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MATHEMATICS **Max Marks: 100**

(SINGLE CORRECT ANSWER TYPE)

This section contains **20 Multiple Choice Questions.** Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

01. The integral $\int_0^{\pi} \frac{(x+3)sinx}{1+3\cos^2 x}dx$ is equal to

1) $\frac{\pi}{2\sqrt{3}}(\pi+4)$ **2)** $\frac{\pi}{\sqrt{3}}(\pi+2)$ **3)** $\frac{\pi}{3\sqrt{3}}(\pi+6)$ **4)** $\frac{\pi}{\sqrt{3}}(\pi+1)$

Key: 3

Sol: $I=\int_0^{\pi}\frac{(x+3)\sin x}{1+3cox^2x}dx$

$$I=\int_0^{\pi/2}\frac{(x+3)\sin x}{1+3\cos^2 x}+\frac{(p-x+3)\sin(\pi-x)}{1+3\cos^2(\pi-x)}dx$$

$$I=\int_0^{\pi/2}\frac{(\pi+6)\sin x}{1+3\cos^2 x}dx$$

$$I-(\pi+6)\int_0^{\pi/2}\frac{\sin x}{1+3\cos^2 x}$$

$$I=(\pi+6)\int_1^0\frac{-dt}{1+3t^2} \quad (\because \cos x=t)$$

$$=\frac{\pi+6}{3}\int_0^1\frac{dt}{\left(\frac{1}{\sqrt{3}}\right)^2+t^2}$$

$$=\frac{\pi+6}{3}.\sqrt{3}.\left(\tan^{-1}\sqrt{3}t\right)_0^1$$

$$=\frac{\pi+6}{\sqrt{3}}.\frac{\pi}{3}$$

02. If for $\theta \in \left[-\frac{\pi}{3}, 0\right]$, the points $(x, y) = \left(3\tan\left(\theta + \frac{\pi}{3}\right), 2\tan\left(\theta + \frac{\pi}{6}\right)\right)$ lie on $xy + \alpha x + \beta y + \gamma = 0$ is equal to

1) 80 **2)** 72 **3)** 96 **4)** 75

Key: 4

Sol: $x = 3\tan\left(\theta + \frac{\pi}{3}\right)$

$$x = 3\frac{\left(\tan\theta + \sqrt{3}\right)}{1 - \sqrt{3}.\tan\theta}$$

$$x - \sqrt{3} \times \tan\theta = 3\tan\theta = 3\tan\theta + 3\sqrt{3}$$

$$\tan\theta\left(3 + \sqrt{3}x\right) = x - 3\sqrt{3}$$

$$\tan\theta = \frac{x - 3\sqrt{3}}{3 + \sqrt{3}x} \quad \rightarrow (1)$$

$$y = 2\tan\left(\theta + \frac{\pi}{6}\right)$$

$$y = 2\left(\frac{\tan\theta + \frac{1}{\sqrt{3}}}{1 - \tan\theta\frac{1}{\sqrt{3}}}\right)$$

$$y = 2\left(\frac{\sqrt{3}\tan\theta + 1}{\sqrt{3} - \tan\theta}\right)$$

$$\sqrt{3}y - \tan\theta.y = 2\sqrt{3}\tan\theta + 2$$

$$\tan\theta\left(2\sqrt{3} + 4\right) = \sqrt{3}y - 2$$

$$\tan\theta = \frac{\sqrt{3}y - 2}{2\sqrt{3} + y} \quad \rightarrow (2)$$

From (1),(2)

$$\frac{x-3\sqrt{3}}{3+\sqrt{3}}=\frac{\sqrt{3}y-2}{2\sqrt{3}y}$$

$$2\sqrt{3}x-18+xy-3\sqrt{3}y=3\sqrt{3}y=3\sqrt{3}y+3xy-6-2\sqrt{3}x$$

$$2xy-4\sqrt{3}x+6\sqrt{3}y+12=0$$

$$xy-2\sqrt{3}x+3\sqrt{x}+3\sqrt{3y}+6=0$$

Compare with

$xy+\alpha.x+\beta y+\gamma=0$

$$\alpha^2+\beta^2+\gamma^2=\left(-2\sqrt{3}\right)^2+\left(3\sqrt{3}\right)^2+(6)^2$$
$$=12+27+36$$
$$=48+27$$
$$=75$$

03. Let $y = y(x)$ be the solution curve of the differential equation $x\left(x^2+e^x\right)dy+\left(e^x(x-2)y-x^3\right)dx=0, x>0$, passing through the point (1, 0). Then y (2) is equal to

1) $\frac{4}{4-e^2}$ **2)** $\frac{2}{2+e^2}$ **3)** $\frac{4}{4+e^2}$ **4)** $\frac{2}{2-e^2}$

Key: 3

Sol:

$$x\left(x^2+e^x\right)dy+\left(e^x(x-2)y\right)-x^3dx=0$$

$$\frac{dy}{dx}+e^x(x-2)y=\frac{x^3}{x\left(x^2+e^x\right)}$$

I.F. $= e^{\int \frac{e^x(x-2)}{x\left(x^2+e^x\right)}dx}$

$$=e^{\int\frac{x.e^x-2e^x}{x\left(x^2+e^x\right)}}$$

$$= e\int \frac{e^x + 2x^2 - 2e^x - 2x^2}{x\left(x^2 + e^x\right)}$$

$$= e^{\int \frac{e^x + 2x}{e^x + x^2} - \frac{2}{x}}$$

$$= e^{\ln\left(e^x + x^2\right) - \frac{2}{x}}$$

$$= e^{\ln\frac{e^x + x^2}{x^2}} = \frac{e^x + x^2}{x^2}$$

Solution

$$y\left(\frac{e^x + x^2}{x^2}\right) = \int \frac{x^3}{x\left(x^2 + e^x\right)} \cdot \frac{e^x + x}{x^2}$$

$$y\left(\frac{e^x + x^2}{x^2}\right) = x + c$$

Use $x = 1, y = 0 \quad \Rightarrow c = -1$

$$y\left(\frac{e^x + x^2}{x^2}\right) = x - 1$$

Put $x = 2$

$$y\left(\frac{e^2 + 4}{4}\right) = 2 - 1 = 1$$

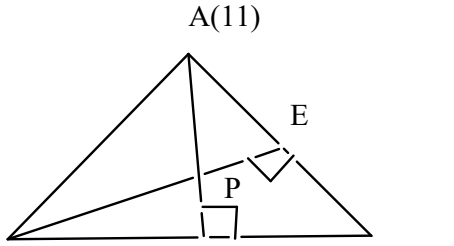
$$y = \frac{4}{4 + e^2}$$

04. Let ABC be the triangle such that the equations of the lines AB and AC be $3y$-x=2 and x+y=2, respectively, and the points B and C lie on x-axis. If P is the orthocenter of the triangle ABC, then the area of the triangle PBC is equal to

1) 4 **2)** 8 **3)** 6 **4)** 10

Key: 3

Sol:



$A=(1,1)$

$B=(2,0)$

$C=(-2,0)$

Slope of BC=0

$\Rightarrow \ AD=\tan\pi/2$

Equation of AD is x=1

Slope of AC = $\frac{1-0}{1+2}=\frac{1}{3}$

Slope of $\overline{BE}$ is "-3"

Equation of $\overline{BE}$

$y-0=-3(x-2)$

$y=-3x+6$

Solve $\overline{AD},\overline{BE},(1,3)P$

Area of ΔCPB

$=\frac{1}{2}.4.3$

$=6$

05. Let C_1 be the circle in the third quadrant of radius 3, that touches both coordinate axes. Let C_2 be the circle with centre (1, 3) that touches C_1 externally at that point (α,β). If $(\beta-\alpha)^2=\frac{m}{n}$, $\gcd(m,n)=1$, then $m+n$ is equal to

1) 22 **2)** 9 **3)** 13 **4)** 31

Key: 1

Sol:

$c_1=(-3,-3), r_1=3$

$c_2=(1,3),\ r_2=c_1c_2-3=\sqrt{52}-3$

$$(\alpha,\beta)=\left(\left(\frac{3\times1+\left(\sqrt{52}-3\right)(-3)}{\sqrt{52}}\right),\frac{33+\left(\sqrt{52}-3\right)(-3)}{\sqrt{52}}\right)$$

$$(\alpha,b)^2=\left(\frac{3-9}{\sqrt{52}}\right)^2=\frac{36}{52}=\frac{9}{13}=\frac{m}{n}$$

$m+n=22$

06. Let A be a 3×3 matrix such that $\left|adj\left(adj\left(adj\ A\right)\right)\right|=81$. If

$S=\left\{n\in\mathbb{Z}:\left(\left|adj\left(adj\ A\right)\right|\right)^{\frac{(n-1)^2}{2}}=|A|^{\left(3n^2-5n-4\right)}\right\}$, then $\sum_{n\in S}\left|A^{\left(n^2+n\right)}\right|$ is equal to

1) 866 **2)** 820 **3)** 732 **4)** 750

Key: 3

Sol:

$\left|adJ\left(adJ\left(adJA\right)\right)=81\right|$

$\Rightarrow\quad |A|^2=3$

Also $\left|adJ\left(adJA\right)\right|^{\left(\frac{n-1}{2}\right)^2}=|A|^{3n^{2-5n-4}}$

$\Rightarrow\quad 2(n-1)^2=3n^2-5n-4$

$\Rightarrow\quad n^2-n-6=0,n=3,-2$

$\sum_{n\in s}\left|A^{n^2+n}\right|=\left|A^{9+3}\right|+\left|A^{4-2}=3^63=732\right|$

07. Let x_1,x_2,x_3,x_4 be in progression. If 2,7, 9, 5 are subtracted respectively from x_1,x_2,x_3,x_4, then the resulting numbers are in an arithmetic progression. The the value of $\frac{1}{24}\left(x_1,x_2,x_3,x_4\right)$ is:

1) 72 **2)** 216 **3)** 18 **4)** 36

Key: 2

Sol:

$let\ x_1=a$

$x_2=ar$

$x_3 = ar^2$

$x_4 = ar^3$

$a-2,\ ar-7,\ ar^2-9,\ ar^3-5$ *is A.P.*

$\Rightarrow ar-7-a+2 = ar^2-ar-2 = ar^3-ar^2+4$

$\Rightarrow ar^2-2ar+a+3=0 \quad \rightarrow (1)$

$ar^3-2ar^2+a+6=0 \ \rightarrow 2$

$r(-3)+6=0 \ \rightarrow r=2$

From (1)

$a\times 4-4a+a+3=0$

$a=-3$

$\Rightarrow\ x_1=-3,\ x_2=-6,\ x_3=-12,\ x_4=-24$

$\therefore \dfrac{x_1x_2x_3x_4}{24} = \dfrac{(-3)(-6)(-12)(-24)}{24}$

=216

08. If the shortest distance between the lines $\dfrac{x-1}{2}=\dfrac{y-2}{3}=\dfrac{z-3}{4}$ and $\dfrac{x}{1}=\dfrac{y}{\alpha}=\dfrac{z-5}{1}$ is $\dfrac{5}{\sqrt{6}}$, then the sum of all possible values of α is

