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04-Apr-2025_Jee-Main_2025_Shift-02_Q.Paper, Key and Solutions

MATHEMATICS **Max Marks: 100**

(SINGLE CORRECT ANSWER TYPE)

This section contains **20 Multiple Choice Questions.** Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. Let $A=\{-3,-2,-1,0,1,2,3\}$ & R be a relation on A defined by xRy if and only if $2x-y\in\{0,1\}$. Let l be number of elements in R. Let m & n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations respectively then $l+m+n=$

1) 18 2) 15 3) 17 4) 16

Ans: 3

Sol: $2x-y\in(0,1)$ where $x,y\in(-3,-2,-1,0,1,2,3)$

$2x-y=0\Rightarrow y=2x, x=-1$ then $y=-2$, $x=0$ then $y=0$, $x=1$ then $y=2$

(3) cases

$2x-y=1\Rightarrow y+1=2x, x=-1$ then $y=-3$, $x=0$ then $y=-1$, $x=1$ then $y=1$

$x=2$ then $y=3$ (4) cases

$l=7$, $m=5$, $n=5$ $\Rightarrow l+m+n=17$

2. The sum of the infinite series $\cot^{-1}\left(\frac{7}{4}\right)+\cot^{-1}\left(\frac{19}{4}\right)+\cot^{-1}\left(\frac{39}{4}\right)+\cot^{-1}\left(\frac{67}{4}\right)....$ is

1) $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{2}\right)$ 2) $\frac{\pi}{2}+\cot^{-1}\left(\frac{1}{2}\right)$

3) $\frac{\pi}{2}-\cot^{-1}\left(\frac{1}{2}\right)$ 4) $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\right)$

Ans: 4

Sol: Required $=\tan^{-1}\left(\frac{4}{7}\right)+\tan^{-1}\left(\frac{4}{19}\right)+\tan^{-1}\left(\frac{4}{39}\right)+...\infty$

$=\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{4}{4r^2+3}\right)$

$=\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{1}{r^2+\frac{3}{4}}\right)=\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{1}{1+r^2-\frac{1}{4}}\right)$

$$=\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{\left(r+\frac{1}{2}\right)-\left(r-\frac{1}{2}\right)}{1+r^2-\frac{1}{4}}\right)$$

$$=\sum_{r=1}^{\infty}\tan^{-1}\left(r+\frac{1}{2}\right)-\tan^{-1}\left(r-\frac{1}{2}\right)$$

$$=\tan^{-1}\frac{3}{2}-\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{5}{2}-\tan^{-1}\frac{3}{2}+\tan^{-1}\frac{7}{2}-tan^{-1}\frac{5}{2}+...+\infty$$

$$=\tan^{-1}(\infty)-\tan^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{2}-\tan^{-1}\frac{1}{2}$$

3. Let the product of $W_1=(8+i)\sin\theta+(7+4i)\cos\theta$ and $W_2=(1+8i)\sin\theta+(4+7i)\cos\theta$ be $\alpha+i\beta, i=\sqrt{-1}$ let p and q be the maximum and the minimum values of $\alpha+\beta$ respectively then p + q is equal to

1) 140 2) 130 3) 150 4) 160

Ans: 2

Sol: $\omega_1=(8+i)\sin\theta(7+4i)\cos\theta$

$=\sin\theta+7\cos+i(\sin\theta+4\cos\theta)$

$\omega_2=(1+8i)\sin\theta+(4+7i)\cos\theta$

$=(\sin\theta+4\cos\theta)+i(8\sin\theta+7\cos\theta)$

$\omega_1\omega_2=0+i(8^2+1)\sin^2\theta+(7^2+4^2)\cos^2\theta+120\sin\theta\cos\theta$

$=0+i(65+60\sin2\theta)$

$\alpha=0,\beta=65+60\sin2\theta$

$\alpha+\beta=65+60\sin2\theta$

$p=65+60=125, q=65-60=05$

$\boxed{p+q=130}$

4. Let A be the point of intersection of the lines $L_1:\frac{x-7}{1}=\frac{y-5}{0}=\frac{z-3}{-1}$ and $L_2:\frac{x-1}{3}=\frac{y+3}{4}=\frac{z+7}{5}$ Let B and C be the points on the lines L_1 and L_2 respectively such that $AB=AC=\sqrt{15}$. Then the square of the area of the ΔABC is

1) 54 2) 57 3) 63 4) 60

Ans: 1

Sol: $L_1=\frac{x-7}{1}=\frac{y-5}{0}=\frac{z-3}{-1}=t$

$L_2=\frac{x-1}{3}=\frac{y+3}{4}=\frac{z+7}{5}$

General point is line 4 is $(t+7,5,3-t)$

$\frac{t+6}{3}=2=\frac{10-t}{5}$

$\boxed{t=0}$

Point of intersection is $A(7,5,3)$

$$B\left(\sqrt{\frac{15}{2}}+7,5,3-\sqrt{\frac{15}{2}}\right), C\left(7+3\sqrt{\frac{3}{10}},5+4\sqrt{\frac{3}{10}},3+5\sqrt{\frac{3}{10}}\right)$$

$$\text{Area} = \frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$$

$$= \frac{1}{2}\begin{vmatrix} i & j & k \\ \sqrt{\frac{15}{2}} & 0 & -\sqrt{\frac{15}{2}} \\ 3\sqrt{\frac{3}{10}} & 4\sqrt{\frac{3}{10}} & 5\sqrt{\frac{3}{10}} \end{vmatrix} = \frac{1}{2}\left|4\sqrt{\frac{45}{20}}\bar{i},-8\sqrt{\frac{45}{20}}\bar{j}+4\sqrt{\frac{45}{20}}\bar{k}\right|$$

$$= \frac{1}{2}\times 4\sqrt{\frac{45}{20}}\times\sqrt{6}$$

$$= 2\sqrt{\frac{90\times 3}{20}} = 6\sqrt{\frac{3}{2}}$$

$$A^2 = 6^2\times\frac{3}{2} = 36\times\frac{3}{2} = 54$$

5. If $1^2\left({}^{15}C_1\right)+2^2\left({}^{15}C_2\right)+3^2\left({}^{15}C_3\right)+\ldots+15^2\left({}^{15}C_{15}\right) = 2^m.3^n.5^k$ where m, n, $k\in N$ then $m+n+k=$

1) 20 2) 21 3) 18 4) 19

Ans: 4

Sol: Required $= 1^2.{}^{15}C_1+2^2.{}^{15}C_2+3^2.{}^{15}C_3+\ldots+15^2\,{}^{15}C_{15}$

$$= \sum_{r=1}^{15} r^2.{}^{15}C_r$$

$$S = \sum_{r=1}^{15} r^2.\frac{15}{r}.{}^{14}C_{r-1} = 15\sum_{r=1}^{15} r\,{}^{14}C_{r-1} = 15\sum_{r=1}^{15}(r-1+1)\,{}^{14}C_{r-1}$$

$$= 15\left[\sum_{r=1}^{15} r-1\frac{14}{r-1}.{}^{13}C_{r-2}+\sum_{r=1}^{15}{}^{14}C_{r-1}\right]$$

$$= 15\times 14.2^{13}+15.2^{14}$$

$$= 15.2^{13}(16) = 2^{17}.3^1.5^1$$

Then $m=17, n=1, k=1 \Rightarrow m+n+k=19$

6. Let the mean and the standard deviation of the observation 2, 3, 3, 4, 5, 7, a, b be 4 and $\sqrt{2}$ respectively then the mean deviation about the mode of these observations is

1) $\frac{3}{4}$ 2) 2 3) 1 4) $\frac{1}{2}$

Ans: 3

Sol: $2,3,3,4,5,7,a,b$ are the observations

$$\text{Mean} = 4 = \frac{2+6+4+12+a+b}{8} \Rightarrow \boxed{a+b=8}$$

$$\text{Variance} = 2 = \frac{4+18+16+25+49+a^2+b^2}{8} - 4^2 = 2$$

$a^2+b^2 = 32$

$a = b = 4$

Mean deviation about mode

$$= \frac{1}{8}\left(|2-4|+|3-4|+|3-4|+|5-4|+|7-4|\right)$$

$$= \frac{1}{8}(2+1+1+1+3) = 1$$

7. Let the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ satisfy $A^n = A^{n-2} + A^2 - I$ for $n \geq 3$ then the sum of all the elements of A^{50} is

1) 44 2) 39 3) 52 4) 53

Ans: 4

Sol: Given that,

$A^n = A^{n-2} + A^2 - I, n \geq 3$

$A^{50} = A^{48} + A^2 - I$

$= A^{46} + 2\left(A^2 - I\right).....we\,get$

$= 25A^2 - 24I$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \; A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$A^{50} = 25\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} - 24\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$$

Sum of all elements is $25+25+1+1+1 = 53$

8. If $f(x) + 2f\left(\frac{1}{x}\right) = x^2 + 5$ and $2g(x) - 3g\left(\frac{1}{2}\right) = x, x > 0$. If $\alpha = \int_1^2 f(x)dx$ and $\beta = \int_1^2 g(x)dx$. Then value of $9\alpha + \beta$ is

