Statement I and Statement II are true

4) Statement I is true but Statement II is false

Key: 3

Sol: Nylon 6, 6 Polymer clothes has Inter molecular H-bond so it takes more time for dry.

$$\left(NH-(CH_2)_6-\overset{H}{\underset{|}{N}}-\overset{O}{\overset{||}{C}}-(CH_2)_4-\overset{O}{\overset{||}{C}}\right)_4$$

Nylon 6, 6

63. In Dumas' method for estimation of nitrogen 0.4 g of an organic compound gave 60 mL of nitrogen collected at 300 K temperature and 715 mm Hg pressure. The percentage composition of nitrogen in the compound is (Given: Aqueous tension at 300 K = 15 mm Hg)

1) 17.46 % **2)** 7.85 % **3)** 20.95 % **4)** 15.71 %

Key: 4

Sol: $\frac{p_1v_1}{T_1}=\frac{p_2v_2}{T_2}$ $p_{dry\ gas}=p_{most\ gas}-Aqueous\ tension$

$\frac{200\times 60}{300}=\frac{760\times v_2}{273}$ $715-15=700$

$v_2=50.28$

$\%N=\frac{28}{22400}\times\frac{V_{N_2}initial\ at\ S.T.r}{wt\ to\ O.C\ in\ gm}\times 100$

$\frac{28}{22400}\times\frac{50.28}{0.4}\times 100=15.71\%$

64. For electrons in '2s' and '2p' orbitals, the orbital angular momentum values, respectively are:

1) 0 and $\sqrt{6}\frac{h}{2\pi}$ **2)** $\frac{h}{2\pi}$ and $\sqrt{2}\frac{h}{2\pi}$

3) 0 and $\sqrt{2}\frac{h}{2\pi}$ **4)** $\sqrt{2}\frac{h}{2\pi}$ and 0

Key: 4

Sol: $\sqrt{\ell(\ell+1)}\frac{4}{2\pi}$

$25=\frac{oh}{2\pi}=0$

$2p=\sqrt{1(1+1)}\frac{4}{2\pi}$

$=\frac{\sqrt{2}h}{2\times 5}$

65. Identify the diamagnetic octahedral complex ions from below:

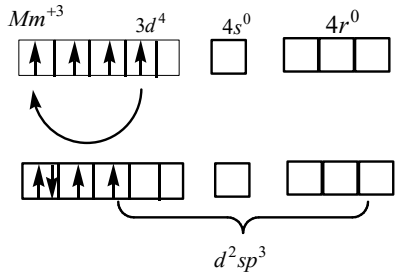
A. $\left[Mn(CN)_6\right]^{3-}$ B. $\left[Co(NH_3)_6\right]^{3+}$ C. $\left[Fe(CN)_6\right]^{4-}$ D. $\left[Co(H_2O)_3F_3\right]$

Choose the correct answer from the options given below:

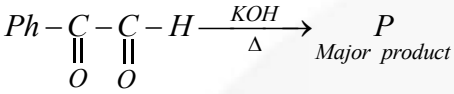
1) A and D only **2)** B and D only **3)** B and C only **4)** A and C only

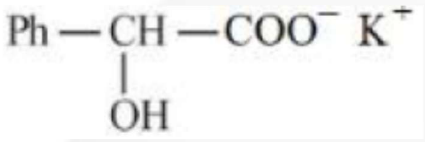
Key: 3

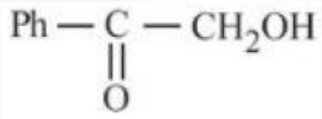
Sol:

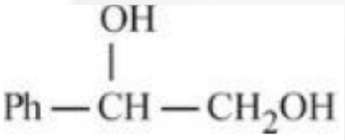


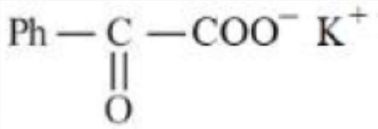
66. The major product (P) in the following reaction is:



1) 

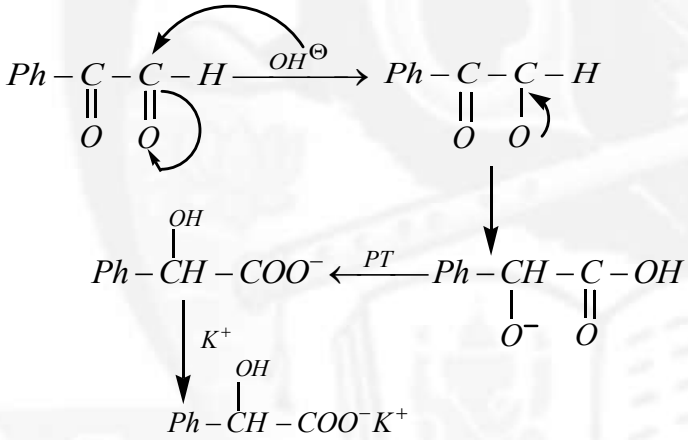
2) 