1) 3 **2)** $\dfrac{3}{2}$ **3)** $-\dfrac{3}{2}$ **4)** -3

Key: 4

Sol:

$A(1,2,3), B(005)$

S.D. = $\dfrac{\begin{vmatrix}1 & 2 & -2\\ 2 & 3 & 4\\ 1 & \alpha & 1\end{vmatrix}}{\begin{vmatrix}1 & J & k\\ 2 & 3 & 4\\ 1 & \alpha & 1\end{vmatrix}}$

$$\left|\frac{3-4\alpha+4-4\alpha+6}{\sqrt{(3-4\alpha)^2+4+(2\alpha-3)^2}}\right|=\frac{5}{\sqrt{6}}$$

$$\left|\frac{13-8\alpha}{\sqrt{20\alpha^2-36\alpha+22}}\right|=\frac{5}{\sqrt{6}}$$

$$6(13-8\alpha)^2=25\left(20\alpha^2-36\alpha+22\right)$$

$$\Rightarrow \quad 116\alpha^2+348\alpha-464=0$$

$$\alpha_1+\alpha_2=\frac{-348}{116}=-3$$

09. The mean and standard deviation of 100 observations are 40 and 5.1 respectively. By mistake one observation is taken as 50 instead of 40. If the correct mean and the correct standard deviation are μ and σ respectively, then $10(\mu+\sigma)$ is equal to

1) 449 **2)** 447 **3)** 445 **4)** 451

Key: 1

Sol:

$$\frac{\sum x_i}{100}=40$$

$$\Rightarrow \sum x_i=4000$$

$$\sum x_i=4000-50+40=3990$$

$$\mu=\frac{3990}{10}=39.9$$

$$(5.1)^2=\frac{\sum x_i^{\,2}}{100}-(40)^2$$

$$\sigma=\sqrt{\frac{\{(5.1)^2+1600\}\times 100-(50)^2+(40)^2}{100}-(39.90)^2}=5$$

$$10(\mu+\sigma)=10(39.90+5)=449$$

10. The remainder when $\left((64)^{(64)}\right)^{(64)}$ is divided by 7 is equal to

1) 3 **2)** 1 **3)** 4 **4)** 6

Key: 2

Sol:

$64^{64} = (63+1)^{64} = 63\lambda_1 + 1$

$\left((64)^{(64)}\right)^{(64)} = (63\lambda + 1)^{64} = 63\lambda_2 + 1$

11. From a group of 7 batsman and 6 bowlers, 10 players are to be chosen for a team, which should include at least 4 batsman and at least 4 bowlers. One batsmen and one bowler who are captain and vice-captain respectively of the team should be included. Then the total number of ways such a selection can made is

1) 145 **2)** 155 **3)** 165 **4)** 135

Key: 2

Sol:

$7 Batsmen$	$6 Bowlers$	
4	6	${}^6C_3.{}^5C_5 = 20$
5	5	${}^6C_4.{}^5C_4 = 75$
6	4	${}^6C_5.{}^5C_3 = 60$
		total=155

12. Let the angle $\theta, 0 < \theta < \frac{\pi}{2}$ between two unit vectors $\hat{a}$ and $\hat{b}$ be $Sin^{-1}\left(\frac{\sqrt{65}}{9}\right)$. If the vector $\bar{c} = 3\hat{a} + 6\hat{b} + 9\left(\hat{a} \times \hat{b}\right)$, then the value of $9\left(\bar{c}.\hat{a}\right) - 3\left(\bar{c}.\hat{b}\right)$ is

1) 31 **2)** 29 **3)** 27 **4)** 24

Key: 2

Sol:

$\bar{c} = 3\hat{a} + 6\hat{b} + 9\left(\hat{a} \times \hat{b}\right);\ \left(\hat{a}, \hat{b}\right) = \theta \quad Sin\theta = \frac{\sqrt{65}}{9},\ Cos\theta = \frac{4}{9}$

$\bar{c}.\hat{a} = 3\hat{a}.\hat{a} + 6\hat{b}.\hat{a} + 9\left(\hat{a} \times \hat{b}\right).\hat{a}$

$= 3 + 6Cos\theta = \frac{17}{3}$

$\bar{c}.\hat{b} = 3\hat{a}.\hat{b} + 6\hat{b}.\hat{b} + 9\left(\hat{a} \times \hat{b}\right).\hat{b}$

$= 3Cos\theta + 6 = \frac{22}{3}$

$9\left(\bar{c}.\hat{a}\right) - 3\left(\bar{c}.\hat{b}\right)$

$= 51 - 22 = 29$

13. $2x+3y+5z=9,$

$7x+3y-2z=8,$

$12x+3y-(4+\lambda)z=16-\mu$

have infinitely solutions. Then the radius of the circle centered at (λ,μ) and touching the line $4x=3y$ is

1) $\frac{21}{5}$ **2)** $\frac{7}{5}$ **3)** 7 **4)** $\frac{17}{5}$

Key: 2

Sol:

$|A|=0$

$$\begin{vmatrix}2 & 3 & 5\\ 7 & 3 & -2\\ 12 & 3 & -(4+\lambda)\end{vmatrix}=0$$

$\lambda=5$

$|\Delta_x|=0$

$$\begin{vmatrix}9 & 3 & 5\\ 8 & 3 & -2\\ 16-\mu & 3 & -9\end{vmatrix}=0$$

$\mu=9$

Centre of the Circle$=(\lambda,\mu)=(5,9)$

The line $4x-3y=0$ touching the circle,

$r=d$

$r=\left|\frac{20-27}{5}\right|=\frac{7}{5}$

14. Let p be the parabola, whose focus is $(-2,1)$ and directrix is $2x+y+2=0$.

Then the sum of the ordinates of the points on P, whose abscissa is -2

1) $\frac{5}{2}$ **2)** $\frac{3}{4}$ **3)** $\frac{3}{2}$ **4)** $\frac{1}{4}$

Key: 3

Sol: We know that SP = PM

$$\sqrt{(x+2)^2+(y-1)^2}=\left|\frac{2x+y+2}{\sqrt{4+1}}\right|$$

$$5\left((x+2)^2+(y-1)^2\right)=(2x+y+2)^2$$

If $x = -2$

$5\left(\left(y-1\right)^2\right)=\left(y-2\right)^2$

$4y^2-6y+1=0$

Sum of the ordinates$=\frac{6}{4}=\frac{3}{2}$

15. Let $x=-1$ and $x=2$ be the critical points of the function $f\left(x\right)=x^3+ax^2+b\log_e\left|x\right|+1, x\neq 0$. Let m and M respectively be the absolute minimum and the absolute maximum values of f in the interval $\left[-2,-\frac{1}{2}\right]$.

Then $\left|M+m\right|$ is equal to (Take $\log_e 2=0.7$):

1) 22.1 **2)** 21.1 **3)** 19.8 **4)** 20.9

Key: 2

Sol:

$f\left(x\right)=x^3+ax^2+b\log_e\left|x\right|+1$

$f'\left(x\right)=3x^2+2ax+\frac{b}{x}$

x = -1, 2 are critical Points

$f'\left(-1\right)=0\Rightarrow 3-2a-b=0$

$f'\left(2\right)=0\Rightarrow 24+8a+b=0$

$a=-\frac{9}{2}, b=12$

$f\left(x\right)=x^3-\frac{9}{2}x^2+12\log_e^{\left|x\right|}+1$

$f\left(-2\right)=-8-18+12\log\left(2\right)+1$

$=-25+12\left(0.7\right)=-16.6$

$f\left(-1\right)=-1-\frac{9}{2}+1=-\frac{9}{2}=-4.5$

$f\left(-\frac{1}{2}\right)=-\frac{1}{8}-\frac{9}{8}+12\log\left(\frac{1}{2}\right)+1$

$=-\frac{1}{4}-12\log 2$

$=-0.25-12\left(0.7\right)=-12.95$

$m=-16.6, M=-4.5$

$\left|m+M\right|=21.1$

16. If the area of the region bounded by the curves $y=4-\frac{x^2}{4}$ and $y=\frac{x-4}{2}$ is equal to α, then 6α equals

1) 240 **2)** 250 **3)** 210 **4)** 220

Key: 2

Sol:

$$\text{Required Area} = \int_{-6}^{4}\left(4-\frac{x^2}{4}\right)-\left(\frac{x-4}{2}\right)dx$$

$$\alpha = \int_{-6}^{4}\left(6-\frac{x^2}{4}-\frac{x}{2}\right)dx$$

$$= \left(6-\frac{x^3}{12}-\frac{x^2}{4}\right)_{-6}^{4}$$

$$\alpha = \frac{125}{3}\ Sq.\ units$$

$$6\alpha = 250$$

17. $\lim\limits_{x\to 0^+}\dfrac{\tan\left(5(x)^{\frac{1}{3}}\right)\log_e\left(1+3x^2\right)}{\left(\tan^{-1}3\sqrt{x}\right)^2\left(e^{5(x)^{\frac{4}{3}}}-1\right)}$ is equal to

1) $\frac{1}{15}$ **2)** 1 **3)** $\frac{5}{3}$ **4)** $\frac{1}{3}$

Key: 4

Sol:

$$\frac{\log\left(\frac{1+3x^2}{3x^2}\right)\times 3x^2\times 5x^{\frac{1}{3}}\times\frac{\tan\left(5x^{1/3}\right)}{5x^{1/3}}}{\left(\frac{\tan 3\sqrt{x}}{3\sqrt{n}}\right)^2\times 9x\times\left(\frac{e^{5x^{4/3}}-1}{5x^{1/3}}\right)\times 5x^{4/3}} = \frac{15}{9\times 5} = \frac{1}{3}$$

18. Among the Statements

(S1): The set $\left\{z\in\mathbb{C}-(-i):|z|=1 \text{ and } \frac{z-i}{z+i} \text{ is purely real}\right\}$ contains exactly two elements, and

(S2): The set $\left\{z\in\mathbb{C}-(-1):|z|=1 \text{ and } \frac{z-1}{z+1} \text{ is purely imaginary}\right\}$ contains infinitely many elements.

1) Only (S2) is Correct **2)** Both are Correct

3) Only (S1) is Correct **4)** Both are incorrect

Key: 1

Sol:

$$\frac{z+i}{z-i}=\frac{x+i(y+1)}{x+i(y-1)}\times\frac{x-i(y-1)}{x-i(y-1)}=purely\,real$$

$$=\frac{\left(x^2+y^2-1\right)+i\left(x(y+1)-x(y-1)\right)}{x^2+(y-1)^2}$$

Imaginary part is zero.