1) 0 2) 10 3) 1 4) 11

Ans: 4

Sol: $f(x) + 2f\left(\frac{1}{x}\right) = x^2 + 5 \ldots. (1)$, x replace by $\frac{1}{x} \Rightarrow f\left(\frac{1}{x}\right) + 2f(x) = \frac{1}{x^2} + 5 \ldots. (2)$

$$2 \times (2) - (1) \Rightarrow 3f(x) = 5 - x^2 + \frac{2}{x^2}$$

Now put $x=\frac{1}{2}$ in $2g(x)-3g\left(\frac{1}{2}\right)=x \Rightarrow g\left(\frac{1}{2}\right)=-\frac{1}{2} \Rightarrow g(x)=\frac{x}{2}-\frac{3}{4}$

$$\therefore 9\alpha+\beta=\int_1^2\left(15-3x^2+\frac{6}{x^2}+x-\frac{3}{2}\right)dx$$

$$=\left[15x-x^3-\frac{6}{x}+\frac{x^2}{2}-\frac{3x}{2}\right]_1^2=18-7=11$$

9. Let for two distinct values of P the lines $y=x+p$ touch the ellipse $E=\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$ at the points A and B. Let the lines y = x intersect E at the points C,D the area of the quadrilateral ABCD is equal to

1) 36 2) 24 3) 48 4) 20

Ans: 2

Sol: $y=x+p$ is tangent to ellipse $\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$

General Tangent to the ellipse

$y=mx\pm\sqrt{a^2m^2+b^2}$

By comparing $\boxed{m=1}$ $y=x\pm\sqrt{a^2+b^2} \Rightarrow y=x\pm5$

$A\left(\frac{-16}{5},\frac{9}{5}\right), B\left(\frac{16}{5},\frac{-9}{5}\right)$

$y=x$ intersects ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, $\frac{x^2}{16}+\frac{x^2}{9}=1 \Rightarrow 25x^2=144$, $x^2=\left(\frac{12}{5}\right)^2$, $x=\pm\frac{12}{5}$

$C\left(\frac{12}{5},\frac{12}{5}\right), D\left(-\frac{12}{5},-\frac{12}{5}\right)$

Area of quadrilateral is $=\frac{1}{2}\begin{vmatrix}x_1-x_3 & y_1-y_3\\ x_2-x_4 & y_2-y_4\end{vmatrix}=\frac{1}{2}\left|\left(\frac{(108+192)4}{25}\right)\right|=\frac{600}{25}=24$

10. If a curve $y=y(x)$ passes through the point $[1,\pi/2]$ and satisfies the differential equation $\left(7x^4\cot y-e^x\operatorname{cosec} y\right)\frac{dx}{dy}=x^5 x\geq 1$ then at x = 2 the value of cosy is

1) $\frac{2e^2-e}{128}$ 2) $\frac{2e^2+e}{64}$ 3) $\frac{2e^2+e}{128}$ 4) $\frac{2e^2-e}{64}$

Ans: 1

Sol: Differential equation

$$\left(7x^4\cot y-e^x\cos y\right)\frac{dx}{dy}=x^5$$

$$\frac{dy}{dx}=\frac{7}{x}\cot y-\frac{e^x}{x^5}\operatorname{cosec} y$$

$$\sin y\frac{dy}{dx}=\frac{7}{x}\cos y-\frac{e^x}{x^5}$$

Let $t=\cos y \Rightarrow dt=-\sin y dy$

$$-\frac{dt}{dx}=\frac{7}{x}t-\frac{e^x}{x^5}$$

$$x^7\frac{dt}{dx}+7x^6=x^2e^x$$

$$x^7t=\left(x^2-2x+2\right)e^x+C$$

$x^7\cos y=\left(x^2-1\right)^2e^x+e^x+C$, Passing through $\left[1,\frac{\pi}{2}\right]$, $0=e+C\Rightarrow\boxed{C=-e}$

$$x^7\cos y=\left(x-1\right)^2e^x+e^x-e$$

$$2^7\cos y=2e^e-e$$

$$\boxed{\cos y=\frac{2e^2-e}{128}}$$

11. If the sum of first 20 terms in the series $\frac{4.1}{4+3.1^2+1^4}+\frac{4.2}{4+3.2^2+2^4}+\frac{4.3}{4+3.3^2+3^4}+\frac{4.4}{4+3.4^2+4^4}.....$ is $\frac{m}{n}$, where m and n are coprime then $m+n$ is equal to

1) 422 2) 423 3) 421 4) 420

Ans: 3

Sol: Required $=\frac{4.1}{4+3.1^2+1^4}+\frac{4.2}{4+3.2^2+2^4}+\frac{4.3}{4+3.3^2+3^{47}}.....20$ terms

$$=\sum_{r=1}^{2}\frac{4r}{4+3r^2+r^4}$$

$$=\sum_{r=1}^{2}\frac{4r}{\left(r^2+2\right)^2-r^2}=2\sum_{r=1}^{20}\frac{2r}{\left(r^2+2-r\right)\left(r^2+2+r\right)}$$

$$=2\sum_{r=1}^{20}\frac{1}{r^2-r+2}-\frac{1}{r^2+r+2}$$

$$=2\left(\frac{1}{2}-\frac{1}{4}+\frac{1}{4}-\frac{1}{8}+...-\frac{1}{422}\right)$$

$$\frac{m}{n}=1-\frac{1}{211}=\frac{210}{211}$$

$$m+n=421$$

12. A line passing through the point $A(-2,0)$ touches the parabola $P:y^2=x-2$ at the point B in the first quadrant. The area of the region bounded by the line AB, parabola 'P' and the x-axis is

1) $\frac{7}{3}$ 2) $\frac{8}{3}$ 3) 3 4) 2

Ans: 2

Sol: $y=m\left(x-2\right)+\frac{1}{4m}$ is the tangent of parabola

$$y^2=x-2$$

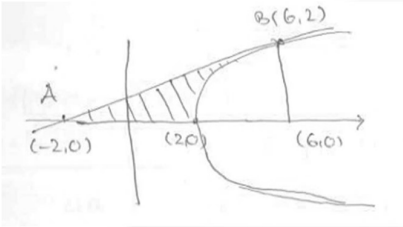
Passing through $A(-2,0)$

$$-4m+\frac{1}{4m}=0\Rightarrow 4m=\pm 1\Rightarrow \boxed{m=\pm\frac{1}{4}}$$

$$my^{2=0}-y+\frac{1}{4}m$$

If $m=\frac{1}{4}$ as $\frac{y^2}{4}-y+1=0\Rightarrow \boxed{y=2}$

$B(6,2)$



Area = area of the triangle – area of the curve

$$\frac{1}{2}\times 8\times 2-\int_2^6\sqrt{x-2}dx$$

$$=8-\frac{2}{3}\left((x-2)^{3/2}\right)_2^6$$

$$=8-\frac{16}{3}=\frac{8}{3}$$

13. The axis of a parabola is the line $y=x$ and its vertex and focus are in the first qudrent at distances $\sqrt{2}$ and $2\sqrt{2}$ units from the origin respectively. If the point (1, k) lies on the parabola then a possible value of k is

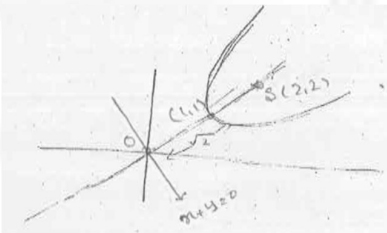
1) 3 2) 9 3) 4 4) 8

Ans: 1

Sol: Given, axis is $y=x$, $VO=\sqrt{2}$, Let vertex is (a,a), $\sqrt{a^2+a^2}=\sqrt{2}$

$\Rightarrow \boxed{a=1}$

$\therefore$ Vertex is $(1,1)$



$\Rightarrow OS=2\sqrt{2}\ S(\alpha,\alpha)\Rightarrow \alpha=2$

Equation of parabola is $(x-2)^2+(y-2)^2=\left(\frac{x+y}{\sqrt{2}}\right)^2$ passes through (1, k)

$$1+(k-2)^2=\left(\frac{1+k}{\sqrt{2}}\right)^2$$

$$\Rightarrow k^2-10x+9=0\Rightarrow k=1,9$$

14. The centre of a circle C is at the centre of the ellipse $E=\frac{x^2}{a^2}+\frac{y^2}{b^2}, a>b$. Let c pass through the foci F_1 and F_2 of E such that the circle C and the ellipse E intersect at four points let P be one of these four points. If the area of the triangle points PF_1F_2 is 30 and length of the major axis of E is 17 then the distance between the foci of E is

1) $\frac{13}{2}$ 2) 12 3) 26 4) 13

Ans: 4

Sol: $E:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ C:x^2+y^2=a^2e^2,\ b^2x^2+a^2y^2=a^2-b^2,\ x^2+y^2=a^2-b^2$

$\left(a^2-b^2\right)y^2=b^2\left(b^2\right),\ y=\frac{\pm b^2}{\sqrt{a^2-b^2}}=\frac{\pm b^2}{ae},\ x^2+\frac{b^4}{a^2-b^2}=a^2-b^2$

$x^2=\frac{a^4-2a^2b^2}{a^2-b^2}\Rightarrow x=\pm a\sqrt{\frac{a^2-2b^2}{a^2-b^2}},\ \boxed{2a=17}$