3) 

4) 

Key: 1

Sol:



67. Match the List –I with List – II

LIST-I (Family)		LIST-II (Symbol of Element)	
A.	Pnictogen (group 15)	I.	Ts
B.	Chalcogen	II.	Og
C.	Halogen	III.	Lv
D.	Noble gas	IV.	Mc

Choose the correct answer from the options given below:

1) $A-III, B-I, C-IV, D-II$ **2)** $A-IV, B-I, C-II, D-III$

3) $A-IV, B-III, C-I, D-II$ **4)** $A-II, B-III, C-IV, D-I$

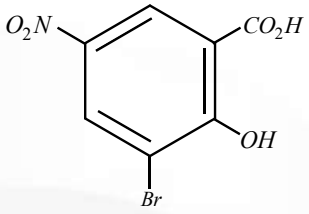
Key: 3

Sol: $A \rightarrow$ Pnictogen [group 15] $\rightarrow$ Moscoviu$[M_c]$ [IV]

$B \rightarrow$ Chalcogen $\rightarrow$ Livenmonium [Lv] – [III

$C \rightarrow$ Halogen $\rightarrow$ Tennesine [Ts] $\rightarrow I$

$D \rightarrow$ Noble gas $\rightarrow$ oganesson [Og] $\rightarrow II$



?

68. What is the correct IUPAC name of

1) 3-Bromo-2-Hydroxy-5-nitrobenzoic acid

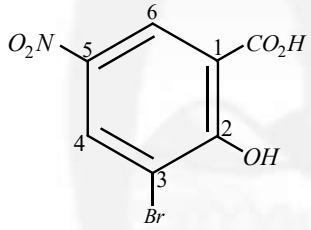
2) 5-Nitro-3-bromo-2-hydroxybenzoic acid

3) 2-Hydroxy-3-bromo-5-nitrobenzoic acid

4) 3-Bromo-4-hydroxy-1-nitrobenzoic acid

Key: 1

Sol:



69. Compounds that should not be used as primary standards in titrimetric analysis are:

A. $Na_2Cr_2O_7$ B. Oxalic acid C. NaOH D. $FeSO_4.6H_2O$

E. Sodium tetraborate

Choose the most appropriate answer from the options given below:

1) C, D and E only **2)** D and E only

3) B and D only **4)** A, C and D only

Key: 4

Sol: Sodium carbonate

Sodium tetraborate

Potassium hydrogenphtholate

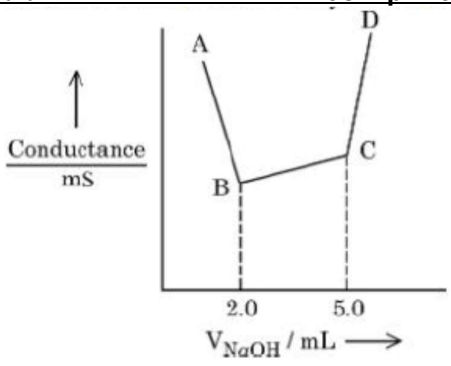
Oxalic acid

Ferrous ammonium sulphate

Primary standards NCERT

$\left.\begin{matrix} NaOH \\ KMnO_4 \end{matrix}\right\}$ secondary standards [NCERT]

70. 40mL of a mixture of CH_3COOH and HCl (aqueous solution) is titrated against $0.1MNaOH$ solution conductometrically. Which of the following statement is correct?



1) Point 'C' indicates the complete neutralization of HCl

2) The concentration of HCl in the original mixture is 0.005 M

3) The concentration of CH_3COOH in the original mixture is 0.005M

4) CH_3COOH is neutralized first followed by neutralization of HCl

Key: 2

Sol: Being as strong and HCl undergoes neutralization with $NaOH$ initially.