$x(y+1)=x(y-1)$ *implies* $x=0$ *and* y *is* infinitely many values

Therefore statement 1 is wrong.

$$\frac{z-1}{z+1}=\frac{x+iy-1}{x+iy+1}=\frac{(x-1)+iy}{(x+1)+iy}\times\frac{(x+1)-iy}{(x+1)-iy}$$

$$=\frac{\left(\left(x^2-1\right)y^2+i\left(y(x+1)-y(x-1)\right)\right)}{(x+1)^2+y^2}$$

Purely imaginary implies real part is zero.

$\therefore x^2+y^2=1$

Lies on infinitely many points

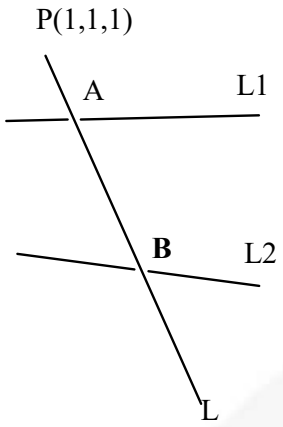
Therefore statement "2" is true.

19. Let the line L pass through (1, 1, 1) and intersect the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z}{1}$. Then, which of the following points lies on the line L?

1) (5, 4, 3) **2)** (7, 15, 13) **3)** (10, -29, -50) **4)** (4, 22, 7)

Key: 2

Sol:



drs of PA $=(2\lambda, 3\lambda-2, 4\lambda)$

drs of PV $=(\mu+2, 2\mu+3, \mu-1)$

$$\frac{2\lambda}{\mu+2}=\frac{3\lambda-2}{2\mu+3}=\frac{4\lambda}{\mu-1}$$

$$2\lambda(2\mu+3)=(3\lambda-2)(\mu+2)$$

$$4\lambda\mu+6\lambda=3\lambda\mu+6\lambda-2\mu-4$$

$$\lambda\mu+2\mu=-4 \quad \rightarrow 1$$

$$2\lambda(\mu-1)=4\lambda(\mu+2)$$

$$2\lambda\mu-2\lambda=4\lambda\mu+8\lambda$$

$$2\lambda\mu+10\mu=0$$

$$2\lambda u+4\mu=8$$

$$10\lambda-4\mu=8$$

$$5\lambda-2\mu=4$$

$$\mu=\frac{5\lambda-4}{2}$$

$$2\lambda\left(\frac{5\lambda-4}{2}\right)+10\lambda=0$$

$5\lambda^2 - 4\lambda + 10\lambda = 0$

$5\lambda^2 + 6\lambda = 0$

$\lambda(5\lambda + 6) = 0$

$\lambda = \frac{-6}{5}$

No. of PA $= (2\lambda, 3\lambda - 2, 4\lambda)$

$= \left(-\frac{12}{5}, -\frac{-18}{5} - 2, \frac{-24}{5}\right)$

$= \left(-\frac{-12}{5}, \frac{-28}{5}, -\frac{-24}{5}\right)$

$= (+12, +28, +24)$

$= (6, 14, 12)$

$\frac{x-1}{6} = \frac{y-1}{14} = \frac{z-1}{12}$

$(x, y, z = 7, 15, 13)$ *satisfied*

20. Let the set of all values of $P \in \mathbb{R}$, for which both the roots of the equation $x^2 - (p+2)x + (2p+9) = 0$ are negative real numbers, be the interval $(\alpha, \beta]$. Then $\beta - 2\alpha$ is equal to

1) 0 **2)** 5 **3)** 20 **4)** 9

Key: 2

Sol:

$\alpha\beta > 0$

$\Rightarrow\ 2p + 9 > 0$

$\Rightarrow\ p > \frac{-9}{2}$

and $\alpha + \beta < 0$

$p + 2 < 0$

$\Rightarrow\quad p < -2$

$(p+2)^2 - 4(2p+9) \geq 0$

$(p+4)(p-8) \geq 0$

$p \in (-\infty, -4) \cup (8, \infty)$

$p \in \left(-\frac{9}{2}, -4\right) = (\alpha, \beta)$

$\beta - 2\alpha = -4 + 9 = 5$

SECTION-II (NUMERICAL VALUE TYPE)

This section contains **5 Numerical Value Type Questions.** The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **Nearest Integer** value (Example i,e. If answer is above **10** and less than **10.5** round off is **10** and If answer is from **10.5** and less than **11** round off is **11).**

Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases..

21. For $n \geq 2$, let S_n denote the set of all subsets of $\{1,2,.....,n\}$ with **no** two consecutive numbers. For example $\{1,3,5\} \in S_6$, Then $\{1,2,4\} \notin S_6$. Then $n(S_5)$ is equal to _____

Key: 13

Sol:

Let's call S(n) the number of such subsets when building a subset with the numbers upto n, you have two options: you either include n in it, or not.

If you include n, you cannot include n,n-1 so you have s(n-2) possible subsets (any subsets of the numbers 1 to n-2, plus n). If you don't include n, then you have S(n-1) possible subsets (formed by the numbers 1 to n-1).

Thus we can write S(n) =S(n-2)+S(n-1). Since S(0)=1 (the empty, only) and S(1)=2(the empty, and {1}) we have that S(n) is the (n+1)the Finonacci numbers : S(0)=1, S(1)=2, S(2)=3, S(3)=5, S(4)=8, S(5)=13, etc.

22. Consider the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ having one of its focus at P(-3,0). If the latus rectum through its other focus subtends a right angle at P and $a^2b^2 = \alpha\sqrt{2} - \beta,\ \alpha, \beta \in \mathbb{N}$, then $\alpha + \beta$ is _____

Key: 1944

Sol:

$$\tan 45 = \frac{\frac{b^2}{a}}{2ac} = \frac{b^2}{2a^2c}$$

$$b^2 = 2a^2c = 2a(ac)$$

$$b^2 = a^2(c^2 - 1) = 6a$$

$$a(c^2 - 1) = 6$$

$$a\left(\frac{9}{a^2} - 1\right) = 6$$

$$\frac{9}{a} - a = 6$$

$$9 - a^2 = 6a$$

$$a^2 + 6a - 9 = 0$$

$$a = \frac{-6 \pm \sqrt{36 + 36}}{2} = \frac{-6 \pm 6\sqrt{2}}{2}$$

$= -3\sqrt{2} - 3$

$b^2 = a^2\left(e^2 - 1\right) = (ae)^2 - a^2 = 9 - a^2$

$= b^2 > 0$

$b^2 = a^2 - 9$

$a^2b^2 = a^2\left(a^2 - 9\right)$

$a^2 = \left(3\left(\sqrt{2} - 1\right)\right)^2 = 9 - 9\left(2 + 1 - 2\sqrt{2}\right) = 27 - 18\sqrt{2}$

$b^2 = 9 - a^2 = 9 - \left(3\left(\sqrt{2} - 1\right)\right)^2 = 18\sqrt{2} - 18 = 18\left(\sqrt{2} - 1\right)$

$a^2b^2 = a^2\left(a^2 - 9\right) = 9 \times 18\left(3 - 2\sqrt{2}\right)\left(\sqrt{2} - 1\right)$

$= 162\left(5\sqrt{2} - 7\right) = 810\sqrt{2} - 1134$

$= \alpha\sqrt{2} - \beta$

$= \alpha\sqrt{2} - \beta \quad \Rightarrow \alpha = 810,\ \beta = 1134$

$\alpha + \beta = 1944$

23. The number of points of discontinuity of the function $f(x) = \left[\frac{x^2}{2}\right] - \left[\sqrt{x}\right],\ x \in [0,4]$, where [.] denotes the greatest integer function, is _____

Key: 8

Sol:

$0 \le x \le 4 \quad \Rightarrow\ 0 \le \frac{x^2}{2} \le 8$

$x = 0, 1, \sqrt{2}, \sqrt{6}, \sqrt{8}, \sqrt{10}, \sqrt{12}, \sqrt{14}, \sqrt{16}$

$f(0) = 0, f\left(0^+\right) = 0,\ f(4) = 6, f(-4) = 6$

24. The number of relations on the set $A = \{1,2,3\}$, containing at most 6 elements including (1, 2), which are reflexive and transitive but not symmetric, is _____

Key: 5

Sol:

R is reflexive R has $\{(1,1),(2,2),(3,3)\}$

$R_1 = \{(1,1),(2,2),(3,3),(1,2)\}$

$R_2 = \{(1,1),(2,2),(3,3),(2,3),(1,2)\}$

$R_3 = \{(1,1),(2,2),(3,1),(1,2)\}$

$R_4 = \{(1,1),(2,2),(3,3),(1,2)\}$

$R_5 = \{(1,1),(2,2),(3,3),(3,1),(3,2),(1,2)\}$

So number of relations which includes $(1,2)$, and containing atmost six elements which are reflexive and transitive but not symmetric=5

25. The number of singular matrices of order 2, whose elements are from the set {2, 3, 6, 9}, is _____

Key: 36

Sol:

$$\begin{vmatrix} a & d \\ b & c \end{vmatrix} = ab - bc = 0 \Rightarrow ad = bc$$

Case 1: exactly 1 number is used $\Rightarrow$ All singular $\Rightarrow$ $4c_1$

Case 2: exactly 2 number is used $4c_2 . 2 \times 2$

Case 3: exactly 3 number is used None will be singular

Case 4: exactly 4 number is used ab = cd $\Rightarrow 2 \times 9 = 3 \times 6$

$$\begin{bmatrix} 9 & \_\_ \\ \_\_ & 2 \end{bmatrix} \quad {}^4C_1 \times 2! \Rightarrow 8 \ matrices$$

$$\Rightarrow 4 + 6 \times 4 + 0 + 8 = 36 \ matrices$$

PHYSICS

SECTION-I (SINGLE CORRECT ANSWER TYPE)

This section contains **20 Multiple Choice Questions.** Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

26. In a hydrogen like ion, the energy difference between the 2nd excitation energy state and ground is 108.8 eV. The atomic number of the ion is:

1) 2 **2)** 1 **3)** 4 **4)** 3

Key: 4

OBJECTIONS:

Q.NO(26): NTA GIVEN KEY Z = 2 $\Rightarrow$ OPTION 1

(CORRECT KEY Z=4) $\Rightarrow$ OPTION 4

Sol:

Given $E_2 - E_1 = 108.8 \ e.v$

$$13.6 \ e.v z^2 \left(1 - \frac{1}{9}\right) = 108.8 ev$$

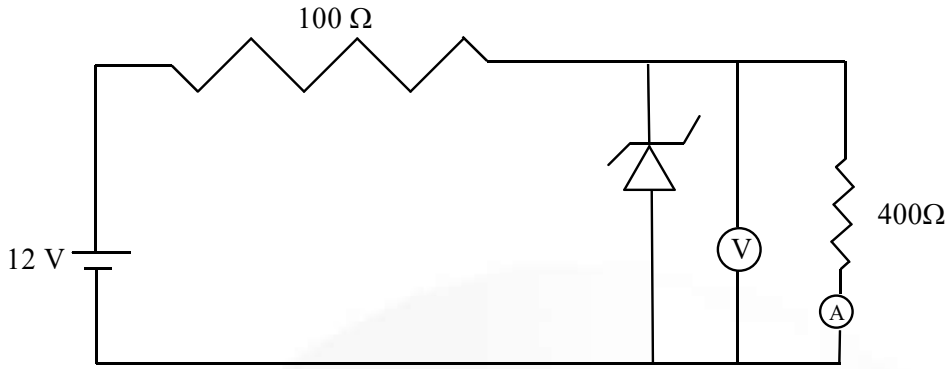
$$13.6 \left(\frac{8}{9}\right) z^2 = 108.8$$

$$z^2 = 9$$

$$z = 3$$

27. In the following circuit, the reading of the ammeter will be

(Take Zener breakdown Voltage = 4 V)



1) 24 mA **2)** 80 mA **3)** 60 mA **4)** 10 mA

Key: 4

Sol: Ammeter reading = $i_L = \frac{4}{400} = 10mA$

28. Two projectiles are fired from ground with same initial speeds from same point at angles $\left(45^o + \alpha\right)$ and $\left(45^o - \alpha\right)$ with horizontal direction. The ratio of their times of flight is

1) $\frac{1+Sin2\alpha}{1-Sin2\alpha}$ **2)** $\frac{1+\tan\alpha}{1-\tan\alpha}$ **3)** 1 **4)** $\frac{1-\tan\alpha}{1+\tan\alpha}$

Key: 2

Sol: $\frac{T_1}{T_2} = \frac{Sin\theta_1}{Sin\theta_2} = \frac{Sin\left(45^o+\alpha\right)}{Sin\left(45^o-\alpha\right)} = \frac{1+\tan\alpha}{1-\tan\alpha}$

29. Two thin convex lenses of focal lengths 30 cm and 10 cm are placed coaxially, 10 cm apart. The power of this combination is:

1) 20 D **2)** 1 D **3)** 5 D **4)** 10 D

Key: 4

Sol:

$$P = P_1 + P_2 - dP_1P_2$$

$$= \frac{100}{30} + \frac{100}{10} - \left(\frac{10}{100}\right)\left(\frac{100}{30}\right)\left(\frac{100}{10}\right)$$

$$= 10\,D$$

30. An ac current is represented as $i = 5\sqrt{2} + 10\cos\left(650\pi t + \frac{\pi}{6}\right) Amp$. The r.m.s value of the current is

1) $50\,Amp$ **2)** $5\sqrt{2}\,Amp$ **3)** $10\,Amp$ **4)** $100\,Amp$

Key: 3

Sol:

$$i_{vms} = \sqrt{a^2 + \frac{b^2}{2}} = \sqrt{\left(5\sqrt{2}\right)^2 + \frac{10^2}{2}} = 10$$

31. Two plane polarized light waves combine at a certain point whose electric field components are

$$E_1 = E_0 Sin\,\omega t$$

$$E_2 = E_0 Sin\left(\omega t + \frac{\pi}{3}\right)$$

Find the amplitude of the resultant wave.

1) 3.4 E_0 **2)** 1.7 E_0 **3)** 0.9 E_0 **4)** E_0

Key: 2

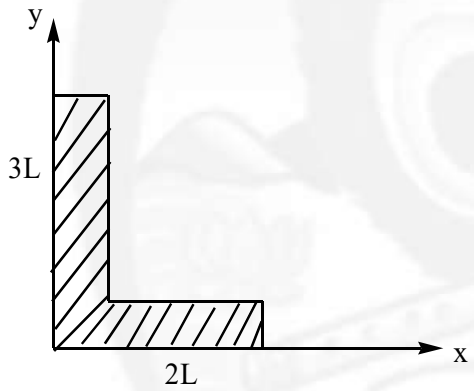
Sol:

Resultant Amplitude $A_R = 2A\,Cos\frac{\theta}{2}$

Given $\theta = 60^o$

$\Rightarrow A_R = \sqrt{3}E_o$

32. A rod of length 5 L is bent right angle keeping one side length as 2 L.



The position of the centre of mass of the system:

(Consider L = 10 cm)

1) $5\hat{i} + 8\hat{j}$ **2)** $4\hat{i} + 9\hat{j}$ **3)** $3\hat{i} + 7\hat{j}$ **4)** $2\hat{i} + 3\hat{j}$

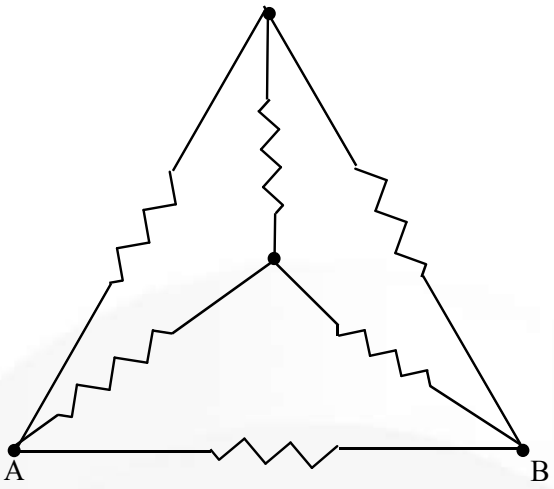
Key: 2

Sol:

$$x_{com} = \frac{(2m)(L) + 0}{2m + 3m} = \frac{2L}{5} = 4\hat{i}$$

$$y_{com} = \frac{(2m)(0) + (3m)(1.5L)}{2m + 3m} = \frac{4.5L}{5} = 9\hat{j}$$

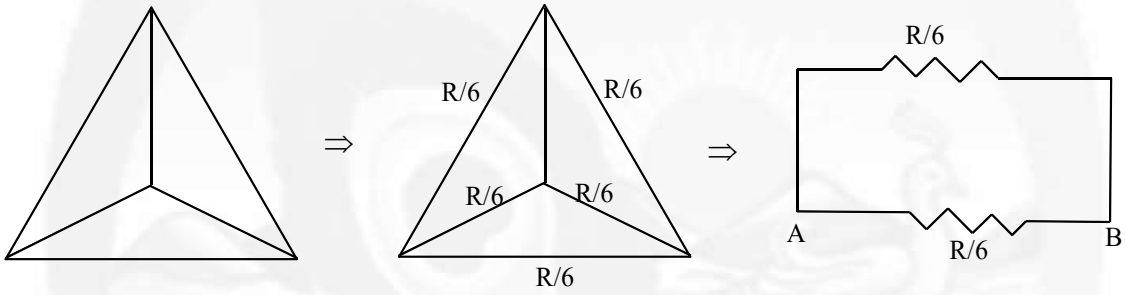
33. A wire of resistance R is bent into a triangular pyramid as shown in figure with each segment having same length. The resistance between points A and B is R/n. The value of n is:



1) 12 **2)** 14 **3)** 16 **4)** 10

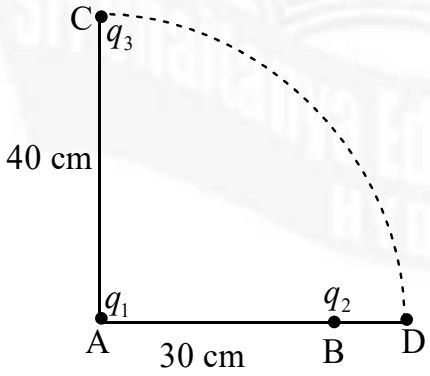
Key: 1

Sol:



$$R_{AB} = \frac{R}{12}$$

34. Two charges q_1 and q_2 are separated by a distance of 30 cm. A third charge q_3 initially at 'C' as shown in the figure, is moved along the circular path of radius 40 cm from C to D. If the difference in potential energy due to movement of q_3 from C to D is given by $\frac{q_3 K}{4\pi \in_0}$, the value of K is:



1) $8q_2$ **2)** $6q_2$ **3)** $6q_1$ **4)** $8q_1$

Key: 1

Sol:

$$U_D - U_C = \left(\frac{KQ_1Q_3}{40\times10^{-2}} + \frac{KQ_2Q_3}{10\times10^{-2}}\right) - \left(\frac{KQ_1Q_3}{40\times10^{-2}} + \frac{KQ_2Q_3}{50\times10^{-2}}\right)$$

$$= KQ_2Q_3\left[\frac{1}{10\times10^{-2}} - \frac{1}{50\times10^{-2}}\right] = \frac{Q_3}{4\pi\in_o}8Q_2$$

35. If $\in_0$ denotes the permittivity of free space and ϕ_E is the flux of the electric field through the area bounded by the closed surface, then dimensions of $\left(\in_0 \frac{d\phi_E}{dt}\right)$ are that of:

1) electric charge **2)** electric potential **3)** electric field **4)** electric current

Key: 4

Sol:

Displacement Current = $\in_o \frac{d\phi_E}{dt}$

36. Two wires A and B are made of same material having ratio of lengths $\frac{L_A}{L_B} = \frac{1}{3}$ and their diameters ratio $\frac{d_A}{d_B} = 2$. If both the wires are stretched using same force, what would be the ratio of their respective elongations?

1) 1 : 12 **2)** 1 : 3 **3)** 3 : 4 **4)** 1 : 6

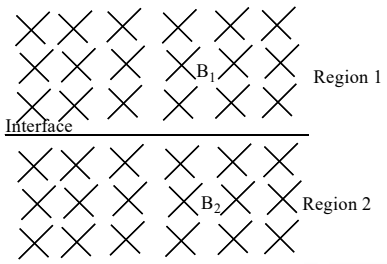
Key: 1

Sol:

$$Y = \frac{Fl}{Ae}$$

Elongation $\Delta l = \frac{Fl}{YA} \Rightarrow \frac{\Delta l_1}{\Delta l_2} = \frac{l_1/l_2}{A_1/A_2} = \frac{1/3}{4/1} = \frac{1}{3}\times\frac{1}{4} = \frac{1}{12}$

37. Uniform magnetic fields of different strengths (B_1 and B_2), both normal to the plane of the paper exist as shown in the figure. A charged particle of mass m and charge q, at the interface at an instant, moves into the region 2 with velocity *v* and returns to the interface. It continues to move into region 1 and finally reaches the interface. What is the displacement of the particle during this movement along the interface?