Area $=30=\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ ae & 0 \\ -ae & 0 \\ x_1 & y_1 \end{vmatrix},\ 30=ae\frac{b^2}{ae}=b^2,\ b^2=30\Rightarrow b=\sqrt{30},\ a^2\left(1-e^2\right)=30$

$\frac{289}{4}\left(1-e^2\right)=30\Rightarrow 1-e^2=\frac{120}{289},\ e^2=\frac{169}{287},\ e=\frac{13}{17}$

$F_1F_2=2ae=13$

15. Consider two sets A and B, each containing three numbers in A.P. Let the sum and the product of the elements of A be 36 and p respectively and the sum and the product of the elements of B be 36 and q respectively. Let d and D be the common differences of AP's in A and B respectively such that $D=d+3$, d > 0. If $\frac{p+q}{p-q}=\frac{19}{5}$, then p – q is equal to

1) 540 2) 450 3) 630 4) 600

Ans: 1

Sol: $a-d,a,a+d$ $a-D,a,a+D$

$\Rightarrow 3a=36\Rightarrow a=12$ $\Rightarrow 3a=36\Rightarrow a=12$

$a\left(a^2-d^2\right)=p$ $a\left(a^2-D^2\right)=q$

Given $D=d+3$

$\Rightarrow \frac{p+q}{p-q}=\frac{19}{5}$ solving we get d = 6

$\therefore p=12(144-36)=12(108)=1296$

$\therefore q=12(144-81)=12(63)=756$

$\therefore p-q=540$

16. Let the sum of the focal distance of the point $P(4,3)$ on the hyperbola $H:\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be $8\sqrt{\frac{5}{3}}$. If for H, the length of the latus rectum is l and the product of the focal distances of the point P is m, then $9l^2+6m$ is equal to

1) 186 2) 185 3) 184 4) 187

Ans: 2

Sol: Given, sum of focal distances of $P(4,3)=8\sqrt{\frac{5}{3}}$

Focal distance are ex_1+a, ex_1-a

$\therefore SP+S^1P=2ex_1=8\sqrt{\frac{5}{3}}$

$\Rightarrow 8e=8\sqrt{\frac{5}{3}}\Rightarrow \boxed{e=\sqrt{\frac{5}{3}}}$

$1+\frac{b^2}{a^2}=\frac{5}{3}\Rightarrow \boxed{\frac{b^2}{a^2}=\frac{2}{3}}$

$(4,3)$ lies on its

$\Rightarrow \frac{16}{a^2}-\frac{9}{b^2}=1$

$\Rightarrow \frac{16}{a^2}-\frac{27}{2a^2}=1$

$\Rightarrow \boxed{a^2=\frac{5}{2}}\Rightarrow \boxed{b^2=\frac{5}{3}}$

$l^2=(L.L.R)^2=\left(\frac{2b^2}{a}\right)^2=\frac{2^2(5/3)^2}{5/2}=\frac{40}{9}$

$m=e^2x_1^2-a^2$

$=\left(\frac{5}{3}\right)(4)^2-\left(\frac{5}{2}\right)$

$=\frac{145}{6}$

$\therefore 9l^2+6m=40+145=185$

17. Let the domain of the function $f(x)=\log_4\log_3\log_7\left(8-\log_2\left(x^2+4x+5\right)\right)$ and $g(x)=\sin^{-1}\left[\frac{7x+10}{x-2}\right]$ be (α,β) and $[\gamma,\delta]$ respectively then $\alpha^2+\beta^2+\gamma^2+\delta^2$ is equal to

1) 15 2) 14 3) 16 4) 13

Ans: 1

Sol: $f(x)=\log_4\log_3\log_7\left[8-\log_2\left(x^2+4x+5\right)\right]$

$\log_3 \log_7 \left[8-\log_2\left(x^2+4x+5\right)\right]>0$

$\log_7 \left[8-\log_2\left(x^2+4x+5\right)\right]>1$

$8-\log_2\left(x^2+4x+5\right)>7$

$1>\log_2\left(x^2+4x+5\right)$

$x^2+4x-5<2 \Rightarrow x^2-4x+3<0 \Rightarrow (x+1)(x+3)<0 \Rightarrow x\in(-3,-1)$

$\alpha=-3, \beta=-1$

$g(x)=\sin^{-1}\left[\frac{7x+10}{x-2}\right],\ -1\le\frac{7x+10}{x-2}\le 1,\ \frac{8x+8}{x-2}\ge 0,\ \frac{6x+12}{x-2}\le 0$

$x\in(-\infty,1)\cup(2,\infty)$ & $x\in(-2,2),\ x\in(-2,-1), \gamma=-2, \delta=-1$

Required $=\alpha^2+\beta^2+\gamma^2+\delta^2=3^2+1^2+2^2+1^2=15$

18. Let a > 0, If the function $f(x)=6x^3-45ax^2+108a^2x+1$ attains its local maximum and minimum value at the points x_1 and x_2 respectively such that $x_1x_2=54$ then $a+x_1+x_2$ is

1) 15 2) 18 3) 13 4) 24

Ans: 2

Sol: $f(x)=6x^3-45ax^2+108a^2x+1$

$f^1(x)=18x^3-90ax+108a^2 <^{x_1}_{x_2}$

$x_1x_2=54$

$\frac{108}{18}a^2=54 \Rightarrow a^2=9$ as $\boxed{a=\pm 3}$; $a>0$

$\boxed{a=3}$

$18x^2-270x+972=0$

$x^2-15x+54=0 \Rightarrow x_1=6, x_2=9$

$a+x_1+x_2=3+6+9=18$

19. Let the value of p for which the shortest distance between the line $\frac{x+1}{3}=\frac{y}{4}=\frac{z}{5}$ and $\vec{r}=\left(p\hat{i}+2\hat{j}+\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$ is $\frac{1}{\sqrt{6}}$ be a, b (a < b) then the length of the latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

1) $\frac{3}{2}$ 2) $\frac{2}{3}$ 3) 9 4) 18

Ans: 2

Sol: $L_1: \frac{x+1}{3}=\frac{y}{4}=\frac{7}{5}$

$L_1: \vec{r}=(-1,0,0)+t(3,4,5),\ L_2: \vec{r}=(P,2,1)+\lambda(2,3,4)$

Shortest distance $=\left|\frac{(P+1,2,1).(-1,2,-1)}{\sqrt{6}}\right|$

$$=\left|\frac{P-2}{\sqrt{6}}\right|=\frac{1}{\sqrt{6}}$$

$P=1\,(\text{or})\,3$, $a=1, b=3$, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Length of Latus rectum is $\frac{2a^2}{b}=\frac{2\times1}{3}=\frac{2}{3}$

20. Let f be a differentiable function on R such that $f(2)=1$, $f'(2)=4$. Let $\lim\limits_{x\to0}\left(f(2+x)\right)^{3/x}=e^{\alpha}$. Then the number of times the curve $y=4x^3-4x^2-4(\alpha-7)x-\alpha$ meet x-axis is

1) 1 2) 0 3) 2 4) 3

Ans: 3

Sol: $\underset{x\to0}{Lt}\left[f(2+x)\right]^{3/x}=e^{x}$ given $f(2)=1, f^{1}(2)=4$

1^{∞} from $e^{\underset{x\to0}{Lt}\left[f(2+x)-1\right]\frac{3}{x}}$ $\left[\because \underset{x\to0}{Lt} f(x)^{(g(x)}=e^{\underset{x\to0}{Lt}(f(x)-1)g(x)}\right]$

$=e^{3\underset{x\to0}{Lt}\frac{f(2+x)-1}{x}}=e^{3\underset{x\to0}{Lt}f^{11}(x+2)}$ $=e^{3\underset{x\to0}{Lt}f^{1}(2)}=e^{3\times4}\times e^{12}$ $\boxed{\alpha=12}$

$\Rightarrow y=4x^3-4x^2-20x-12=0$

By trial & error, $x=-1$ is a root of $y=0$

$4x^2-8x-12=0\Rightarrow x^2-2x-3=0$

$x=-1, x=3$ are the roots

$\therefore$ the given equation having roots -1, -1, 3

$\therefore$ y meets x-axis '2' times only.

SECTION-II (NUMERICAL VALUE TYPE)

This section contains **5 Numerical Value Type Questions.** The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **Nearest Integer** value (Example i,e. If answer is above **10** and less than **10.5** round off is **10** and If answer is from **10.5** and less than **11** round off is **11).**

Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases..