Point B indicates $\rightarrow$ neutralization of HCl

Point C indicate $\rightarrow$ neutralization of AeO_{15}

At point B:

$$n_{NaOH}\left[used\ for\ nitration\ of\ HCl\right] = 0.1\times 2\times 10^{-3}$$
$$= 2\times 10^{-4}$$
$$= {}^{n}\ HCl$$

Equation of $HCl = 2\times 10^{-4}\times\dfrac{1000}{40}$

$$= 0.005$$

At point C

${}^{n}\ NaOH$ [used for titration of AeOH] $= 0.1\times 3\times 10^{-3}$

$$= 3\times 10^{-4}$$
$$= {}^{n}\ AeOH$$

Conc. Of $AeOH = 3\times 10^{-4}\times\dfrac{1000}{40}$

$$= 0.0075$$

SECTION-II (NUMERICAL VALUE TYPE)

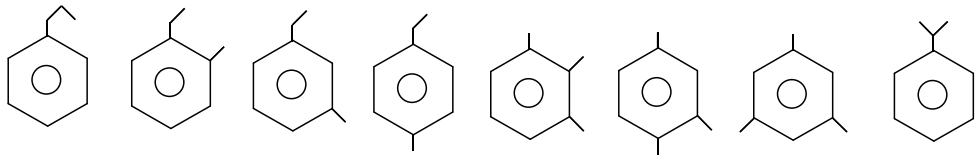
This section contains **5 Numerical Value Type Questions.** The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **Nearest Integer** value (Example i,e. If answer is above **10** and less than **10.5** round off is **10** and If answer is from **10.5** and less than **11** round off is **11).**

Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.

71. The total number of structural isomers possible for the substituted benzene derivatives with the molecular formula C_9H_{12} is_____

Key: 8

Sol: D.O.U=4



72. Among Sc, Mn, Co and Cu, identify the element with highest enthalpy of atomization. The spin only magnetic moment value of that element in its +2 oxidation state is_____ BM (in nearest integer)

Key: 6

Sol: Mn has the highest enthalpy of atomization

$Mn^{+2} \rightarrow n = 5$

$\mu = \sqrt{n(n+2)}Bm$

$= \sqrt{5(5+2)}$

$\simeq 6$

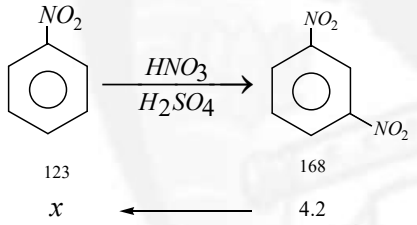
73. X g of nitrobenzene on nitration gave 4.2 g of m-dinitrobenzene.

$X =$ _____ g . (nearest integer)

[Given : molar mass (in $g\ mol^{-1}$) $C:12, H:1, O:16, N:14$]

Key: 3

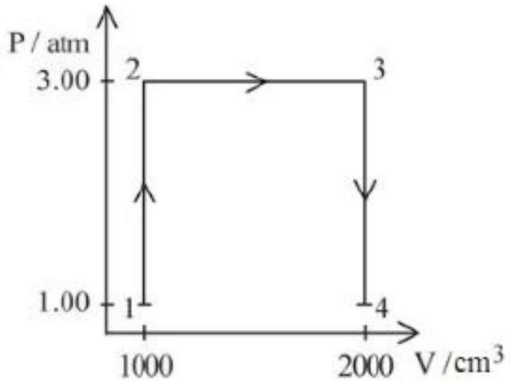
Sol:



$x = \frac{4.2}{168} \times 123 = 3.075$

$\simeq 3$

74.



A perfect gas (0.1 mol) having $\vec{C}_v = 1.50R$ (independent of temperature) undergoes the above transformation from point 1 to point 4. If each step is reversible, the total work done (w) while going from point 1 to point 4 is (-)_____ J (nearest integer)

[Given: $R = 0.082 L\ atm K^{-1} mol^{-1}$]

Key: 304

Sol: $1 \to 2\ \ \omega_{1\to 2} = 0$ (volume constant)

$2 \to 3\ \ \omega_{2\to 3} = p\Delta v = 3 \times 1000$

$= 3000\ L atm$

$3 \to 4\ \omega_{3\to 4=0} = 3000 \times 101.3 J$

$= 304\ kJ$

Work done $= \omega_{1\to 2} + \omega_{2\to 3} + \omega_{3\to 4}$

$= 0 + 304 + 0.\ kJ$

$= 304\ kJ$

75. A sample of n-octane (1.14g) was completely burn t in excess of oxygen in a bomb calorimeter, whose heat capacity is $5\ kJ\ K^{-1}$. As a result of combustion reaction, the temperature of the calorimeter is increased by 5 K. The magnitude of the heat of combustion of octane at constant volume is_____ kJ mol^{-1} (nearest integer).

Key: 2504

Sol: $n = \dfrac{1.14}{114.28} = 0.0099$

$$\Delta H_{combustion} = \frac{C \times \Delta T}{n} = \frac{5 \times 5}{0.0099} \simeq 2504\ kJ$$