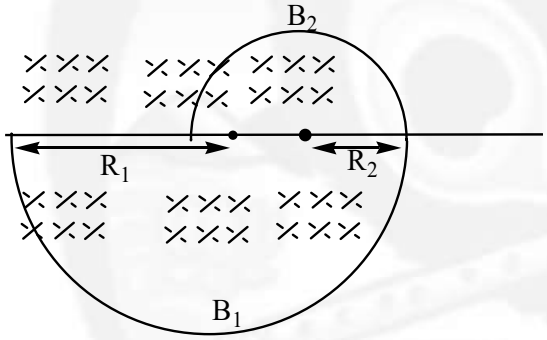
(Consider the velocity of the particle to be normal to the magnetic field and $B_2>B_1$)

1) $\frac{mv}{qB_1}\left(1-\frac{B_1}{B_2}\right)$ **2)** $\frac{mv}{qB_1}\left(1-\frac{B_2}{B_1}\right)\times 2$

3) $\frac{mv}{qB_1}\left(1-\frac{B_1}{B_2}\right)\times 2$ **4)** $\frac{mv}{qB_1}\left(1-\frac{B_2}{B_1}\right)$

Key: 3

Sol:



Displacement $= 2R_1 - 2R_2 = \left(\frac{mV}{B_1Q} - \frac{mV}{B_2Q}\right)\times 2 = \frac{mV}{B_1Q}\left(1-\frac{B_1}{B_2}\right)\times 2$

38. For a hydrogen atom, the ratio of the largest wavelength of Lyman series to that of the Balmer series is

1) 5 : 27 **2)** 3 : 4 **3)** 27 : 5 **4)** 5 : 36

Key: 1

Sol:

$$\frac{1}{\lambda} = Rz^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

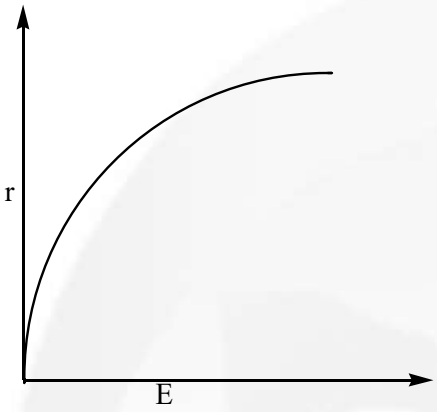
$$\frac{1}{\lambda_1} = Rz^2\left(\frac{1}{1} - \frac{1}{4}\right)$$

$$\frac{1}{\lambda_2} = Rz^2\left(\frac{1}{4} - \frac{1}{9}\right)$$

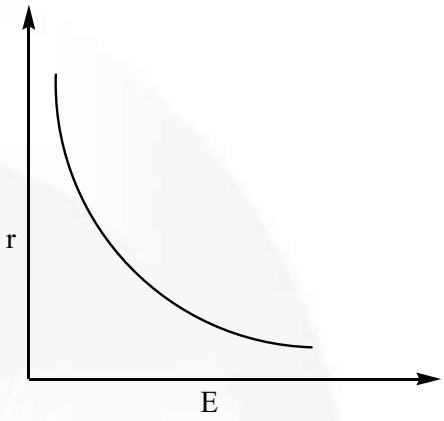
$$\frac{\lambda_1}{\lambda_2}=\frac{5}{27}$$

39. A particle of charge q, mass m and kinetic energy E enters in magnetic field perpendicular to its velocity and undergoes a circular are of radius (r). Which of the following curves represents the variation of r with E?

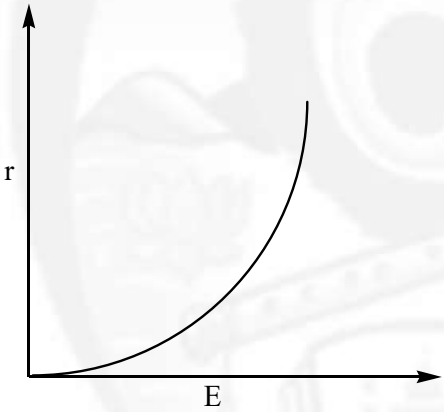
1)



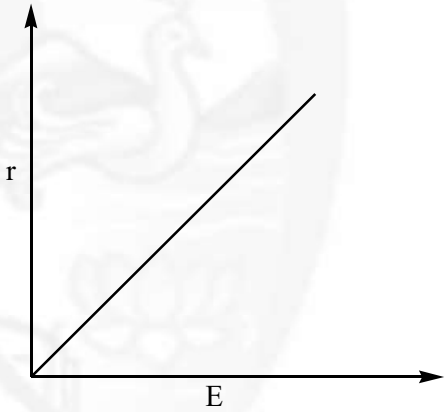
2)



3)



4)



Key: 1

Sol:

$$r=\frac{mv}{qB}=\frac{\sqrt{2mk}}{qB}$$

$$\Rightarrow r=c\sqrt{k}$$

40. The percentage increase in magnetic field (B) when space within a current carrying solenoid is filled with magnesium (magnetic susceptibility $\chi_{Mg}=1.2\times10^{-5}$) is:

1) $\frac{5}{6}\times10^{-4}\%$ **2)** $\frac{5}{3}\times10^{-5}\%$ **3)** $\frac{6}{5}\times10^{-3}\%$ **4)** $\frac{5}{6}\times10^{-5}\%$

Key: 3

Sol:

$$\frac{B^1 - B}{B} \times 100 = \left(\frac{B^1}{B} - 1\right)100$$

$$= (\mu_r - 1)100 \quad \left[\because B^1 = \mu_r B\right]$$

$$= (1 + x - 1)100$$

$$= x(100)$$

$$= 1.2 \times 10^{-3} = \frac{6}{5} \times 10^{-3}$$

41. Match the **LIST-I** with **LIST-II**

	LIST-I		**LIST-II**
A)	Triatomic rigid gas	**P)**	$\frac{C_p}{C_v} = \frac{5}{3}$
B)	Diatomic non-rigid gas	**Q)**	$\frac{C_p}{C_v} = \frac{7}{5}$
C)	Monoatomic gas	**R)**	$\frac{C_p}{C_v} = \frac{4}{3}$
D)	Diatomic rigid gas	**S)**	$\frac{C_p}{C_v} = \frac{9}{7}$

Choose the **correct** answer from the options given below:

1) A-III, B-IV, C-I, D-II

2) A-III, B-II, C-IV, D-I

3) A-IV, B-II, C-III, D-I

4) A-II, B-IV, C-I, D-III

Key: 1

Sol:

$f_{\text{mono}} = 3$

$f_{\text{diatomic rigid}} = 5$

$f_{\text{diatomic non-rigid}} = 7$

$f_{\text{triatomic rigid}} = 6$

$$\gamma = 1 + \frac{2}{f}$$

42. A lens having refractive index 1.6 has focal length of 12 cm, when it is in air. Find the focal length of the lens when it is placed in water.

(Take refractive index of water as 1.28)

1) 355 mm **2)** 288 mm **3)** 555 mm **4)** 655 mm

Key: 2

Sol:

$$\frac{1}{f} = (1.6-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = \frac{6}{10}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

$$\frac{1}{f'} = \left(\frac{1.6}{1.28}-1\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

$$\frac{1}{f'} = \frac{0.32}{1.28}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

$$\frac{\left(\frac{1}{f}\right)}{\left(\frac{1}{f'}\right)} = \frac{6}{10}\times\frac{128}{32} \Rightarrow f' = \frac{6\times128}{10\times32}\times12 = \frac{9216}{320} = 28.8\ cm = 288\ mm$$

43. Two harmonic waves moving in the same direction superimpose to form a wave $x = a\cos(1.5t)\cos(50.5t)$ where t is in seconds. Find the period with which they beat. (close to nearest integer)

1) 2 s **2)** 4 s **3)** 1 s **4)** 6 s

Key: 1

Sol:

$$x = a\,Cos(1.5t)\,Cos(50.5t)$$

$$x = \frac{a}{2}\left[Cos(1.5+50.5)t + Cos(50.5-1.5)t\right]$$

$$\omega_1 = 52;\ \omega_2 = 49$$

Beat frequency $= \frac{1}{2\pi}(52-49) = \frac{3}{2\pi}$

$$T = \frac{2\pi}{3} = 2s$$

44. A cubic block of mass m is sliding down on an inclined plane at 60° with an acceleration of $\frac{g}{2}$, the value of coefficient of kinetic friction is

1) $\frac{\sqrt{2}}{3}$ **2)** $\sqrt{3}-1$ **3)** $1-\frac{\sqrt{3}}{2}$ **4)** $\frac{\sqrt{3}}{2}$

Key: 2

Sol:

$$a = g\sin\theta - \mu g\cos\theta$$

$$\frac{g}{2} = g\left(\sin 60^o - \mu\cos 60^o\right)$$

$$\frac{1}{2} = \frac{\sqrt{3}}{2} - \mu\frac{1}{2} \Rightarrow = \sqrt{3}-1$$

45. An object of mass 1000 g experiences a time dependent force $\vec{F} = \left(2t\,\hat{i} + 3t^2\,\hat{j}\right)N$. The power generated by the force at time t is:

1) $\left(3t^3+5t^5\right)W$ **2)** $\left(2t^2+18t^3\right)W$

3) $\left(2t^2+3t^3\right)W$ **4)** $\left(2t^3+3t^5\right)W$

Key: 4

Sol:

$$P = \bar{F}.\bar{V}$$

$$= \left(2t\hat{i} + 3t^2\hat{j}\right).\left(t^2\hat{i} + t^3\hat{j}\right)$$

$$a = \frac{dv}{dt} \Rightarrow \int dv = \int a\,dt$$

$$\bar{V} = \left(t^2\hat{i} + t^3\hat{j}\right)$$

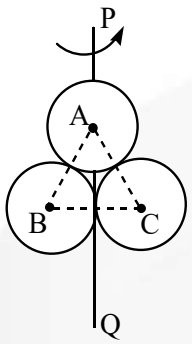
$$P = 2t^3 + 3t^5$$

SECTION-II (NUMERICAL VALUE TYPE)

This section contains **5 Numerical Value Type Questions.** The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **Nearest Integer** value (Example i,e. If answer is above **10** and less than **10.5** round off is **10** and If answer is from **10.5** and less than **11** round off is **11).**

Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.

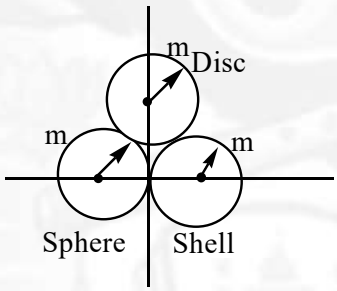
46. A, B and C are disc, solid sphere and spherical shell respectively with same radii and masses. These masses are placed as shown in figure.



The moment of inertia of the given system about PQ axis is $\frac{x}{15}I$, where I is the moment of inertia of the disc about its diameter. The value of x is

Key: 199

Sol:



$I = I_{DISC} + I_{SPHERE} + I_{SHELL}$

$$I = \frac{MR^2}{4} + \frac{7}{5}MR^2 + \frac{5}{3}MR^2$$

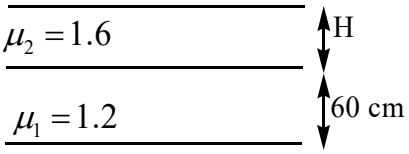
$$= \frac{199}{60}MR^2$$

47. A container contains a liquid with refractive index of 1.2 up to a height of 60 cm and another liquid having refractive index 1.6 is added to height H above first liquid. If viewed from above, the apparent shift in the position of bottom of container is 40 cm. The value of H is _____ cm.