21. A card from a pack of 52 cards is lost. From the remaining 51 cards, n cards are drawn and are found to be spades. If the probability of the lost card to be a spade is $\frac{11}{50}$, then n is equal to ____

Ans: 2

Sol: A $\rightarrow$ spade card , $A^C\rightarrow$ not a spade card

$\Rightarrow P(A)=\frac{13}{52}=\frac{1}{4}, P(A^C)=\frac{3}{4}$

$B\rightarrow$ n cards drawn are spades

$P\left(\frac{B}{A}\right)=\frac{{}^{12}C_n}{{}^{51}C_n}, P\left(\frac{B}{A^C}\right)=\frac{{}^{13}C_n}{{}^{51}C_n}$

By Bayers theorem $P(req)=\dfrac{\frac{{}^{12}C_n}{{}^{51}C_n}\times\frac{1}{4}}{\frac{{}^{12}C_n}{{}^{51}C_n}\times\frac{1}{4}+\frac{{}^{13}C_n}{{}^{51}C_n}\times\frac{3}{4}}=\frac{11}{50}(given)$

$\Rightarrow\dfrac{{}^{12}C_n}{{}^{12}C_n+3.{}^{13}C_n}=\dfrac{11}{50}\Rightarrow n=2$

22. Let m and n (m < n) be two 2 digit number. Then the total numbers of pairs (m, n) such that GCD (m, n) = 6 is

Ans: 64

Sol: Let an = 6a, n = 6b

m < n $\Rightarrow 6a<6b\Rightarrow a<b$

GCD $(6a,6b)=6\Rightarrow GCD(a,b)=1$

$a=2\Rightarrow b=3,5,7,9,11,13,15\rightarrow 7$

$a=3\Rightarrow b=4,5,7,8,10,11,13,14,16\rightarrow 9$

$a=4\Rightarrow b=5,7,9,11,13,15\rightarrow 6$

$a=5\Rightarrow b=6,7,8,9,11,12,13,14,16\rightarrow 9$

$a=6\Rightarrow b=7,11,13\rightarrow 3$

$a=7\Rightarrow b=8,9,10,11,12,13,15,16\rightarrow 8$

$a=8\Rightarrow b=9,11,13,15\rightarrow 4$

$a=9\Rightarrow b=10,11,13,14,16\rightarrow 5$

$a=10\Rightarrow b=11,13\rightarrow 2$

$a=11\Rightarrow b=12,13,14,15,16\rightarrow 5$

$a=12\Rightarrow b=13\rightarrow 1$

$a=13\Rightarrow b=14,15,16\rightarrow 3$

$a=14\Rightarrow b=15\rightarrow 1$

$a=15\Rightarrow b=16\rightarrow 1$

Required number of ordered pairs = 64

23. If $\int\dfrac{\left(\sqrt{1+x^2}+x\right)^{10}}{\left(\sqrt{1+x^2}-x\right)^9}dx=\dfrac{1}{m}\left[\left(\sqrt{1+x^2}+x\right)^n\left(n\sqrt{1+x^2}-x\right)\right]+c$ where c is constant of integration and $m,n\in N$ then $m+n$ is equal to

Ans: 379

Sol: Let $\sqrt{1+x^2}+x=t$

$\sqrt{1+x^2}=t-x$

S.B.S

$1+x^2=t^2+x^2-2t-x$

$2tx=t^2-1$

$x=\dfrac{t^2-1}{2t}=\dfrac{1}{2}\left[t-\dfrac{1}{t}\right]$

$dx=\frac{1}{2}\left[1+\frac{1}{t^2}\right]dt \qquad =\frac{t^2+1}{2t^2}dt$

$\int\frac{\left(\sqrt{1+x^2}+x\right)^{10}}{\left(\sqrt{1+x^2}-x\right)^9}dx \qquad =\int\left(\sqrt{1+x^2}+x\right)^{19}dx=\int t^{19}\frac{\left(t^2+1\right)}{2t^2}dt$

$=\frac{1}{2}\int t^{17}\left(t^2+1\right)dt \qquad =\frac{1}{2}\int\left(t^{19}+t^{17}\right)dt$

$=\frac{1}{2}\left[\frac{t^{20}}{20}+\frac{t^{18}}{18}\right] \qquad =\frac{t^{19}}{4}\left[\frac{t}{10}+\frac{1}{9t}\right]$

$=\frac{t^{19}}{4}\left[\frac{9t}{90}+\frac{10}{90t}\right] \qquad =\frac{t^{19}}{360}\left[9t+\frac{10}{t}\right]$

$=\frac{t^{19}}{360}\left[9\left(x+\sqrt{1+x^2}\right)+\frac{10}{\sqrt{x^2+1}+x}\right]$

$=\frac{t^{19}}{360}\left[9x+9\sqrt{1+x^2}+10\sqrt{x^2+1}-10x\right]$

$=\frac{t^{19}}{360}\left[19\sqrt{x^2+1}-x\right]=\frac{\left(\sqrt{1+x^2}+x\right)^{19}}{360}\left(19\sqrt{x^2+1}-x\right)$

$n=19, m=360\Rightarrow m+n=379$

24. If α is root of the equation $x^2+x+1=0$ and $\sum_{k=1}^{n}\left(\alpha^k+\frac{1}{\alpha^k}\right)^2=20$ then n is equal to

Ans: 11

Sol: $x^2+x+1=0$ is roots ω,ω^2

$\alpha\xrightarrow{root}x^2+x+1=0$

$\alpha=\omega$

$\sum_{k=1}^{n}\left(\alpha^k+\frac{1}{\alpha^k}\right)^2=20$

$\sum_{k=1}^{n}\left(\omega^k+\omega^{2k}\right)^2$

If $n=3\Rightarrow$ then Answer is 1+1+4=6

If $n=6\Rightarrow$ then Answer is (1+1+4)2=12

If $n=9\Rightarrow$ then Answer is (1+1+4)3=18

So $\boxed{n=11}$

25. Let the three sides of a triangle ABC to given by the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$. Let G be the centroid of the triangle ABC. Then $6\left(\left|\overrightarrow{AG}\right|^2+\left|\overrightarrow{BG}\right|^2+\left|\overrightarrow{CG}\right|^2\right)$ is equal to

Ans: 164

Sol: $AB=\sqrt{6}, BC=\sqrt{35}, AC=\sqrt{41}$

We know that $\left(\overline{AG}\right)^2+\left(\overline{BG}\right)^2+\left(\overline{CG}\right)^2=\frac{1}{3}\left(AB^2+BC^2+CA^2\right)=\frac{1}{3}(6+35+41)$

$6\left(\left|\overline{AG}\right|^2+\left|\overline{BG}\right|^2+\left|\overline{CG}\right|^2\right)=2\times 82=164$

PHYSICS **Marks:100**

SECTION-I (SINGLE CORRECT ANSWER TYPE)

This section contains **20 Multiple Choice Questions.** Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

26. Two polarisers P_1 and P_2 are placed in such a way that the intensity of the transmitted light will be zero A third polarize P_3 is inserted between P_1 and P_2 at a particular angle between P_2 and P_3. The transmitted intensity of the light passes the through the polarizes is maximum. The angle between P_2 and P_3 is

1) $\frac{\pi}{3}$ 2) $\frac{\pi}{8}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{6}$

Ans: 3

Sol: Given, that, angle b/w P_1 & P_2 is 90°

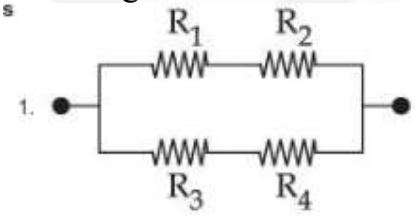
Let angle b/w P_1 and P_3 be θ then

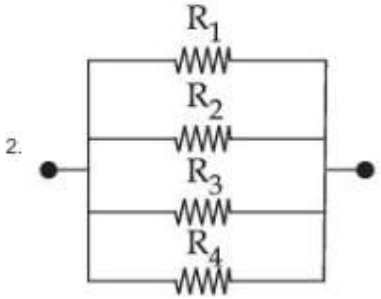
$$I = \frac{I_0}{2} \times Cos^2\theta \times \sin^2\theta = \frac{I_0}{8}\sin^2 2\theta$$

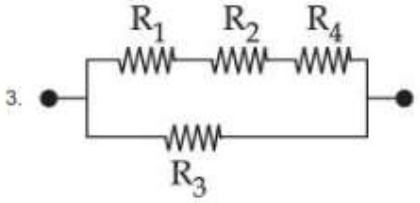
For this to be maximum $\theta = 45^\circ$

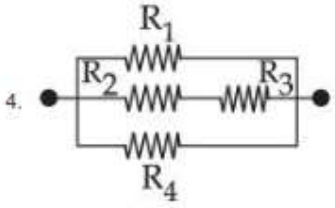
27. From combination of resistance values $R_1 = R_2 = R_3 = 5\Omega$, $R_4 = 10\Omega$ which of the following combination is the best circuit to get an equivalent resistance of 6Ω

s

1. 

2. 

3. 

4. 

Ans: 1

Sol: in option 1 $R_{eq}=\frac{10\times15}{10+15}=6\Omega$

28. Consider a rectangular sheet of solid material of length $l=9cm$ and width $d=4cm$. The coefficient of linear expansion is $\alpha=3.1\times10^{-5}K^{-1}$ at room temperature and one atmosphere pressure. The mass of sheet m = 0.1 kg and the specific heat capacity $C_v=900Jkg^{-1}K^{-1}$. If the amount of heat supplied to the material is $8.1\times10^{2}J$. The change in area of the rectangular sheet is

1) $3.0\times10^{-7}m^2$ 2) $2.0\times10^{-6}m^2$ 3) $4.0\times10^{-7}m^2$ 4) $6.0\times10^{-7}m^2$

Ans: 2

Sol: $Q=mCv\Delta T$

$810=0.1\times900\times\Delta T$

$\therefore \Delta T=9K$

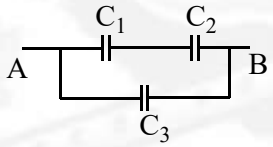
And $\beta=\frac{\Delta A}{A(\Delta t)}$

$\Delta A=2\alpha(A)\Delta t$

$=2\times3.1\times10^{-5}\times36\times10^{-4}\times9$

$=2.0081\times10^{-6}m^2$

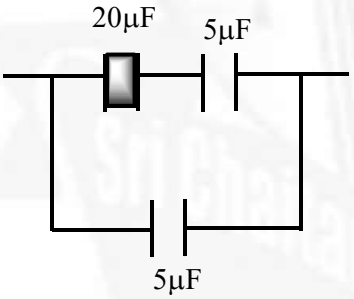
29. Three parallel plate capacitor $C_1=C_2=C_3=5\mu F$ are connected as shown in the figure, the effective capacitance between A and B when the space between the capacitor C_1 is filled with a dielectric medium of K = 4 is



1) $7.5\mu F$ 2) $9\mu F$ 3) $22.5\mu F$ 4) $30\mu F$

Ans: 2

Sol:



$C_1'=5\mu F\times4=20\mu F$

$C_1=\frac{\cancel{20}^{4}\times\cancel{5}}{\cancel{20}+5}+5=9\mu F$

30. Match List – I with List – II

List – I	List - II
A) Isobaric	I) $\Delta Q=\Delta W$
B) Isochoric	II) $\Delta Q=\Delta U$
C) Adiabatic	III) $\Delta Q=zero$

D) Isothermal IV) $\Delta Q = \Delta U + P\Delta V$

ΔQ = Heat supplied

ΔW = Work done by the system

ΔU = Change in internal energy

P = Pressure of the system

ΔV = Change in volume of system

Choose the correct answer from the options given below

1) A – IV, B – I, C – III, D – II 2) A – IV, B – II, C – III, D – I

3) A – IV, B – III, C – II, D – I 4) A – II, B – IV, C – III, D – I

Ans: 2

Sol: A) Isobaric , $\Delta U = \Delta U + P(\Delta V)$

B) Isochoric , $\Delta Q = \Delta U$

C) Adiabatic, $\Delta Q = \text{zero}$

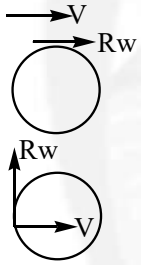
D) Isothermal, $\Delta Q = \Delta W$

31. A wheel is rolling on a plane surface the speed of particle on highest point of rim is 8m/s. The speed of particle on rim of wheel at same level as centre of wheel will be

1) $8\sqrt{2} m/s$ 2) $8 m/s$ 3) $4\sqrt{2} m/s$ 4) $4 m/s$

Ans: 3

Sol:



Given that $V + Rw = 8 m/s$

As it is in pure rolling V=Rw

$V = 4 m/s$

$V = \sqrt{V^2 + (Rw)^2} = \sqrt{(4)^2 + (4)^2}$

$V = 4\sqrt{2}\ m/s$

32. A cylindrical rod of length 1m and radius 4cm is mounded vertically. If subjected to a shear force of $10^5 N$ at the top considering infinitesimally small displacement in the upper edge, the angular displacement θ of the rod axis from its original position would be $(shear\ moduli, \eta = 10^{10} N/m^2)$

1) $1/2\pi$ 2) $1/160\pi$ 3) $1/40\pi$ 4) $1/4\pi$

Ans: 2

Sol: $\eta = \frac{F}{A\theta} \Rightarrow \theta = \frac{F}{\pi r^2 \eta} = \frac{1}{160\pi}$

33. For the determination of refractive index of glass slab a travelling microscope is used whose main scale contain 300 equal divisions equal to 15cm. The vernier scale attached to the microscope has 25 divisions equal to 24 divisions of main scale. The least count of the microscope in (cm)

1) 0.002 2) 0.0005 3) 0.0025 4) 0.001

Ans: 1

Sol: $1MSD = \frac{15}{300}$ and $1VSD = \frac{24}{25} MSD$

$1MSD - 1VSD = LC$

$$\frac{15}{300}-\frac{24}{25}\times\left(\frac{15}{300}\right)=\text{LC}$$

$$\text{LC}=\frac{^{3}\cancel{15}}{\cancel{300}}\times\frac{1}{\cancel{25}_{5}}=\frac{1}{500}\text{cm}$$

34. There are two vessels filled with an ideal gas of volume of one is double the volume of other. The long vessel contain the gas at 8Kpa at 1000 K while the smaller vessel contain 7 Kpa at 500K. If the vessels are connected to each other by a thin tube allowing the gas to flow and the temperature of both vessels is maintained at 600 K at steady state. The pressure in the vessel will be (in kpa)

1) 18 2) 4.4 3) 24 4) 6

Ans: 4

Sol: $n_1+n_2=n$

$$\frac{7\times\cancel{10^{3}}^{200}\times v}{\cancel{R}\times\cancel{500}}+\frac{8\times\cancel{10^{3}}^{100}v(2)}{\cancel{R}\times\cancel{1000}}=\frac{P(3\cancel{v})}{\cancel{R}(\cancel{600})}$$

$$\Rightarrow p=6kPa$$

35. Given below are two statements

S1: The dimension of planck's constant and angular momentum are same.

S2: In Bohr's model electron revolve around the nucleus only in those orbits for which angular momentum are integral multiple of planck's constant.

In the light of the above statements choose the most appropriate answer from the options given below.

1) Statement 1 is incorrect but statement 2 is correct
2) Both Statement 1 and statement 2 are incorrect
3) Statement 1 is correct but statement 2 is incorrect
4) Both Statement 1 and statement 2 are correct.

Ans: 3

Sol: We know that

Dimensional formula of $\vec{L}$ is $\left[ML^2T^{-1}\right]$ and

Dimensional formula of (h) is $\left[ML^2T^{-1}\right]$

From Bohr's Postulates

Angular momentum = $\frac{nh}{2\pi}$

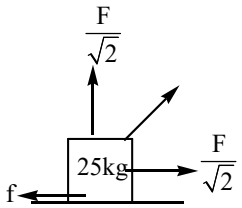
Where , $n=1,2,3,.....$

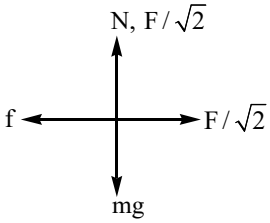
36. A black of mass 25 kg is pulled along a horizontal surface by a force at an angle 45^{o} with the horizontal. The friction coefficient between the block and the surface is 0.25. The block travels at a uniform velocity the workdone by the applied force during a displacement of 5m of the block is

1) 970 J 2) 245 J 3) 735 J 4) 490 J

Ans: 2

Sol:





1) $N + \frac{F}{\sqrt{2}} = mg$

2) $f = \mu N = \mu\left(mg - \frac{F}{\sqrt{2}}\right)$

3) υ is constant so, $f = \frac{F}{\sqrt{2}}$

From 1, 2, 3

$$\mu\left(mg - \frac{F}{\sqrt{2}}\right) = \frac{F}{\sqrt{2}}$$

$$\frac{F}{\sqrt{2}} = 50$$

$$F = 50\sqrt{2}N$$

$$\therefore W = \vec{F}.\vec{x} = F \times x \times \frac{1}{\sqrt{2}} = 50\sqrt{2} \times 5 \times \frac{1}{\sqrt{2}} = 250N$$

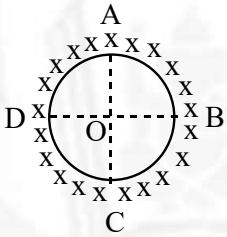
37. An object is kept at rest at a distance of 3R above the earth surface (R = radius of earth). The minimum speed with which it must be projected so that it does not return to earth is

1) $\sqrt{\frac{GM}{2R}}$ 2) $\sqrt{\frac{GM}{R}}$ 3) $\sqrt{\frac{3GM}{R}}$ 4) $\sqrt{\frac{2GM}{R}}$

Ans: 1

Sol: $v_e = \sqrt{\frac{2GM}{R+h}} = \sqrt{\frac{2GM}{4R}} = \sqrt{\frac{GM}{2R}}$

38. A metallic ring is uniformly charged as shown in the figure. AC and BD are two mutually perpendicular diameters. Electric field due to AB at O is E in magnitude the of electric field at O due to arc (ABC)



1) zero 2) E/2 3) $\sqrt{2}E$ 4) 2E

Ans: 3

Sol; $E = \frac{\lambda}{4\pi\varepsilon_0 r} 2\sin 45$

$$E' = \frac{\lambda}{4\pi\varepsilon_0 r} 2\sin 90$$

$$\frac{E'}{E} = \sqrt{2} \Rightarrow E' = \sqrt{2}E$$

39. Displacement of wave is expressed as $x(t)=5\cos\left(628t+\frac{\pi}{2}\right)m$

$(\mu_0=4\pi\times10^{-7}H/m,\in_0=8.85\times10^{-12}F/m,c=3\times10^8 m/s)$. The wavelength of the wave when its velocity is 300 m/s is $(\pi=3.14)$