(Consider liquids are immisible)

Key: 80

Sol:



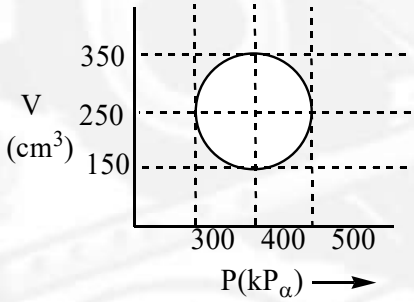
$$\text{Shift} = = \left(1-\frac{1}{\mu_1}\right)60+\left(1-\frac{1}{\mu_2}\right)H$$

$$40 = \left(1-\frac{1}{1.2}\right)60+\left(1-\frac{1}{1.6}\right)H$$

$$40 = \frac{2}{12}\times 60+\frac{6}{16}H$$

$$30 = \frac{6}{16}H \Rightarrow H = 5\times 16 = 80\,Cm$$

48. An ideal gas has undergone through the cyclic process as shown in the figure. Work done by the gas in the entire cycle is _____ $\times 10^{-1} J$



Key: 314

Sol:

W= area of the loop

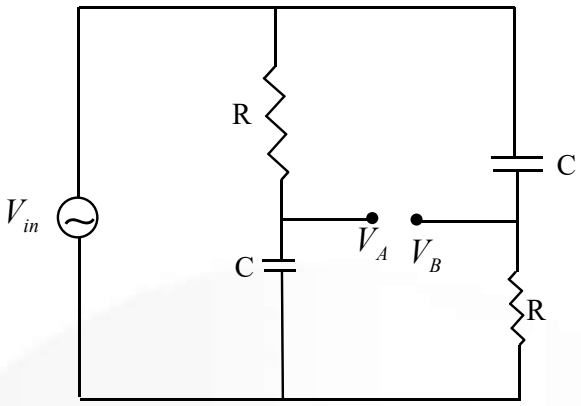
$$= \pi r_1 r_2$$

$$= \pi\left(\frac{v_2 - v_1}{2}\right)\left(\frac{P_2 - P_1}{2}\right)$$

$$= \pi\left(100\times 10^{-6}\right)\left(200\times 10^{3}\right)$$

=31.4 J

49. For the circuit shown in figure, $R = 100k\Omega$ and $C = 100\,pF$ and the phase difference between V_{in} and $\left(V_B - V_A\right)$ is 90^o. The input signal frequency is 10^x rad/sec, where 'x' is _____



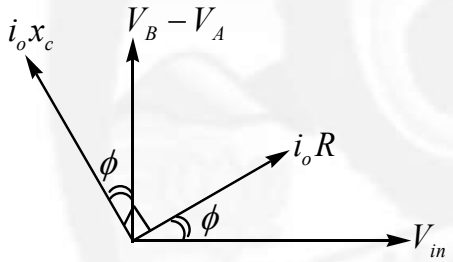
Key: 5

Sol:

$$V_B + i_o Sin(\omega t + \phi) X_c - i_o R\, Sin(\omega t + \phi) = V_A$$

$$V_B - V_A = i_o R\, Sin(\omega t + \phi) - i_o Sin(\omega t + \phi) X_c$$

$$= i_o R\, Sin(\omega t + \phi) + i_o X_c Cos(\omega t + \phi + 90^o)$$



$$= i_o X_c Sin\phi = i_o R Cos\phi$$

$$\frac{X_c}{R} = \frac{Cos\phi}{Sin\phi} = \frac{R}{X_c}$$

$$X_c = R$$

$$\frac{1}{\omega c} = R \Rightarrow \omega = \frac{1}{R_c} = \frac{1}{100 \times 10^3 \times 100 \times 10^{-12}} = 10^5$$

X=5

50. A wire of length 10 cm and diameter 0.5 mm is used in a bulb. The temperature of the wire is 1727°C and power radiated by the wire is 94.2 W. Its emissivity is $\frac{x}{8}$ where x = _____

(Given $\sigma = 6.0 \times 10^{-8} Wm^{-2}K^{-4}$, $\pi = 3.14$ and assume that the emissivity of wire material is same at all wavelength)

Key: 5

Sol:

$$P = \sigma e A T^4$$

$$94.2 = 6\times10^{-8}\times\frac{x}{8}\times 2\pi rl\left(2000\right)^4$$

$$x = 5$$

CHEMISTRY

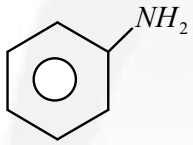
Max Marks: 100

SECTION-I (SINGLE CORRECT ANSWER TYPE)

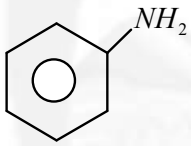
This section contains **20 Multiple Choice Questions.** Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

51. Which of the following amine (s) show (s) positive carbylamines test?

A) 

B) $(CH_3)_2NH$ C) CH_3NH_2 D) $(CH_3)_3N$

E) 

1) A and C only 2) B, C and D only 3) A and E only 4) C only

Key: 1

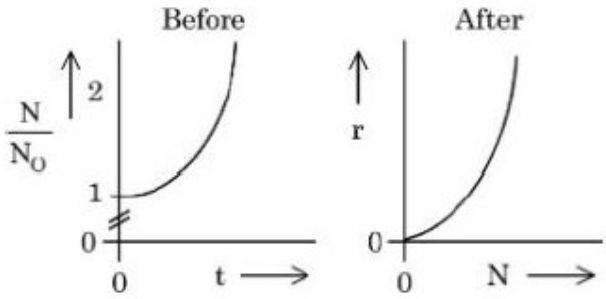
Sol: primary aliphatic and aromatic amines can give positive carbyl amine test

$$RNH_2 + CHCl_3 + 3KOH(alc) \rightarrow RNC + 3KCl + 3H_2O$$

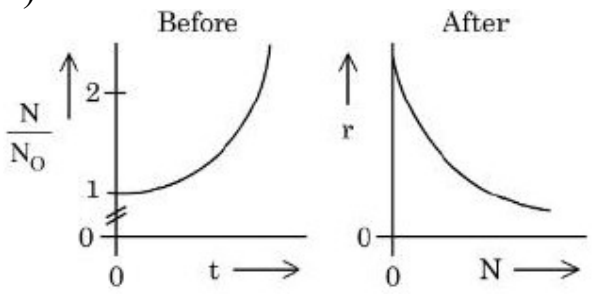
52. Person's wound was exposed to some bacteria and then bacterial growth started to happen at the same place. The wound was later treated with some antibacterial medicine and the rate of bacterial decay(r) was found to be proportional with the square of the existing number of bacteria at any instance. Which of the following set of graphs correctly represents the 'before' and "after" situation of the application of the medicine

[Given: N = No of bacteria, t = time, bacterial growth follows 1st order kinetics]

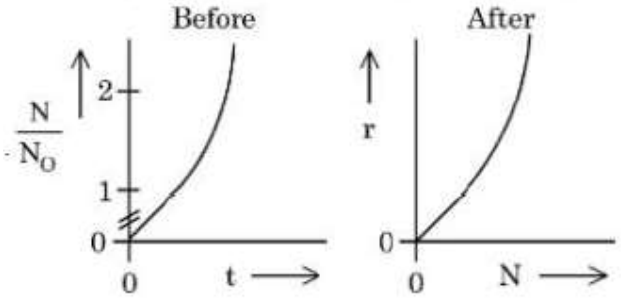
1)



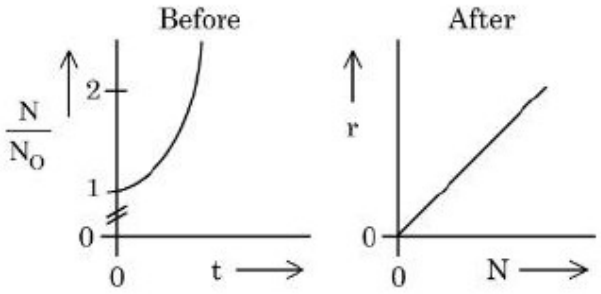
2)



3)



4)



Key.1

Sol. before applying medicine the bacterial growth follows first order kinetics exhibits exponential growth $N = N_0 e^{-kt}$

$$t = 0 \text{ so } \frac{N}{N_0} = 1$$

After applying medicine the rate of bacterial decay is proportional to the square of the existing number of bacteria indicates rapid decrease in bacteria

Rate = $K N^2$

It follows parabola graph

53. An octahedral complex having molecular composition Co. $5NH_3.Cl.SO_4$ has two isomers A and B. The solution of A given precipitate with $AgNO_3$, solution and the solution of B gives white precipitate with $BaCl_2$ solution. The type of isomerism exhibited by the complex is…

1) Linkage isomerism
2) Geometrical isomerism
3) Co-ordinate isomerism
4) Ionisation isomerism

Key.4

Sol. Complex exhibits ionization isomerism

$$[Co(NH_3)_5SO_4]Cl + AgNO_3 \rightarrow AgCl\downarrow$$

White precipitate

$$[Co(NH_3)_5Cl]SO_4 + BaCl_2 \rightarrow BaSO_4\downarrow$$

White precipitate

54. Given below are two statements:

Statement I: Dimethyl ether is completely soluble in water. However, diethyl ether is soluble i water to a very small extent.
Statement II: Sodium metal can be used to dry diethyl ether and not ethyl alcohol.

In the light of given statements. choose the correct answer from the options given below
1. Both Statement I and Statement II are False
2. Both Statement I and Statement II are true
3. Statement I is true but Statement II is false
4. Statement I is false but Statement II are true

Key.2

Sol. Dimethyl ether is water soluble due to H-bonding. Diethyl ether is soluble in water to very small extent due to hydrophobic nature of alkyl groups. Sodium metal is used to dry diethyl ether but not used to dry alcohol because it reacts with alcohol.

55. At the sea level, the dry air mass percentage composition is given as nitrogen gas: 70.0, oxygen gas: 27.0 and argon gas: 3.0. If total pressure is 1.15 atm, then calculate the ratio of following respectively:

(i) partial pressure of nitrogen gas to partial pressure of oxygen gas

(ii) partial pressure of oxygen gas to partial pressure of argon gas

(Given: Molar mass of N. O and Ar are 14, 16 and 40 g mol^{-1} respectively.)
1) 4.26, 19.3 2) 2.59, 1185 3) 2.96, 11.2 4) 5.46, 17.8

Key. 3

Sol.

$$n_{N_2} = \frac{70}{28} = 2.5$$

$$n_{O_2} = \frac{27}{32} = 0.843$$

$$n_{Ar} = \frac{3}{40} = 0.075$$

$$X_{N_2} = \frac{2.5}{2.5 + 0.843 + 0.075} = \frac{2.5}{3.418} = 0.7314$$

$$X_{O_2} = \frac{0.843}{3.418} = 0.246$$

$$X_{Ar} = \frac{0.075}{3.418} = 0.0219$$

$$P_{N_2} = X_{N_2}.P_T = 0.7314 \times 1.15 = 0.8411$$

$$P_{O_2} = X_{O_2}.P_T = 0.246 \times 1.15 = 0.2829$$

$$P_{Ar} = X_{Ar}.P_T = 0.0219 \times 1.15 = 0.025$$

$$\frac{P_{N_2}}{P_{O_2}} = \frac{0.8411}{0.2829} = 2.97$$

$$\frac{P_{O_2}}{P_{Ar}} = \frac{0.2829}{0.025} = 11.3$$

56.