1) 0.5 m 2) 5m 3) 0.33 m 4) 3 m

Ans: 4

Sol: $\lambda=\frac{v}{f}\ \&\ \omega=2\pi f\Rightarrow\lambda=3m$

40. There are n number of identical electric bulbs, each is designed to draw a power P and independently from main supply. They are now joined in series across the main supply. The total power drawn by the combination is

1) $\frac{P}{n^2}$ 2) p/n 3) P 4) np

Ans: 2

Sol: Let Resistance of each bulb be R then power $P=\frac{V^2}{R}$

All are joined in series so

Equivalent Resistance =nR

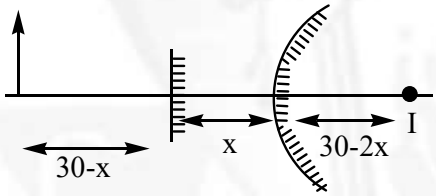
Power dissipated $=\frac{V^2}{Req}=\frac{V^2}{nR}=\frac{P}{n}$

41. A finite size object is placed normal to principal axis at distance of 30cm from convex mirror of f = 30cm. A plane mirror is now placed in such a way that the image produced by both mirrors coincide with each other. The distance between two mirrors is

1) 7.5 cm 2) 45 cm 3) 22.5 cm 4) 15 cm

Ans: 1

Sol: From the mirror formula



$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$

$\frac{1}{30}=\frac{1}{-30}+\frac{1}{30-2x}$

$x=7.5cm$

42. In an electromagnetic system a quantity defined as ratio of electric dipole moment and magnetic dipole moment has dimension of $\left[M^P L^Q T^R A^S\right]$ then P, Q are

1) 0, -1 2) 1, -1 3) -1, 0 4) -1, 1

Ans: 1

Sol: Electric Dipole moment = $[LTA]$

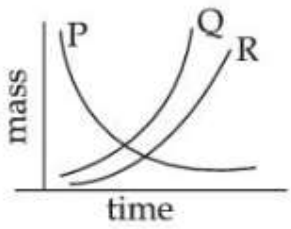
Magnetic Dipole moment =$\left[L^2A\right]$

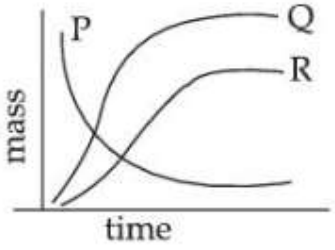
Ratio has a dimensions of $\left[M^{o}L^{-1}TA^{o}\right]$

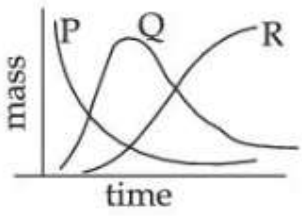
P=O, Q=-1

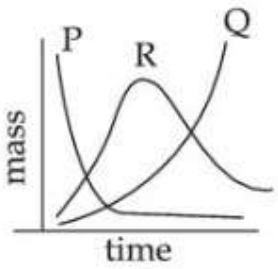
43. The radioactive material p first decay into Q and then Q decay into non-radioactive material R which of the following figure represents time dependence mass of P, Q, R

s

1. 

2. 

3. 

4. 

Ans: 3

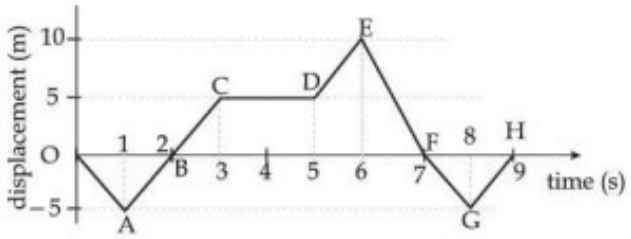
Sol:

at $t=\infty$

$[P]=[Q]=0$

$[R]$ is maximum. Because $P \rightarrow Q \rightarrow R$

44. The displacement verses time graph is shown below



A) The average velocity during 0 to 3s is 10 m/s

B) The average velocity during 3 to 5 s is 0m/s

C) The instantaneous velocity at t = 2s is 5 m/s

D) The average velocity during 5 to 7s and instantaneous velocity at t = 6.5s are equal

E) The average velocity from t = 0 to t = 9 s is zero

Choose the correct answer from the options given below

1) B, C, E only 2) B, C, D only 3) A, D, E only 4) B, D, E only

Ans: 1

Sol;

$$\overline{V_{avg}} = \frac{total\ displacement}{total\ time} = \frac{\overline{x_f} - \overline{x_i}}{t_2 - t_1}$$

45. Consider a n-type semiconductor in which n_e and n_h are number of electrons and holes, respectively

A) Holes are minority carriers

B) The dopant is pentavalent atom

C) $n_e n_h \neq n_i^2$

(where n_i is number of electrons or holes in semiconductor when it is intrinsic form)

D) $n_e n_h \geq n_i^2$

E) The holes are not generated due to the donors

Choose the correct answer from the options below

1) A, C, D only 2) A, B, E only 3) A, C, E only 4) A, B, C only

Ans: 2

Sol: $n_e n_h = n_i^2$

SECTION-II (NUMERICAL VALUE TYPE)

This section contains **5 Numerical Value Type Questions.** The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **Nearest Integer** value (Example i,e. If answer is above **10** and less than **10.5** round off is **10** and If answer is from **10.5** and less than **11** round off is **11**).

Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.

46. A solid sphere with uniform density and radius R is rotating initially with constant angular velocity (ω_1) about its diameter. After some time during the rotation its starts

loosing mass at uniform rate, with no change in its shape. The angular velocity of the sphere when its radius become R/2 is $x\omega_1$. The value of x is _____

Ans: 32

Sol: $I=\frac{2}{5}MR^2=\frac{2}{5}\left(\frac{4}{3}\pi R^3\rho\right)R^2\Rightarrow I\propto R^5$

And $I_1\omega_1=I_2\omega_2$

$\omega_2=\omega\left(\frac{R_1}{R_2}\right)^5=32\omega$

47. In a Young's double slit experiment , two slits are located 1.5 mm apart. The distance of screen from slits is 2 m and the wavelength of the source is 400 mm. If the 20 maxima of the double slit pattern are contained within the central maximum of the single slit diffraction pattern, then the width of each slit is $x\times10^{-3}$, then the value of x is

Ans: 15

Sol:

Given that $20\frac{\lambda D}{d}=\frac{2\lambda D}{a}$

$a\rightarrow$ slit width in single slit

$d\rightarrow$ separation between slits in YDSE

$a=\frac{2d}{20}=\frac{d}{10}=\frac{1.5\times10^{-3}}{10}=15\times10^{-5}m=15\times10^{-3}cm$

48. An inductor of self inductance 1H is connected in series with a resistor of 100π ohm and AC supply of 100π volt 50Hz, maximum current flowing in the circuit is ____A.

Ans: 1

Sol: $V_0=V_{rms}\sqrt{2}=100\pi\sqrt{2}$

$Z=\sqrt{R^2+(\omega L)^2}=100\pi\sqrt{2}$

$I_{max}=\frac{100\pi\sqrt{2}}{100\pi\sqrt{2}}=\frac{\sqrt{2}}{\sqrt{2}}\text{amp}=1\text{amp}$

49. In a optical medium passes a relative permiability of $\frac{10}{\pi}$ and relative permittivity of $\frac{1}{0.0885}$, then the velocity of light is greater in vaccum than that in this medium by ____ times.

$(\mu_0=4\pi\times10^{-7}H/m,\in_0=8.85\times10^{-12}F/m,c=3\times10^8m/s)$

Ans: 6

Sol; $\frac{C}{V}=\sqrt{\mu_r\varepsilon_r}$

$=\sqrt{\frac{10}{\pi}\times\frac{1}{8.85\times10^{-2}}}$

$=\sqrt{\frac{10}{\pi}\times\frac{10^2\times10^{-12}}{\varepsilon_0}}$

$=\sqrt{9\times10^9\times4\times10^{-9}}=6$

50. A particle of charge $1.6\mu C$ and mass $16\mu g$ is present in a strong magnetic field of 6.28T. The particle is then fired perpendicular to magnetic field. Time taken for the particle to rectum to original location for first time is _____s ($\pi=3.14$)

Ans: 0.01

Sol: $T=\frac{2\pi}{qB}m$

$=\frac{2\pi\times16\times10^{-9}}{16\times10^{-7}\times6.28}=10^{-2}$ sec

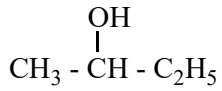
CHEMISTRY **Max Marks: 100**

SECTION-I (SINGLE CORRECT ANSWER TYPE)

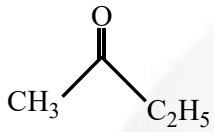
This section contains **20 Multiple Choice Questions.** Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

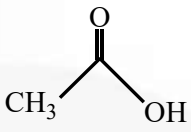
Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

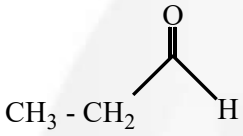
51. Which among the following compounds give yellow solid when reacted with NaOI / NaOH

A) 

B) $CH_3 - CH_2 - CH_2 - OH$

C) 

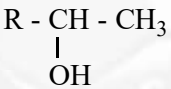
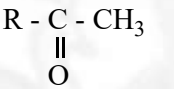
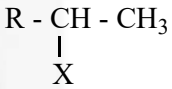
D) 

E) 

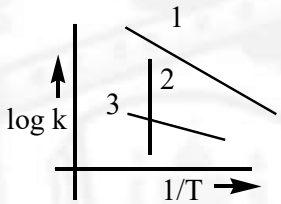
Choose correct

1) B, C, E 2) A, C, D 3) C&D 4) A, C

ANS: 4

Sol: To give iodoform test compound should have  or  ,  groups.

52. Consider the following plots of log of rate constant (logk) vs 1/T for three different reactions. The correct order of activation energy of these reactions are



1) $Ea_3 > Ea_2 > Ea_1$ 2) $Ea_1 > Ea_3 > Ea_2$

3) $Ea_1 > Ea_2 > Ea_3$ 4) $Ea_2 > Ea_1 > Ea_3$

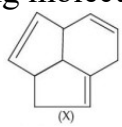
Ans: 4

Sol: $K = Ae^{-Ea/RT}$

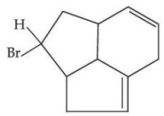
$$\log K = \log A - \frac{Ea}{RT}$$

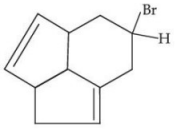
$-\frac{Ea}{R}$ is slope, As negative slope increases activation energy also increases.

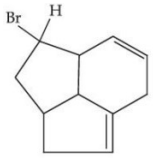
53. Consider the following molecule (X)

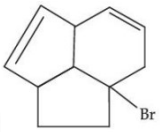
The structure of X is 

The major product formed when the given molecules (X) is treated with HBr (1 eq) is:

1) 

2) 

3) 

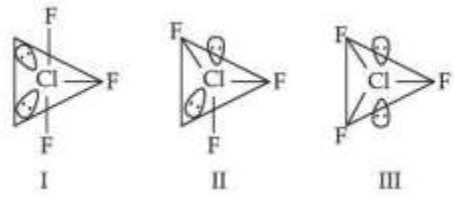
4) 

Ans: 4

Sol: Electrophilic addition reaction occurs on electron rich C = C (or)
C = C which can form most stable cation.

54. Given below are two statements:

Statement (I) : For ClF_3, all three possible structures may be drawn as follows.



Statement (II): Structure III is most stable, as the orbitals having the lone pairs are axial, where the lp – bp repulsion is minimum.

In the light of the above statements, choose the most appropriate answer from the options.

1) Statement I is correct but Statement II is incorrect
2) Both Statement I and Statement II are incorrect
3) Statement I is incorrect but Statement II is correct6
4) Both Statement I and Statement II are correct

Ans: 1

Sol: Based on VSEPR theory structure I has minimum lone pair and bond pairs.

55. Consider the given data:

(a) $HCl(g)+10H_2O(l)\rightarrow HCl.10H_2O \quad \Delta H=-69.01KJmol^{-1}$

(b) $HCl(g)+40H_2O(l)\rightarrow HCl.40H_2O \quad \Delta H=-72.79KJmol^{-1}$

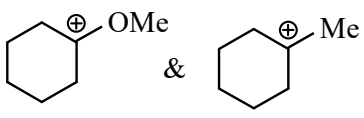
Choose the correct statement:

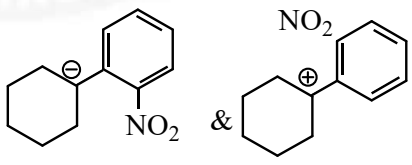
1) The heat of dilution for the $(HCl.10H_2O\,to\,HCl.40H_2O)$ is 3.78 kJ mol^{-1}
2) The heat of solution depends on the amount of solvent.
3) Dissolution of gas in water is an endothermic process.
4) The heat of formation of HCl solution is represented by both (a) and (b)

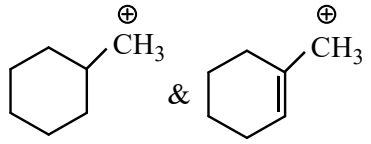
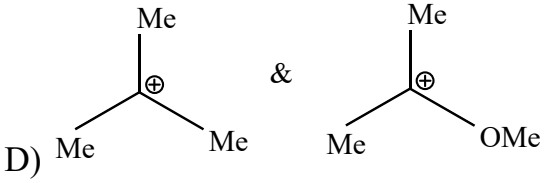
Ans: 2

Sol: Conceptual

56. In which pairs the first ion is more stable than second?

A) 

B) 

C)  D) 

1) B & D only 2) A & C only 3) B & C only 4) A & B only

Ans: 4

Sol: Positive charge is stabilized by donating effect and Negative charge is stabilized by with drawing effect.

57. Match List – I with List – II

List – I (Separation of)	List – II (Separation technique)
A) Aniline from aniline-water mixture	I) Simple distillation
B) Glycerol from spent-lye in soap industry	II) Fractional distillation
C) Different fractions of crude oil in petroleum industry	III) Distillation at reduced pressure
D) Chloroform- Aniline mixture	(IV) Steam distillation

1) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)
2) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
3) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)
4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Ans: 4

Sol: NCERT lines

Chloroform and aniline have high boiling point difference and fractions of crude oil have low boiling point difference.

58. The elements of group-13 with highest and lowest first ionization enthalpies are respectively

1) B & In 2) Ti & B 3) B & Tl 4) B & Ga

Ans: 1

Sol: I.E values of Group – 13 is $B > Tl > Ga > Al > In$

59. A toxic compound "A" when reacted with NaCN in aqueous acidic medium yields an edible cooking component and food preservative "B". "B" is converted to "C" by diborane and can be used as an additive to petrol to reduce emission. "C" upon reaction with oleum at 140° C yields an inhalable anesthetic "D". Identify "A", "B", "C" &"D" respectively.

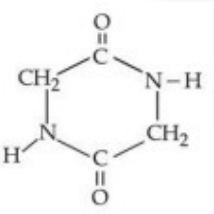
1) Acetaldehyde: 2-hydroxypropanoic acid: propanoic acid: dipropyl either
2) Methanol: acetic acid: ethanol: diethyl ether
3) Ethanol: acetonitrile: ethylamine: ethylene
4) Methanol:" formaldehyde: methyl chloride: chloroform

Ans: 1

Sol: $\underset{A}{CH_3OH} \xrightarrow{NaCN|H^+} CH_3-CN \xrightarrow{H_3O^+} \underset{B}{CH_3COOH} \xrightarrow{B_2H_6}$

$CH_3-\underset{(C)}{CH_2-OH} \xrightarrow{H^+|140^\circ C} \underset{(D)}{CH_3CH_2OCH_2CH_3}$

60. A dipeptide, "x" on complete hydrolysis gives "y" and "z". "y" on treatment with aq. HNO_2 produces lactic acid. On the other hand "z" on heating gives the following cyclic molecule.

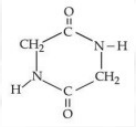


Based on the information given, the dipeptide X is

1) Valine-glycine 2) Alanine-alanine
3) Valine-leucine 4) Alanine-glycine

Ans: 4

Sol: If $y \xrightarrow{H_3O^+} CH_3-CH(OH)-COOH$



Then y is $CH_3-CH(NH_2)-COOH$ and if $z \xrightarrow{\Delta}$

Then z is H_2N-CH_2-COOH

So, x is alanine – glycine.

61. The IUPAC name of the following compound is

$HC\equiv C-CH_2-\underset{\underset{OH}{|}}{C}-CH_2-CH=CH_2$

A) 4 – Hydroxyhept -6-en-1-yne B) 4- Hydroxyhept-1-en-6-yne
C) Hept-6-en-1-yn-4-ol D) Hept-1-en-6-yn-4-ol

Ans: D

7 6 5 4 3 2 1

Sol: $HC\equiv C-CH_2-CH(OH)-CH_2-CH=CH_2$

Functional group > double bond >triple bond

62. Given below are two statements

Statement 1: Alcohols are formed when alkyl chloride are treated with aqueous potassium hydroxide by elimination reaction?

Statement 2: In alcoholic potassium hydroxide alkyl chlorides forms alkenes by abstracting the hydrogen from the β-carbon

In the light of the above statements. Choose the most appropriate answer from the options given below

1) Statement I is correct but Statement II is incorrect
2) Both Statement I and Statement II are correct
3) Both Statement I and Statement II are incorrect

4) Statement I is incorrect but Statement II is correct

Ans: 4

Sol: $R-Cl \xrightarrow[H_2O]{KOH} R-OH$ is substitution not elimination

63. Given below are two statements

Statement I: The first ionization enthalpy of group – 14 elements is higher than the corresponding elements of group-13.

Statement II: Melting point and boiling point of group-13 elements are in general much higher than those of corresponding elements of group 14

In the light of the above statements. Choose the most appropriate answer from the options given below

1) Statement I is incorrect but Statement II is correct

2) Both Statement I and Statement II are incorrect

3) Both Statement I is correct and Statement II are incorrect

4) Both Statement I and Statement II is correct

Ans: 3

Sol: NCERT Lines

Due to small sized and high Z_{eff} in group-14 elements when compare with group-13 elements.