LIST-I (Molecule/ ion)		LIST-II (Bond pair: lone pair (on the central atom)	
A	ICl_2^-	I	4 : 2
B	H_2O	II	4 : 1
C	SO_2	III	2 : 3
D	XeF_4	IV	2 : 2

Choose the correct answer from the options gives below

1) A-III, B-IV, C-I, D-II 2) A-II, B-I, C-IV, D-III

3) A-III, B-IV, C-II, D-I 4) A-IV, B-III, C-II, D-I

Key. 3

Sol. $\underline{ICl_2^-}$

$$H = \frac{1}{2}(V + X - C + A)$$

$$= \frac{1}{2}(7 + 2 - 0 + 1)$$

$= 5(sp^3d)$ 2 : 3

$l.p = H - bp = 5 - 2 = 3$

$\underline{H_2O}\ (sp^3)$

$b.p : l.p = 2 : 2$

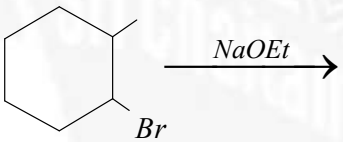
$\underline{SO_2}(sp^2)$

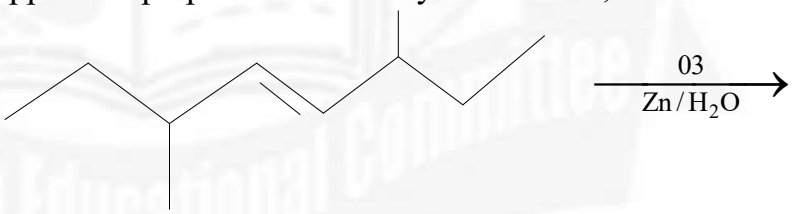
$b.p : l.p = 4 : 1$

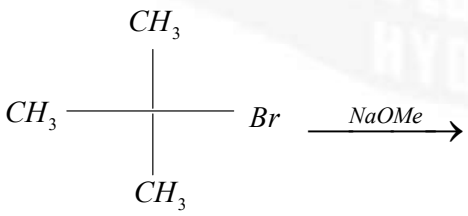
$\underline{XeF_4}(sp^3d^2)$

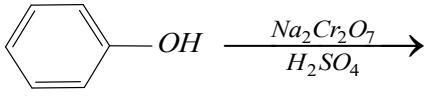
$b.p : l.p = 4 : 2$

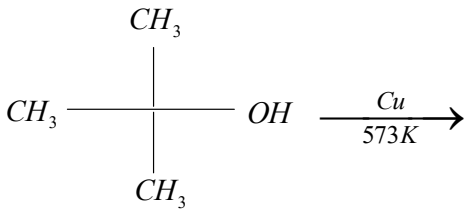
57. The reactions which cannot be applied to prepare an alkene by elimination, are

A) 

B) 

C) 

D) 



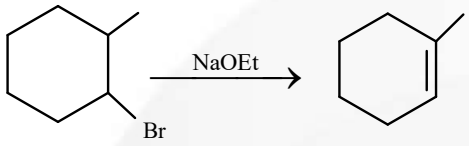
E)

Choose the correct answer from the options gives below:

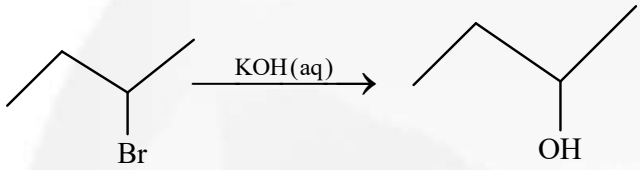
1) B, C & D only 2) B & E only 3) B & D only 4) A, C & D only

Key. 3

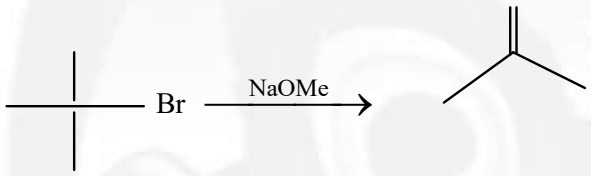
Sol. A.



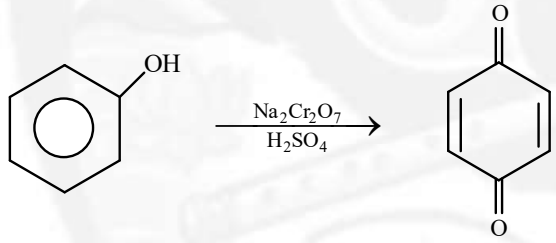
B.



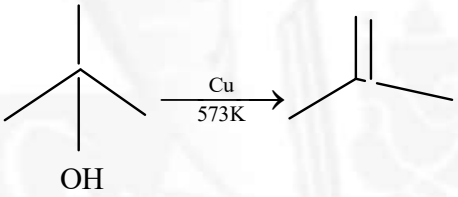
C.



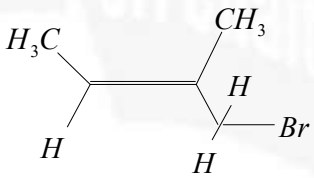
D.



E.



58. Which of the following is the correct IUPAC name of given organic compound (X)?



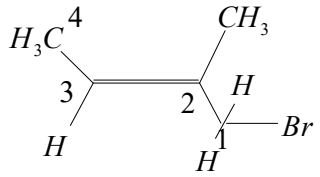
(X)

1) 1-Bromo-2-methylbut-2-ene 2) 4-Bromo-3-methylbut-2-ene

3) 2-Bromo-2-methylbut-2-ene 4) 3-Bromo-3-methylbut-2-ene

Key. 1

Sol.



(X)

59. An aqueous solution of HCl with pH 1.0 is diluted by adding equal volume of water (ignoring dissociation of water). The pH of HCl solution would

1) Reduce to 0.5 2) Increase to 2 3) Remain same 4) Increase to 1.3

Key.4

Sol. Let vol = 100ml

$P^H = 1$

$[H^+] = 10^{-1}$

on dilution $M_1V_1 = M_2V_2$

$10^{-1} \times 100 = M_2 \times 200$

$M_2 = 0.05$

$P^H = -\log[5 \times 10^{-2}]$

$= 1.3011$

60. Given below are two statements:

Statement I: Ozonolysis followed by treatment with Zn, H_2O of cis-2-butene gives ethanal.

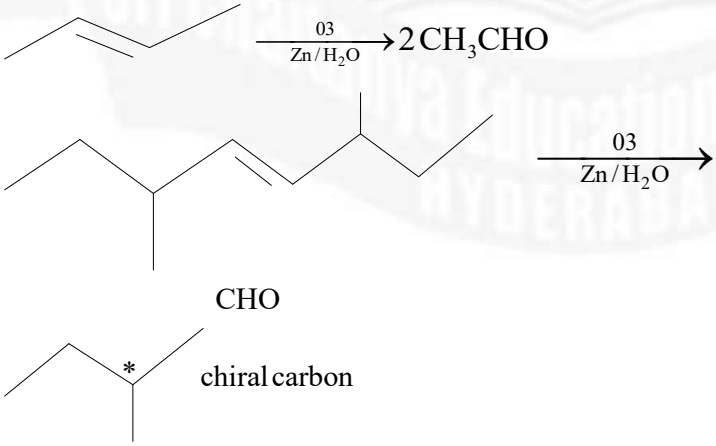
Statement II: The product obtained by ozonolysis followed by treatment with Zn, H_2O of 3, 6-dimethyloct-4-ene has no chiral carbon atom.

In the light of the above statements, choose the correct answer from the options given below

1. Statement I is true but Statement II is false
2. Statement I is false but Statement II are true
3. Both Statement I and Statement II are False
4. Both Statement I and Statement II are true

Key. 1

Sol.



61. The number of valence electrons present in the metal among Cr, Co, Fe and Ni which has the lowest enthalpy of atomisation is

1) 9 2) 6 3) 10 4) 8

Key.2

Sol. Cr has lowest enthalpy of atomisation among given Cr has 6 valance electrons.

62. When a salt treated with sodium hydroxide solution it gives gas X on passing gas X through reagent Y a brown coloured precipitate is formed. X and Y respectively, are

1) $X = NH_4Cl$ and $Y = KOH$ 2) $X = NH_3$ and $Y = HgO$

3) $X = NH_3$ and $Y = K_2HgI_4 + KOH$ 4) $X = HCl$ and $Y = NH_4Cl$

Key.3

Sol. $NH_4Cl + NaOH \rightarrow NH_3(g) + H_2O$

$$NH_3(g) + K_2HgI_4 \xrightarrow{OH^-} HgO.Hg(NH_2)I$$

Brown ppt

63. Given below are two statements:

Statement-I: $D-(+)\text{glucose} + D-(+)\text{fructose} \xrightarrow{-H_2O} \text{sucrose}$

$\text{sucrose} \xrightarrow{\text{hydrolysis}} D-(+)\text{glucose} + D-(+)\text{fructose}$

Statement-II: Invert sugar is formed during sucrose hydrolysis

In the light of the above statements, choose the correct answer from the options given below:

1) Statement-I is false but Statement-II are true

2) Both statement I and Statement II are false

3) Both statement I and Statement II are true

4) Statement-I is true but Statement-II is false

Key.1

Sol.

$C_{12}H_{22}O_{11} + H_2O \rightarrow C_6H_{12}O_6 + C_6H_{12}06$

(Sucrose) $D-(+)-\text{glucose}$ $D-(-)-\text{fructose}$

Sucrose is an invert sugar.

64. $A(g) \rightarrow 2B(g) + C(g)$ is a first order reaction. It was started with pure A

t/min	Pressure of system at time t/mm Hg
10	160
∞	240

Which of the following option is incorrect?