64. Half life of zero order reaction A$\rightarrow$ product in one hour, when initial concentrations of reactants is $2molL^{-1}$. The time required to decrease concentration of A from 0.50 to 0.25 $molL^{-1}$.

1) 15 min 2) 60 min 3) 4 hour 4) 0.5 hour

Ans: 1

Sol: $t_{1/2}=1hr$

$R_0=2$

$t_{1/2}=\frac{R_0}{2k}$

$k=\frac{R_0}{2t_{1/2}}$

$k=\frac{R_0-R_t}{t}$

65. Consider the ground state chromium (Z = 24). How many electrons are with Azimuthal quantum number $l=1$, $l=2$ respectively

1) 12 & 4 2) 16 & 5 3) 12 & 5 4) 16 & 4

Ans: 3

Sol: $Z=24, l=1 \rightarrow p-subshell$

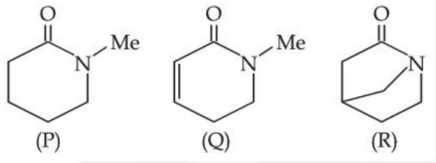
$l=2 \rightarrow d-subshell$

$_{24}Cr=1s^2 2s^2 2p^6 3s^2 3p^6 3d^5 4s^1$

$l=1 \rightarrow p$ no. of electrons is 12

$l=2 \rightarrow d$ no. of electrons is 5

66. The correct order of basicities of the following molecules is



A) P > Q> R B) Q > P > R C) R > Q > P D) R > P > Q

Ans: 3

Sol: In R, lone pair is not involved in resonance where as P & Q have resonance.
In a ketone is in cross conjugation so we observe less delocalization in Q.

67. 'X' is the number of electrons in t_{2g} orbitals of the most stable complex ion among $\left[Fe(NH_3)_6\right]^{3+}$, $\left[FeCl_6\right]^{3-}$, $\left[Fe(C_2O_4)_3\right]^{3-}$ and $\left[Fe(H_2O)_6\right]^{3+}$ the nature of oxide of vanadium of the type V_2O_x is

1) Basic 2) Acidic 3) Amphoteric 4) Neutral

Ans: 3

Sol: $\left[Fe(C_2O_4)\right]^{-3}$ is most stable among given due to chelation $Fe^{+3} \rightarrow t_{2g}^3 eg^2$

NTA Key: 3
Sri Chaitanya Key: 1

68. The correct order of $\left[FeF_6\right]^{3-}$, $\left[CoF_6\right]^{3-}$, $\left[Ni(CO)_4\right]^{2-}$, $\left[Ni(Cl)_4\right]^{2-}$ complex species based on the number of unpaired electrons present is

1) $\left[CoF_6\right]^{3-} > \left[FeF_6\right]^{3-} > \left[Ni(CO)_4\right] > \left[Ni(CN)_4\right]^{2-}$

2) $\left[FeF_6\right]^{3-} > \left[CoF_6\right]^{3-} > \left[Ni(CN)_4\right]^{2-} > \left[Ni(CO)_4\right]$

3) $\left[Ni(CN)_4\right]^{2-} > \left[FeF_6\right]^{3-} > \left[CoF_6\right]^{3-} > \left[Ni(CO)_4\right]$

4) $\left[FeF_6\right]^{3-} > \left[CoF_6\right]^{3-} > \left[Ni(CN)_4\right]^{2-} = \left[Ni(CO)_4\right]$

Ans: 4

Sol: $Fe^{+3} \rightarrow 5$ unpaired electrons

$Co^{+3} \rightarrow 4$ unpaired electrons

Ni^{+2} in square planar complex have no unpaired electron

$Ni^{o} \rightarrow$ no unpaired electrons.

69. The incorrect relationship in the following pair in relation to ionization enthalpies is

1) $Fe^{+2} < Fe^{+3}$ 2) $Mn^{+2} < Fe^{+2}$ 3) $Mn^{+} < Mn^{+2}$ 4) $Mn^{+} < Cr^{+}$

Ans: 2

Sol:

$Fe^{+2} \rightarrow d^6 s^o$

$Fe^{+3} \rightarrow d^5 s^o$

$Mn^{+2} \rightarrow d^5 s^o$

$Mn^{+} \rightarrow d^6 s^o$

$Cr^{+} \rightarrow d^5 s^o$

Based on exactly half-filled stable electronic configuration.

70. Given below are two statements:

Statement I: Molar depression constant K_f is given by $\frac{M_1 R T_f}{\Delta S_{fus}}$, where symbols have their usual meaning

Statement II: K_f for benzene is less than the K_f for water

1) Statement I is incorrect but Statement II is correct
2) Both Statement I and Statement II are correct
3) Both Statement I and Statement II are incorrect
4) Statement I is correct but Statement II is incorrect

Ans: 4

Sol: $SI - K_f = \frac{RT_f^2 M}{\Delta H_{vap}} = \frac{RT_f{}^2 M}{\Delta S_{vap} \times T_f} = \frac{RT_f M}{\Delta S_{vap}}$

S-I correct.

$SII: K_f$ of benzene > K_f of water

(Molecular weight of benzene > molecular weight of water)

SECTION-II (NUMERICAL VALUE TYPE)

This section contains **5 Numerical Value Type Questions.** The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **Nearest Integer** value (Example i,e. If answer is above **10** and less than **10.5** round off is **10** and If answer is from **10.5** and less than **11** round off is **11).**

Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.

71. The molar conductance of an indefinitely dilute solution of ammonium chloride was found to be 185 S $cm^2\ mol^{-1}$ and the ionic conductance of hydroxyl and chloride ions are 170 and 70 S $cm^2\ mol^{-1}$ respectively. If molar conductance of 0.02 M solution of ammonium hydroxide is 8505 s $cm^2\ mol^{-1}$, its degree if dissociation is given by $x \times 10^{-1}$ The value of x is _____ (nearest integer)

Ans: 3

Sol: $\wedge^{\infty}_{NH_4Cl} = 185$

$\lambda_{OH^-} = 170$

$\lambda_{Cl^-} = 70$

$\wedge^{\infty}_{NH_4OH} = \lambda^{\infty}_{NH_4Cl} + \lambda^{\infty}_{OH^-} - \lambda^{\infty}_{Cl^-}$

$\alpha = \frac{\wedge_m}{\wedge_\infty}$

72. Sea water, which can be considered as a 6 molar (6M) solution of NaCl, has a density of $2gml^{-1}$. The concentration of dissolved oxygen (O_2) in sea water is 5.8 ppm then concentration of dissolved oxygen (O_2) in sea water is $x \times 10^{-4} m(molal)$. The value of X is ____ (nearest integer)

Given: Molar mass of NaCl is $58.5gmol^{-1}$

Molar mass of O_2 is $32gmol^{-1}$

NTA Key: 3

Sri Chaitanya: 2.19

Sol: $6M \rightarrow$ 6 Mol NaCl in 1000 ml H_2O

$6 \times 58.5gm$ in 2000 gm

$? = 10^6 gram$

$= \frac{6 \times 58.5 \times 10^6}{2 \times 10^3}$

$= 0.175 \times 10^6 gm$ NaCl

Weight of pure water = $10^6 gm - 0.175 \times 10^6 gm$

$= 10^6 \times 0.825$

$m = \frac{wt}{GMW} \times \frac{10000}{a}$

$= 2.19 \times 10^{-4} m$

73. A metal complex with a formula $MCl_4 3NH_3$ is involved in sp^3d^2 hybridization of upon reaction with excess of $AgNO_3$ solution gives 'X' moles of AgCl. Consider π is equal the no. of lone pairs of electrons present in central atom of BrF_5. Then the number of geometrical isomers exhibited by the complex is

Ans: 2

Sol: $MCl_4.3NH_3 \rightarrow sp^3d^2$

$\left[M\left(NH_3\right)_3 Cl_3\right]Cl + AgNO_3 \rightarrow AgCl\downarrow$

$BrF_5 \rightarrow$ 1 lone pair of electrons

1 mole AgCl is precipitate.

Structure of complex is $\left[M\left(NH_3\right)Cl_3\right]^+ = Ma_3b_3$

Fac & mer isomers.

No. of isomers = 2

74. x mg of $Mg(OH)_2$ (molar mass: 58) is required to be dissolved in 1.0 L of water to produce a pH of 10.0 at 298 K. The value of x is _____ mg (Nearest integer) (Given: $Mg(OH)_2$ is assumed to dissociate completely in H_2O)

Ans: 63

Sol: $pH = 10$

$pOH = 4$

$\left[OH^-\right] = 10^{-4} N$

$N = M \times n_f$

$M = \frac{weight}{GMW} \times \frac{1}{V(in\,lit)}$

75. The amount of Calcium oxide produced on heating 150 kg lime stone (75% pure) is ___ kg (Nearest integer) (Given: Molar mass (in $gmol^{-1}$ of Ca – 40, O – 16, C – 12)

Ans: 2

Sol: $\underset{100gm}{CaCO_3} \rightarrow \underset{56gm}{CaO} + CO_2 \uparrow$ Weight of pure $CaCO_3 = \frac{150 \times 75}{100} = 1.5 \times 75kg$

$1.5 \times 75kg \rightarrow ?$

$= \frac{56 \times 1.5 \times 75}{100} = 63grams$