1) Rate constant of the reaction is 1.693 min^{-1}

2) Partial pressure of A after 10 minute is 40 mm Hg

3) The reaction never goes to completion

4) Initial pressure of A is 80 mm Hg

Key.1

Sol. $A(g) - 2B(g) + C(g)$

$t = 0 \quad p_0$

$t = 10 \quad p_0 - p \quad 2_P \quad p$

$t = \infty \quad p_0 - p_0 \quad 2p_0 \quad p_0$

$0 \quad 2p_0 \quad p_0$

At $t = 10\,\text{min} \quad p_t = p_0 + 2p = 160$

pt $t = \infty \quad pt = 2p_0 + p_0 = 240$

$3p_0 = 240$

$p_0 = 80\,\text{m mHg}$

$p_0 + 2p = 160$

$2p = 160 - 80 = 80$

$p = 40\,\text{m mHg}$

$$K = \frac{2.30}{t}\log\frac{(p_0)A}{(p_0 - p)A}$$

$$\frac{2303}{10}\log\frac{80}{40}$$

$$\frac{2.303}{10} \times 0.3010$$

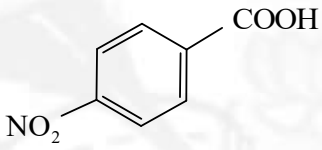
0.0693

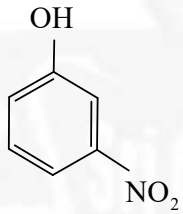
At $t = 10\,\text{min} \quad p_A = p_0 - p = 80 - 40 = 40\text{mmHg}$

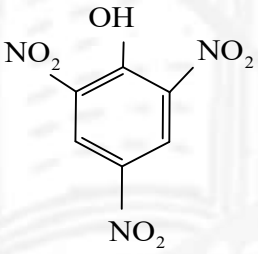
First order reactions never goes to completion.

65. Which of the following compounds is least likely to give effervescence of CO_2 in presence of aq. $NaHCO_3$?

1) $Ph - \overset{(+)}{N}H_3 \; \overset{(-)}{Cl}$

2) 

2) 

4) 

Key. 1

Sol. Acids which are stronger than HCO_3^- only liberating CO_2 from $NaHCO_3$

66. The group 14 elements A and B have the first ionisation enthalpy values of 708 and 715 kJ mol^{-1} respectively. The above values are lowest among their group members. The nature of their ions A^{2+} and B^{4+} respectively is

1) both reducing
2) both oxidising
3) oxidising and reducing
4) reducing and oxidising

Key.4

Sol. Sn^{+2} reducing agent

pb^{+4} oxidising agent

67. Given below are two statements:

Statement I: Mohr's salt is composed of only three types of ions-ferrous, ammonium and sulfate.

Statement II: If the molar conductance at infinite dilution for ferrous, ammonium and sulfate ions are x_1, x_2 and x_3 S cm^2, respectively then the molar conductance for Mohr's salt solution at infinite dilution would be given by $x_1 + x_2 + 2x_3$

In the light of the given statements, choose the correct answer from the options given below:

1) Both Statements I and Statement II are false

2) Both Statements I and Statement II are true

3) Statement I is true but Statement II are false

4) Statement I is false but Statement II is true

Key.3

Sol. mohr's salt has one ferrous, two ammonium and two sulphate ions

68. The first transition series metal 'M' has the highest enthalpy of atomisation in its series. One of its aquated ion (M^{n+}) exists in green colour. The nature of oxide formed by the above M^{n+}

1) neutral 2) amphoteric 3) acidic 4) basic

Key. 4

Sol. V has the heighest enthalpy of atomisation in 3d series

V^{+3} green in colour

V_2O_3 basic in nature

69. Total enthalpy change for freezing 1 mol of water at $10^\circ C$ to ice at $-10^\circ C$ is…….

(Given : $\Delta_{fus}H = x\,kJ/mol$

$C_p[H_2O(l)] = y\,J\,mol^{-1}K^{-1}$

$C_p[H_2O(s)] = z\,J\,mol^{-1}K^{-1}$

1) $x - 10y - 10z$ 2) $10(100x + y + z)$ 3) $-x - 10y - 10z$ 4) $-10(100x + y + z)$

Key. 4

Sol.

$$H_2O_{(L)} \xrightarrow{\Delta H_1} H_2O_{(L)} \xrightarrow{\Delta H_{fusion}} H_2O_{(s)} \xrightarrow{\Delta H_2} H_2O_{(s)}$$

$283\,K \qquad 273K \qquad 273K \qquad 263K$

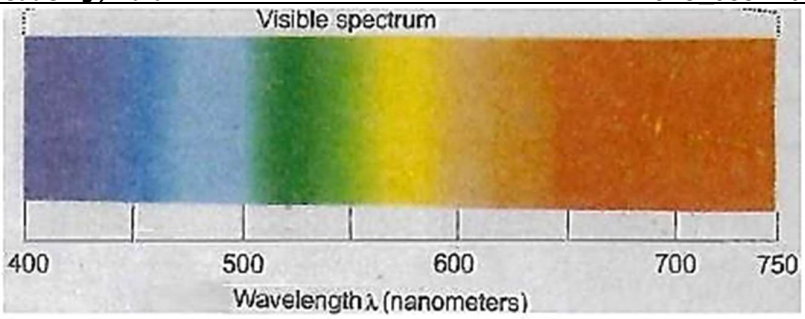
$\Delta H_1 = ncp\Delta T \qquad \Delta H_f = 1000x\,J \qquad \Delta H_2 = -z10J$

$= -10y\,J$

$\Delta H_{net} = -10y - 1000x - z10$

$= -10(100x + y + z)$

70.



Which of the following statements are correct, if the threshold frequency of caesium is 5.16×10^{14} Hz?

A. When Cs is placed inside a vacuum chamber with an ammeter connected to it and yellow light is focused on Cs, the ammeter shows the presence of current.

B. When the brightness of the yellow light is dimmed, the value of the current in the ammeter is reduced.

C. When a red light is used instead of the yellow light, the current produced is higher with respect to the yellow light.

D. When a blue light is used, the ammeter shows the formation of current.

E. When a white light is used. the ammeter shows formation of current.

Choose the correct answer from the options given below:

1) A, C, D and E only
2) A, B, D and E only
3) B, C and D only
4) A, D and E only

Key.2

Sol. $\nu=\frac{c}{\lambda}$

$=\frac{3\times10^{8}}{580\times10^{-9}}$

$=5.17\times10^{14}\,H.z$

A) ν yellow $>\nu_0$ of Cs photo electric effect occur.

B) Dimming light decrease number of photons so no. of ejected electrons are less

D) ν blue $>\nu_0$ of Cs photo electric effect occur.

E) white light contains range of frequency so photo electric effect is seen.

SECTION-II (NUMERICAL VALUE TYPE)

This section contains **5 Numerical Value Type Questions.** The Answer should be within **0 to 9999.** If the Answer is in **Decimal** then round off to the **Nearest Integer** value (Example i,e. If answer is above **10** and less than **10.5** round off is **10** and If answer is from **10.5** and less than **11** round off is **11**).
Marking scheme: +4 for correct answer, 0 if not attempt and -1 in all other cases.

71. An organic compound weighing 500 mg. produced 220 mg of CO_2. On complete combustion. The percentage composition of carbon in the compound is…….% (nearest integer) (Given molar mass in g mol^{-1} of C : 12, O : 16)

Key: 12

Sol: $C\% = \frac{12}{44} \times \frac{\text{wt of } CO_2}{\text{wt of O.C}} \times 100$

$= \frac{12}{44} \times \frac{220}{500} \times 100$

$= 12$

72. The percentage dissociation of a salt (MX_3) solution at given temperature (van't Hoff factor i = 2).....is % (Nearest integer)

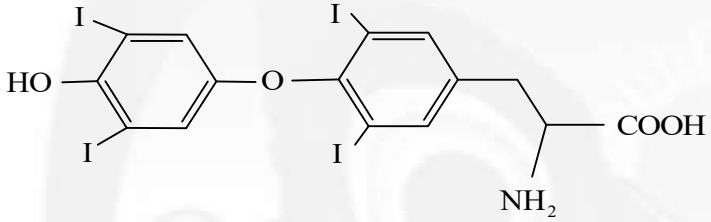
Key. 33

Sol. $MX_3 \rightarrow M^{+3} + 3X^-$

$\%\alpha = \frac{i-1}{n-1} \times 100$

$= \frac{2-1}{4-1} \times 100 = 33\%$

73. Thyroxine, the hormone has given below structure



The percentage of iodine in thyroxine is.....% (nearest integer)
(Given molar mass in g mol^{-1} C: 12, H: 1, O: 16, N: 14, I: 127)

Key. 65

Sol. $M.F = C_{15}H_{11}O_4NI_4$

$GMW = 777, \quad \text{wt of } I = 508$

$\%I = \frac{\text{wt of I}}{\text{total weight}} \times 100$

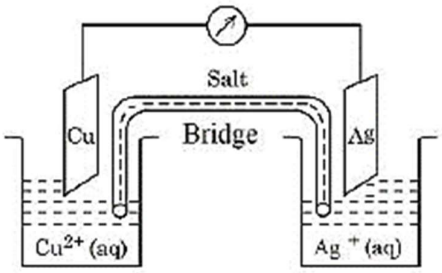
$= \frac{508}{777} \times 100 = 65\%$

74. The number of paramagnetic complexes among $[FeF_6]^{3-}, [Fe(CN_6]^{3-}, [Mn(CN)_6]^{3-}, [Co(C_2O_4)_3]^{3-}, [MnCl_6]^{3-}$, and $[CoF_6]^{3-}$, which involved d^2sp^3 hybridization is.....

Key. 2

Sol. $[Fe(CN)_6]^{-3}$ & $[Mn(CN)_6]^{-3}$ are d^2sp^3 and paramagnetic.

75. 1 Faraday electricity was passed through Cu^{2+} (1.5 M, 1 L) / Cu and 0.1 Faraday was passed through Ag^+ (0.2 M, 1 L) / Ag electrolytic cells. After this two cells were connected as shown below to make electrochemical cell. The emf of the cell thus formed at 298 K is.....mV (nearest integer)



Given : $E^{\circ}Cu^{2+}/Cu = 0.34V$

$E^{\circ}Ag^{+}/Ag = 0.06V$

$\frac{2.303RT}{F} = 0.06V$

Key. 400 mV

Sol. $Cu^{+2} + 2e^{-} \rightarrow Cu$

$(2F)$

$Cu^{+2} + 1F \rightarrow 0.5\,Cu$

Left moles of $Cu^{+2} = 1$

$\left[Cu^{+2}\right] = 1\,M$

$Ag^{+} + 1F \rightarrow Ag$

$0.1F \rightarrow 0.1\,Ag$

Left moles of $Ag^{+} = 0.1$

$\left[Ag^{+}\right] = 0.1\,M$

$$E_{cell} = (E_R - E_L) - \frac{0.06}{n}(\log\frac{[Cu^{+2}]}{[Ag^{+}]^2}$$

$$= (0.8 - 0.34) - \frac{0.06}{2}\log\left[\frac{1}{\left(10^{-1}\right)^2}\right]$$

$$= 0.46 - \frac{0.06}{2}\log(10^2)$$

$$= 0.46 - \frac{0.06 \times 2}{2}$$

$$= 0.46 - 0.06$$

$$E_{cell} = 0.40\,V$$

$$= 0.40 \times 1000 = 400\,mV$$